



Content

- Bitwise operators & properties.
- Check-Bit
- Unique Element
- Single Element - II
- Single Element - III

Bitwise Operators $\Rightarrow \&, |, ^, \sim, \ll, \gg$

Truth table

same same 'puppy shame'

a	b	$a b$	$a \& b$	a^b	$\sim a$
0	0	0	0	0	0
0	1	1	0	1	0
1	0	1	0	1	1
1	1	1	1	0	1

$(2^0) \quad (2^1) \quad (2^2) \quad (2^3) \quad (2^4) \quad (2^5) \quad (2^6) \quad (2^7)$

7
6
5
4
3
2
1
0

$a = 29 : \underline{0} \quad \underline{0} \quad \underline{0} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{0} \quad \underline{1}$

$b = 18 : \underline{0} \quad \underline{0} \quad \underline{0} \quad \underline{1} \quad \underline{0} \quad \underline{0} \quad \underline{1} \quad \underline{0}$

$a \& b : \underline{0} \quad \underline{0} \quad \underline{0} \quad \underline{1} \quad \underline{0} \quad \underline{0} \quad \underline{0} \quad \underline{0} = (16)$.

$a|b : \underline{0} \quad \underline{0} \quad \underline{0} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} = (31)$

$a^b : \underline{0} \quad \underline{0} \quad \underline{0} \quad \underline{0} \quad \underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{1} = (15)$

Properties

$$a \& b = b \& a$$

$$a | b = b | a$$

$$a \wedge b = b \wedge a$$

// Commutative Property.

$$a \& b \& c = b \& a \& c = c \& a \& b$$

$$a | b | c = b | a | c = c | a | b$$

$$a \wedge b \wedge c = b \wedge a \wedge c = c \wedge a \wedge b$$

// Associative Property

$$\begin{array}{r} a=10 : \quad 1 \ 0 \ 1 \ 0 \\ & \& 0 \ 0 \ 0 \ 1 \\ \hline & \quad 0 \ 0 \ 0 \ 0 \end{array}$$

$$\begin{array}{r} a=11 : \quad 0 \ 0 \ 1 \ 1 \\ & \& 0 \ 0 \ 0 \ 1 \\ \hline & \quad 0 \ 0 \ 0 \ 1 \end{array}$$

$$\left[\begin{array}{l} a \& 1 = 0 \quad \{ \text{last bit of } a \text{ is '0'} \} \Rightarrow \text{No. is even} \\ a \& 1 = 1 \quad \{ \text{last bit of } a \text{ is '1'} \} \Rightarrow \text{No. is odd.} \end{array} \right]$$

$$a \& 0 = 0$$

$$a \& a = a$$

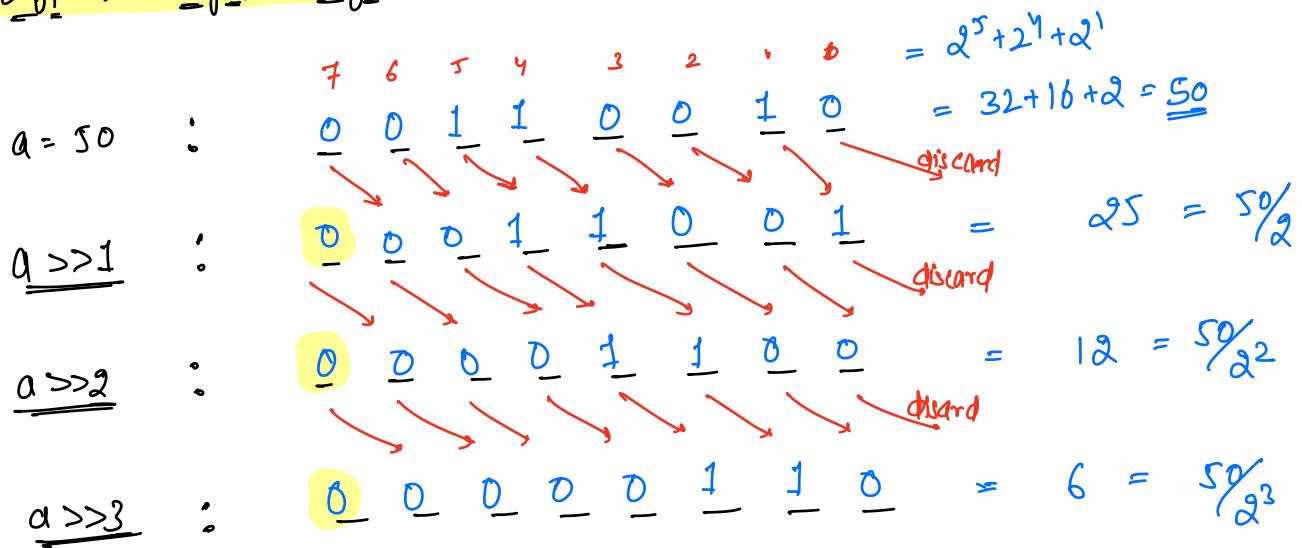
$$a | 0 = a$$

$$a | a = a$$

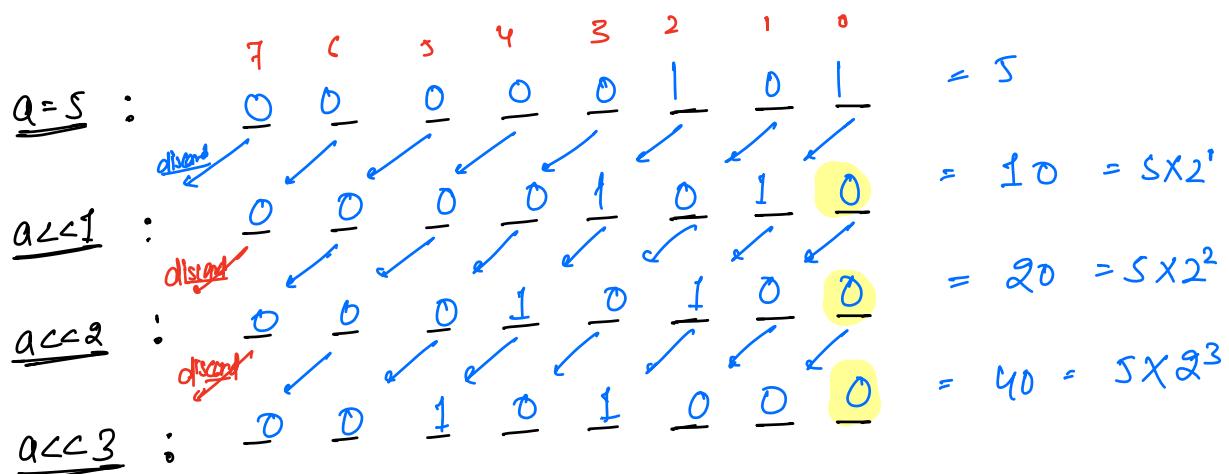
$$a \wedge 0 = a$$

$$a \wedge a = 0$$

Left & Right Shift →



$$[a \gg n = a / 2^n]$$



$$[a \ll n = a * 2^n]$$

$$[1 \ll n = 1 * 2^n = 2^n]$$

Q) Given a number N & i. Check if ith-bit is set.

set \rightarrow 1, unset \rightarrow 0

N=53: $\begin{array}{ccccccc} 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 & \underline{1} & 0 & 1 \end{array}$, $i=2 \rightarrow$ set.
 $i=3 \rightarrow$ unset

$N>>1$ $\begin{array}{cccccc} 0 & 0 & 1 & 1 & 0 & 1 0 \end{array}$

$N>>2$ $\begin{array}{ccccc} 0 & 0 & 0 & 1 & 1 0 1 \end{array}$

$i=0$

$(N \& 1) == 1$ { 0th bit is set }

$i=1$

$((N>>1) \& 1) == 1$ { 1st bit is set }

$i=2$

$((N>>2) \& 1) == 1$ { 2nd bit is set }

1

1

1

1

$((N>>i) \& 1) == 1$ { ith bit is set }

boolean checkBit (N, i) {
 if $((N>>i) \& 1) == 1$ {
 return true;
 }
 else {
 return false;
 }
}

T.C $\rightarrow O(1)$
S.C $\rightarrow O(1)$

{ To do \rightarrow
try with left shift operator }

Q) Given N elements , every element repeats twice except one.
find unique element .

arr[7]: { 3 2 3 7 2 8 7 } : ans = 8.

// ans = XOR of all the elements.

$\begin{bmatrix} \text{T.C} \rightarrow O(N) \\ \text{S.C} \rightarrow O(1) \end{bmatrix}$

Pseudocode:

```
val = 0
for( i=0; i < N; i++ ) {
    val = val ^ arr[i]
}
return val
```

Q) Every element repeats thrice except one. Find unique element.

Constraints : $1 \leq N \leq 10^6$, $1 \leq arr[i] \leq 10^9$

arr[13]: { 5 7 5 4 7 11 11 9 11 7 5 4 4 }

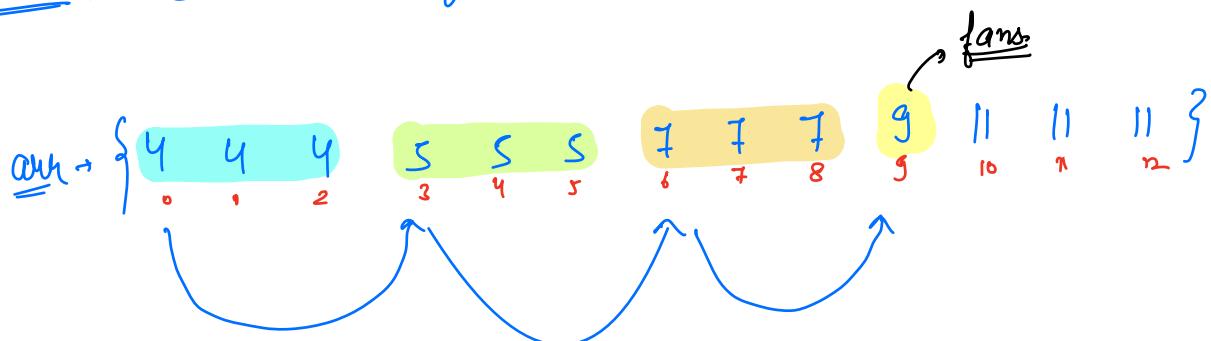
Idea-1 for every element, iterate & find its frequency.

T.C $\rightarrow O(N^2)$, S.C $\rightarrow O(1)$

Idea-2: for every element, store its frequency in hashmap.

T.C $\rightarrow O(N)$, S.C $\rightarrow O(N)$

Idea-3: Sort the array + { iterate & check every 3 elements }



T.C $\rightarrow O(N \log N)$, S.C $\rightarrow O(1)$

arr[13]: { 5, 7, 5, 4, 7, 11, 11, 9, 11, 7, 5, 4, 4 }

	3	2	1	0
5:	0	1	0	1
7:	0	1	1	1
5:	0	1	0	1
4:	0	1	0	0
7:	0	1	1	1
11:	1	0	1	1
11:	1	0	1	1
9:	1	0	0	1
11:	1	0	1	1
7:	0	1	1	1
5:	0	1	0	1
4:	0	1	0	0
4:	0	1	0	0

[Count of set-bits at every position]

unique = { 3rd ↓ 2nd ↓ 1st ↓ 0th ↓ }
{ 1 0 1 0 }

4 9 6 10.

$$y_3 = 1 \quad y_3 = 0 \quad y_3 = 0 \quad 10 \cdot y_3 = 1.$$

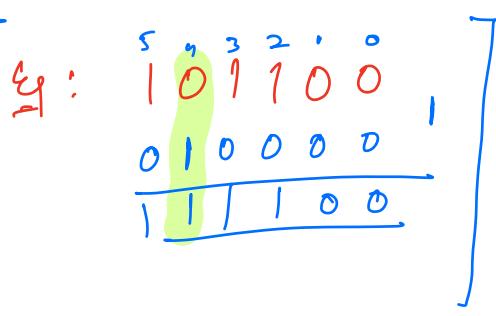
$$\text{ans} \rightarrow \{ 1 \ 0 \ 0 \ 1 \} = 9.$$

idea: At every bit position, iterate on array elements and find count of set-bits for that bit-position.
if (count \cdot 2 $= 0$) { In unique element, ith-bit will be unset }
else { In unique element, ith-bit will be set }

pseudo-code

```
ans = 0 [Initially all bits are '0']  
for( i = 0 ; i < 32 ; i++ ) {  
    // find count of set-bits at ith bit position  
    // of all the elements  
    count = 0  
    for( j = 0 ; j < N ; j++ ) {  
        if ( checkBit( arr[j], i ) ) {  
            count++  
        }  
        if ( count % 3 == 1 ) { // extra bit.  
            // set ith-bit in your 'ans'-  
            ans = ans + 2i ] Revise &  
            ans = ans | ( 1 << i ) ] Check.  
    }  
}  
return ans;
```

no of iterations = $32 \cdot N$
T.C $\rightarrow O(N)$
S.C $\rightarrow O(1)$

Eg: 
$$\begin{array}{r} 101100 \\ 010000 \\ \hline 111100 \end{array}$$

Some variations

① Every element repeats thrice except 1.

② Every element repeats thrice & one element is repeating twice.

$$[CY.3 == \underline{2}]$$

③ Every element is repeating 5 times except 1.

$$[CY.5 == 1]$$

④ Every element is repeating 5 times & one element is repeat twice.

$$[CY.5 == \underline{2}]$$

⑤ Every element is repeating 5 times & one element is repeat thrice

$$[CY.5 == \underline{\underline{3}}].$$

Q1 Given N elements every element repeats twice except 2 unique elements. Find two unique elements.

Ex: arr(GT) $\rightarrow \{3, 6, 4, 4, 3, 8\}$ unique elements = 6, 8
arr(U) $\rightarrow \{4, 9, 9, 8\}$ unique elements = 4, 8

Idea-1. for every element, iterate and find the frequency.

T.C $\rightarrow O(N^2)$, S.C $\rightarrow O(1)$

~~out~~

Idea-2. for every element, get frequency using hashmap.

T.C $\rightarrow O(N)$, S.C $\rightarrow O(N)$

Idea-3. Sort the array + iterate and get unique elements.

arr $\rightarrow \{3, 3, 4, 4, 6, 8\}$

T.C $\rightarrow O(N \log N)$, S.C $\rightarrow O(1)$

Idea-4: XOR of all the elements.

$$\{ \cancel{1} \cancel{6} \cancel{1} \cancel{4} \cancel{1} \cancel{3} \cancel{8} \} = \{6^8\} = 14$$

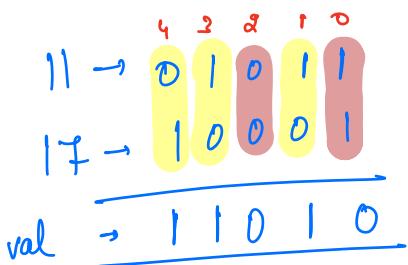
[How to extract 2-unique elements from this value.]

arr →

0	1	2	3	4	5	6	7	8	9	10	11
10	8	8	9	12	9	6	11	10	6	12	17
1010	1000	1000	1001	1100	100	0110	1011	1010	0110	1100	10001

Split the array based on 1st bit position.

$$val = 11 \wedge 17$$



1st bit.



$$\begin{cases} 10, 6, 11, 10 \\ 6 \end{cases}$$

$$\begin{cases} 8, 8, 9, 12 \\ 9, 12, 17 \end{cases}$$

s1

s2

observation: bits are different

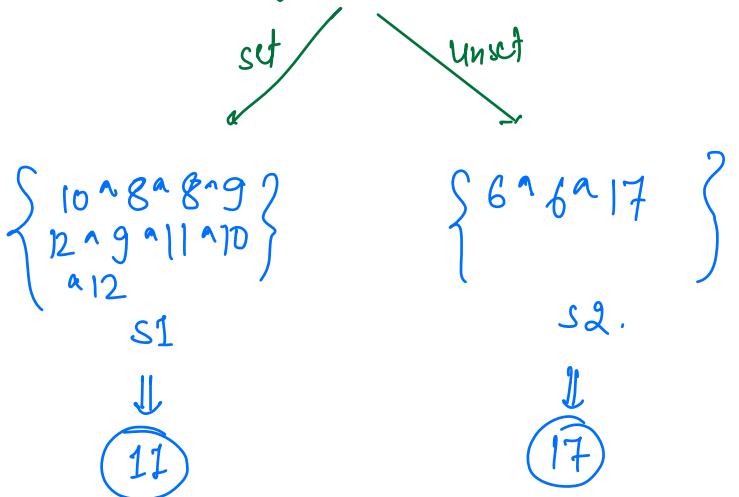
at pos → [1, 3, 4].

for 2 unique elements.

$$\text{xor of s1} = 11$$

$$\text{xor of s2} = 17$$

Split the array on 3rd bit position



pseudo code

Step 1: Take XOR of all the elements

```

val = 0, pos = -1
for (i = 0; i < N; i++) {
    val = val ^ arr[i]
}

```

] N

Step 2: get a set-bit position in val

```

for (i = 0; i < 32; i++) {
    if (checkBit(val, i)) {
        pos = i
        break
    }
}

```

] 32

Step 3: Split the array elements on basis of posth bit

```

s1 = 0, s2 = 0
for (i = 0; i < N; i++) {
    if (checkBit(arr[i], pos)) {
        s1 = s1 ^ arr[i]
    } else {
        s2 = s2 ^ arr[i]
    }
}

```

] N

Step 4: print (s1 and s2)

T.C $\rightarrow O(N)$
S.C $\rightarrow O(1)$

Q1) Given N array elements . Array contains all elements from $[1 \text{ to } N+2]$ except 2 elements . find 2 missing elements.

$\text{arr}[4]: \{ 3 \ 6 \ 1 \ 4 \}$

missing elements
 $[2, 5]$

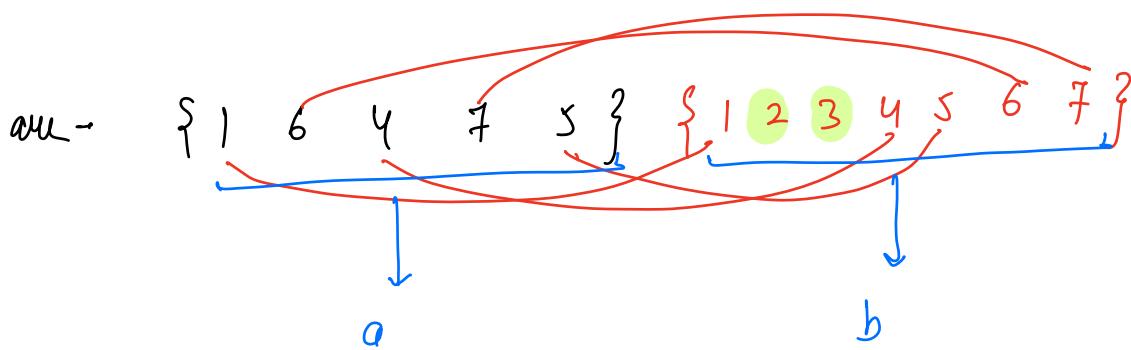
$\text{arr}[5]: \{ 1 \ 6 \ 4 \ 7 \ 5 \}$

missing elements
 $[2, 3]$

ideas. →

- nested loop
- hashmap
- sort
- Swap elements to keep them at their correct position

Todo



{XOR of all elements from
1 to $N+2$ }

$[a \wedge b = \text{XOR of all the elements}]$

$b = 0$
 for($i=1$; $i \leq N+2$; $i++$) {
 }
 $b = b \wedge i$;
 }

$a = 0$
 for($i=0$; $i < N$; $i++$) {
 }
 $a = a \wedge arr[i]$

$$val = a \wedge b$$

→ Remaining steps of previous question. {#todo}

$$\left\{ T.C \rightarrow O(N) \quad S.C \rightarrow O(1) \right\}$$

→ Repeat & Missing Number. } → very similar to above question.