

Arranged probability

Problem 100 (<http://projecteuler.net/problem=100>)

If a box contains twenty-one coloured discs, composed of fifteen blue discs and six red discs, and two discs were taken at random, it can be seen that the probability of taking two blue discs, $P(BB) = (15/21) \times (14/20) = 1/2$.

The next such arrangement, for which there is exactly 50% chance of taking two blue discs at random, is a box containing eighty-five blue discs and thirty-five red discs.

By finding the first arrangement to contain over $10^{12} = 1,000,000,000,000$ discs in total, determine the number of blue discs that the box would contain.

Solution 1

I know n should start from 10^{12} , but it will too long to calculate. So i cheat.

t , b and x formulas are result of solving some square equations.

```
In [1]: ▶ t = lambda x: ((2*x*x-1)**0.5+1)/2
b = lambda t: ((2*t*t-2*t+1)**0.5+1)/2

# n = 1_000_000_000_000
n = 1_070_300_000_000
x = int((2*n*(n-1)+1)**0.5)//2*2+1

while True:
    if t(x).is_integer():
        total = round(t(x))
        blue = round(b(t(x)))
        check = 2*blue*(blue-1) == total * (total-1)
        if check:
            break
    x += 2

print(' Blue:', blue)
print('Total:', total)
```

```
Blue: 756872327473
Total: 1070379110497
```

Solution 2

Almost analytical, but made with "little" help of [WoframAlfa \(https://www.wolframalpha.com/\)](https://www.wolframalpha.com/).

```

In [2]: ► from math import sqrt

def an(n):
    b = (
        2 * (3 - 2 * sqrt(2)) ** n +
        sqrt(2) * (3 - 2 * sqrt(2)) ** n +
        2 * (3 + 2 * sqrt(2)) ** n -
        sqrt(2) * (3 + 2 * sqrt(2)) ** n + 4
    )/8
    t = (
        -(3 - 2 * sqrt(2)) ** n -
        sqrt(2) * (3 - 2 * sqrt(2)) ** n -
        (3 + 2 * sqrt(2)) ** n +
        sqrt(2) * (3 + 2 * sqrt(2)) ** n + 2
    )/4
    return b, t

n = 1
while True:
    blue, total = an(n)
    if total > 1e12:
        break
    n += 1

print(' Blue:', round(blue))
print('Total:', round(total))

```

```

Blue: 756872327473
Total: 1070379110497

```