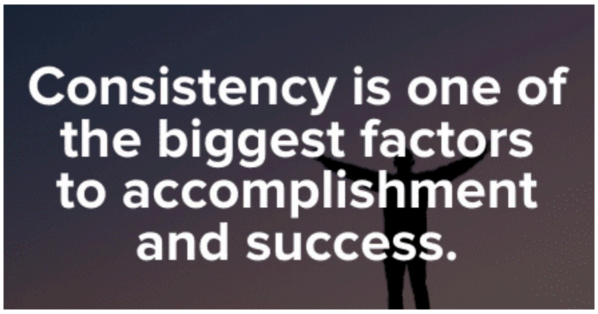
Today's Quote



Content → Basics of Bit Manipulation.

Number System

→ **Decimal Number System** ⇒ $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ Base → 10

$743 \rightarrow 7 \times 100 + 4 \times 10 + 3$

$\rightarrow 7 \times 10^2 + 4 \times 10^1 + 3 \times 10^0$

$2568 \rightarrow 2 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 8 \times 10^0$.

→ **Binary Number System.** $\{0, 1\}$ → base = 2.

$110 \rightarrow 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \Rightarrow 6.$

$\underset{3\,2\,1\,0}{1011} \rightarrow (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \Rightarrow 11$

0	10	20	30		90	100
1	11					
2	12					
3	13					
4				- - - -		
5						
6						
7						
8						
9	19	29	39		99	

0 → 0	10 → 2	100 → 4	1000 → 8
1 → 1	11 → 3	101 → 5	
		110 → 6	
		111 → 7	

Binary to Decimal

Eg. 1 0 1 1 0

4 3 2 1 0

$0 \times 2^0 = 0$

$1 \times 2^1 = 2$

$1 \times 2^2 = 4$

$0 \times 2^3 = 0$

$1 \times 2^4 = 16$

22

$\therefore (10110)_2 = (22)_{10}$

Eg: 1 0 1 1 0 1 0

6 5 4 3 2 1 0

$0 \times 2^0 = 0$

$1 \times 2^1 = 2$

$0 \times 2^2 = 0$

$1 \times 2^3 = 8$

$1 \times 2^4 = 16$

$0 \times 2^5 = 0$

$1 \times 2^6 = 64$

90

1011010

6543210

$2^6 + 2^4 + 2^3 + 2^1$

$64 + 16 + 8 + 2 = 90.$

$\therefore (1011010)_2 = (90)_{10}$

Eg: $(1020110)_2 = (?)_{10}$ Invalid

Decimal to Binary

$$(20)_{10} = (\underset{4}{1}\,\underset{3}{0}\,\underset{2}{1}\,\underset{1}{0}\,\underset{0}{0})_2 .$$

		rem.
2	20	
2	10	→ 0
2	5	→ 0
2	2	→ 1
2	1	→ 0
	0	→ 1

$$\Downarrow$$

$$2^4 + 2^2 = 16 + 4 = \underline{\underline{20}} .$$

$$(45)_{10} = (\underset{5}{1}\,\underset{4}{0}\,\underset{3}{1}\,\underset{2}{1}\,\underset{1}{0}\,\underset{0}{1})_2$$

2	45	
2	22	→ 1
2	11	→ 0
2	5	→ 1
2	2	→ 1
2	1	→ 0
	0	→ 1

$$\Downarrow$$

$$2^5 + 2^3 + 2^2 + 2^0 = \underline{\underline{45}} .$$

$$\therefore (45)_{10} = (101101)_2 .$$

Addition.

$$\begin{array}{r} \ {}^{1}\ \\ 3\ 6\ 7 \\ +\ 4\ 2\ 9 \\ \hline 7\ 9\ 6 \\ \hline \end{array}$$

$0 + 0 \rightarrow 0$

$0 + 1 \rightarrow 1$

$1 + 0 \rightarrow 1$

$1 + 1 \rightarrow 10$

$(2)_{10} = (10)_2$

$$\begin{array}{r} 1\ \ 1\ 1\ \\ 1\ 0\ 1 \\ +\ 0\ 1\ 1 \\ \hline 1\ 0\ 0\ 0 \\ \hline \end{array} \quad \begin{array}{l} \rightarrow (5) \\ \rightarrow (3) \\ \Rightarrow (8) \end{array}$$

$$\begin{array}{r} 1\ 1\ 1\ \\ 1\ 0\ 1 \\ +\ 1\ 1\ 1 \\ \hline 1\ 1\ 0\ 0 \\ \hline \end{array} \quad \begin{array}{l} \rightarrow (5) \\ \rightarrow (7) \\ \hline (12) \end{array}$$

Eg:

$$\begin{array}{r} 1\ 1\ \\ 1\ 0\ 1\ 1\ 0 \\ +\ 0\ 0\ 1\ 1\ 1 \\ \hline 1\ 1\ 1\ 0\ 1 \\ \hline \end{array} \quad \begin{array}{l} \rightarrow (22) \\ \rightarrow (7) \\ \rightarrow (29) \end{array}$$

Bitwise Operators

$(42)_{10} \rightarrow (101010)_2$ [1 byte → 8 bits]

integer → 4 bytes of space → $\boxed{32 \text{ bits}}$

$42 \Rightarrow \{ \underset{31}{0}\ \underset{30}{0}\ \underset{29}{0}\ \underset{28}{0}\ \underset{27}{0} \ldots \underset{5}{1}\ \underset{4}{0}\ \underset{3}{1}\ \underset{2}{0}\ \underset{1}{1}\ \underset{0}{0} \}$

$\Downarrow$

$(0 \times 2^{31}) + (0 \times 2^{30}) + (0 \times 2^{29}) + (0 \times 2^{28}) + \ldots (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$

$= 42.$

Operators → { AND, OR, NOT, XOR, $\boxed{\text{Left Shift, Right Shift}}$ }

& | !/~ ^ << >>

A	B	A & B	A | B	A ^ B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

0 → (unset)
1 → (set)

↳ { same same puppy shame }

Bitwise operations on numbers

$\boxed{5 \,\&\, 6 = 4}$

$5 \rightarrow 1\ 0\ 1$
$6 \rightarrow 1\ 1\ 0$ &
$4 \rightarrow 1\ 0\ 0$

$A = 20, \; B = 45$

$A \mid B = 61$

$A \rightarrow 0\ 1\ 0\ 1\ 0\ 0$
$B \rightarrow 1\ 0\ 1\ 1\ 0\ 1$
$A|B \rightarrow 1\ 1\ 1\ 1\ 0\ 1$

$5\ 4\ 3\ 2\ 1\ 0$

$\Downarrow$

$2^5 + 2^4 + 2^3 + 2^2 + 2^0 = 61$

$A = 20, \; B = 45$

$\boxed{A \wedge B = 57}$

$A \rightarrow 0\ 1\ 0\ 1\ 0\ 0$
$B \rightarrow 1\ 0\ 1\ 1\ 0\ 1$
$A \wedge B \rightarrow 1\ 1\ 1\ 0\ 0\ 1$

$5\ 4\ 3\ 2\ 1\ 0$

$\Downarrow$

$2^5 + 2^4 + 2^3 + 2^0 = 57$

Properties

① $A \& 1 = ?$

$A = 10 \rightarrow$

$$\begin{array}{r} 1\,0\,1\,0 \\ \& \; 0\,0\,0\,1 \\ \hline 0\,0\,0\,0 \end{array}$$

$A = 9 \rightarrow$

$$\begin{array}{r} 1\,0\,0\,1 \\ \& \; 0\,0\,0\,1 \\ \hline 0\,0\,0\,1 \end{array}$$

Conclusion.

$A \& 1$ →
- 0, if last-bit of A is 0 ⇒ A is even
- 1, if last-bit of A is 1 ⇒ A is odd

1 0 1 1 0 1

5 4 3 2 1 | 0

↓ Even + ↓ odd = $\boxed{\text{odd.}}$

$2^5 + 2^3 + 2^2,$

1 0 1 1 1 0

5 4 3 2 1 0

↓ Even + 0 = Even.

1 1 1 1 1 1 1 _

7 6 5 4 3 2 1 0

$2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1$

⇓

$128 + 64 + 32 + 16 + 8 + 4 + 2$

② $A \,\&\, 0 = 0$

③ $A \,\&\, A = A$

④ $A \,|\, 0 = A$

$$\begin{array}{rl} A \rightarrow & 101 \\ & 000 \\ \hline A|0 \rightarrow & 101 \end{array} \qquad \begin{array}{rl} A \rightarrow & 101 \\ A \rightarrow & 101 \\ \hline A|A \rightarrow & 101 \end{array}$$

⑤ $A \,|\, A = A$

⑥ $A \wedge 0 = A$

$$\begin{array}{rl} A \rightarrow & 101 \\ & 000 \;\wedge \\ \hline A\wedge 0 \Rightarrow & 101 \end{array} \qquad \begin{array}{rl} A \rightarrow & 101 \\ A \rightarrow & 101 \\ \hline A\wedge A \rightarrow & 000 \end{array}$$

*** ⑦ $\underline{A \wedge A = 0}$

↳ XOR of two same values = 0.

$\boxed{\text{Break till 10:42}}$

$$\begin{array}{rl} A \rightarrow & 101101 \\ 0 \rightarrow & 000000 \\ \hline A\wedge 0 \rightarrow & 101101 \end{array} = A \qquad \boxed{A\wedge 0 = A}$$

⑧ <u>Commutative</u> <u>Property</u>.

$a \,\&\, b = b \,\&\, a$

$a \,|\, b = b \,|\, a$

$a \wedge b = b \wedge a$

$\underbrace{(a \,\&\, b)}_{x} \,\&\, c = c \,\&\, \underbrace{(a \,\&\, b)}_{x}$

$x \,\&\, c = c \,\&\, x$

$\left[\begin{array}{l} c\,|\,\underline{a\,|\,b} = \underline{b\,|\,a}\,|\,c \\ = c\,|\,b\,|\,a = a\,|\,b\,|\,c = b\,|\,c\,|\,a \end{array}\right]$

⑨ <u>Associative Property</u>

$$(a \& b) \& c = a \& (b \& c)$$
$$(a \mid b) \mid c = a \mid (b \mid c)$$
$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

$$\begin{bmatrix} b \& a \& c \\ a \& b \& c \\ a \& c \& b \\ c \& a \& b \end{bmatrix}$$

They all are equal.

Q → $a \wedge b \wedge d \wedge b \wedge c \wedge c \wedge a$

$$= \underbrace{0 \wedge a \wedge}\ \underbrace{b \wedge b}\ \wedge\ \underbrace{c \wedge c}_{1} \wedge d$$
$$= 0 \wedge 0 \wedge 0 \wedge d$$
$$= 0 \wedge d = \underline{\underline{d}}.$$

Q. $1 \wedge 3 \wedge 5 \wedge 3 \wedge 2 \wedge 1 \wedge 5$

$$= \underbrace{1 \wedge 1}\ \wedge\ \underbrace{3 \wedge 3}\ \wedge\ \underbrace{5 \wedge 5} \wedge 2$$
$$= 0 \wedge 0 \wedge 0 \wedge 2 = 0 \wedge 2 = \underline{\underline{2}}.$$

Q.) Given an integer arr[N]. All the no's are present twice in the array except only one. Find the no. which is only present once.

arr[N] : [6 9 6 5 9 8 5]

(indices: 0 1 2 3 4 5 6)

$\Rightarrow 6 \wedge 9 \wedge 6 \wedge 5 \wedge 9 \wedge 8 \wedge 5 = 8$

$\Rightarrow \underbrace{6 \wedge 6} \wedge \underbrace{9 \wedge 9} \wedge \underbrace{5 \wedge 5} \wedge 8$

$0 \wedge 0 \wedge 0 \wedge 8 = 0 \wedge 8 = \underline{8}.$

<u>pseudo code</u> →

```
int fun( arr, N) {
    ans = 0
    for( i → 0 to N-1) {
        ans = ans ^ arr[i]
    }
    return ans
}
```

$\boxed{\begin{array}{l} T.C \rightarrow O(N) \\ S.C \rightarrow O(1) \end{array}}$

arr → {6, 9, 9, 8, 6}

ans = ~~6~~ ~~15~~ ~~6~~ ~~14~~ 8

$0 \wedge 6 = 6$

$6 \wedge 9 = 15$

$15 \wedge 9 = 6$

$6 \wedge 8 = 14$

$14 \wedge 6 = 8$

0 1 1 0 → 6

1 0 0 0 → 8

1 1 1 0 → $6 \wedge 8$

1 1 1 0 → 14

0 1 1 0 → 6

1 0 0 0 → $14 \wedge 6$

Left - shift.

int → 4 bytes ⇒ 32-bits.

Assumption. {int → 8 bits.} , just for explaination.

	7	6	5	4	3	2	1	0	
$a = 45$	0	0	1	0	1	1	0	1	⇒ {45}
$(a \ll 1)$ discard	0	1	0	1	1	0	1	0	⇒ {90}
$(a \ll 2)$ discard	1	0	1	1	0	1	0	0	⇒ {180}
$(a \ll 3)$ discard	0	1	1	0	1	0	0	0	⇒ $\boxed{104}$

*2, *2

360

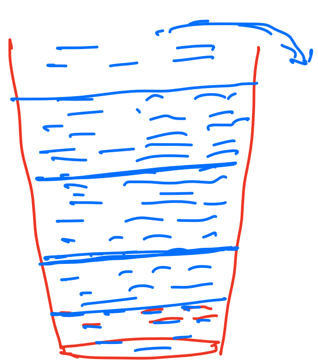
Why?

$2^6 + 2^5 + 2^3 = 64 + 32 + 8 = 104$

$$\boxed{\begin{aligned}(a \ll n) &= a * 2^n \\ (1 \ll n) &= 2^n\end{aligned}}$$

360 is too large to be stored in 8-bits.

⇒ Overflow condition



Capacity → 10 lt.

$1\,lt \xrightarrow{*2} 2\,lt \xrightarrow{*2} 4\,lt \xrightarrow{*2} 8\,lt$]*2

Right Shift operator

	7	6	5	4	3	2	1	0		
$a = 20$	0	0	0	1	0	1	0	0	discard.	20
										%2
$a >> 1$	0	0	0	0	1	0	1	0	discard	10
										%2
$a >> 2$	0	0	0	0	0	1	0	1		5
										%2
$a >> 3$										2
										%2
$a >> 4$										1
										%2
$a >> 5$										0

$$\boxed{a >> n = a / 2^n}$$

No overflow condition.

Doubts.

$2^{22} \rightarrow \boxed{2^{40}}$

long a = 1

a << 40 ✓

long a = (long) a[i] * (long)(i+1) * (long)(n-i).

overflow for int.

⇓

[take long data-type instead.]

$10^{9} * 10^{15} * 10^{15}$

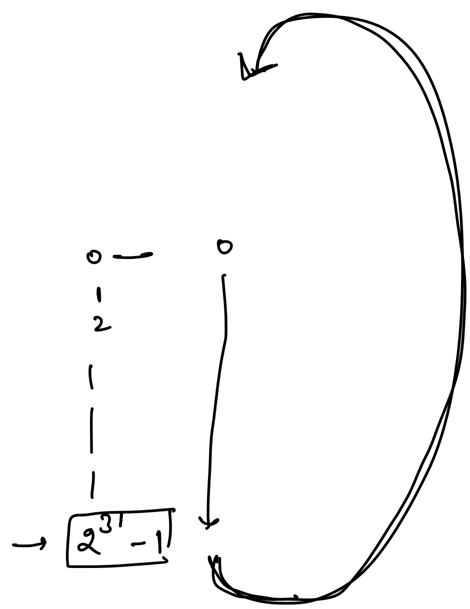
int int int.

$\Rightarrow \boxed{10^{39}}$

smaller value.

int ± * / int ⇒ int.

long ± * / long ⇒ value.



$2^{31}-1+1 \Rightarrow \cancel{\boxed{2^{31}}} \Rightarrow -2^{31}$

table.

i	j	k

```
for( i=0; i<n; i++){
    for(j=i; j<=n; j++){
        long sum=0
        for(k=1; k<=j; k++){
        }
    }
}
```

$\boxed{TC \to 2.}$

after 1hr.