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A MULTIVARIATE ANALYSIS OF INDUSTRIAL BOND RATINGS

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I. Introduction

Industrial corporate bonds have been assigned quality ratings since the early 1900s. Each year private organizations, such as Moody's and Standard & Poor's, assign ratings to a portion of new bonds issued that year. The purpose of these ratings ". . . is to provide the American investor with a simple system of graduation by which the relative investment qualities of bonds may be noted" [21, p. v]. Bond ratings are based, in part, on available statistics depicting a firm's operating and financial condition. In addition to quantifiable data, the rater's qualitative judgment concerning the future ability of a firm to make interest and principal payments also influences the bond ratings. Several recent studies have met with limited success in attempting to predict bond ratings for both corporate [16, 29, 35] and municipal bonds [5, 17]. The purpose of this study was to develop and test a factor analysis/multiple discriminant model for predicting industrial bond ratings.

A brief review of bond ratings and previous research is presented in section II, while the multiple discriminant technique is considered in section III. The factor analysis/multiple discriminant model is developed in section IV and evaluated in section V. The final section presents implications and conclusions.

II. Bond Ratings

Bond ratings attempt to provide a simple measure of the relative investment quality of these securities. Moody's employs nine different ratings (Aaa to C) while Standard & Poor's employs a total of twelve ratings (AAA to D). Virtually all bonds rated by either Moody's or Standard & Poor's fall in the first six classifications. Medium and high grade bonds (Aaa, Aa, A and Baa for Moody's and AAA, AA, A and BBB for Standard & Poor's) are considered "investment" class bonds. Under present commercial bank regulations only

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bonds rated in these four classifications are eligible for bank investment. Due to the small number of Aaa bonds issued during the time period of this study, only bonds rated Aa, A, Baa, Ba or B by Moody's are examined.

A number of previous studies on bond ratings have related them to frequency of default. Harold [13], Hickman [14], and Atkinson and Simpson [3] all indicate that some relationship exists between bond ratings and historical records of bond default. In another study, Fisher [9] hypothesized that the risk premium on corporate bonds was a function of default and marketability risk. Only three studies, Horrigan [16], West [35], and Pogue and Soldofsky [29] have specifically attempted to predict corporate bond ratings based on the financial and/or statistical characteristics of the bonds and issuing firms.

Horrigan attempted to predict the top six bond classifications for both Moody's and Standard & Poor's employing a multiple regression model. He concluded that a model containing six variables (subordination, total assets, working capital/sales, net worth/total debt, sales/net worth, and net operating profit/sales) could predict approximately 58 per cent of Moody's ratings and 52 per cent of Standard & Poor's ratings. West made use of the model developed by Fisher and attempted to predict the first six Moody's bond ratings. Employing four variables (the logarithms of earnings variability, period of solvency, market value of stock/debt, and market value of all bonds outstanding) in a multiple regression model he was able to predict approximately 62 per cent of the actual ratings. Pogue and Soldofsky employed a regression model with a dichotomous (0-1) dependent variable to predict which of two ratings (i.e., Aaa or Baa) a bond should be assigned. Limiting their analysis to investment quality bonds (Aaa, Aa, A or Baa), as rated by Moody's, they developed a model employing five different variables (debt/total capital, net income/total assets, coefficient of variation of net income/total assets, net total assets and net income plus interest/interest). This model was employed to classify a number of subsets of bonds with particularly good results when they paired high (Aaa) and low (Baa) rated bonds. These three studies have some overlap in data; however, much of the data employed in each study is unique. In the present study virtually all of these variables, along with a number of additional variables, were considered in developing the predictive model.

III. Multiple Discriminant Analysis

The approach employed in developing the model involved: 1) initial screening of variables accomplished via factor analysis; 2) followed by the use of multiple discriminant analysis to develop the final predictive model. Multiple discriminant analysis (MDA) can be employed as both a descriptive and predictive technique. Descriptive uses include the investigation of mean group differences and the overlaps among groups, while predictive uses center around the formation of classification schemes to assign objects (bonds in this instance) to appropriate groups. The basic assumption of MDA are that: 1) the groups are discrete and known; 2) each observation in each group is described by a set of measurements on n characteristics or variables; and

3) the n variables arise from multivariate normal populations.[1] MDA is applicable when the within groups variance-covariance matrix for the variables in the observation vector is assumed to be the same for each group, but the mean vectors are assumed to be different.

MDA has the advantage of reducing the space dimensions from a number of different independent variables to M-1 dimensions, when M is the number of original *a priori* groups. In the two group case this results in a single discriminant function of the form $y = f_1 x_1 + f_2 x_2 + \ldots + f_n x_n$ where f_1, $f_2, \ldots f_n$ are the discriminant coefficients, and $x_1, x_2, \ldots x_n$ are the independent variables. MDA computes the discriminant coefficients which have the property of providing the "best" linear function distinguishing between the two groups by maximizing the ratio of the between group variance of y to the pooled sample within group variance of y.

The extension from two groups to M groups results in more than one discriminant function (M-1) depending on the number of *a priori* groups (M). This set of functions is obtained in the multivariate-multigroup case by maximizing the ratio of the matrix of between groups deviation sums of squares, to the matrix of the pooled within groups deviation sums of squares. The first discriminant function has the largest possible discriminant-criterion value, and each of the others has a conditionally maximal discriminant-criterion value. The first discriminant function indicates the dimension along which the maximum group differentiation occurs; the second discriminant function indicates the largest group differentiation not accounted for by the first dimension, etc. The five variable MDA model to be developed has a maximum of four significant discriminant functions; however, it is possible that less than that will be statistically significant. Multiple discriminant analysis has previously been employed in finance for both two groups [1, 8, 26] and more than two groups [32, 36].

IV. Development of the Model

Sample Selection

All industrial corporate bonds rated B or above listed in the new issue section of *Moody's Bond Survey* [20] from January 1, 1967 to December 31, 1968 were initially selected for the study. After eliminating duplicate firms (those issuing more than one bond during the period) and verifying that all desired financial information was available, a total of 186 bonds were available for analysis. This total was subsequently reduced to 180 firms by eliminating all six Aaa rated bonds. These were eliminated due to the very small number of Aaa bonds issued during this time period. The 180 bonds were randomly assigned to one of two groups—132 firms forming the *original* sample and 48 firms forming a *holdout* sample. The distributions of these samples are presented in Table 1.

Financial data were collected and/or calculated on thirty-five different variables. The basic types of variables are: 1) those related strictly to the bond

1. For a more extensive mathematical derivation than that presented in this section see, for example, Anderson [2], Cooley and Lohnes [6], Morrison [23], Rao [30], or Tatsuoka [33].

TABLE 1
SAMPLE SIZE OF NEWLY RATED INDUSTRIAL CORPORATE BONDS*

Moody's Bond Rating	Original Sample Number	Original Sample Per cent	Holdout Sample Number	Holdout Sample Per cent
Aa	14	10.6	4	8.3
A	26	19.7	9	18.8
Baa	25	19.0	9	18.8
Ba	44	33.3	17	35.3
B	23	17.4	9	18.8
TOTAL	132	100.0	48	100.0

* A list of the firms is available from the authors.

(subordination, issue size, etc.); 2) one year variables of financial characteristics (i.e., total assets, net income/total assets); 3) five year average variables (i.e., long term debt/net worth, sales/total assets); and 4) coefficients of variation (i.e., coefficient of variation of net income, long term debt/total assets). Data distributions for all variables were plotted; outliers were standardized where necessary; and a common log transformation was applied to a number of the variables to improve normality and reduce the heteroscedasticity of the distributions. The variables, standardization procedures and respective transformations are presented in the Appendix.

Factor Analysis

The financial data were screened by factor analyzing the data. Factor analysis groups together the variation in a data matrix into distinct dimensions (or patterns) while trying to account for as much of the total variation in the data as possible. The objectives in factor analyzing the data were to: 1) gain a better understanding of the regularity and order in the data; and 2) identify basically independent dimensions of the data. The financial and bond-related data were subjected to an R factor analysis which produces groupings (or dimensions) of the variables (financial data) in terms of the cases (industrial firms). The variables contained in the results of an R factor analysis are associated with numerical weights called factor loadings which show the degree of involvement the variables (financial data) have in a particular factor pattern. Thus, the results of the R factor analysis identify basically independent dimensions (patterns) of financial data for these industrial firms.[2]

Factor loadings delimiting the basic data dimensions are presented in Table 2. Seven factor patterns were identified which accounted for 63 percent of the variation in the data matrix. The seven factors appear to represent: 1) size; 2) financial leverage; 3) long-term capital intensiveness; 4) return on invest-

2. The principal component solution employing the varimax rotation technique on the product moment correlation matrix of the data was performed. The communality estimate was the squared multiple correlation coefficient and the computer program employed was BMD 03M [7]. See Rummel [31] for a discussion of factor analysis. The number of factors to be extracted was determined by a dual criteria: 1) the eigenvalues of all factors to be reported had to be equal to or greater than 1.00; and 2) the results were checked for discontinuity so that no factor slightly below a reported factor but with an eigenvalue slightly below 1.00 was omitted. Further analysis of the financial dimensions of firms is presented in [28].

ment; 5) short-term capital intensiveness; 6) earnings stability; and 7) debt and debt coverage stability. These factors (or dimensions) provide an indication of the underlying structure of these industrial firms, and assist in determining which financial data pattern closely together in n-dimensional space. By limiting the MDA model to only one variable from each factor dimension a low correlation between variables can be insured. The final MDA model contains variables associated with only five of these dimensions. Thus, while all seven of the factors identified in Table 2 were important in describing the dimensionality of the data, variables from only five of the dimensions appear to be important for predicting industrial bond ratings.

TABLE 2
FACTOR PATTERNS FOR INDUSTRIAL FIRMS ISSUING NEWLY RATED CORPORATE BONDS*

Financial Data	Factor Loading 1	2	3	4	5	6	7
Total Assets	−.95						
Net Working Capital	−.66						
Sales	−.87						
(X_3) Issue Size	−.81						
Number of Shares of Common Stock	−.89						
Total Assets: Mean	−.93						
Long Term Debt/Total Assets		−.88					
Long Term Debt/Net Worth: Mean		−.78					
(X_5) Long Term Debt/Total Assets: Mean		−.77					
Long Term Debt/Net Worth		−.91					
Sales/Net Worth			−.90				
Sales/Total Assets			−.95				
Sales/Total Assets: Mean			−.95				
Net Income/Sales				.61			
Net Income/Net Worth				.60			
Net Income/Total Assets: Mean				.81			
(X_6) Net Income/Total Assets				.86			
Net Working Capital/Sales					.76		
Net Worth/Total Assets					−.71		
Price Earnings Ratio						.55	
(X_2) Years of Consecutive Dividends						−.60	
Earnings Per Share						−.57	
Net Income: Coefficient of Variation						.70	
Net Income/Total Assets: Coefficient of Variation						.57	
(X_4) Net Income + Interest/Interest: Mean							−.58
Long Term Debt/Total Assets: Coefficient of Variation							−.71
Net Income + Interest/Interest: Coefficient of Variation							−.78
Market Value of Common Stock/Long Term Debt: Mean							−.82

* Only variables with factor loadings of .55 or greater were reported.

MDA Model

The MDA model that performed best among those tested incorporated six variables: X_1—subordination, X_2—years of consecutive dividends, X_3—issue size, X_4—net income + interest/interest: five year mean, X_5—long term debt/total assets: five year mean, and X_6—net income/total assets.[3] Of the six variables three (X_2, X_3 and X_4) had a common log transformation, two (X_5 and X_6) are incorporated in their normal form while subordination is represented by a dichotomous (0-1) variable.

X_1—Subordination (0-1). Subordination represents the legal status of the bonds. For all of the bonds studied, no bonds rated Aa were subordinated and all bonds rated Ba and B were subordinated. Inclusion of this variable is consistent with the findings of Horrigan.[4]

X_2—Years of Consecutive Dividends (log 10). This variable is indicative of the stability of the earnings of the firm as indicated by their ability to pay cash dividends. In Table 2 this variable loaded in the factor representing earnings stability and is related to the coefficient of variation of net income/total assets employed by Pogue and Soldofsky.

X_3—Issue Size (log 10). The inclusion of this variable is an indication that bond ratings and the size of the bond issue are related. There is a high correlation ($r = .80$) between issue size and total assets indicating that large corporations tend to issue large bond issues. Both Horrigan, and Pogue and Soldofsky included total assets in their models.

X_4—Net Income + Interest/Interest: Five Year Mean (log 10). This measure of debt coverage provides an idea of the ability of the firm to meet its debt obligations. This variable was employed by Pogue and Soldofsky and, as indicated in Table 2 loads in the same factor as market value of common stock/long-term debt employed by West.

X_5—Long Term Debt/Total Assets: Five Year Mean. This ratio is a measure of the capital structure of the firms under consideration. Prior studies by Horrigan, and Pogue and Soldofsky incorporated measures closely related to this ratio. The five year mean of long term debt/total assets is important to bond ratings since, *ceteris paribus,* the lower the debt, the lower the probability of bond impairment.

X_6—Net Income/Total Assets. This ratio corresponds closely to a six year average employed by Pogue and Soldofsky and is a measure of the ability of management to earn a satisfactory return on investment.

3. Operationally, a number of different strategies were employed in determining the final multiple discriminant model. Models based on all of the following were calculated: 1) stepwise MDA (BMD 07M); 2) variables employed in other studies; 3) variables identified by factor analysis; and 4) variable selection based on judgment. Three different stepwise methods for determining which variables should enter the model first were employed. These three options are: 1) enter the variable with the largest F value; 2) enter the variable which when partialed on the previously entered variables has the highest multiple correlation with the groups; and 3) enter the variable which, after entering, maximizes the smallest F ratio between pairs of groups. While many different models were run, there is no claim regarding optimality of the final MDA model since the process is essentially iterative.

The independence of these six variables can be seen by analyzing the r^2's between variables. Exclusive of X_1, the two highest r^2's are .110 between variables X_2 and X_3 and .103 between variables X_4 and X_6.

4. The use of a dichotomous variable in MDA violates a basic assumption of the technique—that is, a normal distribution. However, Gilbert [11] indicates that the violation of this assumption with 0-1 variables does not appear to negate the entire model. See, also, Hills [15] and Linhart [19]. All but two of the subordinated bonds were also convertible.

The overall discriminating power of the model was determined by testing the equality of the group means. The calculated F value for the MDA model is $F_{24, 426} = 17.225$. Since the tabled $F_{24, \infty}$ value (.001) is 2.13, the calculated F value permits rejection of the null hypothesis that the bonds come from the same population. With the overall conclusion that the *a priori* groups of bonds are significantly different, the six variables entering the final MDA can be examined. The individual discriminating ability of each of the variables is indicated in Table 3. Variables X_2, X_3, and X_4 are significantly different

TABLE 3
VARIABLE MEANS AND TEST OF SIGNIFICANCE

	Bond Rating					
Variable	Aa	A	Baa	Ba	B	F Ratio
X_1	0.000	0.077	0.520	1.000	1.000	—
X_2	1.634	1.581	1.260	1.058	0.468	25.45***
X_3	1.869	1.657	1.275	1.354	1.250	13.97***
X_4	1.138	0.606	0.560	0.511	0.707	6.05***
X_5	0.091	0.162	0.154	0.151	0.215	4.06**
X_6	0.099	0.075	0.066	0.075	0.069	2.68*

$F_{4,\infty}$ (.001) $= 4.62$ *** Significant at .001 level.
$F_{4,\infty}$ (.01) $= 3.32$ ** Significant at .01 level.
$F_{4,\infty}$ (.05) $= 2.37$ * Significant at .05 level.

beyond the .001 level, X_5 is significant beyond the .01 level, and variable X_6 is significant beyond the .05 level; thus, significant differences exist between some of the group means for each of the variables. On a strictly univariate basis it would be concluded that X_2—years of consecutive dividends (with the highest F ratio and properly ordered group means) is the most important variable. Also on a univariate basis the order of importance of the other variables is: X_3—issue size, X_4—net income + interest/interest:mean, X_5—long term debt/total assets:mean, and X_6—net income/total assets.

In order to examine the relative importance of the six variables as they contribute to the MDA model (instead of on a univariate basis) it is necessary to examine the four discriminant functions obtained from the analysis. In Table 4 the four discriminant functions, eigenvalues, portion of cumulative dispersion, V statistic and degrees of freedom are presented. In order to determine how many of the discriminant functions contribute significantly to group differentiation Bartlett's V statistic is employed [33, pp. 164-170]. This statistic is distributed approximately as chi-square and may be employed to test each discriminant function for significance after removing (or "partialling out") the effect of prior discriminant functions. As indicated in Table 4, only the first three discriminant functions are statistically significant and they account for 99 per cent of the total dispersion. The fourth discriminant function is immaterial for differentiation since the differences along this dimension can be attributed to sampling error. Of the three significant discriminant functions, the first is the most important since it accounts for 90 per cent of the total dispersion explained by the MDA model.

The contribution of the original variables to each of the three significant

TABLE 4

DISCRIMINANT FUNCTIONS AND TESTS OF SIGNIFICANCE FOR MULTIPLE DISCRIMINANT MODEL

Discriminant Functions	Eigenvalues	Cumulative Portion of Total Dispersion	Bartlett's V Statistic	d.f.
$Y_1 = -.329X_1 + .107X_2 + .100X_3 + .005X_4 - .270X_5 + .893X_6$	5.19	.90	$V = 296.54$*	24
$Y_2 = .046X_1 + .218X_2 - .212X_3 - .264X_4 - .505X_5 - .762X_6$	.34	.95	$V - V_1 = 67.68$*	15
$Y_3 = -.128X_1 - .044X_2 - .138X_3 + .001X_4 + .320X_5 - .928X_6$	.23	.99	$V - V_1 - V_2 = 30.91$*	8
$Y_4 = -.017X_1 + .013X_2 - .172X_3 + .123X_4 - .604X_5 - .768X_6$	.04	1.00	$V - V_1 - V_2 - V_3 = 4.96$	3

* Significant at .001 level.

discriminant functions is presented in Table 5. To eliminate the spurious effects of different measurement units, the discriminant coefficients are multiplied by the square root of the diagonal element of the W matrix to arrive at the scaled coefficients. An examination of Table 5 indicates that variables X_1—subordination, X_2—years of consecutive dividends and X_3—issue size, are the three most important variables in the first discriminant function. For the second discriminant function X_4—net income and interest/interest:mean and X_2—years of consecutive dividends, are the most important variables. Finally, in

TABLE 5
SCALED COEFFICIENTS FOR THE THREE STATISTICALLY SIGNIFICANT DISCRIMINANT FUNCTIONS

	Discriminant Function					
	One		Two		Three	
Variable	Scaled Coefficient	Rank	Scaled Coefficient	Rank	Scaled Coefficient	Rank
X_1	−.936	1	.131	6	−.365	2
X_2	.528	2	1.073	2	−.216	5
X_3	.360	3	−.758	3	−.493	1
X_4	.023	6	−1.284	1	.006	6
X_5	−.283	5	−.529	4	.335	4
X_6	.327	4	−.280	5	−.340	3

the third discriminant function X_3—issue size, is the most important followed by X_1—subordination, X_6—net income/total assets, and X_5—long term debt/total assets:mean. Realizing that the first dimension accounts for 90 per cent of the total explained dispersion it becomes apparent that the single most important variable in the MDA model is subordination. The order of importance of the remaining variables (over the three discriminant functions) appears to be X_2—years of consecutive dividends, X_3—issue size, X_4—net income and interest/interest:mean, X_6—net income/total assets, and X_6—long term debt/total assets:mean. This appears to suggest that bond raters are very concerned about specific provisions of the bond issue and the stability of the firm, while the financial performance of the firm is of lesser importance.

V. EVALUATION OF THE MODEL

Original Sample

The MDA model developed in the last section was employed to classify the original test group from which the model was developed. It should be noted that this is not a predictive use of the model since the data set remains the same one from which the model was developed. The classification procedure employed was to classify bonds based on the probability of group membership.[5]

5. Rao [30, pp. 316-318] has shown that when the assumptions of MDA are met, the surfaces of the constant likelihood ratios are defined in terms of what might be called a linear discriminant score (L_m). The specific algorithm employed in this study follows Rao and yields five (M) sets of coefficients and constants for calculating linear discriminant scores along with the four (M-1) discriminant functions. A constant likelihood function corresponds to a constant difference in the discriminant scores. If the *a priori* probabilities are $\pi_1, \pi_2, \ldots \pi_m$ for M groups then the procedure

This procedure assigns each bond to the group for which its probability of membership, after considering the *a priori* probability, is highest. The results of applying this classification rule to the MDA model for the original sample group are presented in Table 6. The total number of bonds correctly rated is obtained by summing the main diagonal entries in Table 6. The MDA model correctly rated 92 of the 132 bonds. The percentage correctly predicted of 69.70 (92/132) is analogous to the coefficient of determination (R^2) in regression analysis, which measures the per cent of the variation in the dependent

TABLE 6
CLASSIFICATION OF ORIGINAL BOND SAMPLE

	Predicted Rating				
Actual Rating	Aa	A	Baa	Ba	B
Aa	10	4	0	0	0
A	2	22	0	2	0
Baa	0	8	4	13	0
Ba	0	0	0	39	5
B	0	0	0	6	17

variable explained by the independent variables. In examining the classification results of the MDA model it is evident that: 1) the model produced results that corresponded fairly accurately with the ratings for Aa, A, Ba, and B bonds since it correctly rated (for the four ratings) 88 of the 107 ratings; and 2) Baa bonds were correctly rated only four out of 25 times. If interest is in the ability of the model to rate within one classification (either higher or lower) of the actual rating, the model performed very accurately by classifying all but two of the bonds within one rating of the actual rating. The inability of the model to correctly rate Baa bonds will be examined subsequently.

Validation of the Holdout Sample

When the original sample is reclassified by the MDA model the resulting accuracy of the classification is biased upward by: 1) sampling errors in the original sample; and 2) search bias (10). Therefore, it is important to apply the MDA model to classify other bonds than those employed to develop the model. The essence of the procedure is to develop the model based on one

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is to assign a bond to that group for which $L_m + \log_e \cdot \pi_m$ is a maximum. If the *a priori* probabilities are unknown ($\pi_1 = \pi_2 \ldots \pi_m$), the maximum likelihood method leads to the rule of assigning a bond to that group for which L_m is largest. The appropriateness of this specific approach is dependent upon the data meeting the assumption of equal variance—covariance matrices between the groups. Since this assumption was not specifically tested, the appropriateness of this (or any linear) classification rule may be questioned.

Testing established the fact that classification based on the probability of group membership provided the same results obtained by employing the linear discriminant score (adjusted for *a priori* probabilities) discussed by Rao [30, pp. 316-317]. Classification based on chi-square scores, also investigated, resulted in fewer correct classifications. Serious consderation was given to two related questions considering the *a priori* probability of group membership: 1) should a group be assigned equal weights; and 2) if not, what weights should be assigned? It is maintained that bond raters do have some general standards which they follow concerning the approximate relative percentage of bonds rated in different groups; therefore, equal weighting would be out of the question. The prior probabilities of group membership employed in this study are those based on the relative frequency distributions of the original sample presented in Table 1.

set of data and then validate the model on subsequent samples. In this study a holdout sample of 48 bonds was employed.

The results of applying the MDA model to the holdout sample are presented in Table 7. The MDA model correctly rated 31 of 48 bonds (64.58 per cent) in the holdout sample and rated all bonds within one rating higher or lower than the actual rating. In order to reject the null hypothesis of no differences between the groups and substantiate that the model does possess discriminating power when applied to a secondary sample, a Z statistic was calculated.[6] With a Z value of 6.71, the null hypothesis that the results are due to chance is rejected and leads to the conclusion that search bias does not appear to exist. Examination of Table 7 indicates that the MDA model has fairly high predictive ability except for Baa and B rated bonds. Thus, the earlier inability of the MDA model to perform satisfactorily for certain bond ratings is upheld when the model is applied to the holdout sample.

TABLE 7
CLASSIFICATION OF HOLDOUT SAMPLE

	Predicted Rating				
Actual Rating	Aa	A	Baa	Ba	B
Aa	1	3	0	0	0
A	0	9	0	0	0
Baa	0	2	0	7	0
Ba	0	1	0	15	1
B	0	0	0	3	6

Further Validation of the MDA Model

In order to further validate the MDA model a stratified random sample of 48 companies who issued bonds during the first six months of 1969 was gathered. The MDA model (developed from a sample of newly rated bonds issued in 1967 and 1968) was employed to predict the ratings on the new bonds issued in 1969 (Table 8). Twenty-seven of the 48 bonds were rated

TABLE 8
CLASSIFICATION OF NEW BONDS ISSUED IN THE FIRST HALF OF 1969

	Predicted Rating				
Actual Rating	Aa	A	Baa	Ba	B
Aa	2	2	0	0	0
A	1	8	0	0	0
Baa	0	4	0	5	0
Ba	0	0	0	13	4
B	0	0	0	5	4

6. The Z test when group sizes are unequal, is presented [24]. For the holdout sample with a mean of 11.417 and variance of 8.082 the test statistic is:

$$Z = \frac{|(1+9+0+15+6) - 11.417| - .5}{\sqrt{8.8082}} = 6.71$$

An analysis of the bonds correctly and incorrectly rated failed to identify any patterns or tendencies of significance.

correctly indicating that the model possesses future predictive ability.[7] The model again performed poorly for Baa and B rated bonds; in fact, none of the ten Moody's Baa rated bonds were classified Baa by the MDA model.[8]

Subordination

The subordination status of a corporate bond (represented by a 0-1 variable) is the most important variable effecting bond ratings examined in this study. As set forth in Table 9, all Aa bonds examined in this study were nonsubordinated and 33 of 35 A bonds were also nonsubordinated. Similarly, all B rated bonds were subordinated and 60 of 61 Ba bonds were subordinated. Only Baa bonds exhibited a large proportion of both subordinated and nonsubordinated bonds with 20 of the bonds falling into the former category and 14 into the latter. If one was only interested in rating bonds as investment quality (Aa, A and Baa) or non-investment quality (Ba and B), the best single predictor is the subordinated status of the bond. Based on this variable alone, correct ratings (investment versus non-investment quality) would have resulted 88.6 (117/132) per cent of the time for the original sample and 83.3 (40/48) per cent of the time for the holdout sample.

TABLE 9

SUBORDINATED STATUS OF BONDS IN THE ORIGINAL AND HOLDOUT SAMPLES

Sample	Status Subordinated (S) or Non-Subordinated (NS)	Bond Rating Aa	A	Baa	Ba	B
Original	S	0	2	13	44	23
	NS	14	24	12	0	0
		14	26	25	44	23
Holdout	S	0	0	7	16	9
	NS	4	9	2	1	0
		4	9	9	17	9

Multiple Range Tests

In an attempt to ascertain why the MDA model performed poorly in dis-

7. $$Z = \frac{|(2+8+0+13+4) - 11.417 - .5|}{\sqrt{8.8082}} = 5.30$$

8. An analysis of the previous bond rating studies was also undertaken. This was accomplished by incorporating the variables specified in these studies with the data and technique (multiple discriminant analysis) in this study. The percentage of correct predictions was: 1) Horrigan—holdout sample — 48.98, new 1969 bonds — 53.06; West—holdout sample — 46.94, new 1969 bonds — 28.57, and; 3) Pogue and Soldofsky—holdout sample — 46.94, new 1969 bonds — 40.82. Most of the variables specified in these previous studies with the exception of West, were available in our study. Therefore, the MDA model presented in this paper appears to possess greater predictive ability than previous models suggested for rating corporate bonds.

criminating Baa bonds from A or Ba bonds, Duncan's new multiple range tests were calculated. This test is the most stringent procedure for determining *which* of a group of means differ significantly.[9] The results of this analysis indicate that no significant differences (at the .01 level) exist between the group means of a number of the variables included in the MDA model. Specifically, variable X_2 has two homogeneous subsets—(Aa, A, Baa) and (Baa, Ba); X_3 has two homogeneous subsets (Aa, A) and (Baa, Ba, B); X_4 has a homogeneous group (A, Baa, Ba, B); X_5 has two homogeneous subsets (A, Baa, Ba, B) and (Aa, A, Baa, Ba); and X_6 has two homogeneous subsets (Aa, A, Ba, B), (A, Baa, Ba, B). All five of the variables (excluding subordination) in the MDA model have means that are not significantly different (that is, they are members of homogeneous subsets) for bonds rated Baa and Ba. In addition, bonds rated A and Baa are members of homogeneous subsets for four of the five variables in the MDA model. Thus, the reason the specific MDA model presented earlier in the study performs poorly in predicting Baa rated bonds is because the specific variables included in the model do not, in general, discriminate very effectively between bonds rated Baa and bonds in adjacent rating groups.

Duncan's new multiple range tests were also run for all of the other variables considered for inclusion in the MDA model. An analysis of these tests indicated that none of the other twenty-nine variables considered for inclusion in the MDA model had significant differences between Baa and Ba bonds. Four variables had significant differences between A and Baa bonds—total assets, sales, number of shares of common stock, and total assets:mean. A number of MDA models incorporating various combinations of variables were calculated; however, none performed better than the model presented earlier.

In analyzing the results of the multiple range tests, it is evident that *very few* statistically significant differences exist between bonds in many of the categories. No variable considered for inclusion in the MDA model had a significant difference between the means of Baa and Ba rated bonds. In addition, many of the variables considered did not assist in predicting bond ratings because of inconsistent group means between the bond rating groups. Subordination, as represented by a 0-1 dichotomous variable was the most important variable for predicting bond ratings. The next most important variables for predicting bond ratings are X_2—years of consecutive dividends and X_3—issue size. The factor analysis results presented in Table 2 indicates that X_2—years of consecutive dividends, is related to other variables measuring earnings stability, and that X_3—issue size, is a measure of firm size. Only after the legal status of the bond as evidenced by subordination, earnings stability and firm size are taken into account, are variables related to financial leverage (X_5), debt and debt coverage stability (X_6), or return on investment (X_4) considered. Thus the results of this analysis indicate that many traditional financial considerations are fairly insignificant in the bond rating process.

9. The specific computer program employed is BMD 07V. For a discussion of Duncan's new multiple range test see, for example, Bruning and Kintz (4, pp. 115-117).

VI. Implications and Conclusions

Implications

In recent years the adequacy of bond ratings has been questioned.[10] This questioning is based in part on the apparently heavy reliance on qualitative factors taken into consideration by the bond raters. Moody's, for example, states that:

> Since ratings involve a judgment about the future, on the one hand, and since they are used by investors, as a means of protection on the other, the effort is made when assigning ratings, to look at 'worst' potentialities in the 'visible' future rather than solely at the past record and status of the present. Investors using the ratings should not, therefore, expect to find them a reflection of statistical factors alone. They are not statistical ratings but an appraisal of long term risks, such appraisal giving recognition to many statistical factors. [21, p. v]

Albert C. Esokait, senior vice president of Moody's, says that bond rating:

> . . . is not a number game. You couldn't rate bonds on a computer. It would blow a gasket. Bond-rating is a comprehensive analysis of the position of a company in whatever industry it is in. [34, p. 19]

While bond ratings can be compared to the odds that a firm will pay back the interest and principal fully and on time, no satisfactory evidence has been found to suggest that the present rating systems provide a substantially better estimate of this probability than could be obtained by looking only at a firm's operating and financial characteristics.

There are, however, a number of factors suggesting that the present rating schemes are the process of a system that was designed to perform in altogether different circumstances. For example, an official of Moody's recently was quoted as saying that rating corporate bonds has become even harder: ". . . due to scientific advances, a large scale expansion of corporate debt, more complex corporate structures and rapidly changing industrial patterns" [22, p. 2]. In addition, many analysts question whether ". . . the rating agencies are devoting sufficient manpower and technological resources to keep abreast of the work" [22, p. 2]. Finally, the decision by Moody's to charge for its services may subject them to even more pressure for "good" ratings. While no one questions the honesty of this agency, the fact remains that it is in the unusual position of providing objective ratings even though the principals (firms or underwriters) in the offering are the ones footing the bill for the evaluation report. Conversations with numerous corporate executives indicate that extensive negotiations are sometimes undertaken in order to secure a desired rating. In addition, certain municipalities have requested (and been granted) higher bond ratings [25].

The MDA model developed earlier in this study is able to correctly predict approximately two-thirds of the actual ratings. The inability to predict more of the actual ratings may be due to one of two reasons: 1) quantifiable data not incorporated in the model; or 2) qualitative factors causing actual bond

10. This has been particularly true with municipal bond ratings. See, for example, (12) and (27). Further information about the rating procedure for corporate bonds is provided in (18).

ratings to be different (higher or lower) than the quantifiable data justifies. While certain unusual circumstances may occasionally occur where rater judgment is essential, it appears that present bond ratings rely fairly heavily on qualitative factors.

Conclusions

The purpose of this study was to develop and test a model for predicting industrial bond ratings. The joint application of factor analysis and M-group multiple discriminant analysis, in a financial context, was found to be both viable and essential in developing and understanding the model for predicting industrial bond ratings. The final MDA model incorporating six variables correctly predicted 69.70 per cent of the actual ratings in the original sample, and predicted approximately 60 per cent of the ratings for a holdout sample and another sample of newly rated bonds. The best replications of Moody's ratings were obtained when variables relating to earnings stability, size, financial leverage, debt and debt coverage stability, return on investment, along with subordination, were considered. The model performed very poorly for Baa rated bonds. An analysis of the multiple range tests indicates that the inability of the MDA model to accurately predict Baa rated bonds appears to be due to a lack of statistically significant differences in the quantifiable variables considered for inclusion in the model.

These findings suggest that further research in this area could concentrate on two different aspects of the bond rating process. First, further attempts could be made to replicate existing bond ratings by focusing on other quantifiable variables not included in this study. The second possible area of future research is to develop new bond rating systems which more effectively reflect the probability of financial impairment for individual bond issues.

APPENDIX
VARIABLES AND TRANSFORMATIONS

Variable Number	Variable Name	Transformations*
1	Subordination	(0–1)
2	Total Assets	Log_{10}
3	Net Working Capital/Sales	Standardized
4	Net Worth/Total Assets	Standardized
5	Sales/Net Worth	
6	Net Income/Sales	
7	Recent Market High of Debt	Log_{10}
8	Price/Earnings	Firms with negative ratios replaced with 100; Log_{10}
9	Years of Consecutive Dividends	Log_{10}
10	Long Term Debt/Total Assets	
11	Sales/Total Assets	
12	Net Income/Net Worth	Standardized
13	Net Working Capital	Log_{10}; Standardized
14	Sales	Added .75; Log_{10}
15	Earnings Per Share	
16	Issue Size	Log_{10}
17	Number of Shares of Common Stock	Log_{10}
18	Market Value of Common Stock/Long Term Debt	Added 1.00; Log_{10}; Standardized
19	Net Income: Coefficient of Variation	Log_{10}
20	Long Term Debt/Net Worth: Five Year Mean	Standardized
21	Net Income/Total Assets: Five Year Mean	Standardized
22	Net Income/Total Assets: Coefficient of Variation	Standardized
23	Total Assets: Five Year Mean	Log_{10}
24	Net Income and Interest/Interest: Five Year Mean	Firms with low interest or none replaced with 100; added 1.00; Log_{10}
25	Long Term Debt/Total Assets: Five Year Mean	
26	Sales/Total Assets: Five Year Mean	
27	Long Term Debt/Total Assets: Coefficient of Variation	
28	Net Income and Interest/Interest: Coefficient of Variation	
29	Market Price Change: Coefficient of Variation (Six Years)	Standardized
30	Earnings Per Share Change: Coefficient of Variation (Six Years)	Standardized
31	Market Value of Common Stock/Long Term Debt: Five Year Mean	Added 1.00; Log_{10}; Standardized
32	Sales Percentage Change: Six Year Mean	Standardized
33	Net Income Percentage Change: Six Year Mean	Standardized
34	Long Term Debt/Net Worth	Standardized
35	Net Income/Total Assets	Standardized

* Since extreme values can influence both factor analysis and multiple discriminant analysis, outliers were standardized to approximately ± 2.00 standard deviations from the mean.

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