Y. B. KAFAI, M. L. FRANKE, C. C. CHING and J. C. SHIH

GAME DESIGN AS AN INTERACTIVE LEARNING ENVIRONMENT FOR FOSTERING STUDENTS' AND TEACHERS' MATHEMATICAL INQUIRY

ABSTRACT. Many learning environments, computer-based or not, have been developed for either students or teachers alone to engage them in mathematical inquiry. While some headway has been made in both directions, few efforts have concentrated on creating learning environments that bring both teachers and students together in their teaching and learning. In the following paper, we propose game design as such a learning environment for students and teachers to build on and challenge their existing understandings of mathematics, engage in relevant and meaningful learning contexts, and develop connections among their mathematical ideas and their real world contexts. To examine the potential of this approach, we conducted and analyzed two studies: Study I focused on a team of four elementary school students designing games to teach fractions to younger students, Study II focused on teams of pre-service teachers engaged in the same task. We analyzed the various games designed by the different teams to understand how teachers and students conceptualize the task of creating virtual game learning environment for others, in which ways they integrate their understanding of fractions and develop notions about students' thinking in fractions, and how conceptual design tools can provide a common platform to develop meaningful fraction contexts. In our analysis, we found that most teachers and students, when left to their own devices, create instructional games to teach fractions that incorporate little of their knowledge. We found that when we provided teachers and students with conceptual design tools such as game screens and design directives that facilitated an integration of content and game context, the games as well as teachers' and students' thinking increased in their sophistication. In the discussion, we elaborate on how the design activities helped to integrate rarely used informal knowledge of students and teachers, how the conceptual design tools improved the instructional design process, and how students and teachers benefit in their mathematical inquiry from each others' perspectives. In the outlook, we discuss features for computational design learning environments.

INTRODUCTION

A central issue in mathematics education focuses on engaging teachers and students in mathematical inquiry. Teacher development has focused on engaging teachers in reflecting on the theoretical underpinnings of their decision making (Cobb et al., 1991, 1992), engaging teachers in understanding the development of children's thinking (Carpenter, Fennema and

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Franke, 1996; Schifter and Fosnot, 1993) and developing model classroom environments that foster students' understanding (Ball, 1993). Research on students' thinking and learning has focused on understanding the development of children's mathematical thinking within particular mathematical domains (Grouws, 1992), identifying authentic mathematical tasks (Lave, 1988), and engaging students in both sharing and challenging their problem solving solutions (Ball, 1993; Fennema, Franke, Carpenter and Carey, 1993). For the most part, the research on teachers and the research on students occurs independently; yet, within both areas the research suggests that engaging either teachers or students in following programs or routines does not support mathematical inquiry that leads to sophisticated understanding (Brown and Campione, 1996). Mathematical inquiry leading to the development of understanding requires that both students and teachers operate on principled ideas in their learning and teaching. Our work explores the potential of creating integrated opportunities for both teachers and students that highlight principled ideas rather than routines.

Recently, Hiebert and his colleagues discussed what we consider the essence of principled ideas by introducing and elaborating *problematizing*. According to Hiebert et al. (1996) problematizing asks teachers to "draw on two resources: knowledge of the subject to select the tasks that encourage students to wrestle with key ideas and knowledge of student thinking to select tasks that link with student experience and for which students can see the relevance of ideas and skills they already possess" (p. 16). For students, problematizing means "to wonder why things are, to inquire, to search for solutions and to resolve incongruities" (p. 12). Problematizing, defined as such, sets the parameters for a learning environment where teachers and students engage in mathematical inquiry that leads to the creation of on-going learning opportunities. Instructional design has been used to create such problematizing learning activities and opportunities. Traditionally, instructional design activities are created by those outside the classroom, like curriculum designers and content specialists. Here, we suggest that students and teachers become instructional designers themselves, where instructional design becomes the forum for problematizing.

In considering a context for instructional design, educational games as such do not come naturally to mind but are salient in other aspects. Games, especially video games, have become a significant part of children's culture in the late 20th century (Provenzo, 1991). A considerable number of research studies point out that teachers need to link their instructional practice to culturally relevant materials and activities (Resnick, 1987).

Many teachers have used educational games for their motivational benefits. We intend to build on these motivating and culturally relevant aspects of games but with a distinct difference by having teachers and students construct their own educational games – and to reexamine their mathematical thinking in the process. In the world of educational game design, such constructionist approaches have received far less attention than their instructional counterparts, yet we believe making games hold potential for not only engaging teachers and students in mathematical inquiry but also for integrating technology as an expressive mathematical medium.

In the present paper, we report on two studies attempting the first steps of creating such a "problematizing" environment; here we focus on game making activities for teaching fractions. In Study I, we examine a design session in which four elementary-school students designed computer games to teach fractions to younger students. In Study II, we present work with four teams of pre-service teachers who designed a series of computer games to help children learn about equivalent fractions. In contrast to previous research situated in the same framework (Kafai, 1995), students and teachers only created games on paper and in discussions but did not proceed to implement them on the computer. In the results section, we present how teachers and students conceptualize the task of creating virtual game learning environments for others, in which ways they integrate their understanding of fractions and develop notions about students' thinking in fractions, and how conceptual design tools can provide common platforms to develop meaningful fraction contexts. Based on the students' and teachers' game designs, we discuss how the conceptual design tools afforded opportunities for teachers and students to problematize their own learning and teaching. We conclude with an outlook of how computational environments can support game making activities for mathematical learning.

BACKGROUND

Student and Teacher Knowledge

The starting point for our research was the recognition that game making activities as a problematizing context needed to address not only students' problems with learning fractions but also those of teachers' providing instructional guidance. There is a substantial body of literature that analyzes the fundamental constructs underlying students' rational number knowledge (Behr et al., 1993; Harel and Confrey, 1994; Hiebert and Behr, 1988; Kieren, 1976, 1988, 1993). This theoretical work provides a solid conceptual basis for identifying students' fundamental rational number problems and an analysis of how children solve them (Behr et al., 1993). Much of the early empirical rational number research focused on children's errors and misconceptions (Kerslake, 1986; Post, 1981); yet, a growing body of research suggests that children possess intuitive knowledge of basic constructs underlying rational number concepts (Baker, 1994; Confrey, 1996; Kieren, 1988; Mack, 1990). In particular, children's development of fair sharing concepts (Streefland, 1991, 1993) seems to provide a promising starting place for developing rational number understanding.

Concurrent to students' problems with understanding fractions are teachers' problems with understanding fractions and developing problematizing teaching practices. The research literature suggests that teachers' knowledge orchestrates teachers' planning, teaching and interpretations of classroom interactions and activities (Brophy, 1991; Leinhart and Smith, 1985, Fennema and Franke, 1992; Lehrer and Franke, 1992; Loef, 1991; Munby, 1982; Solas, 1992; Pope and Keen, 1981). In her earlier work, Franke (Loef, 1991) found that teachers differed in the organization of their knowledge about the teaching and learning of mathematics, particularly as it pertained to the development of children's mathematical thinking. Teachers who successfully implemented a new model of teaching based on the development of children's mathematical thinking, not only integrated the development of children's thinking into their existing knowledge structures, but also restructured their knowledge so that the principles underlying the new model drove the organization of their knowledge. This and other research indicates that teachers can make use of research-based knowledge about children's mathematical thinking in their classroom practice (Fennema, Carpenter, Franke, Levi, Jacobs and Empson, 1996). But the key becomes how knowledge becomes integrated and organized for the teachers, and how teachers make connections between knowledge of children's thinking, knowledge of the mathematics, knowledge of pedagogy and how this knowledge organization plays out in their classrooms.

Educational Game Design

One way teachers motivate students for mathematical learning has been to use educational games on and off the computer (Block and King, 1987; Malone and Lepper, 1987). The large number of educational games to teach fractions found in teacher journals (e.g., Bride and Lamb, 1991; Fennell, Houser, McPartland and Parker, 1984; Priester, 1984) and on the commercial market (e.g., *MathBlaster*™) is another indicator of their widespread appeal and use. Yet, the particular instructional benefits of playing educational games are unclear; in spite of the general recognition of game playing's importance for children's intellectual, social, and emotional development (e.g., Bruner, Jolly and Sylva, 1976; Piaget, 1951; Sutton-Smith, 1986). There are few research studies that examine the ways in which the instructional objectives of educational games (such as the content, its placement in the course of instruction and its taxonomic level) are achieved. Bright, Harvey and Wheeler (1985) concluded that educational games could be used not just to teach basic operation skills but also to involve more complex cognitive processes. There are few studies that used and examined computer game playing for learning mathematical concepts and strategies. Some examples are graphing (Dugdale, 1981) and transformational geometry (Edwards, 1991). These studies also reported positive results. More recent research has examined children's learning of mathematical concepts and strategies while playing games (e.g., Saxe and Bermudez, 1996). While the educational benefits of playing educational games received some research attention, the potential of learning while making games remains open territory. Kafai (1995) investigated a class of fourth-grade students who were involved in programming computer games to teach fractions to younger students. A core assumption in educational game making was that students would construct their own fraction representations and in this process establish better connections between different fraction representations (such as written, symbolic, graphic) and connections to everyday objects. While students significantly increased their understanding of fractions, one of the problematic aspects in Kafai's study was the integration of fraction content and game ideas. With the exception of one game designer, all students developed games with extrinsic fraction integration. Extrinsic integration describes a context in which game idea and fraction content are unrelated (e.g., the game player is shooting rockets and solving fraction problems when missing rockets), whereas intrinsic integration describes a context in which game ideas and fraction content are related (e.g., the game player has to assemble fractions of a map to progress through the game). This distinction begins to sketch out a taxonomy of learning affordances, or the problematizing potential, of particular tasks: when designing an extrinsically integrated game, there are few incentives for the designers (and also for the learner) to think about the content matter.

Instructional Game Design for Teachers and Students

Previous research studies point out the learning benefits of engaging students and teachers in instructional design (Baylor, 1997; Harel, 1991). But this work also indicates that while students definitely benefit from the process of instructional design, the quality of products created did not equal or represent in most cases the quality of the instructional process.

Students within these instructional design projects often followed well-trained routines in learning fractions by representing decontextualized problems with a focus on understanding fraction operations rather than concepts. Our current studies attempt to address the imbalance – "good process but weak product" – found in traditional instructional design for learning. As teachers and students place themselves in the role of the teacher and the learner, we expect them to examine their instructional principles, reflect on how they incorporate both children's mathematical thinking and the mathematical content, and consider how they can provide explanatory representations and activities (Carpenter, Fennema and Franke, 1996; Wood et al., 1991). In this process teachers and students construct mathematical representations, strategies, and contexts that become available for examination by others in their design of educational games (Lampert, 1989; Streefland, 1991). Furthermore, we anticipate that our approach will engage teachers and students in generative change. Throughout the process of instructional game design teachers and students develop a set of principled ideas rather than a set of routines. Brown and Campione state that "without adherence to first principles, surface procedures tend to be adapted and ritualized in such a way that they cease to serve the thinking function they were originally designed to foster" (1996, p. 291). Within the design process the designers must blend different aspects of their knowledge and then structure the knowledge in a way that allows them to design. Here, the continual organization and reorganization of the designers' thinking, presses the designer to focus on the key ideas across the various aspects of their knowledge. In this way the designer – student or teacher – becomes focused on the principled ideas that allow for continued learning and growth.[1]

METHOD

To examine students' and teachers' educational game design environments for learning and teaching fractions, we conducted two studies, hereafter referred to as *Study I: Students' Game Designs* and *Study II: Teachers' Game Designs*. It should be noted that Study I preceded Study II, and that we used insights gained from the research with students in Study I for the design of conceptual design tools with teachers in Study II. For the purposes of this paper, we describe the participants and procedures used in Studies I and II, respectively, and then detail the methods used to merge and analyze the two data sets.

Study I: Students' Game Designs

Overview. Within Study 1, fifth grade students worked in teams to design educational games that would teach fractions to younger children.[2] Our goal was to document the different aspects of the learning environment and the ways in which the dimensions of instructional design contributed to or impeded students' mathematical explorations. We studied carefully one team of students throughout the entire design process because we were particularly interested in understanding the relationship amongst the different aspects of the game design process and the products produced.

Participants. A team of four girls served as the participants for our case analyses: Caren, Emily, Melanie, and Rachel.[3] The girls, ages 10 to 11, came from a fifth grade class of a suburban elementary school with an ethnically mixed student population. The entire fifth grade class participated in the project; the four girls were selected for focused analyses based on their productive conversation.

Method. The design session took place in the after-school room, around a table with five chairs and lasted approximately one hour. Large pieces of butcher paper on the table served as a way for students to draw and record their ideas; the paper also served as a way for the researcher to note students' ideas on the large board in the room. The fifty-minute session included three significantly different phases during which the researcher introduced design tools and focused students' thinking.

Phase 1: Sharing. In the first part of the design session, students presented the games they had designed for homework the night before. The homework worksheet asked students to describe game ideas, game characters, and fraction involvement. During this initial sharing phase design tools were not introduced. The researcher directed turn taking for sharing and at times asked clarification questions. This phase of the session lasted approximately seven minutes.

Phase 2: Emergent design tool. A significant shift in the design session occurred with the emergence of what became a design tool: using fractions to describe a real-life situation with multiple actors in a game scene. We call this design tool "emergent" because it was not brought into the environment by researchers but rather emerged from students' ideas and discourse. Following is the sequence in which the second design tool emerged from Caren's idea of a game like Jeopardy™, where the user picks categories for fraction questions. The introduction of categories initiated the idea of player choice in the design of the students' games.

Caren: And there's different categories, so if you want to do fractions with, like, TV shows, like Power Rangers. And you have to answer questions. Like there's a picture of the Power Rangers on the screen and it asks, "what part of the Power Rangers are standing?"

Melanie: Or, if you pick animals then do you get a fraction question about animals?

Researcher: So what happens after you've picked what you want, like animals?

Caren: So you pick animals, and then some animals show up.

Researcher: Can you all draw a scene showing how that would look on the screen?

Caren's idea for a game in which the user picked categories for fraction questions provided an opportunity for the researcher to engage other students in similar thinking. This phase of the session, during which students worked on using fractions to describe real-life scenes, lasted approximately 12 minutes.

Phase 3: Posing a challenge. In phase 3, the researcher posed a challenge to the students. Using a conceptual design tool, the researcher posed the question, "Can you create a game without asking questions?" Following is the discussion the students and researcher had about the meaning of this task.

Researcher: This is my challenge for you. Every time you think about questions, you think about asking. Now what I want you to think about is a different way to learn about fractions when you're not asking questions. This is a tough thing to do, but I want you to sit down for a moment and think about if you can come up with ideas or examples.

Melanie: So, like, think of something where kids in 4th grade wouldn't have to ask the teacher how to do it?

Caren: Well, you could put the answer and ask 'what's the question?'

Researcher: Okay, but not asking questions. A different way.

Caren: I thought that was the idea – asking questions!

Researcher: That's one way of doing fractions. Maybe you can think of another way, when you're not asking 'how many?' We have lots of examples of how to do that.

Students worked on the idea of creating a game without asking questions in this third phase of the session for the remaining 30 minutes.

Study II: Teachers' Game Designs

Drawing from the insights gained from the research with students in Study I, we examined the game designs and the influence of conceptual design tools when we engaged teachers in designing games. While the design tools in the student session emerged from the shared discourse, within the teacher sessions the researchers systematically introduced design tools at critical junctures in the design process.

Participants. Sixteen pre-service teachers enrolled in the first year of a teacher certificate program participated in the game design activities as part of their weekly discussion meetings. All of the preservice teachers had completed a mathematics methods course that focused on the development of children's mathematical thinking. Of the sixteen participating teachers, thirteen were women. The average age of the teachers was 24 years, most of them recent college graduates.

Method. The preservice teachers participated in three phases of game design, each occurring on separate days. The three sessions with the teachers took place during a six-week period, approximately two weeks apart.

Phase 1: Initial designs. In the first game design session, the sixteen teachers were divided into four teams. Each team was then asked to design a video game to teach fourth grade students about equivalent fractions. No design tools were presented. After twenty minutes, each team presented their final game designs. Each presentation was followed by a class discussion in which the preservice teachers evaluated each other's game designs and raised questions about the game designs and their potential effectiveness.

Phase 2: Introduction of conceptual design tools. In the second game design session, the teachers again designed games but within this process they engaged with two conceptual design tools. The first conceptual design tool shared was a blank page of empty computer screen frames, two to a page, on which they could draw and annotate their game designs. The second design tool was a question; we asked: "Can you create a game without asking questions?". After the teams spent approximately thirty minutes on generating game ideas, each team presented their final designs to the whole class. Teachers then discussed and evaluated each other's designs by comparing the game design results from the second session with those of the first session.

Phase 3: Extending conceptual design tools. The third game design session introduced teachers to another conceptual design tool. This particular tool focused teachers on the issue of dynamic representations by sharing with them an example from the students' designs where the students created a dynamic representation to teach fractions. During this session the teachers built on and extended their game designs developed during phase 2 by attempting to include dynamic representations. As in the other phases, after the teachers worked for 20 minutes, each team presented their final designs, and a class discussion ensued.

Analyses

Procedures for Study I. In analyzing the student games we first transcribed the 50-minute session. We then identified points within the discourse where students introduced and elaborated specific game ideas. While it was obvious that within the short time frame of one design session students would not be able to generate many full-fledged games, it became apparent that the game ideas proposed by students were of different specificity. To create interpretative units, we distinguished between game ideas and game scenes. We classified game ideas as ideas that described a general context in which the game would take place and provided some specification in regard to characters and development. We characterized game scenes as specific computer screens developed within a game context, they often provided more graphical detail in regard to fraction integration, user involvement, and feedback. In our analyses we treated the game ideas and scenes as distinct but equal units because the generality of the game ideas was compensated for by the specificity of the game scenes. We then evaluated all game ideas developed in each phase of the session using the coding scheme described below.

Procedures for Study II. All three of the teachers' game design sessions were video taped. In addition, we followed one team of game designers throughout all of the design sessions by recording all of their group interactions and discussions. We transcribed all of the videotapes and annotated the actual game designs using the teachers' own words from their descriptions of their games. Using the transcription of the group presentations and the ensuing class discussions, the twelve games created by the teachers were evaluated using the coding scheme described below.

TABLE I

Game Design Coding Scheme: Studies I and II Codes Operational Definition

Codes	Operational Definition
Integration	
extrinsic	Game idea and fraction content are separated
intrinsic	Game idea is merged with fraction content
constructivist	Game facilitates players' making of their own fraction content
Representation	
none	Student does not see or have access to any fraction representations
symbolic	Student has access to fraction symbols
fixed pictorial	Student has access to a pictorial fraction representation but cannot manipulate
fixed multiple	Student has access to multiple fixed representations where the pictures differ
free multiple	Student has access to multiple representations that can be manipulated
Consideration of User's Thinking	
Teachers	
none	Issues of children's thinking contexts are not apparent in the game
problems	Problems are designed to afford opportunities for children's thinking
strategies	Design includes thought to different strategy possibilities as well as problems
Students	
punitive	Game provides fraction questions as punishment for errors in unrelated action – no consideration of user's thinking
evaluative	Game is designed to test user's existing knowledge
constructive	Game is designed to provide opportunities for user's thinking about fractions and building new knowledge

Coding Scheme. We analyzed each game idea developed by the students and the teachers according to the following categories: (1) degree of fraction integration into the game's narrative or theme, (2) type(s) of fraction representation used in the game, and (3) degree of consideration displayed by the designers in accommodating the users' thinking processes. Table I provides the categories, the attributes within each category, and a brief description of the coding definitions, all of which will be further explained in the appropriate sections. It should be noted that the goal within the coding scheme was not to find ways to differentiate the games but rather to address issues of concern within this study. Each game for each session was coded separately by two coders.

RESULTS

In creating parallel conceptualizations of the student and teacher data sets, we employed the same coding scheme and applied it to all of the designers' games. In order to highlight the relationships and patterns across the student and teacher game designs we present the results for each group within each coding category. We examine the student games and then the teacher games within each of the following sections: fraction integration, fraction representation, and consideration of user's thinking.

Fraction Integration

One way to document development in students' and teachers' thinking about fractions is to examine the integration of fractions into the game context (Kafai, 1995). We defined three categories of fraction integration in the students' and teachers' games: extrinsic, intrinsic, and constructivist. *Extrinsic* integration of fractions describes the type of games the designers initially had in mind; games center mostly around arcade-style action, and fraction questions are asked occasionally but are unrelated to the game's theme or objective. For example, in Study I, Melanie described a maze game in which different things were chasing the player character and in case he/she bumped into them, the user had to answer a question. Another example is the pyramid game designed by teachers in Study II, in which users build a pyramid by correctly answering questions about fractions.

Intrinsic integration describes game units in which fractions are an inherent part of the scenery, narrative, or game objective. An example of a student-designed game where fractions are intrinsically integrated described a beach in which various fractions of people move in and out of the water (this scene will be treated in more detail in the following section). Another example is a scene from a game designed by teachers in Study II in which the user must divide up pizza among several friends at a party.

Finally, *constructivist* game ideas allow the user to actively create his or her own fractions from what is provided. Rather than constantly quizzing, the constructivist integration allows the user to formulate his or her own questions. An example is the game designed by Rachel, a student in Study I. In Rachel's game the player could create his or her own fractions by coloring in different pieces of a given animal. The intrinsic and constructivist fraction games represent a sophisticated departure from both the drill-and-practice routine of traditional education and the non-mathematical focus of most computer games.

Study I: Students' Fraction Integration

We found that as the 50-minute session progressed and design tools were introduced, the girls' ideas became more focused on intrinsic and constructivist integration. Extrinsic integration was observed occasionally, later in the session, when a new game or story-line idea was introduced, but when the girls revised those ideas to include fractions, they were usually intrinsically integrated.

Before design tools. In the first part of the game design session, lasting approximately 10 minutes, all four girls presented their game ideas developed as part of their homework. In all of these games, fractions were extrinsically integrated and very ill specified (see Table II). Fraction questions were presented as either a necessary hurdle to the progression of unrelated action, as in Emily's race track game ("if you get the right answer, you can move forward"), or as punishment for the player doing something wrong, as in Melanie's exploring rocket ("if you look in the wrong place, you get a fraction"). Since the students developed these games at home, without the influence of the researcher or other students, it can be argued that these games represent students' prior understandings of educational computer games, which consist of either question-answer formats or game events unrelated to fraction learning.

After design tools. After the introduction of the first (emergent) design tool, Caren's categories idea, students managed to integrate fractions intrinsically into all of their subsequent game scenes. All the game scenes developed during that second phase of the session used fractions to describe real-life situations with multiple actors, as in Caren's playground scene (Figure 1).

TABLE II

Students' games classified by fraction integration

	Extrinsic Integration	Intrinsic Integration	Constructivist Integration
Phase 1	Chase Maze Leprechaun Exploring Rocket Race Track Danger Race Track		
Phase 2		Fraction Categories Raccoons Beach Fractions Playground	
Phase 3	True or False Obstacle Course	Wizard Fraction Land Hidden Fractions Fraction Screen Fraction World Divided into Fractions! Ice Skating Fractions Fractions Baseball	Make Your Own Fractions! Color the Mouse Two-Player Game

When the researcher asked students to design a game without asking questions (the second design tool), students found themselves challenged. They had difficulty thinking of an educational context involving fractions where there would not be questions involved. A few even tried to come up with loopholes or ways of posing questions that did not have "wh" words in them, as in the following segment:

Emily: They will give you this fraction question and an answer. Then you have to put true or false.
Researcher: But this is still asking a question.
Melanie: Okay, this is easy. You say, "write this fraction in lowest terms."
Researcher: Is that a question or not?
Caren: It's a statement.
Melanie: "PLEASE write this fraction in lowest terms!" (laughing)

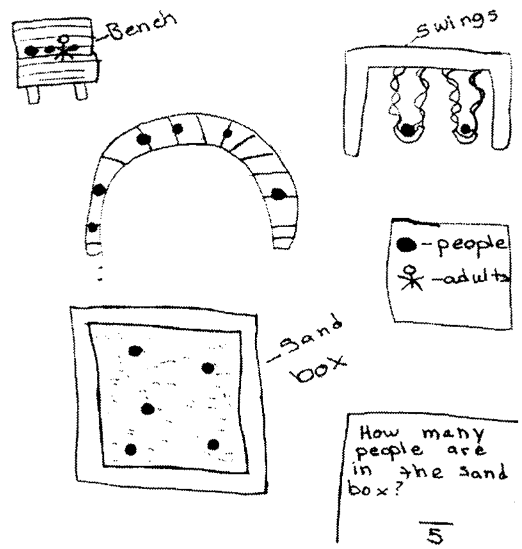

Figure 1. Caren's playground scene.

This interaction clearly expresses a prior understanding that the students brought to the task of game making. Instructional games, on or off the computer, have questions and answers as their main format. Eventually, however, students created three game ideas in which the user would create his or her own fractions. In Rachel's constructivist game, the player used the mouse to color in stripes on an animal and then complete a description telling what fraction of the animal is shaded. Rachel's game idea drew on Melanie's earlier game scene of a raccoon divided into multiple body parts. The "challenge" of creating a game without asking questions forced students to consider notions of fractions in non-traditional contexts that had been explored previously and reappropriate them for constructivist purposes. This task proved very difficult, however, and overall students ideas during the last phase of the session were restricted to intrinsic, but not constructivist integration.

Study II: Teachers' Fraction Integration

The teachers' games are characterized by a shift from extrinsic integration to intrinsic integration after the introduction of the design tools. The design tools also improved the game designs within the intrinsic integration classification; these games included better narrative sequence and richer contexts.

Before design tools. In the first game design session, teachers either created extrinsically integrated games or intrinsically integrated games that lacked

TABLE III

Teachers' games classified by fraction integration

	Extrinsic Integration	Intrinsic Integration	Constructivist Integration
Phase 1	– Find the Equivalent Door / Castle Building – Build a Pyramid	– Bricks and Pizza – Concentration	
Phase 2		– Indiana Jones & the Quest for Equality – Fantasy Day – Choose Your Own Adventure- Day at Jefferson Elementary – Immigrant Journey	
Phase 3		– Fair Share the Treasure – Choose Your Own Adventure- the Dance – Man Contexts	– Fantasy Day Expansion Set

a meaningful game context (see Table III). One of the game designs centered on building a castle and finding an equivalent door (see Figure 2). In this game, fraction questions were presented as an obstacle to the progression of the game. The player must identify the correct equivalent fraction to enter the door and continue the game. The other section of the game involved identifying equivalent fractions to build a castle.

The games that were intrinsically integrated illustrated equivalence ideas without the addition of a game narrative or context. For example, Bricks and Pizza did not include a game player, but rather illustrated equivalence using bricks in a building or with slices of pizza. Similarly, in the classic game Concentration, the objective of the game was to make matches using different representations of equivalent fractions as cards. In other words, the contexts of the phase 1 games either served as a motivator to continue the game or as an attempt to provide concrete representation for the children playing the game – but often the representations were confusing.

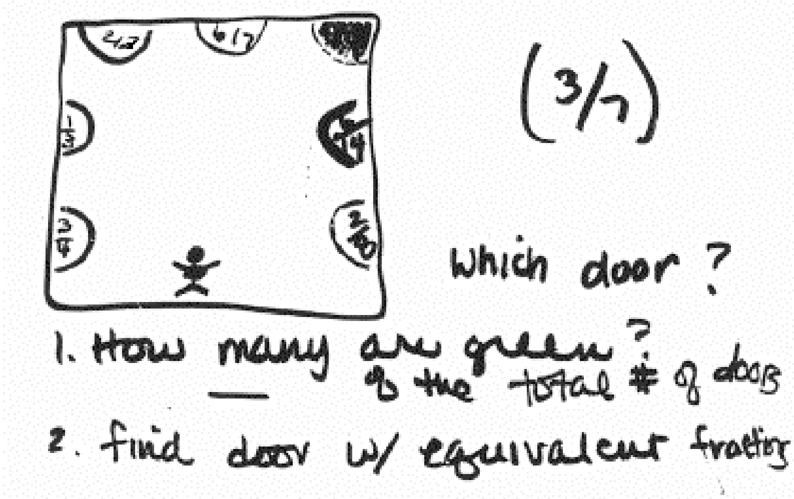

Figure 2. Teachers' Castle / Equivalent Door game. The player attempts to identify the correct equivalent fraction in order to enter the door and continue the game.

After design tools. After the introduction of the design tools, the teachers integrated fractions intrinsically into all of their subsequent game designs. After posing the question "Can you design a game without asking questions?" and providing game screens for each team, each game had a story-based theme that not only provided the impetus for playing the game but used fractions in meaningful situations. For example, one of the games engaged students in becoming the captain of a ship and set them off on an adventure where they would prepare for and journey across the Atlantic Ocean. The mathematical problems found in the game included ensuring that "each passenger on the trip has an equal number of supplies" and allocating inventory given different fractional constraints.

In the final design session, the teachers continued to develop games that integrated fractions intrinsically. They built upon the contexts of their previous games or thought of new contexts for their fraction games. In one case, 'Fantasy Day', the player was able to actively manipulate his or her own fractions within the game. In the beginning of the game, the player decides how many friends to bring along on the 'Fantasy Day' and is able to vary the number of sharers (friends) and objects to be shared (e.g., cookies, pizza) throughout the game.

Fraction Representation

Fraction representation is an issue critical to the development of children's thinking and often one that reflects the way in which the designer thinks about student thinking. We categorized the level at which the designer

TABLE IV

Student games classified by fraction representation

	Phase 1	Phase 2	Phase 3
Representation			
none	2	–	1
symbolic	3	–	1
fixed pictorial	–	3	4
fixed multiple	–	1	–
dynamic pictorial	–	1	–
free pictorial	–	–	2
free multiple	–	–	4

provided opportunities for fraction representation. At the lowest level the designer provided no opportunities related to fraction representation. In the symbolic representation category the designers engaged the player with numeric representation only, whereas in the fixed pictorial category the player had access to a single pictorial representation and no control over the representation. Within the fixed multiple category the player had access to multiple representations (symbolic and pictorial), but they were structured within the game sequence so that at any one time users had access to a single representation that they were asked to use. Again the player here has no control over the representation or the choice of representation.

Study I: Students' Game Design

One of the most striking developments in students' thinking about fractions was in the area of fraction representations. Changes which took place in students' employment of sophisticated fraction representations demonstrate the power of the design tools.

Before design tools. All games in the first phase of the design session dealt with either symbolic representations of fractions or no representations at all (see Table IV). Symbolic representations were common in games such as Emily's race track, where in order to make progress in the game, the user must answer questions such as "what is 6/8 in lowest terms?" In the case of "no representations," fractions took such a back seat to game action or plot development that no attention was devoted to them and they received generic treatment, such as in Melanie's game where the user receives an unspecified "fraction question" for not finding a leprechaun in a field of clover.

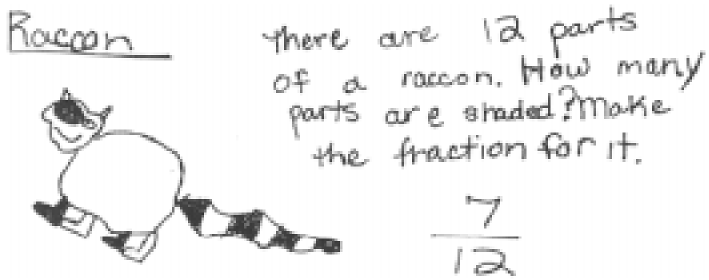

Figure 3. Student game: Raccoons.

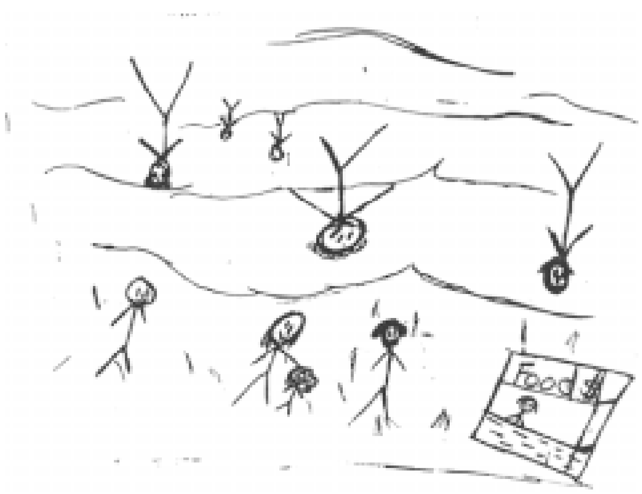

Figure 4. Student dynamic fraction representation: Beach scene.

After design tools. After students were introduced to the design tools, their representations of fractions became more complex and more diverse. Games in the second phase, in which students used fractions to describe everyday situations, consisted mostly of fixed pictorial representations with a single representation for each scene. Other students generally used the model put forth in Caren's original idea of using fractions to describe part/whole relationships in sets of objects. In Melanie's raccoon scene, however, a single raccoon could function as a 1/5 of a set of raccoons or as a whole divided into 12 body parts (see Figure 3).[4]

A particularly interesting and sophisticated shift occurred in phase 2 when Rachel created a dynamic, animated scene at the beach with people moving in and out of the water (see Figure 4). During the animation the user would be asked a question about what fraction of people were on land or in the water. Rachel's beach contained a dynamic fraction representation which could change in two directions: the ratio of people inside and

TABLE V

Teachers' games classified by fraction representation

	Phase 1	Phase 2	Phase 3
Representation			
none	–	–	–
symbolic	4	–	–
fixed pictorial	–	3	1
fixed multiple	–	–	1
dynamic pictorial	–	–	1
free pictorial	–	–	–
free multiple	–	1	1

outside the water could constantly change, but also the total number of objects in the set was flexible, as people could walk on or off the screen. We hypothesize that the context of creating a computer game contributed to the development of such a representation, since most computer games require animation. These animated scenes afford a more flexible view of how fractions can describe "real" situations, in which the parts and wholes are not fixed in time and space.

Games in the third phase, where students attempted to "create a game without asking questions," included six rather than one instance of complex representations. Students also developed two different types of free representations. The first type of free representation involved game ideas in which users could manipulate the fraction representations on the screen chosen by the designers but could not create their own fractions. In the second type, the user was not presented with problem-solving in any structured sense, but rather was invited to find out how to create her own fractions through interacting in an environment with 'fraction potential.' The pattern of representation type included in the games shifted from none or symbolic representation only to more designs including multiple or complex representations.

Study II: Teachers' Game Designs

Before design tools. Each design team created initial game designs that primarily made use of symbolic fraction representations (see Table V). For example, when building a pyramid (see Figure 5), the game player is first asked to identify which blocks have symbolic equivalencies on them. The second part of the game has the player work with a visual representation of

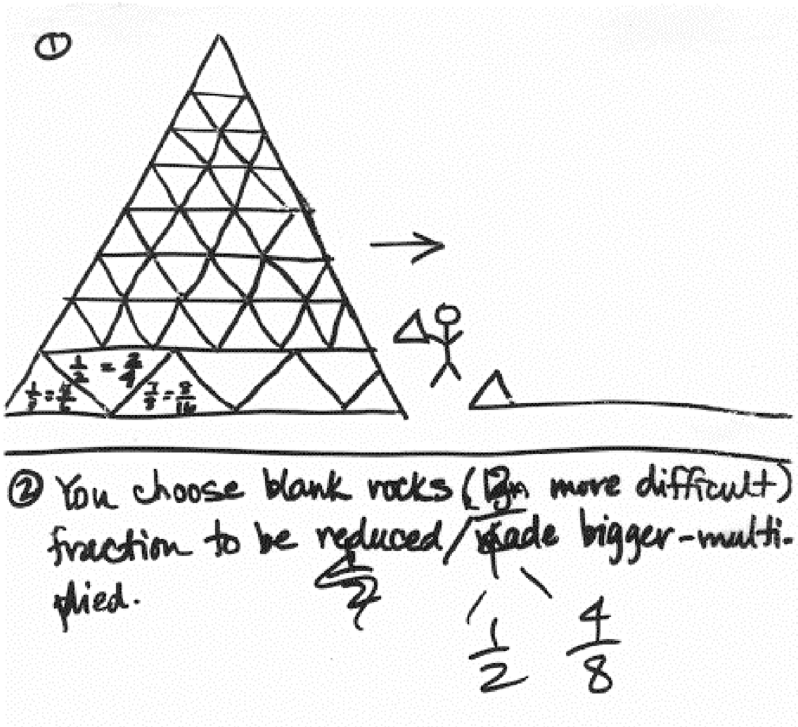

Figure 5. Teachers' Pyramid game. The player identifies which blocks on the pyramid contain correct equivalent statements in order to build a second pyramid.

the size of the blocks but then uses symbols to alter the size of the blocks to fit into the pyramid.

After design tools. After the teachers were introduced to the design tools, their representations grew more complex and diverse. Games in the second design phase consisted mainly of fixed pictorial representations with the game player moving through a better-developed context and story. For example, in 'Indiana Jones and the Quest for Equality', the problems are sequenced so that the player must move through them in a given order to reach the goal, freeing Indy's father. The problems are written so that they often require a particular strategy. For instance, in one room the player must balance the weights in the statues' hands to pass through (see Figure 6). In balancing the weights on the statues' hands, the player must solve the situation by clicking on the appropriate gold bar. This pictorial representation of the weights is in direct contrast with the symbols found on the blocks in the pyramid game from the first design phase. Furthermore, the representation of the weights is fixed. The player does not have

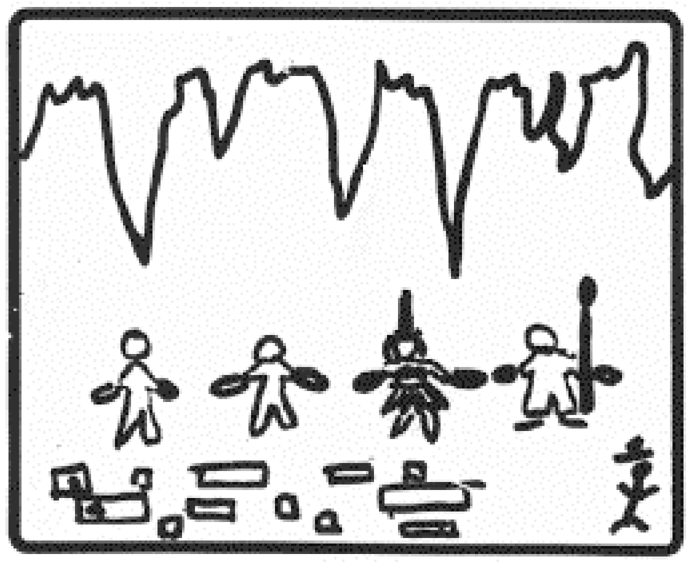

Figure 6. Teachers' Indiana Jones and the Quest for Equality game. The player must balance the weights in each of the statue's hands in order to advance to the next room.

the option of telling the computer how much of each bar they would need without having to go through the process of setting the separate gold bars on the statues' hands.

One game stood out in terms of fractions representation. The object of 'Fantasy Day' is to experience as many things as you can in one day with your best friends. With a wide variety of contexts and fair sharing situations, this game offered multiple representations which could be freely manipulated by the game player. 'Fantasy Day' has fair sharing situations which not only include a varying number of sharers but a wide variety of items that are shared. Many discrete items such as cookies, teammates, and seats on a roller coaster are shared in addition to continuous items such as pizza and sandwiches. There are multiple, variable representations found in the game which children can use in the ways that make sense to them. Unlike most of the other games in this design phase, in addition to having access to both symbolic and pictorial representations, the player in 'Fantasy Day' is free to manipulate the representations. Here the player can overlay representations on top of one another to compare the fractional representations and make choices of which representation to employ. In 'Indiana Jones' the representation is provided, but there is no choice and little if any manipulation on the player's part.

Further, 'Fantasy Day' includes more advanced fair sharing and includes thought to different children's strategies in addition to problems to afford opportunities for children's thinking. The player can change the problem at any time by changing the number of his or her traveling companions or the number of things to be shared. The players also have the option of leaving the problem and coming back to it at another point. Together, these features create multiple entry points into the problems for the player. 'Fantasy Day' also provides multiple solution path possibilities. Often the player can create the representation and choose from different possible strategies as they manipulate the representations. In 'Indiana Jones' the representations are provided in a fixed way and with no choices so the player's strategy choices become fixed.

Games in the final phase, when teachers were asked to include dynamic representations of fractions, primarily focused on the creation or further development of game contexts. In one case, the teachers created a game which included a wide variety of different fair sharing contexts including an aquarium, a stadium, a store and a city. Another game added a dynamic representation with a varying number of sharers in each scene. Overall, after exposure to the design tools, the teachers moved beyond the symbolic representation of fractions in their games to more sophisticated and diverse representations.

Consideration of User's Thinking

One of the most valuable (and most difficult) features of educational game design for mathematical development is its focus on the user. Game design requires designers, whether students or teachers, to decenter from their own perspective as creators of games and focus on their users instead (Kafai, 1995). When teachers are the designers, one might expect them to be considering pedagogical issues as a matter of course, particularly preservice teachers who are studying learning and mathematical development. Students, on the other hand, do not have information about mathematical development at their disposal to help them design their games. Without explicit knowledge of "how children learn fractions," they must consider their own successful (or failed) learning experiences and employ that knowledge in the process of design. In the teacher designs, we wanted to see how they would employ their knowledge of children's mathematical thinking. For the students, we were curious as to which models for an educational task involving fractions they would rely on in their designs.

Study I: Student Game Designs

We defined three ways of approaching the pedagogical function of user consideration for students: punitive, evaluative, and constructive. *Punitive* games administered fraction questions as punishment for user errors in

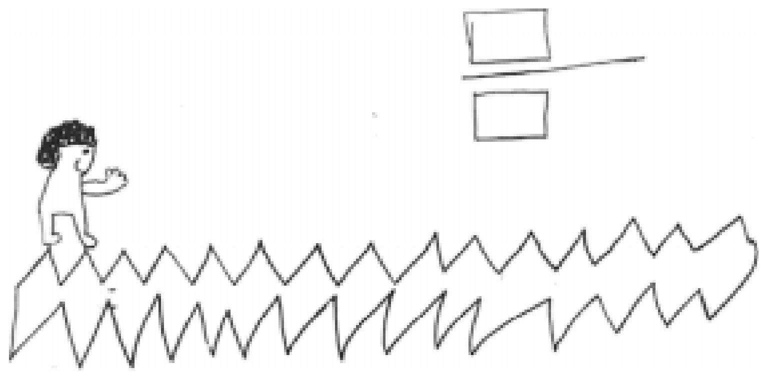

Figure 7. Student punitive strategy: Obstacle Course.

unrelated game action. One example of this type of game is the 'Obstacle Course' (see Figure 7). If the user is unsuccessful in manipulating the player down the crooked course, he or she must answer an equivalent fraction question by filling in a blank fraction symbol in the corner of the screen. This strategy of administering fractions as punishment may have more to do with students' negative attitude toward fractions than any genuine pedagogical thought on their part. In any case, designing punitive games requires little consideration of the user's thinking about fractions.

Evaluative games closely resemble the sort of drill-and-practice software children are familiar with through commercial models. Progress in the game is dependent on correctly answering de-contextualized fraction questions. A good example of this game type is Emily's 'Race Track' game, in which cars are positioned around a track, and in order to move forward, the user must answer fraction questions more quickly than his opponent. Because this type of game relies on quick and correct answers, the evaluative games tend to assess what users already know about fractions rather than providing opportunities for learning something new. Thus, the only attention to users' thinking here would be how to best *test* the user's thinking about fractions.

Constructive games differ in that their primary goals focus on engaging the user with fractions in meaningful ways and providing activities which afford opportunities for building on the players' existing knowledge. This type of game requires designers to take the perspective of the user and contemplate what might be the most productive context for learning about fractions. Constructive games also require attention to how fractions could be integrated into a particular game environment where the user would interact with them, making this type of game difficult to design.

TABLE VI

Students' consideration of users' thinking

	Phase 1	Phase 2	Phase 3
Consideration of Users' Thinking			
punitive	4	–	1
evaluative	1	5	4
constructive	–	–	7

Before design tools. Phase 1 of the design session contained little consideration of users' thinking. Four of the students' games created during phase 1 used fractions as punishment, and one, the race track game, evaluated students' accurate and quick recall of fraction facts (see Table VI).

After design tools. During phase 2 of the design session, students abandoned punitive games and created five evaluative games. These games differed from the race track game in phase 1 in that fractions in the phase 2 games were more integrated into the scenery and more varied representations were used; however, the main format was still question-and-answer.

Phase 3 of the design session saw a significant development in students' consideration of users' thinking. The task of creating a game without asking questions did not allow students to continue considering question-and-answer formats as an option. Without this alternative, students moved toward more constructive strategies for game design. Within the confines of designing a game without questions, student designers employed one of two constructive strategies. One way was to have the user "make their own fractions" with resources provided by the game; this strategy received full treatment in the section on fraction integration. The other option was to have users interact with a game environment where fractions were inherent in the scenery. An example of this strategy is Caren's 'Hidden Fractions' game in which the user applies fractions as a lens to view the surrounding setting and describes objects in fractional terms. As Caren said, "they could say that 3/5 of the cats in this picture are on the ground and 2/5 are in the trees."

Study II: Teachers' Game Designs

The category of consideration of the user for the teachers allowed us to examine the ways in which the designers took into account knowledge related to the development of children's mathematical thinking. We were particularly interested in their use of the development of fair sharing ideas,

TABLE VII

Teachers' consideration of users' thinking

	Phase 1	Phase 2	Phase 3
Consideration of Users' Thinking			
none	4	1	1
problems	–	2	2
strategies	–	1	1

in terms of the problems they might engage students in, and the strategies they use in solving these problems (Streefland, 1991). The fair sharing problems and strategies not only provide particulars about children's thinking but taken together provide a context for thinking about the big ideas students might engage in as they learn about fractions. Our categorization, very simply, focused on whether problems or strategies related to the development of fair sharing ideas were evident in the game designs. The problem category includes instances where the designer included fair sharing problems in their game design. The strategy category included instances of both the problems and the strategy options possible given the problem posed.

The teachers' games are characterized by a shift from no consideration of the user's thinking to using problems to afford opportunities for user's thinking and learning after the introduction of the design tools (see Table VII). In one game, different solution strategies as well as different problems developed the thinking and learning of the user.

Before design tools. The initial game designs had no connection to children's thinking. They did not include problems that would provide opportunities for children to demonstrate different types of strategies nor did they use what they had previously learned about children's thinking in fractions to design possible solution paths. Instead, the games were based upon practicing and illustrating equivalence skills. For example, in the 'Pyramid' game, the player uses equivalence to find matching fractions symbols. The context being developed did not lend itself to building upon the players' understanding of the mathematics.

After design tools. The games designed after the introduction of the design tools showed improvement in consideration of the user's thinking. The games offered contexts that the children could relate to and were also mathematically powerful.

In 'Choose Your Own Adventure-Day at Jefferson Elementary', the user is asked to "Split up the pizzas so that the three girls get the same amount of pizza." and "Set it up so that everyone plays an equal amount of time" on the football field. Especially when compared to the 'Pyramid' game created in the first design session, this context and these types of problems are easier for children to relate to and help to elaborate ideas of fractions and fair sharing.

'Fantasy Day' allows the user to vary the number of sharers in the game by changing the number of friends who will come along on the trip. The user is also able to vary the amount shared for a wide variety of items, ranging from cookies to pizza to peanut butter and jelly sandwiches. These more advanced fair sharing ideas offer opportunities for children to build upon their natural strategies when addressing sharing situations. Again, the introduction of the design tools facilitated the teachers' development of games that better considered the thinking of the user of the game.

DISCUSSION

In the following discussion, we evaluate what we have learned from game design activities for transforming students and teachers into designers of interactive learning environments. Across the various categories used to judge game design quality, both teachers' and students' games became more sophisticated. This may not be surprising; with time and experience we would expect more sophisticated game designs. However, what surprised us was how quickly teachers and students developed their ideas and the magnitude of the changes that occurred. We also became intrigued with the parallel changes in the teachers' and students' game designs.

Overall patterns showed that both teachers and students shifted from extrinsic game designs, where fraction content and game idea was separate, to intrinsic game designs, where idea and content was integrated. In a few cases the game designs even allowed the player to create, and thus, naturally blend content and game idea. The shifts in representations employed in the game designs showed both teachers and students moving from either no productive representations or only symbolic representations to the use of pictorial representations, and in some cases to multiple representations where the player could choose their representation from some given alternatives. In consideration of the user, although expectations for how teachers and students would think about the user differed, in both cases designing a more sophisticated game required the designer to consider alternatives for a player's thinking and tailoring the game to the player's thinking. Neither all of the teachers or all of the student designers were successful at reaching this level of sophistication. Yet this level was reached by some students and some teachers.

Although we noted a series of parallels in the development of the game designs for teachers and students it is helpful to also note the differences in their game design sequences. While the students drew on their experience with game playing the teachers drew on their knowledge of the development of children's thinking. The games designed by the teachers included design elements that took into account what research tells us about the different ways to vary problems and the range of solution paths students may use in addressing those problems. The teachers designed games that to varying degrees accounted for the players' development and learning.

In examining the ways in which the teachers and the students initially designed games and their adaptations of these designs, we learned (a) what making use of informal knowledge might mean in practice, (b) how conceptual design tools combine a focus on process with a focus on product and (c) the potential of engaging both teacher and student and designer and player concurrently. These three ideas are in no way intended to be thought of as independent but rather their potential comes from the ways in which they build on and connect to one another. We will discuss each and its relationship to the others.

Informal Knowledge

Within mathematics education there are a number of ways to think about informal knowledge. Here we focus on the knowledge the teachers and students develop outside the context of formal schooling. For instance, we saw in our work with the preservice teachers that their varied experiences outside of school with video games provided them with informal knowledge about games, the use of technology and even about learning in these contexts (Franke, Kafai and Shih, 1997). We have seen in our interactions with the preservice teachers and the students that in coming to the design task each group brought knowledge that both contributed to and limited their responses.

By opening access to the designers' existing other relevant information (such as knowledge of fair sharing, or other ways of learning) opportunities for developed understanding increased. Increasing the knowledge that the designers considered useful in approaching the task increased the possibilities for more conversation amongst the designers, as there were more points of contact among them, and also increased the opportunities for the designers knowledge to become interconnected.

Both the preservice teachers and the students possessed culturally rich knowledge about games and the playing of games. As we reflect on the knowledge accessed in the design process by the designers we saw that their knowledge at times was useful and at other times potentially hindered the process. For instance, the students' and preservice teachers' informal knowledge, as expressed in the initial designs, reflected a narrow and limiting understanding of educational games. Their ideas included games as drill-and-practice and accumulating points. They conceived of teaching in this environment as asking 'yes/no type' questions and learning as providing answers to those questions. The discussion after the initial game design session indicated that these stereotypical views of educational games made it very difficult for the preservice teachers to articulate students' learning, identify students' development in the learning process and consider the integration of students' thinking, problem solving, and the mathematical content within game design. However, the preservice teachers also possessed some quite useful knowledge about fair sharing in fractions and the development of children's thinking, which they initially did not consider in designing their games.

We came to see the purpose of problematizing as not only engaging the designer in creating new knowledge, but also as a way to push the designers to access existing knowledge that initially they may not have seen as relevant to their game designs. Problematizing led teachers and students to other types of information they possessed; information they used in different contexts under different circumstances that would be helpful to their game designs. The quick and significant changes in the designers' games demonstrated that challenging them to access existing knowledge while pushing them to think differently about certain aspects of design engaged them in design that focused on problem posing (the problem space) and student solution (solution space) concurrently.

Concurrent Focus on Both Process and Product as a Function of Design Tools

Often within instructional design we see a shift away from the focus exclusively on the product to a focus on process. However, what occurs then is that the product becomes lost, and what becomes produced is disappointing even though the process "worked." The use of conceptual design tools allowed us to engage the designer in thinking about the process while still maintaining sight of the product. Essentially we asked the designer to try and think concurrently about the process and the product; they thought about designing a game for someone else to play. But the task itself, designing the game, was not enough to insure that the designer remained focused on both, this occurred as a function of the conceptual design tools. The story screens provided a concrete way for the designers to work through their design and served as a reminder that the sequence of the game was critical for the player. Designing a game without asking a question focused the designers on the relationship between their design and what the player would be doing and learning.

The conceptual design tools pushed the designers to think about both process and product in three specific ways. First the conceptual toolbox included tools that enabled the students and teachers to create externalizable and shareable representations (game design screens). Second, the toolbox included tools that encouraged a focus on learning and the learner (Lave, 1996). Rather than thinking about how to design a game that teaches, the designers turned it around to 'how can I design a game in which the student will have the opportunity to learn?' Here then, new design space was created; the designer created room for the player to negotiate the design space using their existing knowledge and skills. And third, the tools led the teachers to think about principles rather than to focus on the procedures. The question posed to the preservice teachers about designing a game 'without asking a question' pressed teachers to come to think about how to engage students in understanding rather than practicing. As they focused on the principled ideas, the designer looked beyond routines to create opportunities for learning big ideas. The conceptual tools taken together problematized the situation for the designers, pressed them to problematize situations for the player and thus connected a rich process with a dynamic product.

Reciprocity Between Teachers and Students

Within the game design process created here, both the preservice teachers and the students became designers and players in an ongoing way. Whether the designer was a teacher or a student, within this context, they had to be thinking both about designing and playing concurrently. The designer must at the same time be a teacher and a learner. From our perspective this affords opportunity for powerful professional development for teachers. We are beginning to recognize within professional development that we cannot leave the students out of the process. What the game design context provides moves us beyond more typical instructional design and more typical professional development to a place where teachers and students learn together. Although we did not do it here, these parallel studies point out the potential benefit of engaging teachers and students in game design together, where each plays the role of coach and critic, where each must be a teacher and a learner, where they learn from each other. Common ground becomes created where each has problems to solve that are interconnected, where each brings their own knowledge and expertise, and where this knowledge and expertise can be valued and built upon. The conceptual design tools become a vehicle that presses both teacher and student to think. The game design task becomes a forum for that thinking where informal knowledge exists in ways that foster communication, collaboration and developed understanding.

OUTLOOK

In the current paper we discussed game design activities on paper for engaging students and teachers in mathematical thinking to understand better some of the principles that make it work. The conceptual design tools that we developed point toward applications for computer-based design environments. One line of research that has concentrated on creating computational design environments is by Fischer (Fischer and Lemke, 1987/88). In his view, design learning environments provide not only the syntactic components of design (which describe the problem space of the design) but also semantic support in how to go about putting things together. Designers might not know all the objects they need and in which combination they need to be put together. For that reason, Fischer implemented design critics or coaches in his environments. We see conceptual design tools also providing semantic and syntactic support. For example, the conceptual design tool of fair sharing is evidently a semantic tool as it instructs the designer to focus within the game design task on particular operations and objects. The conceptual design tool of making games without asking questions is more of syntactic nature as it is not tied to a particular domain. One could easily imagine Fischer's critics or coaches use these conceptual design tools to provide assistance to the designer.

Another line of research has focused on developing so called construction tool kits. Commercial installations have been *Klick & Play*™ (Maxis Inc., 1994) that provide the designer with a game design environment. In this construction tool kit, the designer is given all the building blocks for making games but the environment is generic and allows the designer to build any game within any context. A construction tool kit of more specific nature is the *Pinball Construction Set*™ (Electronic Arts, 1983) which allowed the game designer to build only pinball games. Here the designer is given all the elements of a pinball game such as levers, balls, and can decide which ones to use where and customize certain features. The game design environment we envision lies in the middle of the *Klick & Play*™ and the *Pinball Construction Set*™ continuum and would incorporate the critics or coaches developed by Fischer. One could imagine a computational game design environment in which the user could load fraction design tools. What this would entail is a set of objects that could be used for fractions. Alternatively, the user could use graphic tools to create fraction objects. Then the user would also have different tools to operate on fractions such as Confrey's (1996) splitting tool (which engages the player is using 'splitting' as a strategy for considering fractions) or Streefland's (1991) fair sharing tool (which focuses on situations in which fair sharing activities can occur). Design support could not only come in the form of computational coaches or critics but also from the designers themselves. We found that conversations and discussions among the teachers and students were essential in helping the designers build more sophisticated representations. Electronic discussion forums would be instrumental in allowing the designers to share, annotate and modify their designs. Our research with the preservice teachers indicated that the reflections on finished game designs were particularly helpful in providing opportunities for the teachers to question and revise their pedagogical decisions. These examples highlight the types of design specifications for computational design learning environments developed in our work with students and teachers.

CONCLUSIONS

Game design in fractions afforded opportunities for teachers and students to think about contexts that were mathematically powerful and to engage in ongoing reflection about the teaching and learning of mathematics. Game design provided a situation that naturally combined issues of practice and theory, and reflection on those relationships and game design provided opportunities for discussion, reflection, and collaboration within a meaningful context.

Taken together our results, the focus on educational games and the use of conceptual design tools allowed us to see how game design can become a powerful learning context for teachers and students. Not only does game design carry this potential, but the parallels in teacher and student thinking suggest that working together in classrooms, where teachers and students engage in game design can enhance learning, creating classrooms that are learning communities for both teachers and students.

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NOTES

[1] Generative change has been recognized as instrumental in the domain of teacher change (Franke, Carpenter, Fennema, Ansell and Behrend, in press).

[2] The *Students' Game Designs* study was conducted as part of a larger effort to develop a prototype for educational interactive television in elementary school mathematics (Miller, 1994). The development plan foresaw the involvement of young students as game design consultants for professional designers, programmers, and content specialists who would then create and implement a commercial version.

[3] Pseudonyms are used for all participants in both Study 1 and Study 2.

[4] Note that actually the raccoon includes 13 parts (counting two eyes, four feet and seven tail parts). Though we recognize that this representation has some difficulties in terms of the correct number of parts and the size of the different parts, it represents a shift in the student's thinking.

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UCLA
Graduate School of Education & Information Studies
405 Hilgard Ave
Mailbox 951521
Los Angeles, CA 90095-1521
U.S.A.