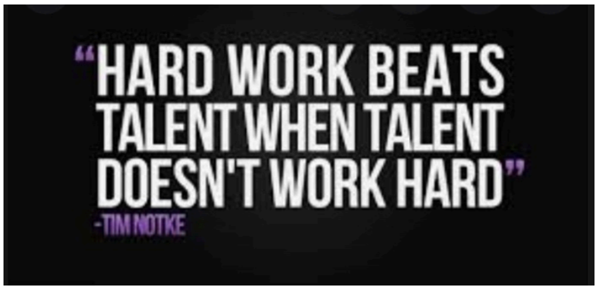
Good Morning Friends ☺

Today's Quote



Today's Content

→ % operator

→ Modular arithmetic

→ 1 Hard problem

Range

int → $[-2 \times 10^9, 2 \times 10^9]$

long → $[-9 \times 10^{18}, 9 \times 10^{18}]$

$$\left[\underset{\text{dvd}}{100} = (\underset{\text{quo}}{14} \times \underset{\text{div}}{7}) + \underset{\text{rem}}{2} \right.$$

% Basics

n % a = Remainder when n is divided by a.

100 % 7 = 2 = [100 - greatest mult. of 7 <= 100] = 2.

Dividend = div * quo + remainder.

$$\boxed{\text{Remainder} = \text{dividend} - (\text{div} * \text{quo})}$$

↳ { greatest mult of div <= dvd }

Quizes.

rem = dvd − (div * quo)   //

$150 \% 11 = 150 - (\underbrace{\text{greatest mult of } 11 <= 150}_{[143]}) = 7$

$100 \% 7 = 100 - (\underbrace{\text{greatest mult of } 7 <= 100}_{98}) = 2$

$-40 \% 7 = -40 - (\underbrace{\text{greater mult of } 7 <= -40}_{-42})$

$= -40 - (-42) = -40 + 42 = 2$

$-60 \% 9 = -60 - (\underbrace{\text{greatest mult of } 9 <= -60}_{[-63]})$

$= -60 - (-63) = -60 + 63 = 3$

$-40 \% 9 = -40 - (\underbrace{\text{greatest mult of } 9 <= -40}_{[-45]})$

$= -40 - (-45) = -40 + 45 = 5$

	Python		Java / C / C++ / C#
-40 % 7	2	$\xleftarrow{+7}$	-5
-60 % 9	3	$\xleftarrow{+9}$	-6
-40 % 9	5	$\xleftarrow{+9}$	-4

Java / C / C++ / C# → Doubt session / Extra content

in these languages.

if (a < 0){
//correct ans.
a % p + p ;

3

Why % : limit our input data in required range

- Hashing : {upcoming classes}
- Consistent Hashing : {HLD, LLD}

$$\left.\begin{matrix} -50{,}000 \\ \\ 50{,}000 \end{matrix}\right\} \% \, 10 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} = [0, 9]$$

$$\left.\begin{matrix} -\infty \\ \\ +\infty \end{matrix}\right\} \% \, p = [0, p-1]$$

% + {+, -, *, /}

Modular Arithmetic

{[0, p-1] + [0, p-1]} > p % p ⇒ [0, p-1]

$$\boxed{(a+b) \% p = (a \% p + b \% p) \% p}$$

a	b	p		
8	6	10	4	8 % 10 + 6 % 10 = (8 + 6) % 10 = 14 % 10 = 4
5	4	6	3	5 % 6 + 4 % 6 = (5 + 4) % 6 = 9 % 6 = 3

$$\boxed{(a * b) \% p = (a \% p * b \% p) \% p}$$

a	b	p		
8	6	10	8	$(8\%10 * 6\%10)\%10 = (48)\%10 = 8$
5	4	6	2	$(5\%6 * 4\%6)\%6 = (20)\%6 = 2.$

$(a-b)\%p$

$(a/b)\%p$

$\rightarrow$ In Advanced module. $\rightarrow$ { Inverse modulo }

① $(a \% p) \% p = a \% p$

$\underbrace{(a\%p)}_{[0 - p-1]} \% p = \underbrace{a \% p}_{[0, p-1]}$

$[0 - p-1] \% p = [0, p-1]$

$6 \% 10 = 6$

$\underbrace{(6 \% 10)} \% 10 = \underline{\underline{6}}.$

② $(a\%p * b) \% p = (a * b) \% p$

$\underline{x = a\%p \quad , \quad y = b.}$

$(x * y) \% p = (x\%p * y\%p) \% p$

$= (\underbrace{(a\%p)\%p} * b\%p) \% p$

$= (a\%p * b\%p) \% p$

$= (a * b) \% p$

[Break till → 11:45 AM]

Divisibility Rules

$\% 3 \to$ sum of all digits should be divisible by 3.

$\% 9 \to$ sum of all digits should be divisible by 9

$\% 4 \to$ last 2 digits should be divisible by 4.

$\% 8 \to$ last 3 digits should be divisible by 8.

1, 2, 3, 4, 5, 6, $\boxed{7}$, 8, 9

6 → 2, 3

7 → TODO

Proof %3

$2475 \% 3 = (2*10^3 + 4\times10^2 + 7\times10^1 + 5) \% 3$

$= [(2*10^3)\%3 + (4\times10^2)\%3 + (7\times10^1)\%3 + (5\times10^0)\%3]\%3$

$= [2\%3 + 4\%3 + 7\%3 + 5\%3]\%3$

$= (2+4+7+5)\%3$

observation.

$10^0 \% 3 = 1$	$10^0 \% 9 = 1$
$10^1 \% 3 = 1$	$10^1 \% 9 = 1$
$10^2 \% 3 = 1$	$10^2 \% 9 = 1$
$10^3 \% 3 = 1$	$10^3 \% 9 = 1$
$10^x \% 3 = 1$	$10^x \% 9 = 1$

Proof %4.

$2457 \% 4 = (2400 + 57)\%4$

$= ((2400\%4) + (57\%4))\%4$

$= (57\%4)\%4 = \underline{57\%4}$

observation

$10^2 \% 4 = 0$

$10^3 \% 4 = 0$

Any multiple of 100 will be divisible by 4.

Eg: $2743 \% 4 \to (\underline{2700} + \underline{43})\%4$

Q) Given a, n, p. Calculate $a^n \% p$ without inbuilt functions.

Constraints $1 <= a <= 10^9$, $2 <= p <= 10^9$, $1 <= n <= 10^5$

Eg: $a=3$, $n=4$, $p=7$

① $a << n \% p \Rightarrow a * 2^n \% p$ [x].

②

```
fun( a, n, p){
    for( i = 1 ; i <= n ; i++){
        a = a * a
    }
    return a % p
}
```

[a, n=4 ⇒ $a^4 \% p$.]

i	value	
1	a^2	[a*a]
2	a^4	[$a^2 * a^2$]
3	a^8	[$a^4 * a^4$]
4	a^{16}	[$a^8 * a^8$]

$a^{16} \% p$.

③.

```
fun( a, n, p){
    long ans = 1
    for( i = 1 ; i <= n ; i++){
        ans = (ans * a )% p
    }
    return ans ;
}
```

a	n	p	
2	30	47	$2^{30} \% 47$
2	60	47	$2^{60} \% 47$
2	100	47	$2^{100} \% 47$

$ans = (ans * a) \% p$

$[0, p-1] * [a] = 10^9 \times 10^9 \approx 10^{18} \Rightarrow$ long can hold this

dry-run. // Given a, $n=4$, p.

ans [long]	i	ans = (ans * a) % p
1	1	ans = $a \% p$ ⇒ No overflow.
$a \% p$	2	ans = $(a\%p * a)\%p$
		$= (\underbrace{(a\%p)\%p} * a\%p)\%p$
		$= (a\%p * a\%p)\%p$
		$= (a*a)\%p = a^2\%p$ ↳ no overflow
$a^2 \% p$	3	ans = $(\underbrace{a^2\%p}_{10^9} * \underbrace{a}_{10^9})\%p$
		ans = $a^3\%p$ ⇒ No overflow
$a^3\%p$	4	ans = $(\underbrace{a^3\%p}_{10^9} * \underbrace{a}_{10^9})\%p$
		$= a^4\%p$ ⇒ No overflow

Q) Given 1 number in arr[] format. Calculate arr[] % p

Note → arr[i] represents a single digit of number.

Google

Constraints:

$$\left[\begin{array}{l} 1 <= N <= 10^5 \\ 0 <= arr[i] <= 9 \\ 2 <= p <= 10^9 \end{array}\right]$$

Eg: arr[5]:

7	2	6	4	3
0	1	2	3	4

, p = 50 [ans → 43]

↳ { 72643 % 50 = 43 }

Eg: arr[4]:

2	3	7	5
0	1	2	3

, p = 16 [ans → 7]

↳ { 2375 % 16 = 7 }

ideas:
① Convert arr[] → number % p

N=2   9 9 $= 10^2 - 1$

N=3.   9 9 9 $= 10^3 - 1$

N=4   9 9 9 9 $= 10^4 - 1$

⋮

$N = 10^5$ — $= 10^{10^5} - 1$

very. very large.

② Calculating divisibility rules.

For every value of P, it is not possible to have divisibility rule.

Hint: Split the no. digit by digit & then try to calculate the answer.

ans:

0	1	2	3	4
6	7	3	4	5

$5\times10^0\,]\ \%\ p$

4×10^1

3×10^2

7×10^3

$[\,6\times10^4$

$$= [6\times10^4 + 7\times10^3 + 3\times10^2 + 4\times10^1 + 5\times10^0]\ \%\ p$$

$(5\times10^0)\%p\ ]\ \%\ p$

$+$

$t = 1$

$5*t$

$(4\times10^1)\%p$

$+$

$t = (t*10)\%p$

$ans = ans + 4*t$

$(3\times10^2)\%p$

$+$

$t = (t*10)\%p$

$ans = ans + 3*t$

$(7\times10^3)\%p$

$t = (t*10)\%p$

$[\,(6\times10^4)\%p$ $+$

$t = (t*10)\%p$

pseudo code:

```
fun( arr[], N, p){
    long ans = 0
    long t = 1      //10^0 = 1
    for( i = n-1 ; i >= 0 ; i--){
        ans = (ans + arr[i] * t ) % p
        t = (t * 10) % p
    }
    return ans;
}
```

$$\boxed{\begin{array}{l} TC \rightarrow O(N) \\ S.C \rightarrow O(1) \end{array}}$$

Doubts

$ans = (ans + arr[i] * t) \% p$

$[0, p-1] + [0-9] * [0, p-1]$

$10^9 + 9 \times 10^9 = 10^{10}$

$t = (t * 10) \% p$

$[0, p-1] * 10$

$\Rightarrow 10^9 * 10 = 10^{10}$

1
$\downarrow$
10
$\downarrow$
10^2
$\downarrow$
10^3
$\downarrow$

$10^{18} \rightarrow 10^{19} \rightarrow 10^{20} \rightarrow 10^{21}$ —— 10^{100}

$$100 \% 7 = \underbrace{100 - (7 \times 14)} = \underline{\underline{2}}.$$

$$= 100 - (\text{greatest mult of } 7 * 7)$$

$$= 100 - \left(\left\lfloor \frac{100}{7} \right\rfloor * 7\right)$$

$$-40 \% 7 = -40 - (7 * \text{greatest mult of } 7 <= -40)$$

$$= -40 - \left(7 * \left\lfloor \frac{-40}{7} \right\rfloor\right)$$

$$= -40 - (7 * -5) = -40 + 35 = \underline{\underline{-5}}.$$

$$-60 \% 9 = -60 - (9 * \text{greatest mult of } 9 <= -60)$$

$$= -60 - \left(9 * \left\lfloor \frac{-60}{9} \right\rfloor\right)$$

$$= -60 - (9 * (-6)) = -60 + 54 = \underline{-6}.$$

<u>% python</u>

$$\frac{-60}{9} = -6.666$$

$$\frac{20}{7} = 2.8 \ldots$$

$$6/4 = 1.5$$

$$\text{floor}\left(-60/9\right) = \text{floor}\left(\underline{-6.666\ldots}\right)$$

$$= -7$$

$$-60 \% 9 = -60 - [9 * \text{greatest mult of } 9 <= -60]$$

$$= -60 - \left[9 * \text{floor}\left(\frac{-60}{9}\right)\right]$$

$$= -60 - [9 * -7] = -60 + 63 = \underline{\underline{3}}.$$