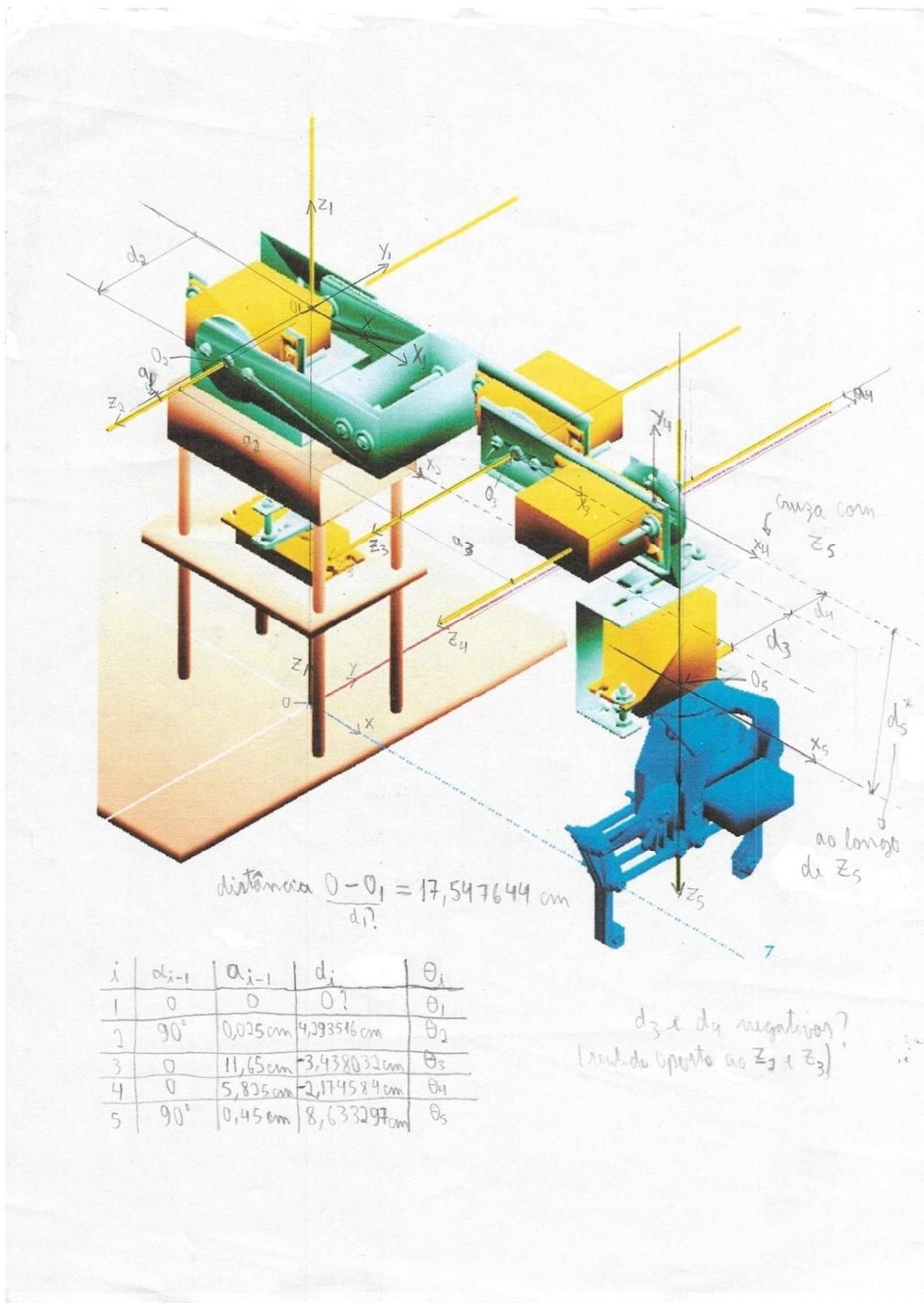
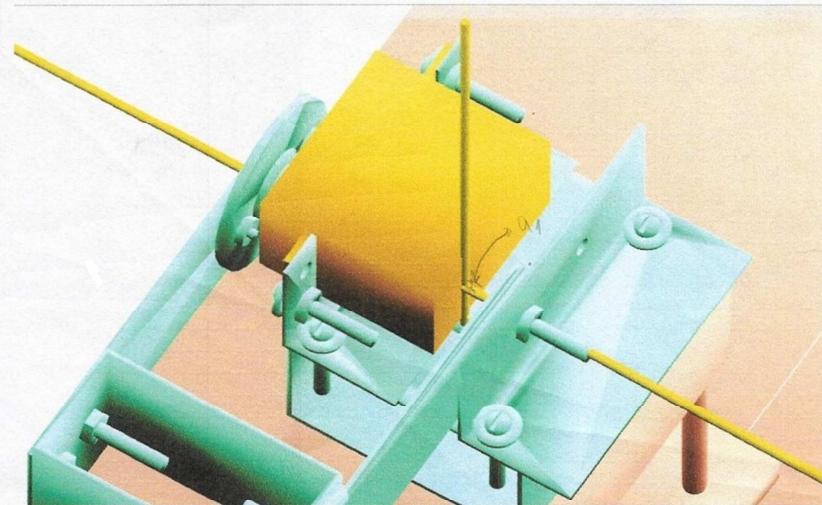
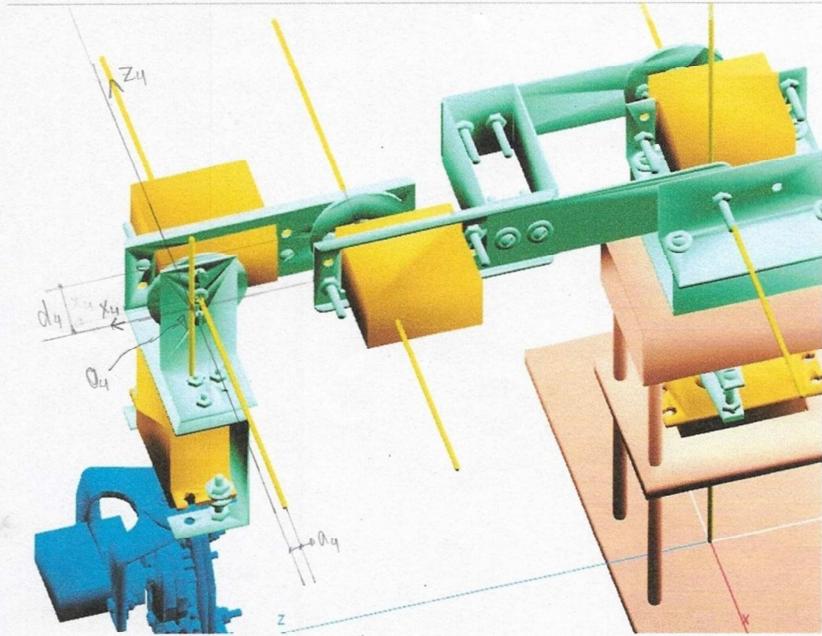


Parâmetros de Denavit-Hartenberg (DH)



Detalhes do a₄ e do a₁



Cálculos da Cinemática Direta

$$\begin{aligned}
 {}^0T = & \begin{bmatrix} c_1 & -\lambda_1 & 0 & 0 \\ \lambda_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T = \begin{bmatrix} c_2 & -\lambda_2 & 0 & a_2 \\ 0 & 0 & -1 & -d_2 \\ \lambda_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 {}^2T = & \begin{bmatrix} c_3 & -\lambda_3 & 0 & a_3 \\ \lambda_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3T = \begin{bmatrix} c_4 & -\lambda_4 & 0 & a_4 \\ \lambda_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 {}^0T = & \begin{bmatrix} c_1c_2c_3 - c_1\lambda_2\lambda_3 & -c_1c_2\lambda_3 - c_1\lambda_2c_3 & \lambda_1 & c_1c_2a_2 + \lambda_1d_3 + (c_1a_1 + \lambda_1d_2) \\ \lambda_1c_2c_3 - \lambda_1\lambda_2\lambda_3 & -\lambda_1c_2\lambda_3 - \lambda_1\lambda_2c_3 & -c_1 & \lambda_1c_2a_2 - c_1d_3 + (\lambda_1a_1 - c_1d_2) \\ \lambda_2c_3 + c_1\lambda_3 & -\lambda_2\lambda_3 + c_1c_3 & 0 & \lambda_2a_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 = & \begin{bmatrix} c_1c_{23} & -c_1\lambda_{23} + c_3 & \lambda_1 & \lambda_1(d_2 + d_3) + c_1(a_1 + c_2a_2) \\ \lambda_1c_{23} & -\lambda_1\lambda_{23} & -c_1 & \lambda_1(a_1 + c_2a_2) - c_1(d_2 + d_3) \\ \lambda_{23} & c_{23} & 0 & \lambda_2a_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3T = \begin{bmatrix} c_4 & -\lambda_4 & 0 & a_3 \\ \lambda_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 {}^0T = & \begin{bmatrix} c_1c_{23}c_4 - c_1\lambda_{23}\lambda_4 & -c_1c_{23}\lambda_4 - c_1\lambda_{23}c_4 & \lambda_1 & c_1c_{23}a_3 + \lambda_1d_4 + \lambda_1(d_2 + d_3) + c_1(a_1 + c_3a_3) \\ \lambda_1c_{23}c_4 - \lambda_1\lambda_{23}\lambda_4 & -\lambda_1c_{23}\lambda_4 - \lambda_1\lambda_{23}c_4 & -c_1 & \lambda_1c_{23}a_3 - c_1d_4 + \lambda_1(a_1 + c_3a_3) - c_1(d_2 + d_3) \\ \lambda_{23}c_4 + c_{23}\lambda_4 & -\lambda_{23}\lambda_4 + c_{23}c_4 & 0 & \lambda_{23}a_3 + \lambda_2a_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 = & \begin{bmatrix} c_1c_{23}c_4 & -c_1\lambda_{23}\lambda_4 & \lambda_1 & \lambda_1(d_2 + d_3 + d_4) + c_1(a_1 + c_2a_2 + c_3a_3) \\ \lambda_1c_{23}c_4 & -\lambda_1\lambda_{23}\lambda_4 & -c_1 & \lambda_1(a_1 + c_2a_2 + c_{23}a_3) - c_1(d_2 + d_3 + d_4) \\ \lambda_{23}c_4 & c_{23}c_4 & 0 & \lambda_{23}a_3 + \lambda_2a_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^4T = \begin{bmatrix} c_5 & -\lambda_5 & 0 & a_4 \\ \lambda_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 {}^0T = & \begin{bmatrix} c_1c_{23}c_5 + \lambda_1\lambda_5 & -c_1c_{23}c_5 + \lambda_1c_5 & c_1\lambda_{234} & c_1c_{23}a_4 + c_1\lambda_{234}d_5 + \lambda_1(d_2 + d_3 + d_4) + c_1(a_1 + c_3a_3 + c_{23}a_3) \\ \lambda_1c_{23}c_5 - c_1\lambda_5 & -\lambda_1c_{23}c_5 - c_1c_5 & \lambda_1\lambda_{234} & \lambda_1c_{23}a_4 + \lambda_1\lambda_{234}d_5 + \lambda_1(a_1 + c_3a_3 + c_{23}a_3) - c_1(d_2 + d_3 + d_4) \\ \lambda_{23}c_5 & -\lambda_{23}\lambda_5 & -c_{234} & -c_{234}a_4 - c_{234}d_5 + \lambda_{23}a_3 + \lambda_2a_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 {}^0T = & \begin{bmatrix} c_1c_{23}c_5 + \lambda_1\lambda_5 & -c_1c_{23}c_5 + \lambda_1c_5 & c_1\lambda_{234} & \lambda_1(d_2 + d_3 + d_4) + c_1(a_1 + c_2a_2 + c_{23}a_3 + c_{23}a_4 + \lambda_{234}d_5) \\ \lambda_1c_{23}c_5 - c_1\lambda_5 & -\lambda_1c_{23}c_5 - c_1c_5 & \lambda_1\lambda_{234} & \lambda_1(a_1 + c_2a_2 + c_{23}a_3 + c_{23}a_4 + \lambda_{234}d_5) - c_1(d_2 + d_3 + d_4) \\ \lambda_{23}c_5 & -\lambda_{23}\lambda_5 & -c_{234} & d_1 + \lambda_2a_2 + \lambda_{23}a_3 + \lambda_{234}a_4 - c_{234}d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

Cinemática direta

Cálculos da Cinemática Direta – Resultado e Otimizações de Fórmulas

$$\begin{matrix} {}_5^0T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_{x0} \\ r_{21} & r_{22} & r_{23} & p_{y0} \\ r_{31} & r_{32} & r_{33} & p_{z0} \\ 0 & 0 & 0 & 1 \end{bmatrix} & c_i = \cos(\theta_i) & \lambda_{23} = \lambda_2 c_3 + c_2 \lambda_3 \\ & s_i = \sin(\theta_i) & c_{23} = c_2 c_3 - \lambda_2 \lambda_3 \\ & i = 1, \dots, 5 & c_{234} = c_{23} c_4 - \lambda_{23} \lambda_4 \\ & & \lambda_{234} = \lambda_{23} c_4 + c_{23} \lambda_4 \end{matrix}$$

$$r_{11} = c_1 c_{234} c_5 + \lambda_1 \lambda_5$$

$$r_{21} = \lambda_1 c_{234} c_5 - c_1 \lambda_5$$

$$r_{31} = \lambda_{234} c_5$$

$$r_{12} = \lambda_1 c_5 - c_1 c_{234} \lambda_5$$

$$r_{22} = -c_1 c_5 - \lambda_1 c_{234} \lambda_5$$

$$r_{32} = -\lambda_{234} \lambda_5$$

$$r_{13} = c_1 \lambda_{234}$$

$$r_{23} = \lambda_1 \lambda_{234}$$

$$r_{33} = -c_{234}$$

$$d_{234} = d_2 + d_3 + d_4$$

$$f = a_1 + a_2 c_2 + a_3 c_3 + a_4 c_{234} + d_5 \lambda_{234}$$

$$p_{x0} = d_{234} \lambda_1 + f c_1$$

$$p_{y0} = f \lambda_1 - d_{234} c_1$$

$$p_{z0} = d_1 + a_2 \lambda_2 + a_3 \lambda_{23} + a_4 \lambda_{234} - d_5 c_{234}$$

Cálculos da Cinemática Inversa – θ_1 e θ_5

$${}^0T = {}^0T^{-1} = \left[\begin{array}{c|cc} {}^0R^T & -{}^0R^T {}^0P_{ORG} \\ \hline 0 & 1 \end{array} \right] \quad {}^0T = \left[\begin{array}{cccc} c_1 & -\lambda_1 & 0 & 0 \\ \lambda_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{array} \right] = \left[\begin{array}{c|cc} {}^0R & {}^0P_{ORG} \\ \hline 0 & 1 \end{array} \right]$$

$${}^0T^{-1} = {}^1T = \left[\begin{array}{ccc|c} c_1 & \lambda_1 & 0 & [-c_1 & -\lambda_1 & 0] \\ -\lambda_1 & c_1 & 0 & \lambda_1 & -c_1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 1 \end{array} \right] = \left[\begin{array}{cccc} c_1 & \lambda_1 & 0 & 0 \\ -\lambda_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & -d_1 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad {}^0T = \left[\begin{array}{cccc} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$${}^0T^{-1} {}^0T = {}^0T^{-1} ({}^0T {}^1T {}^2T {}^3T {}^4T {}^5T) = {}^1T {}^2T {}^3T {}^4T {}^5T$$

$${}^0T^{-1} {}^0T = \left[\begin{array}{cccc} c_1r_{11} + \lambda_1r_{21} & c_1r_{12} + \lambda_1r_{22} & c_1r_{13} + \lambda_1r_{23} & c_1p_x + \lambda_1p_y \\ -\lambda_1r_{11} + c_1r_{21} & -\lambda_1r_{12} + c_1r_{22} & -\lambda_1r_{13} + c_1r_{23} & -\lambda_1p_x + c_1p_y \\ r_{31} & r_{32} & r_{33} & p_z + d_1 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$${}^1T {}^2T {}^3T {}^4T {}^5T = \left[\begin{array}{cccc} t_{11} & t_{12} & t_{31} & v_x \\ t_{21} & t_{22} & t_{32} & v_y \\ t_{31} & t_{32} & t_{33} & v_z \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$t_{11} = c_1^2 c_{234} f_5 + \lambda_1^2 A_5 + \lambda_1^2 c_{234} f_5 - \lambda_1 f_5 = c_{234} f_5 (c_1^2 + \lambda_1^2) = c_{234} f_5$$

$$t_{21} = -\lambda_1 c_1 b_{234} f_5 + \lambda_1^2 A_5 + (\lambda_1 c_1 b_{234} f_5 - c_1^2 A_5) = -A_5$$

$$t_{31} = \lambda_1 b_{234} f_5$$

$$t_{12} = -c_1^2 c_{234} A_5 + \lambda_1^2 A_5 - \lambda_1^2 c_{234} A_5 - \lambda_1 f_5 = -c_{234} A_5$$

$$t_{22} = \lambda_1 c_1 b_{234} A_5 - \lambda_1^2 c_5 - (\lambda_1 c_1 b_{234} A_5 - c_1^2 c_5) = -c_5$$

$$t_{32} = -\lambda_1 b_{234} A_5$$

$$t_{13} = c_1^2 \lambda_1 b_{234} + \lambda_1^2 \lambda_1 b_{234} = \lambda_1 b_{234}$$

$$t_{23} = -\lambda_1 c_1 \lambda_1 b_{234} + \lambda_1 \lambda_1 \lambda_1 b_{234} = 0$$

$$t_{33} = -b_{234}$$

$$v_x = c_1 \lambda_1 d_{234} + c_1^2 f + \lambda_1^2 f - \lambda_1 c_1 d_{234} = f$$

$$v_y = -\lambda_1^2 d_{234} - \lambda_1 c_1 f + (\lambda_1 c_1 f - c_1^2 d_{234}) = -d_{234}$$

$$v_z = \lambda_1 a_2 + \lambda_2 a_3 + \lambda_3 a_4 - c_{234} d_5$$

② θ_5

$$\begin{cases} -\lambda_1 r_{11} + c_1 r_{21} = -A_5 \\ -\lambda_1 r_{12} + c_1 r_{22} = -c_5 \end{cases}$$

$$\begin{cases} A_5 = \lambda_1 r_{11} - c_1 r_{21} \\ c_5 = \lambda_1 r_{12} - c_1 r_{22} \end{cases}$$

$$\theta_5 = \text{atan2}(A_5, c_5)$$

① θ_1 Solução 1 para θ_1
 $\lambda_1 p_x + c_1 p_y = -d_{234}$

$$\theta_1 = \text{atan2}(-p_x, p_y) \pm \text{atan2}(\sqrt{p_y^2 + p_x^2 - d_{234}^2}, -d_{234}) \quad (\text{2 soluções})$$

Solução 1 para θ_1 :

$$-\lambda_1 r_{13} + c_1 r_{23} = 0$$

$$\theta_1 = \text{atan2}(r_{23}, r_{13})$$

$$\theta_1 = \text{atan2}(-r_{23}, -r_{13})$$

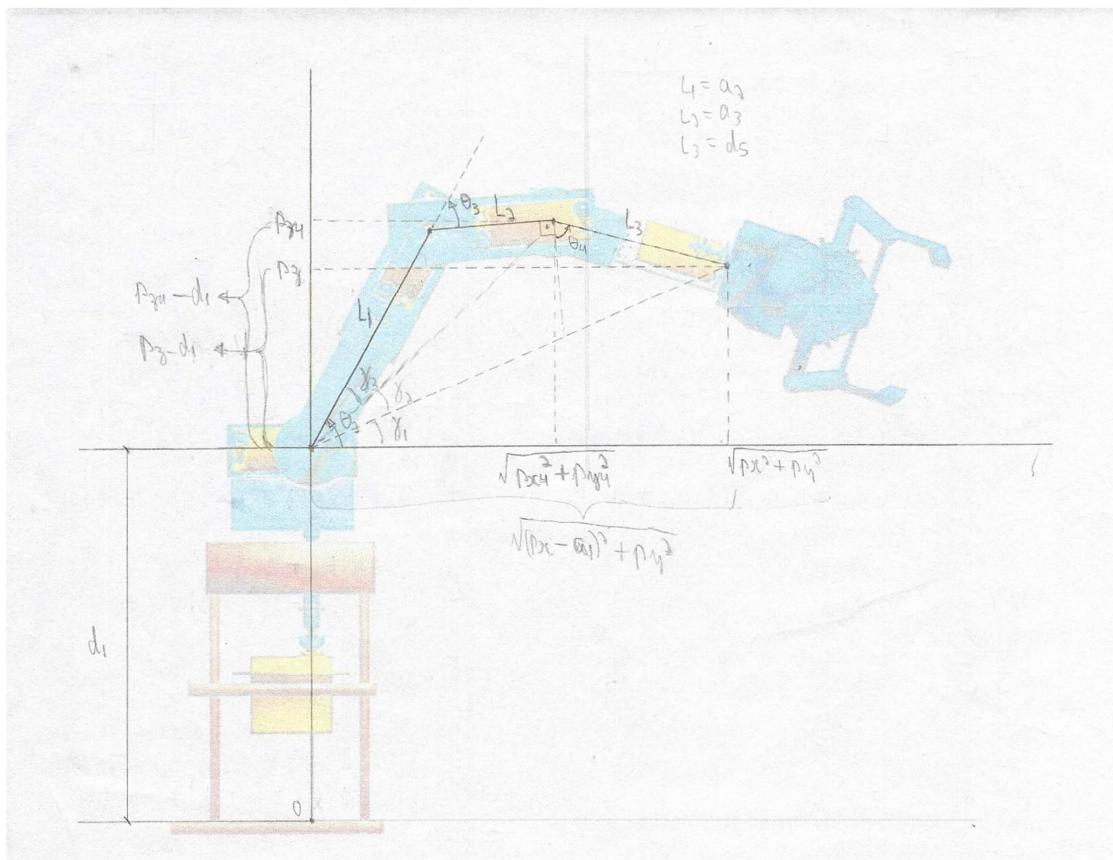
③ θ_{234}

$$\lambda_{234} = c_1 r_{13} + \lambda_1 r_{23}$$

$$c_{234} = -\lambda_{23}$$

$$\theta_{234} = \text{atan2}(\lambda_{234}, c_{234})$$

Cinemática Inversa – Observação sobre a abordagem geométrica



O erro cometido na implementação anterior da cinemática inversa do braço robô se deve a uma abordagem geométrica feita de forma errada. O erro se deve justamente pela cinemática inversa anterior considerar as coordenadas do pulso da garra como sendo na junta 3 (onde se encontra o θ_4). Esse engano foi cometido com base na abordagem geométrica utilizada no braço robô Yasukawa do livro do Craig, que considera as coordenadas do pulso da garra como coincidentes com a junta 4. Esta abordagem é correta para o Yasukawa, já que no caso dele considera-se a distância d_5 como sendo zero. No nosso caso, a distância d_5 não é nula, o que dificulta e muito a abordagem geométrica.

Desta forma, sempre que se fazia cinemática direta e inversa alternadamente no software de controle, a garra tendia sempre a ir para trás, ocasionando a situação em que a garra era posicionada para dentro da base fixa na simulação do braço robô.

Cálculos da Cinemática Inversa – θ_2

$${}^1T^{-1} = \left[\begin{array}{cc|c} {}^1R^T & -{}^1R^T P_{20NG} \\ \hline 0 & 0 & 0 \end{array} \right] = \left[\begin{array}{ccc|c} C_2 & 0 & \lambda_2 & [-C_2 & 0 & -\lambda_2] \\ -\lambda_2 & 0 & C_2 & [\lambda_2 & 0 & -C_2] \\ 0 & -1 & 0 & [0 & 1 & 0] \\ \hline 0 & 0 & 0 & 1 \end{array} \right] \begin{bmatrix} \alpha_1 \\ -d_2 \\ 0 \end{bmatrix}$$

$${}^1T^{-1} = \begin{bmatrix} C_2 & 0 & \lambda_2 & -C_2 \alpha_1 \\ -\lambda_2 & 0 & C_2 & \lambda_2 \alpha_1 \\ 0 & -1 & 0 & -d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T^{-1} {}^0S T = {}^1T^{-1} {}^1T {}^1T {}^2T {}^3T {}^4T {}^5T$$

$$\begin{bmatrix} C_2 \pi_{11} + \lambda_2 \pi_{31} & C_2 \pi_{12} + \lambda_2 \pi_{32} & C_2 \pi_{13} + \lambda_2 \pi_{33} & C_2 \rho_x + \lambda_2 \rho_y - C_2 \alpha_1 \\ -\lambda_2 \pi_{11} + C_2 \pi_{31} & -\lambda_2 \pi_{12} + C_2 \pi_{32} & -\lambda_2 \pi_{13} + C_2 \pi_{33} & -\lambda_2 \rho_x + C_2 \rho_y + \lambda_2 \alpha_1 \\ -\pi_{21} & -\pi_{22} & -\pi_{23} & -\rho_y - d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} t_{11} & t_{12} & t_{13} & \nu_x \\ t_{21} & t_{22} & t_{23} & \nu_y \\ t_{31} & t_{32} & t_{33} & \nu_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$t_{11} = C_1 C_2 C_{234} \varsigma_5 + \lambda_1 C_2 \lambda_5 + \lambda_2 \lambda_3 \varsigma_4 \varsigma_5$$

$$t_{21} = -C_1 \lambda_2 C_{234} \varsigma_5 - \lambda_1 \lambda_2 \lambda_5 + C_2 \lambda_3 \varsigma_4 \varsigma_5$$

$$t_{31} = -\lambda_1 \lambda_3 \varsigma_4 \varsigma_5 + C_1 \lambda_5$$

$$t_{12} = -C_1 C_2 \lambda_{234} \lambda_5 + \lambda_1 C_2 \varsigma_5 - \lambda_2 \lambda_{234} \lambda_5$$

$$t_{22} = C_1 \lambda_2 \lambda_{234} \lambda_5 - \lambda_1 \lambda_2 \lambda_5 - C_2 \lambda_{234} \lambda_5$$

$$t_{32} = \lambda_1 C_{234} \lambda_5 + C_1 \varsigma_5$$

$$t_{13} = C_1 C_2 \lambda_{234} - \lambda_2 \lambda_{234}$$

$$t_{23} = -C_1 \lambda_2 \lambda_{234} + \lambda_2 \lambda_{234}$$

$$t_{33} = -\lambda_1 \lambda_{234}$$

$$\nu_x = \lambda_1 C_2 (d_2 + d_3 + d_4) + C_1 C_2 (a_1 + C_2 \alpha_2 + C_3 \alpha_3 + C_{234} \alpha_4 + \lambda_{234} \lambda_5) + \lambda_2 (d_1 + \lambda_2 \alpha_2 + \lambda_{23} \alpha_3 + \lambda_{234} \alpha_4 - C_{234} \lambda_5)$$

$$-C_2 \alpha_1$$

$$\nu_y = -\lambda_1 \lambda_2 (d_2 + d_3 + d_4) - C_1 \lambda_2 (\alpha_1 + C_2 \alpha_2 + C_3 \alpha_3 + C_{234} \alpha_4 + \lambda_{234} \lambda_5) + C_2 (d_1 + \lambda_2 \alpha_2 + \lambda_{23} \alpha_3 + \lambda_{234} \alpha_4 - C_{234} \lambda_5)$$

$$+ \lambda_2 \alpha_1$$

$$\nu_z = -\lambda_1 (\alpha_1 + C_2 \alpha_2 + C_{23} \alpha_3 + C_{234} \alpha_4 + \lambda_{234} \lambda_5) + C_1 (d_2 + d_3 + d_4) - d_2$$

$$\begin{aligned} c_2 \lambda_{13} + \lambda_2 \lambda_{33} &= c_2 c_1 \lambda_{234} - \lambda_2 \lambda_{234} \\ c_2 (\lambda_{13} - c_1 \lambda_{234}) + \lambda_2 (\lambda_{33} + \lambda_{234}) &= 0 \end{aligned}$$

$$\theta_3 = \operatorname{atan} 2(\lambda_{13} - c_1 \lambda_{234}, -\lambda_{33} - \lambda_{234})$$

or

$$\theta_3 = \operatorname{atan} 2(-\lambda_{13} + c_1 \lambda_{234}, \lambda_{33} + \lambda_{234})$$

$$\theta_{34} = \theta_{234} - \theta_3$$

Cálculos da Cinemática Inversa – θ_3 e θ_4

$${}^3T = {}^3T {}^4T$$

$${}^5T = {}^4T {}^5T$$

$$\begin{aligned} {}^3T = \begin{bmatrix} c_4 & -\lambda_4 & 0 & a_3 \\ \lambda_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_5 & -\lambda_5 & 0 & a_4 \\ 0 & 0 & -1 & -d_5 \\ \lambda_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} &= \begin{bmatrix} c_4 c_5 - c_4 \lambda_5 & \lambda_4 & a_4 c_4 + d_5 \lambda_4 + a_3 \\ \lambda_4 c_5 - \lambda_4 \lambda_5 & -c_4 & a_4 \lambda_4 - d_5 c_4 \\ \lambda_5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ {}^3T = \begin{bmatrix} c_3 & -\lambda_3 & 0 & a_3 \\ \lambda_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 c_5 - c_4 \lambda_5 & \lambda_4 & a_4 c_4 + d_5 \lambda_4 + a_3 \\ \lambda_4 \lambda_5 - \lambda_4 \lambda_5 & -c_4 & a_4 \lambda_4 - d_5 c_4 \\ \lambda_5 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} &= \begin{bmatrix} c_3 c_4 (c_5 - \lambda_3 \lambda_4 c_5) - (c_3 c_4 \lambda_5 + \lambda_3 \lambda_4 \lambda_5) & c_3 \lambda_4 + \lambda_3 c_4 & c_3 (a_4 (c_4 + d_5 \lambda_4 + a_3) - \lambda_3 (a_4 \lambda_4 - d_5 c_4) + a_3) \\ \lambda_3 c_4 c_5 + c_3 \lambda_4 c_5 - \lambda_3 c_4 \lambda_5 - c_3 \lambda_4 \lambda_5 & \lambda_3 \lambda_4 - c_3 c_4 & \lambda_3 (a_4 c_4 + d_5 \lambda_4 + a_3) + c_3 (a_4 \lambda_4 - d_5 c_4) \\ \lambda_5 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ {}^3T = \begin{bmatrix} c_3 c_4 c_5 & -c_3 c_4 \lambda_5 & \lambda_{34} & a_2 + a_3 c_3 + a_4 c_3 c_4 + d_5 \lambda_{34} \\ \lambda_3 c_4 c_5 & -\lambda_3 c_4 \lambda_5 & -c_{34} & a_3 \lambda_3 + a_4 \lambda_{34} - d_5 c_{34} \\ \lambda_5 & 0 & 0 & d_3 + d_4 \end{bmatrix} & \end{aligned}$$

$${}^0T^{-1} = \left[\begin{array}{c|cc} {}^0RT & -{}^0RT P_{\text{ORG}} \\ \hline 0 & 1 & \\ 0 & 0 & 1 \end{array} \right] = \left[\begin{array}{ccc|ccc} c_1 c_2 & \lambda_1 c_2 & \lambda_2 & -c_1 c_2 & -\lambda_1 c_2 & -\lambda_2 & [c_1 a_1 + \lambda_1 d_2] \\ -c_1 \lambda_2 & -\lambda_1 \lambda_2 & c_2 & c_1 \lambda_2 & \lambda_1 \lambda_2 & -c_2 & [\lambda_1 a_1 - c_1 d_2] \\ \lambda_1 & -c_1 & 0 & -\lambda_1 & -c_1 & 0 & d_1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$${}^0T^{-1} = \left[\begin{array}{ccc|ccc} c_1 c_2 & \lambda_1 c_2 & \lambda_2 & -a_1 c_1^2 c_2 - d_2 \lambda_1^2 c_2 - a_1 \lambda_1^2 c_2 + d_2 a_1 c_1 c_2 - d_1 \lambda_2 \\ -c_1 \lambda_2 & -\lambda_1 \lambda_2 & c_2 & a_1 c_1^2 \lambda_2 + d_2 \lambda_1^2 \lambda_2 + a_1 \lambda_1^2 \lambda_2 - d_2 \lambda_1 \lambda_2 \lambda_2 - d_1 c_2 \\ \lambda_1 & -c_1 & 0 & -a_1 \lambda_1^2 c_2 - d_2 \lambda_1^2 \lambda_2 + a_1 \lambda_1^2 \lambda_2 - d_2 c_1^2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$${}^0T^{-1} = \left[\begin{array}{ccc|ccc} c_1 c_2 & \lambda_1 c_2 & \lambda_2 & -a_1 c_2 - d_1 \lambda_2 \\ -c_1 \lambda_2 & -\lambda_1 \lambda_2 & c_2 & a_1 \lambda_2 - d_1 c_2 \\ \lambda_1 & -c_1 & 0 & -d_2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$${}^0T^{-1} {}^0T = {}^3T$$

$$\begin{bmatrix} C_{32}\lambda_{11} + \lambda_1 C_2\lambda_{21} + \lambda_2 C_3\lambda_{31} & C_1\lambda_{12} + \lambda_1 C_2\lambda_{22} + \lambda_2 C_3\lambda_{32} & C_1C_2\lambda_{13} + \lambda_1 C_3\lambda_{23} + \lambda_2 C_3\lambda_{33} & C_1C_2P_X + \lambda_1 C_3P_Y + \lambda_2 P_Z - d_1\lambda_3 - d_1\lambda_2 \\ -C_2\lambda_{11} - \lambda_1 C_2\lambda_{21} + C_3\lambda_{31} & -C_1\lambda_{22} - \lambda_1 C_3\lambda_{22} + C_2\lambda_{32} & -C_1C_3\lambda_{13} - \lambda_1 C_3\lambda_{23} + C_2C_3\lambda_{33} & -C_1C_3P_X - \lambda_1 C_3P_Y + C_2P_Z + \lambda_1 \lambda_3 - d_1(\lambda_3) \\ \lambda_1\lambda_{11} - C_1\lambda_{21} & \lambda_1\lambda_{12} - C_1\lambda_{22} & \lambda_1\lambda_{13} - C_1\lambda_{23} & \lambda_1P_X - C_1P_Y - d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{34}C_5 & -C_3\lambda_5 & \lambda_{34} & a_2 + a_3C_3 + a_4C_4 + d_5\lambda_{34} \\ \lambda_{34}C_5 & -\lambda_{34}\lambda_5 & -C_3\lambda_4 & a_3\lambda_{33} + a_4\lambda_{34} - d_5C_{34} \\ \lambda_5 & C_5 & 0 & d_3 + d_4 \\ 0 & 0 & 0 & \end{bmatrix}$$

Considerar: $\theta_1, \theta_2, \theta_{34}$

$$(C_2P_X + \lambda_1 C_3P_Y + \lambda_2 P_Z - a_1C_3 - d_1\lambda_3) \equiv a_2 + a_3C_3 + a_4C_4 + d_5\lambda_{34}$$

$$-a_3C_3 = a_2(C_1P_X + \lambda_1 P_Y - a_1) + \lambda_2(P_Z - d_1) - a_2 - a_4C_4 - d_5\lambda_{34}$$

$$C_3 = \frac{a_2(C_1P_X + \lambda_1 P_Y - a_1) + \lambda_2(P_Z - d_1) - a_2 - a_4C_4 - d_5\lambda_{34}}{a_3}$$

$$-C_1\lambda_2P_X - \lambda_1\lambda_2P_Y + C_2P_Z + a_1\lambda_3 - d_1C_3 = a_3\lambda_3 + a_4\lambda_{34} - d_5C_{34}$$

$$\lambda_3 = \frac{-\lambda_2(C_1P_X + \lambda_1 P_Y - a_1) + C_2(P_Z - d_1) - a_4\lambda_{34} + d_5C_{34}}{a_3}$$

$$\boxed{\theta_3 = \operatorname{atan2}(C_3, \lambda_3)}$$

$$\boxed{\theta_4 = \theta_{34} - \theta_3}$$