

Today's Content

- Max consecutive 1's by
 - a) atmost 1 replace
 - b) atmost 1 swap
- Count triplets → {Goldman Sachs hiring challenge}
- Josephus Problem

{ans → private chat }
{ques → public chat. }

O's and 1's

Q) Given a binary arr[], we can atmost replace a single 0 with 1. find the maximum consecutive 1's we can get in given arr[].

Eg1 { 1 | 0 | 1 | 1 | 0 | 1 } ans=5

Eg2 { 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 }
 ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
 4 6 5 3

Eg3 { 1 | 1 | 1 | 1 | 1 } ans=6

Eg4 { 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 }
 ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
 ③ + 1 + ② ↓ ← ↓ ↓ ← ② ↓ ← ②
 = 6 7 5 3 5.

idea: For every 0

→ no. of consecutive 1's on l.h.s = (l)

→ no. of consecutive 1's on r.h.s = (k)

→ if ($l+k+1 > \text{ans}$) $\text{ans} = l+k+1$

Edge case: if all 1's are there $\Rightarrow \underline{N}$.

pseudo-code :

```
int maxConsecutiveOnes ( arr, N ) {  
    count = 0  
    for ( i → 0 to N-1 ) {  
        if ( arr[i] == 1 ) count += 1  
    }  
    if ( count == N ) return N;  
    if ( count == 0 ) return 1;  
  
    ans = 0  
  
    for ( i → 0 to N-1 ) {  
        if ( arr[i] == 0 ) {  
            // count of consecutive 1's on left  
            l = 0  
            for ( j → i-1 to 0 ) {  
                if ( arr[j] == 1 ) { l += 1 }  
                else { break }  
            }  
            // count of consecutive 1's on right  
            r = 0  
            for ( j → i+1 to N-1 ) {  
                if ( arr[j] == 1 ) { r += 1 }  
                else { break }  
            }  
            if ( l+r+1 > ans ) { ans = l+r+1 }  
        }  
    }  
    return ans;  
}
```

T.C → O(n)
S.C → O(1)

TC : Discussion

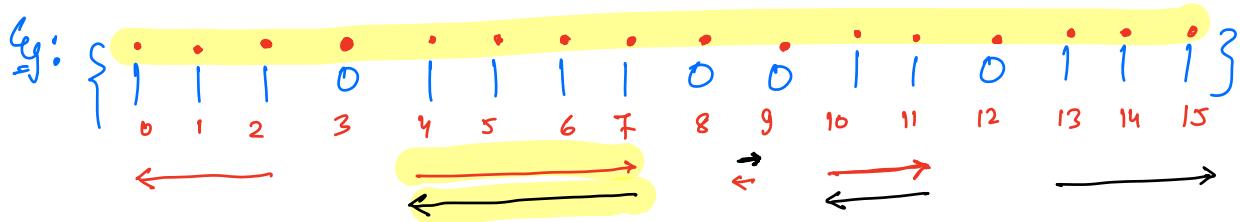
```

for ( i=1 ; i<=3 ; i++) {
    for( j=1 ; j<=n ; j++) {
        =
    }
}
  
```

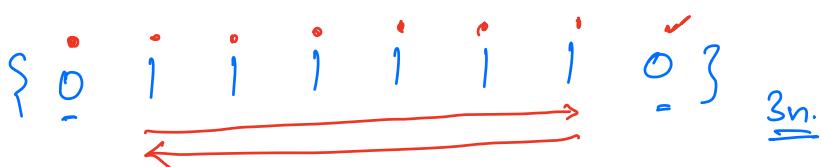
i	j	iterations
1	[1,n]	n
2	[1,n]	n
3	[1,n]	n

3n.

$$T.C \rightarrow O(n)$$



At max, every element of the array is touched 3 times.



Note. → If break is present in nested loops, be careful while calculating time complexity.

Q) Given a binary arr[], find max no. of consecutive 1's we can get by atmost 1 swap. [we can swap with values present in the array]

Eg: { 0 1 1 0 1 1 0 1 } [ans=6]
 l: 2
 r: 3
 $c = l+r+1 = 6$

l: 3
 r: 1
 $c = l+r+1 = 5$

Eg: { 0 1 1 1 0 1 1 0 0 } [ans=5]
 l: 0
 r: 3
 $c = l+r+1 = 4$
 $c = l+r = 5$

l: 2
 r: 0
 $c = l+r+1 = 3$

l: 0
 r: 0
 $c = l+r+1 = 1$

count = 12

Eg: { 1 1 1 0 1 1 0 0 1 1 1 1 0 1 1 1 }
 l: 3
 r: 2
 $c = l+r+1$
 $\Rightarrow \underline{8}$

Pseudo-code :- int maxConsecutiveOnes (arr, N) {

 count = 0

 for (i = 0 to N - 1) {

 if (arr[i] == 1) count += 1

 if (count == N) return N ;

 if (count == 0) return 0 ;

 ans = 0

 for (i = 0 to N - 1) {

 if (arr[i] == 0) {

 // count of consecutive 1's on left .
 l = 0 ✓

 for (j = i - 1 to 0) {

 if (arr[j] == 1) { l += 1 }

 else { break }

 // count of consecutive 1's on right

 r = 0 ✓

 for (j = i + 1 to N - 1) {

 if (arr[j] == 1) { r += 1 }

 else { break }

 K = l + r

 if (l + r < count) { K = r + 1 }

 if (K > ans) { ans = K }

 }

return ans ;

}

T.C $\rightarrow O(N)$
S.C $\rightarrow O(1)$

Q) No. of Triplets

Given $\text{arr}[N]$ elements, calculate no. of triplets i, j, k
 such that $i < j < k$ and $\text{arr}[i] < \text{arr}[j] < \text{arr}[k]$

Eg: $\text{arr}[5] : \{2, 6, 9, 4, 10\} \Rightarrow \boxed{\text{ans} = 5}$

$i < j < k$			$\text{arr}[i] < \text{arr}[j] < \text{arr}[k]$		
0	1	2	2	6	9
0	1	4	2	6	10
1	2	4	6	9	10
0	2	4	2	9	10
0	3	4	2	4	10

Eg2: $\text{arr}[6] : \{4, 1, 2, 6, 9, 7\} \quad [\text{ans} \rightarrow 9]$

$i < j < k$			$\text{arr}[i] < \text{arr}[j] < \text{arr}[k]$		
0	3	4	4	6	9
0	3	5	4	6	7
1	2	3	1	2	6
1	2	4	1	2	9
1	2	5	1	2	7
1	3	4	1	6	9
1	3	5	1	6	7
2	3	4	2	6	9
2	3	5	2	6	7

idea1: Consider every triplet & check if it satisfy the condns.

```
int countTriplets( arr , N ) {  
    count = 0  
    for( i → 0 to N-1 ) {  
        for( j → i+1 to N-1 ) {  
            for( k → j+1 to N-1 ) {  
                if (arr[i] < arr[j] and arr[j] < arr[k]) {  
                    count += 1  
    }  
    return count  
}
```

T.C → O(N³)
S.C → O(1)

Hint for optimisation:

arr[6] = { 4 1 2 6 9 7 }
 . . . 3 4 5

In how many triplets, 6 will be middle element?
{ index-3 }

left: { 4 } middle: 6 right: { 9 }
 { 1 } [j] { 7 }

triplets:
left * right = 3 * 2 = 6.

Idea & pseudo-code:

arr [6] : { 4 1 2 6 9 7 }
 0 1 2 3 4 5

l : 0 0 1 3 4 4
r : 3 4 3 2 0 0

Count: 0 + 0 + 3 + 6 + 0 + 0 = g. total triplets.

int countTriplets (arr, N) {

 ans = 0

 for(j → 0 to N-1) {

 // find count of smaller elements on l-h.s

 l = 0, r = 0

 for(i → 0 to j-1) {

 if(arr[i] < arr[j]) { l = l+1 }

 // find count of greater elements on r-h.s

 for(k → j+1 to N-1) {

 if(arr[j] < arr[k]) { r = r+1 }

 ans = ans + (l * r)

 return ans

}

T.C → O(N²)

S.C → O(1)

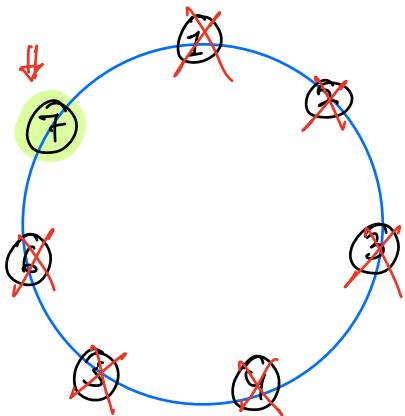
Josephus Problem

N people are standing in a circle. Person 1 has knife, he kills next person in clockwise direction and passes on the knife to next person in clockwise.

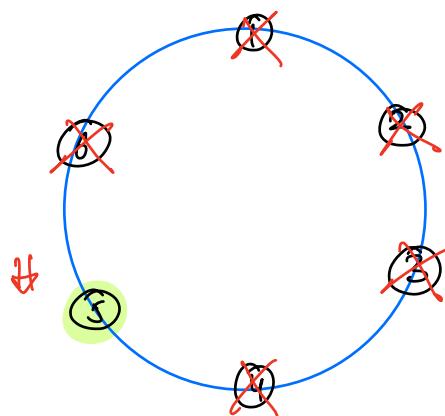
Repeat the process until a **single** person is alive.

Find the last person standing.

$N=7$



$N=6$

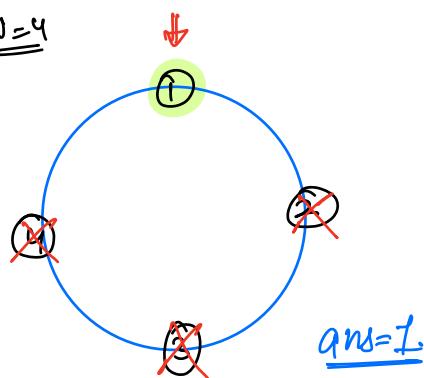


Observations

① if n is odd $\Rightarrow N$
 n is even $\Rightarrow N-1$

② largest prime $< N \Rightarrow p$

$N=4$

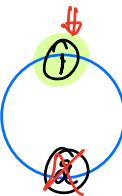


ans=1

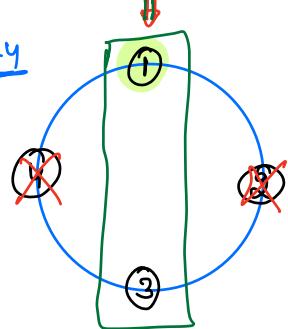
N=1



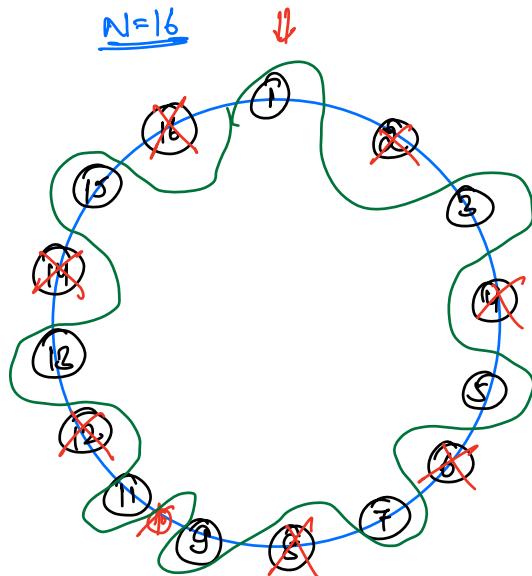
N=2.



N=4



N=16

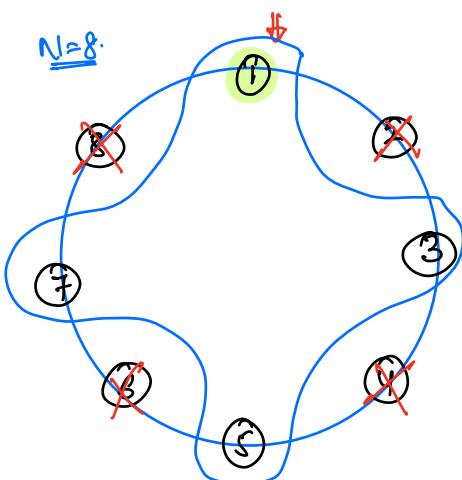


ans=1.

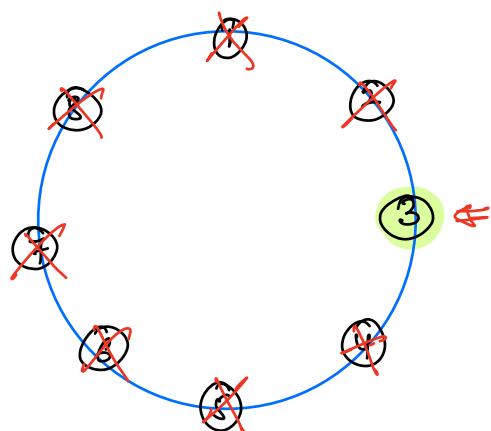
N=32. I has Knife $\Rightarrow 1$

[observation: If N is a power of 2,
I starts the game , $ans=1$]

N=8.

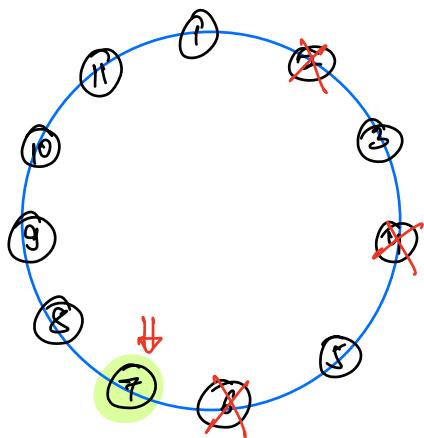


N=8

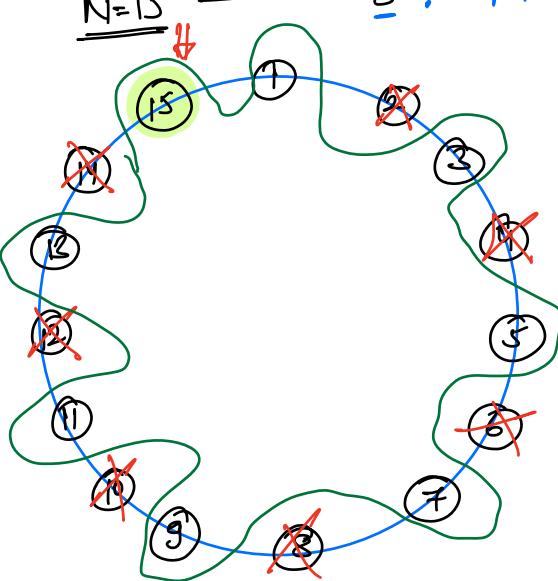


[observation 2.: If N is a power of 2,
whoever starts the game will
win the game.]

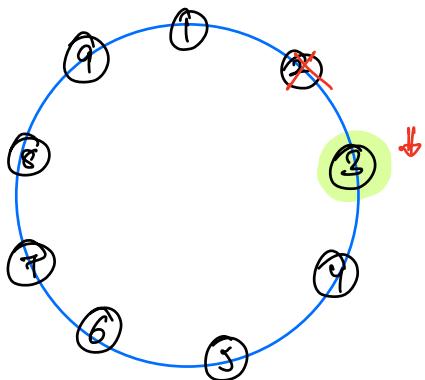
N = 11



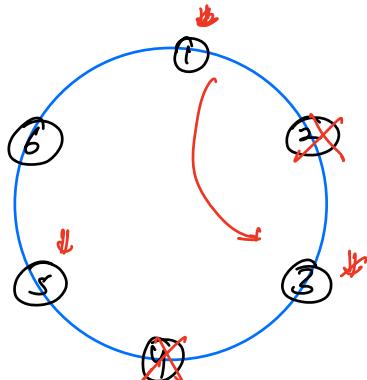
$$\underline{N=15} \xrightarrow{7 \text{ kills}} \underline{8} : 1 + (7 \times 2) = \underline{15}$$



$$\underline{N=9.} \xrightarrow{1 \text{ kill}} \underline{8} : 1 + (1 \times 2) = \underline{3.}$$



$$\underline{N=6.} \xrightarrow{2 \text{ kills}} \underline{4} : 1 + (2 \times 2) = \boxed{5}$$



N = 100

nearest power of 2 which is less than 100 $\rightarrow \underline{64}$.
no. of kills we need to make in order to attain 64 $\Rightarrow \underline{36}$.

$$1 + (36 \times 2) = \boxed{\underline{73.}}$$

pseudo-code: To do:

Step 1. → Find nearest power of 2, which is less than or equal to N

Step 2. → $\text{kills} = N - \frac{np}{\uparrow}$

Step 3. → $\text{ans} = 1 + (2 * \text{kills})$,

Doubts :

→ Generalized: → s.p can be any index.

↳ skip k person. $\{(k+1)^{\text{th}} \text{ person}\}$

$\max \text{Profit} = 11$

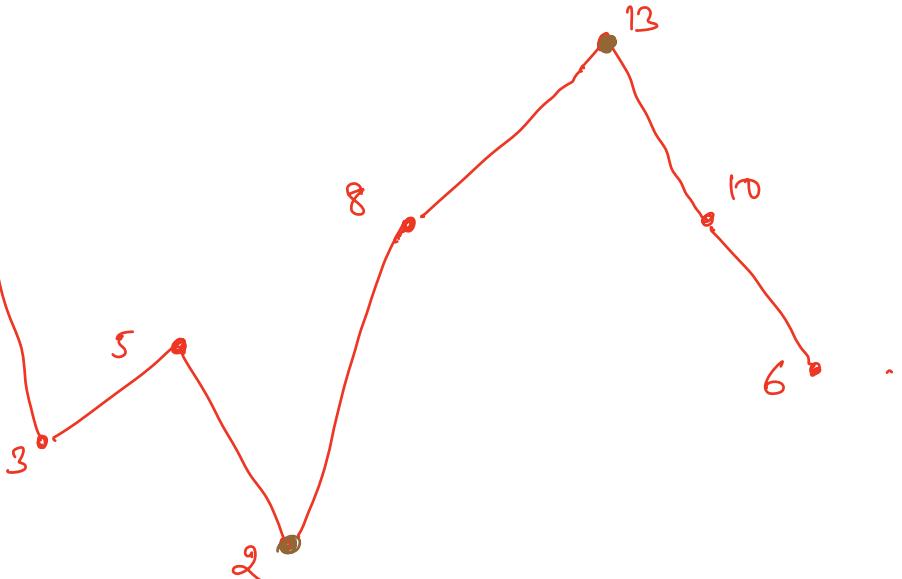
$\text{profit} = \max_{i=1}^m$

Stocks :

$$\max = \frac{10}{13}$$

arr → [7 11 3 5 2 8 13 10 6]

5 0 0 1 2 3 4 5 6 7 8



[Buy → Sell]

Q) mat[N][N] ⇒ SXs.

$$i+j = m+1$$

0	1	2	3	4	
0	1	2	4	7	8
1	3	5	6	9	14
2	10	11	12	13	15
3	16	17	19	22	24
4	18	20	21	23	25

$$m=3.$$

$$i+j = m+1$$

$$i=j$$