

### Time Complexities

#### Q1 Time Complexities and their Order

Consider the following operations and their time complexity.

- Operation 1 complexity:  $O(n \log n)$
- Operation 2 complexity:  $O(n)$
- Operation 3 complexity:  $O(\log n)$

The overall complexity of executing all three operations sequentially is:

OPTIONS:

- $O(n)$
- $O(\log n)$
- $O(n \log n)$
- $O(n^2)$

**Max is the answer.**

- ✓  $O(n) \rightarrow O(n \log n)$
- ✓  $\rightarrow O(n)$
- ✓  $\rightarrow O(\log n)$

Eg Answers, Study

## Data Structures & Algorithms

W) - 4  
Data Structures

- 1) Graphs
- 2) Lists  $\rightarrow$  Study @works.
- 3) DIT

### Algorithms

- 1) Searching
- 2) Sorting
- 3) Traversing

Time Complexities: Judge of how

well an algorithm performs.

↳ Execution Time

↳ Computation Power

#### Q2 Arrange in descending Order

- 1)  $3n + \log n \rightarrow n$
- 2)  $(\log n)^2 \rightarrow (\log n)^2$
- 3)  $\log(\log n) \rightarrow \log(\log n)$
- 4)  $100 \log n \rightarrow \log n$
- 5)  $6n \log n \rightarrow n \log n$

$$\begin{aligned} n &= 100 \\ 100 &\stackrel{(2)}{\rightarrow} 2^2 = 4 \\ 2^2 &\stackrel{(3)}{\rightarrow} \log_{10}(2) = 0.3 \\ \log_{10}(2) &\stackrel{(5)}{\rightarrow} \log n = 2 \\ \log n &\stackrel{(4)}{\rightarrow} 100(2) = 200 \end{aligned}$$

$$5, 1, 2, 4, 3 \leftarrow$$

$$5, 2, 3, 1, 4 \leftarrow$$

$$5, 1, 2, 4, 3$$

Op1  
 $n + \log(n)$

Op2  
 $2n \log(n)$

\* When, have  $> 1$  operations  
take the max.

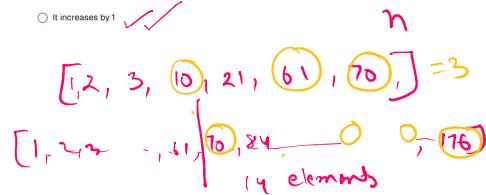
#### Q2 Binary Search

When using the **binary search algorithm** on a sorted list, consider the **maximum number of comparisons** it might take in the worst-case scenario to find an item or confirm its absent.

If the total number of items in this sorted list is doubled, how does this **maximum number of comparisons** change?

- It doubles
- It remains the same
- It becomes half
- It increases by 1

\* **Binary Search always work on Sorted arrays.**



Working of Binary Search

Doubled the size  $\Rightarrow$  Max compare increases by 1

Quad. the size  $\Rightarrow$  Increase by 2.

**Size  $\times n \Rightarrow$  Increase by  $\log_2 n$**

Search for 67  
1) Go to the middle and compare  
2) If middle  $\rightarrow$  go right else go left

3 compare.

### Insertion Sort

#### Q3 Sorting Algos

Consider the following function for Insertion sort.

```
1 def insertionsort(L):
2     n = len(L)
3     if n < 1:
4         return(L)
5     for i in range(n):
6         j = i
7         while(j > 0 and L[j] < L[j-1]):
8             (L[j], L[j-1]) = (L[j-1], L[j])
9             i -= 1
```

$O(n^2) \rightarrow n = 1,000,000 > 10^5$   
 $n^2 \rightarrow 10^{10}$   
 $O(n \log n)$

```

3 n = len(L)
4 if n < 1:
5     return(L)
6 for i in range(n):
7     j = i
8     while(j > 0 and L[j] < L[j-1]):
9         (L[j], L[j-1]) = (L[j-1], L[j])
9     j = j-1
return(L)

```

Under which scenario does the insertion sort exhibit its worst-case time complexity of  $O(n^2)$ , and why does this specific input trigger that behavior?

OPTIONS:

- When the input array is already sorted; the algorithm still performs redundant checks.
- When the input array is sorted in reverse (descending) order; each element needs to be compared against and shifted past all previously sorted elements.
- When the input array contains many duplicate elements; handling duplicates requires extra comparisons.
- When the input array elements are randomly distributed; the lack of order maximizes the average number of shifts required.

1) 1, 9, 9, 27, 35  $\rightarrow O(n)$

2) 35, 27, 9, 4, 1  $\rightarrow O(n^2)$  ✓  
1, 2, 3, 4

3) 1, 9, 2, 2, 5

4) 27, 9, 35, 1, 9  $\rightarrow O(n^2)$

Q4 Merge Sort

Merge Sort's stability (preserving the relative order of equal elements) is an important property for certain applications. How is stability typically ensured during the critical merge step of the algorithm?

OPTIONS:

- Stability is automatically guaranteed by the divide-and-conquer approach itself. ✓
- The algorithm uses a secondary key comparison if the primary keys are equal.
- During the merge step, when comparing an element from the left subarray ( $L[i]$ ) and the right subarray ( $R[i]$ ), if  $L[i] == R[i]$ , the element from the right subarray ( $R[i]$ ) must be chosen first to be placed into the merged array.
- During the merge step, when comparing an element from the left subarray ( $L[i]$ ) and the right subarray ( $R[i]$ ), if  $L[i] == R[i]$ , the element from the left subarray ( $L[i]$ ) must be chosen first to be placed into the merged array.

### Stable Algorithm

3, 1, 2, 5, 2  
 $\downarrow$   
Stable

1, 2, 2, 3, 5

\* Merge Sort is a Stable algorithm

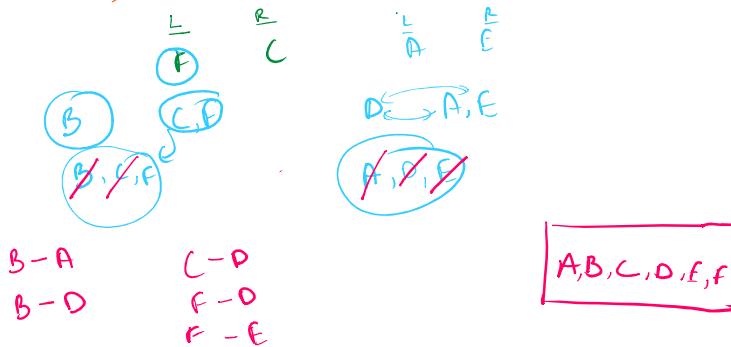
(2, 5)      (2, 7)

In this case  
give priority  
to left.

2, 2

Merge  $\Rightarrow$  Take the left element, compare with the 1st right element.

2) if less append, else append the right element



Q5 Quick Sort

Which of the following scenarios would guarantee that Quick Sort exhibits its worst-case complexity of  $O(n^2)$ ?

OPTIONS:

- Choosing a median value as the pivot for each recursive call.
- Choosing either the smallest or the largest element as the pivot at each recursive call.
- Choosing either the first or the last position element as the pivot at each recursive call.
- Partitioning the array into two roughly equal halves at each recursive call.

Solve  $\rightarrow$  MoM (Week 8)

### Insertion Sort

$n \rightarrow 10$   
 $O(n \log n)$

27, 35, 4, 1, 9

Step

1) Start from 2nd element

2) Check the left/previous element

3) Swap if current < prev.

27, 4, 35, 1, 9

4, 27, 35, 1, 9  
 $\sim \sim \sim \sim$  35

1, 4, 27, 35, 9  
 $\sim \sim$  35

1, 4, 9, 27, 35

B, C, F | D, A, E

B | F, C

D | A, E

B | F | C

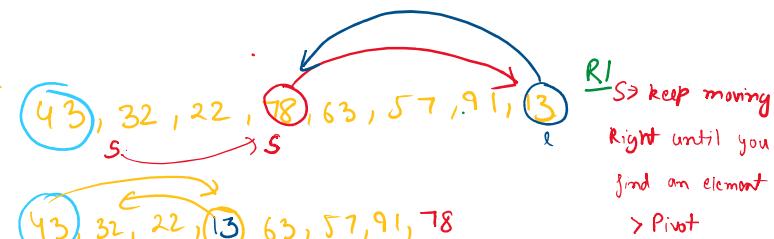
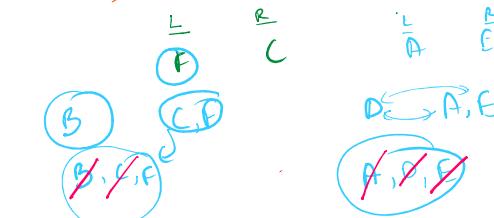
D | A | E

F | C

A | E

Merge  $\Rightarrow$  Take the left element, compare with the 1st right element.

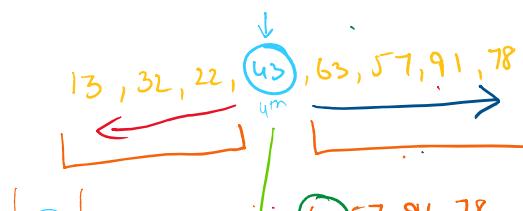
2) if less append, else append the right element

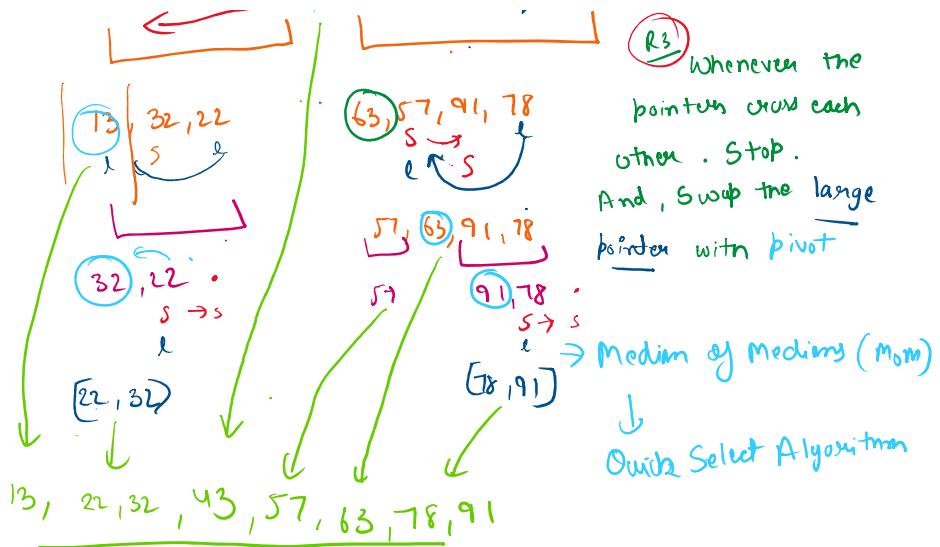


R1  $\Rightarrow$  keep moving  
Right until you  
find an element  
> Pivot

R2  $\Rightarrow$  keep moving  
Left until to find  
an element < Pivot

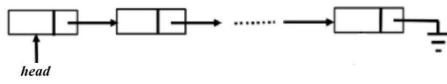
R3 Whenever the  
left is to the right





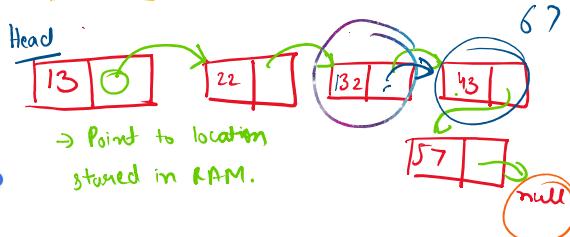
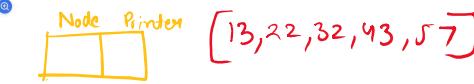
#### Q6 Linked Lists

You are given a linked list where each node of the linked list contains data and a reference to the next node. The linked list has a head pointer that points to the first node of the linked list.



Given a pointer `p_current_node` that points to an arbitrary node in the middle of a large linked list, which of the following actions is impossible to perform directly using only `p_current_node` without traversing from the list's head?

- Modify the data stored within `p_current_node`. @
- Delete the node immediately following `p_current_node` (assuming `p_current_node` is not the tail). @
- Insert a new node just after the `p_current_node`. ✓
- Modify the data of the node that immediately precedes `p_current_node` in the list. ✓



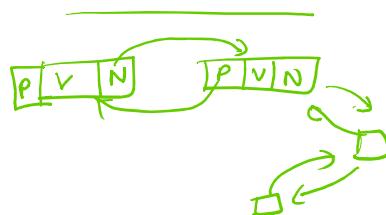
#### Linear Linked List



#### Circular Linked List



#### Double Linked List



1) Doable

2) Change the pointer

3) 32 → 43

32 → 67 → 43

4) We can't go back.

$x = 10$

#### Q7 Stack and Queue

Start with an empty stack `s` and an empty queue `q`. Perform the following sequence of operations in the given order:

1. `q.Enqueue(10)`
2. `q.Enqueue(20)`
3. `s.Push(5)` → Push → Put in Stack
4. `q.Enqueue(30)` → Pop → Take out of Stack
5. `x = q.Dequeue()`, then `s.Push(x)`
6. `s.Push(15)`
7. `y = s.Pop()`, then `q.Enqueue(y)`
8. `q.Enqueue(40)`
9. `z = q.Dequeue()`, then `s.Push(z)`
10. `w = q.Dequeue()` (the returned value is discarded)
11. `w = q.Dequeue()`, then `s.Push(w)`

After all these operations are completed, what is the value of the element at the top of stack `s`?

(A) 10, 20, 30, 15, 40



$x = 10$

$y = 15$

$z = 20$

$w = 30$

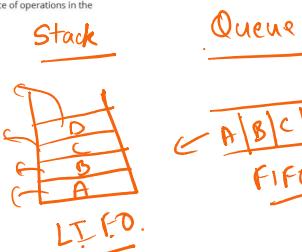
#### Q8 BFS

When BFS is performed on an unweighted, undirected graph starting from a source node `s`, it defines a BFS tree where edges connect nodes to their "discoverers." Consider an edge  $(u, v)$  from the original graph. If this edge  $(u, v)$  is not part of the BFS tree (i.e., it's a non-tree edge), what is the relationship between the levels  $\text{Level}(u)$  and  $\text{Level}(v)$ ? Let  $\text{Level}(x)$  be the distance of node `x` from `s`.

OPTIONS:

- $\text{Level}(u)$  and  $\text{Level}(v)$  must either be the same or differ by exactly 1 (i.e.,  $|\text{Level}(u) - \text{Level}(v)| \leq 1$ ). @
- One of  $u$  or  $v$  must be `s`, and the other is at  $\text{Level} 1$ . @

#### Stack



#### Queue



#### Queue

#### BFS (Queue)

#### A, B, C, D, E, F



→ Put children of current in Queue

→ When take out, put in the children in Q.

- OPTIONS:
- Level( $u$ ) and Level( $v$ ) must either be the same or differ by exactly 1 (i.e.,  $|Level(u) - Level(v)| \leq 1$ ). @
  - One of  $u$  or  $v$  must be  $s$ , and the other is at Level 1. @
  - Level( $u$ ) and Level( $v$ ) must differ by more than 1 (e.g., Level( $v$ )  $\geq$  Level( $u$ ) + 2). @
  - Level( $u$ ) and Level( $v$ ) must be the same, and  $u$  and  $v$  must have been discovered from different parent nodes in the BFS tree. @

#### Q9 Hashing

Consider a hash table of size  $M=10$  (indices 0 through 9). The hash function used is  $h(k) = k \pmod{10}$ .

Collisions are resolved using **linear probing** with a probe step of 1 (i.e., if  $h(k)$  is occupied, try  $(h(k) + 1) \pmod{M}$ , then  $(h(k) + 2) \pmod{M}$ , and so on).

The following keys are inserted into the initially empty hash table in this exact order:

12, 42, 35, 28, 63, 55

After all these keys have been inserted, at what index in the hash table will the key 23 be stored?

- OPTIONS:
- 3
  - 4
  - 6
  - 7

#### Q10 BFS Traversal and DFS Traversals

Consider the following undirected graph represented by its adjacency list:

- 0: [1, 2]
- 1: [0, 4, 5]
- 2: [0, 4]
- 3: [4, 5]
- 4: [1, 2, 3, 5]
- 5: [1, 3, 4]

A Breadth-First Search (BFS) is performed on this graph starting from source node 0. If multiple unvisited adjacent vertices are available from the current vertex being processed, the BFS algorithm always explores the vertex with the smaller label/ID first.

Which of the following edges from the original graph will NOT be part of the BFS traversal tree?

- OPTIONS:
- (1,4)
  - (2,4)
  - (3,4)
  - (3,5)
  - (4,5)
  - (1,5)

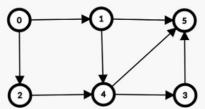
#### Q11 DFS

Consider a complete directed graph  $K_5$  (a graph with 5 nodes where every distinct pair of nodes is connected by a unique edge). If you perform a Depth-First Search (DFS) starting from an arbitrary node, say Node 1, what is the maximum number of edges that will be classified as "back edges" during this single DFS traversal, assuming the traversal explores all reachable nodes from Node 1?

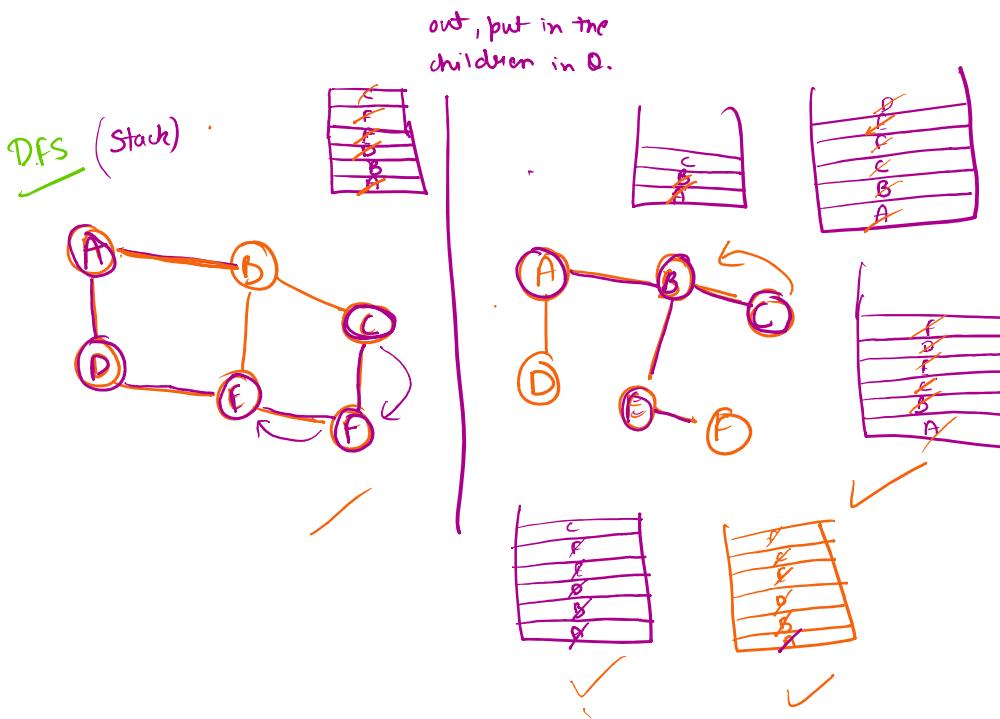
Note: A back edge connects a node to an ancestor in the DFS tree (an already visited node that is still on the recursion stack).

#### Q12 Topological Ordering

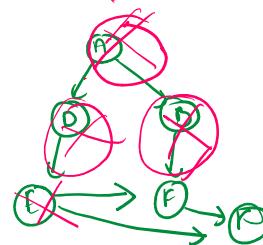
Consider the following directed graph.



The number of topological orderings of the vertices of the given graph is \_\_\_\_



#### Topological Ordering



→ Take the node without any parent/ incoming

→ Remove it → Repeat.

$A, B, D, E, F, K$

$A, D, E, B, F, K$

$A, D, B, E, F, K$