Discrete-Set

Amar Panji Senjaya

[2022-01-28 Fri 09:15]

Source And Learning Material:
https://www.youtube.com/playlist?list=PLHXZ9OQGMqxersk8fUxiUMSIx0DBqsKZS
https://www.amazon.com/Discrete-Mathematics-Applications-Susanna-Epp/dp/1337694193/ref=sr_1_2?keywords=discrete+math&qid=1645043639&sr=8-2

1 What is Set

A **set** is a collection of objects. Individual object of set is element $\in$. Individual object of element that is not in The set is called not in element denoted with $\notin$.

1.1 Examples

- A = Employees in a company
  John $\in$ A

- B = {1,3,4,7}
  3 $\in$ A
  $\pi \notin$ B

- Z = Integers
  3 $\in$ Z

1.2 Order And Repetition Don't Matter

```
{1, 2, 3, 4, 5, 6, 7} = {2, 5, 4, 3, 1, 7, 6}
                      = {2, 2, 2, 2, 5, 4, 3, 1, 1, 7, 6, 6, 6}
```

2 Subsets

- Every Element of A is also in B = A $\subseteq B$

2.0.1 Example 1

A = {1, 3}
B = {1, 3, 4, 6}
A $\subseteq B$

2.0.2 Example 2

A = {1, 8}
B = {1, 3, 4, 6}
A $\nsubseteq B$
$because \in$ 8 is not in B

3 Set-Roster Notion

Replace ... with continuos clear pattern
Example:
Positive even integers
{0, 2, 4, 6, ...}
all even integers
{..., -6, -4, -2, 0, 2, 4, 6, ...}

4 Set-Builder Notation

General form: { x | P(x) }
x variable
"|" such that
P(x) Property is true
Example even integers
= { x | x = Twice an integer }
Example square root
{ x | $\sqrt{x} \in$ Z }

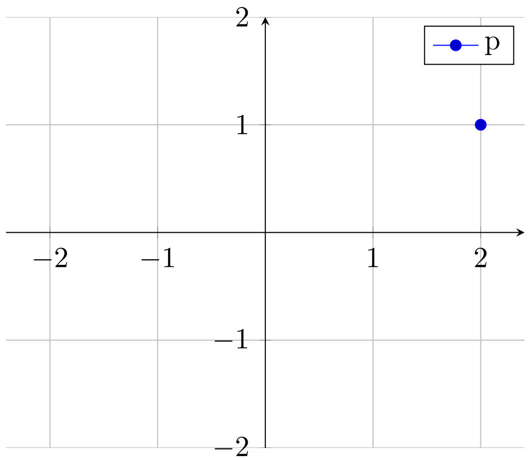
5 Empty Set

Notion: {} or $\emptyset$
{$\emptyset$}
Is $\emptyset \subset$ {1,2,3}?
Recall: A $\subset$ B means:
if x $\in$ A, then x $\in$ B
This is *vacuously* true!

6 Ordered Pairs (a, b)

- Order Matters
- (a,b) = (c,d) of a=c & b=d
- a&b could come from different sets

Defn: The cartesian product A $\times$ B is the set of all ordered pairs (a,b) where a $\in$ A and b $\in$ B



p = (2,1) $\in$ R x R
first component is x axis and second component is y axis

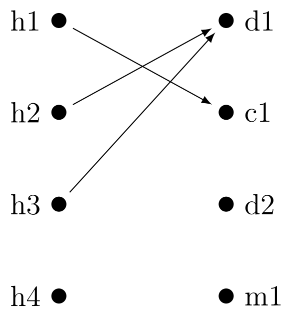
Example:
{a,b} $\times$ {0,1}
A $\times$ B = {(a,1), (a,0), (b,0), (b1)}

7 Relations

Ex: a $<$ b
compares two integers
some pairs have this relationship 2 $<$ 5

some pairs don't $5 \not< 2$

note:
h: human
d: dog
c: cat
m: monkey

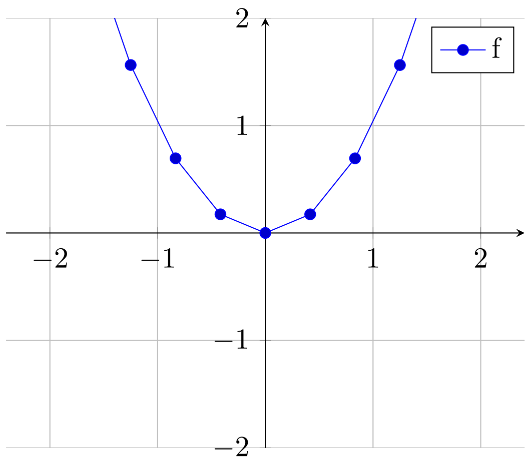


<u>defn</u>: A Relation R between A and B is a subset of A $\times$ B

ie ordered pairs
(a,b) $\in$ A $\times$ B

8 Functions

ex: $f(x) = x^2$



function do something to every input in my domain and produce output for each input

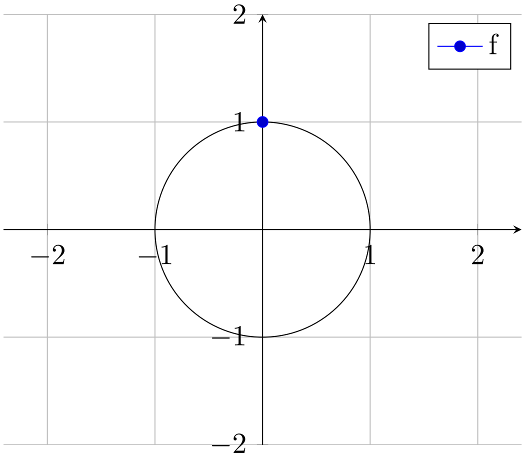
domain: set of all possible input
range: set of all possible output

<u>Defn</u>: A function F between A and B
is a relation between A and B such that:
subset of A $\times$ B

1. For every x $\in$ A there is an element y $\in$ B such that (x,y) $\in$ F
   Which means
   Fo every input x, there is some ouput y, F(x)=y

2. If (x,y) $\in$ F and (x,z) $\in$ F then y = z

Example: Is this relation?
Consider the relation C where (x,y) $\in$ C if $x^2 + y^2 = 1$. Is this a function?



This is relation but not a function because there is more than one output asociated with one input

9 Statement

A statement is a sentence that is either true or false

Examples:
p: "5 > 2" = True
q: "2 > 5" = False
r: "x > 2" = Not a statement because we don't know what x is

9.1 New statement from old

$\neg$ p means not p
p $\wedge$ q means p and q
p $\vee$ q means p or q

Example:
"My shirt is gray but my shorts are not"
p = My shirt is gray
q = My shorts are gray
p$\wedge$ $\neg$ q

9.2 Truth Table for (¬ p) ∨ (¬ q)

p	q	¬ p	¬ q	
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Def: Two statements are logically equivalent if they have the same truth table

p	¬ p	¬ (¬ p)
T	F	T
F	T	F

Def: A tautology t is a statement that is always true
Example:
That dog is a mammal

t: tautology
p: some statement

t	p	t ∨ p
T	T	T
T	F	T

Def: A contrandiction c is a statement that is always false
Example:
That dog is a reptile

c: contrandiction
p: some statement

c	p	c ∧ p
F	T	F
F	F	F

so c ∧ p is a contrandiction

10 Demorgan's Law & Logical Equivalent

p: Trefor is a unicorn
q: Trefor is a goldfish
$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$?
It's NOT the case that Trefor is either a unicorn OR a goldfish.
is equivalent to:
Trefor is NOT a unicorn AND is NOT a goldfish.

p	q	$\neg$ p	$\neg$ q	p $\vee$ q	$\neg$ (p $\wedge$ q)	($\neg$ p) $\wedge$ ($\neg$ q)
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Demorgan's Laws:
$\neg (p \vee q) \equiv (\neg p) \wedge (\neg q)$
$\neg (p \wedge q) \equiv (\neg p) \vee (\neg q)$

Double Negative:
$\neg(\neg p) \equiv p$

Identity Laws:
$p \vee c \equiv p$
$p \wedge t \equiv p$

Universal Bound Laws:
$p \vee t \equiv t$
$p \wedge c \equiv c$

Example:
$(\neg(p \vee \neg q)) \wedge t$
via DeMorgan's
$\equiv (\neg p \wedge \neg (\neg q)) \wedge t$
via Double Negative
$\equiv (\neg p \wedge q) \wedge t$
Via Identity
$\equiv \neg p \wedge q$

So $(\neg(p \vee \neg q)) \wedge t \equiv \neg p \wedge q$

11 Conditional Statement

Def: $p \Rightarrow q$ means:
"if p is TRUE then q is TRUE"
Whenever the hipotesis p is true then the conclution is also true.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$\neg$ p	$\neg$ p $\lor$ q
F	T
F	F
T	T
T	T

$p \Rightarrow q \equiv \neg p \lor q$

Example:
If i study hard, then i will pass
p = i study hard
q = i will pass
$p \Rightarrow q$

Either I don't study hard, or i pass
$\neg$ p = I don't study hard
q = i pass

11.1 When the hypothesis is false, the statement is vacuously true.

vacuously true meant the statement is true but true in a sort of unimportant or unintresting or vacuous set.

Example:
If Trefor is a unicorn, then everyone get's an A
p = Trefor is a unicorn
q = everyone get's an A
$p \Rightarrow q$

Example:
Either Trefor is not a unicorn, or everyone get's an A
$\neg$ p = Trefor is not a unicorn
q = everyone get's an A
$\neg p \lor q$

11.2 Negating a conditional

$\neg$ (p $\Rightarrow$ q) $\equiv$ $\neg$ ($\neg$ p $\lor$ q)
$\equiv$ ($\neg$ $\neg$ p $\land$ $\neg$ q) Demorgan's law
$\equiv$ p $\land$ $\neg$ q

11.3 Contrapositive of a conditional:

p $\Rightarrow$ q $\equiv$ $\neg$ q $\Rightarrow$ $\neg$ p
p $\Rightarrow$ q $\equiv$ $\neg$ p $\lor$ q
$\neg$ q $\Rightarrow$ $\neg$ p $\equiv$ q $\lor$ $\neg$ p

If i study hard, then i will pass p $\Rightarrow$ q
Either I don't study hard, or i pass $\neg$ p $\lor$
If i don't pass, then i didn't study hard $\Rightarrow$ $\neg$ p
Either i pass, or I didn't study hard q $\lor$ $\neg$ p

11.4 The converse and the inverse of a statement

11.4.1 The converse statement

p $\Rightarrow$ q is the statement q $\Rightarrow$ p
The converse statement is not logically equivalent

Example:
Not Logically equivalent
p $\Rightarrow$ q = If it's a dog, then it's a mammal = True
q $\Rightarrow$ p = If it's a mammal, then it's a dog = False

11.4.2 The inverse statement

p $\Rightarrow$ q is the statement $\neg$ p $\Rightarrow$ $\neg$ q
The inverse statements is not logically equivalent

So inverse $\equiv$ converse

Example
Not Logically equivalent
p $\Rightarrow$ q = If it's a dog, then it's a mammal = True
$\neg$ p $\Rightarrow$ $\neg$ q = If it's not a dog, then it's not a mammal = False

11.5 Biconditional statement

The Biconditional p $\iff q$
$meansthatbothp \Rightarrow$ q and q $\Rightarrow$ p

Example
If i study hard, then I will pass = p $\Rightarrow$ q
AND if I pass, then I studied hard = q $\Rightarrow$ p

i will pass **if and only if** i study hard
if and only if = $\Longleftrightarrow$

11.6 Valid and Invalid Arguments

A valid argument is a list of premises from which the conclusion follows.

Example Argument:
If I do the dishes, then my wife will be happy with me.
I do the dishes.
Therefore, my wife is happy with me.

If **p**, then **q**.
p.
Therefore, **q**.

11.6.1 Modus Ponens

Modus Ponens is an argument of the form:
premise1 = If **p**, then **q**.
premise2 = **p**.
conclusion = Therefore, **q**.

Variables

p	q
T	T
T	F
F	F
F	T

Premises

p $\Rightarrow$ q	p
T	T
F	T
T	F
T	F

Conclusion

q
T
X
X
X

11.6.2 Modus Tollens

Modus Tollens is an argument of the form:
if **p**, then **q**.
$\neg$ q.
Therefore, $\neg$ p.

example argument:
If i'm POTUS then i'm an American citizen.
I'm not an American citizen.
Therefore, I'm not the POTUS.

p = i'm POTUS
q = i'm an American citizen.
$\neg$ q = I'm not an American citizen.
$\neg$ p = I'm not the POTUS.

11.6.3 Generalization

Generalization is an argument of the form:
p.
Therefore, p $\vee$ q.

Example:
i'm a canadian
Therefore, I'm a canadian or i'm a unicorn.

11.6.4 Specialization

Specialization is an argument of the form:
p $\wedge$ q.
Therefore, p.

Example:
I'm a canadian and I have a PhD
Therefore, I'm a Canadian.

11.6.5 Contradiction

Contradiction is an argument of the form:
$\neg$ p $\Rightarrow$ c
Therefore, p.

Example:
If i'm skilled at poker, then I will win.
I won money playing poker.
Therefore, I'm skilled at poker.

p = i'm skilled at poker
q = I will win.
q = I won money playing poker.
p = I'm skilled at poker.

p $\Rightarrow$ q
q
p

This argument is invalid argument because it's use converse statement.
We know converse statement is not logically equivalent.

11.7 Predicates and Quantified Statements

Recall: A statement is either TRUE or FALSE

11.7.1 Predicate

a **predicate** is a sentence depending on variables which becomes a sttemnt upon substituting values in the domain.

Example:
P(x): x is a factor of 12 with domain Z^+