1) $\binom{6}{5}$ for subsets from set $\{m, e, t, i, n, g\}$

a) + sets with two e's $= \binom{5}{4} \Rightarrow \binom{6}{5} + \binom{5}{4} = 6 + 5 = \boxed{11}$

b) 785 - strings when e's swap places $\Rightarrow 785 - 10(5^3) = 125$

$= \boxed{1270}$

2) 52 cards
4 of each kind ; 13 kinds
two pairs for each kind of card $\Rightarrow 13 \times 2 = 26$ pairs

a) $\binom{13}{2}\binom{4}{2}\binom{12}{}\binom{4}{2}(11)(4) = \boxed{247{,}104}$

b) $(6)(2)(5)(2)(11)(2) = \boxed{2{,}640}$

3) Best player on red team
2nd player on blue team $\Rightarrow \binom{11}{6}$

plus swap the player with any
1st other teams players $\Rightarrow (6)(5) = \binom{11}{5} + 35 = 492(2)$ for swap of best players

$\boxed{984}$

4) $a + b + c + d + e + f + 1 = 16$
$a + b + c + d + e + f = 16$

* | *** | ***** | ** | * * * * | 20 spots, 5 couples

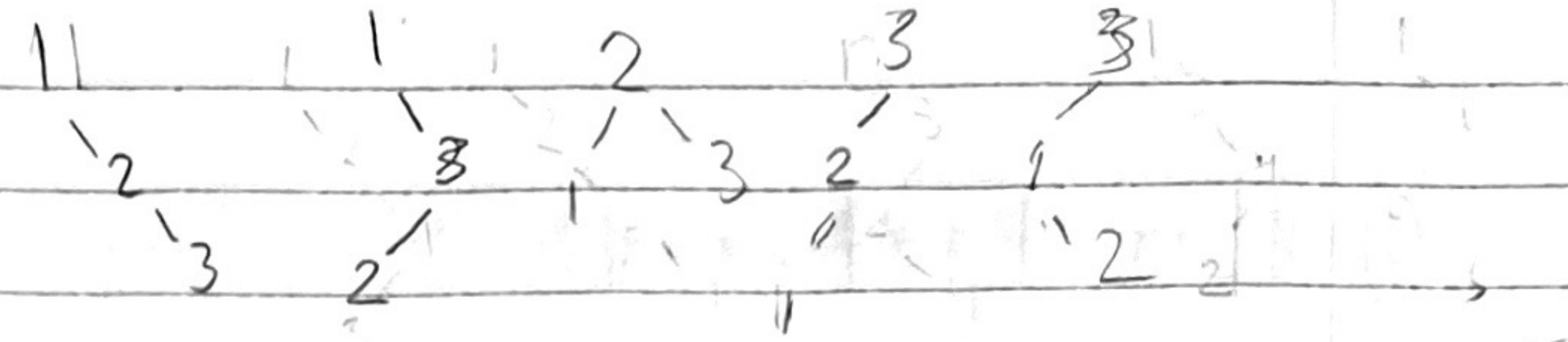
$\binom{20}{5} = 15{,}504\ (16)$

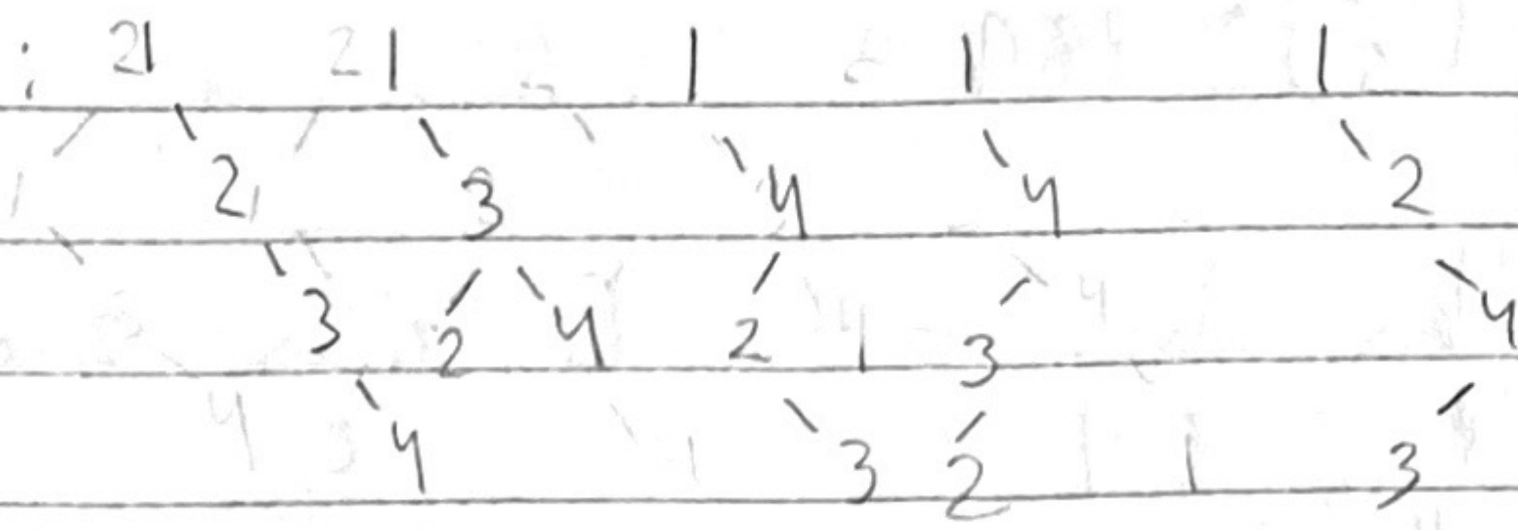
for 16 ways the first couple hears 1 song $= 248{,}064$

+ if couple hears no songs, so one more to end group
$248{,}064 + (15{,}504)(5) = \boxed{325{,}584}$

1234
1243
1324
1342
1423
1432

5) 2nodes = 2
3nodes = 5
4nodes = 14

n=3


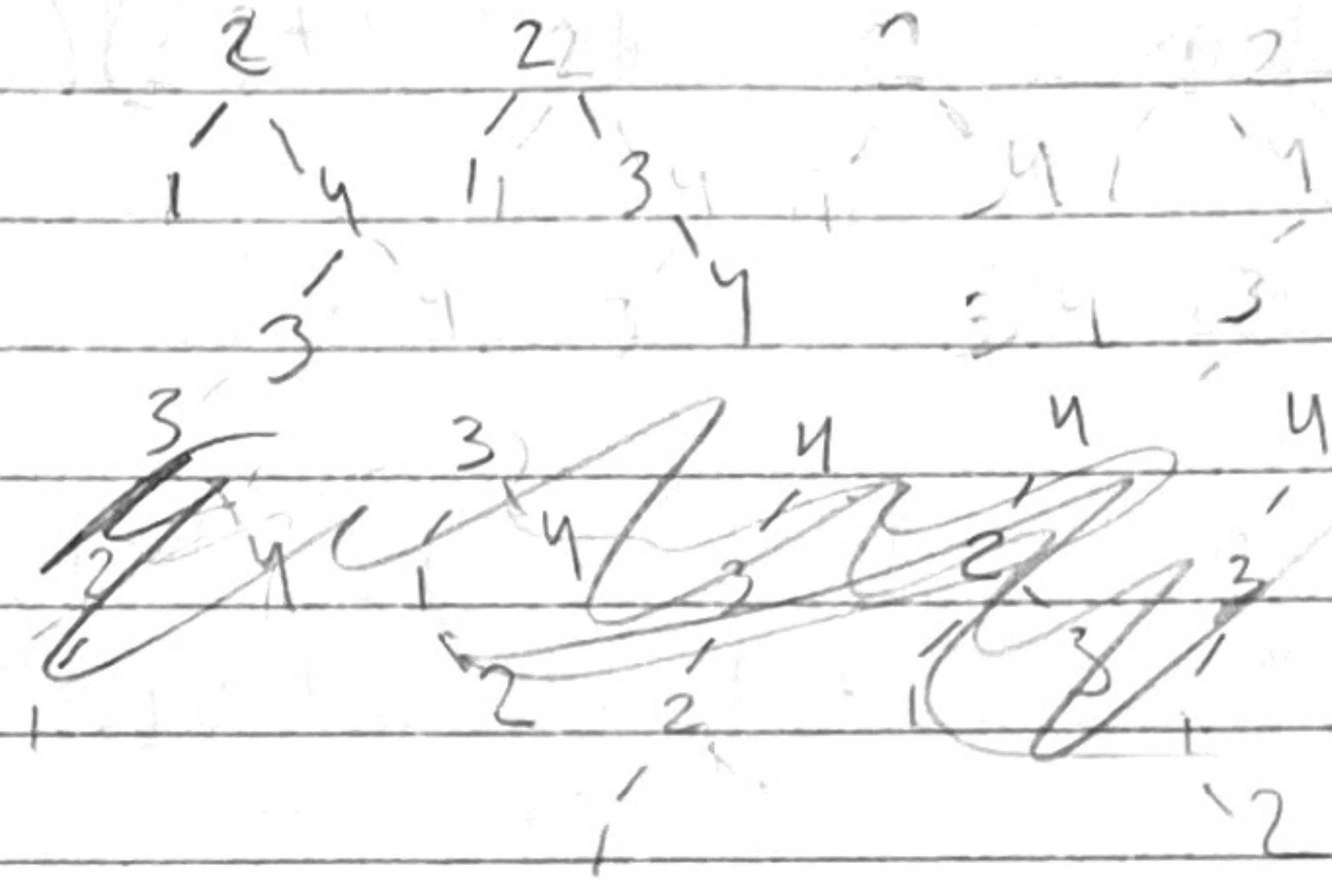
n=4


2n - 3

2143
2134
2314
2341
2413
2431

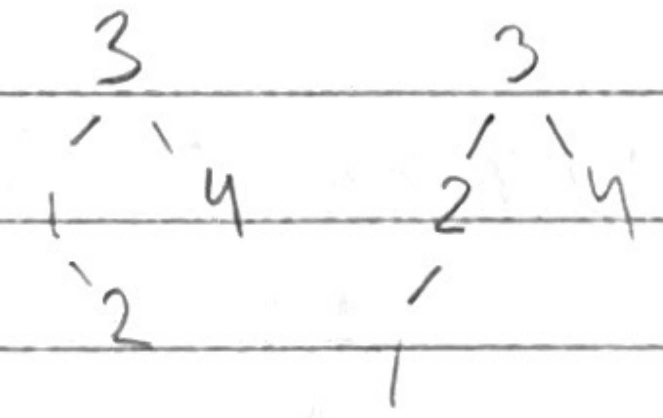
✓2

-6

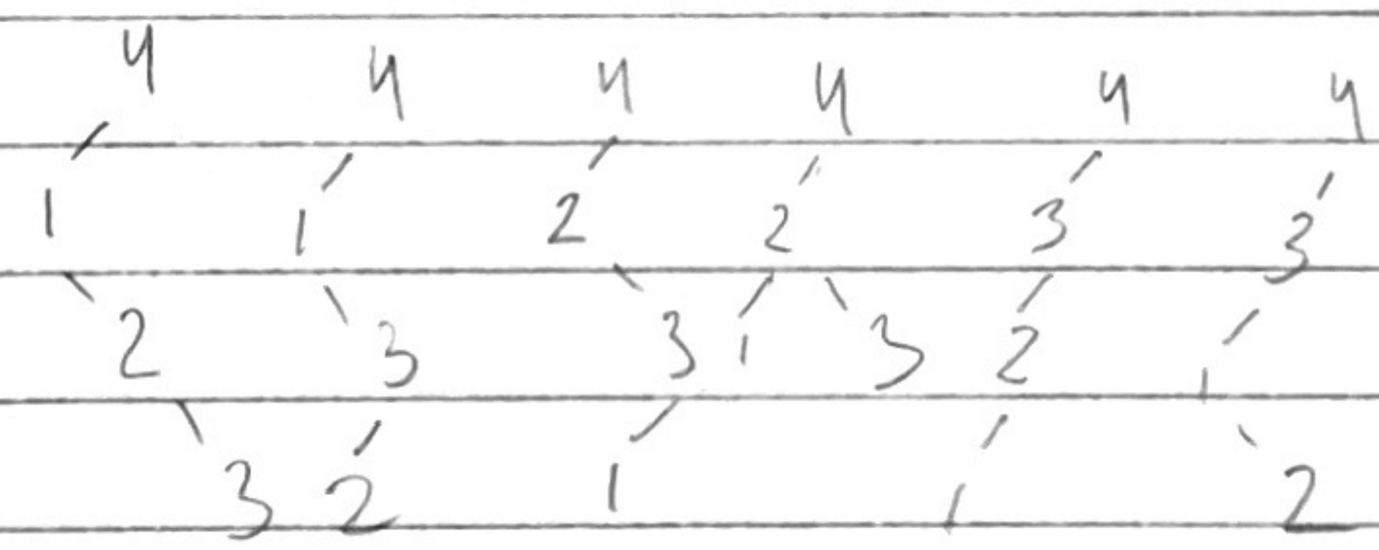


2n - 6

3124
3142
3214
3241
3412
3421



2n-6



2n - 2

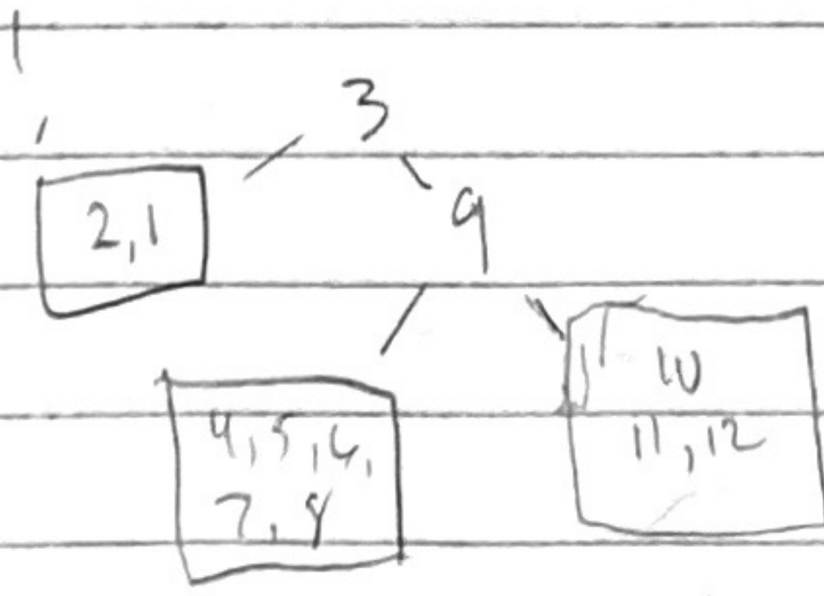
4123
4132
4213
4231
4312
4321

8n - 18 = 14

→ (2n)! , so full set => n=12 => 208,012

(n+1)! n!

n=2 (2) => $\boxed{2,1}$



= 3nodes = 5
n=3

n=5
= $\boxed{42}$

2(42)(?) => $\boxed{420}$

6) $* | * * * | * * | * * * *$

$\binom{13}{3} = 286$ - case where any nurse serves 0 people

Ways where one nurse serves 2 or patients:

4^{10} (no restrictions) $- 4(3^{10}) + \binom{4}{2}(2^{10}) + \binom{4}{1} = 1{,}28{,}168{,}746$

Ways one nurse is on a break:

$4(3^{10}-9) + 4(3^{10} - \binom{3}{1}(2^{10}) + 2(1^{10}) - 6)$
$+ 4(3^{10} - \binom{3}{}2^{10} + 1^{10} - 4) + 4(3^{10} - 2^{10} - 2) = 67107840$

$128{,}168{,}746 + 67107840 = \boxed{34{,}651{,}802}$

1). Probability that a student will get called more than once 13 questions:

$$1 - [P[X=1] + P[X=0]]$$

$$1 - \left[\binom{13}{0}(1/21)^0(20/21)^{13} + \binom{13}{1}(1/21)^1(20/21)^{12}\right] = 1 - [.53 + .344]$$

$= .126 \Rightarrow P[X=0]\ \binom{13}{0}(.126)^0(1-.126)^{13} = .174$

2) # of nums = 100,000

# of odd nums = 50,000

# of 2 even digits start = $(5)(4)(8 P 3)(1/2) = 3360$

$P[x_i] = 3360/100,000 = .0336$

$$\boxed{P[X=7] = \binom{10}{7}(.0336)^7(.9664)^3 = .5 \times 10^8}$$

3) $P[A] = 1 - [P[X=0] + P[X=1]] = 1 - [(.5)^3 + (.5)^2] = .75$

$P[B] = (1)(1/6)(1/6) = .028$

$P[A \cap B]$

$|S| = 6^3 = 216$ $\quad 6/216 = .278$

$|E| = 6$ (1 triplet for each #)

Since $P[A \cap B] \neq P[A]P[B]$, they $\boxed{\text{are not}}$ independent events.

4) All possible hands $= \binom{52}{5} = 2,598,960$

$$P[\text{Straight}] = \frac{10\binom{4}{1}^5 - 40}{\binom{52}{5}} = .0039$$

use geometric distribution $\Rightarrow$ $P[X=i] = p(1-p)^{i-1}$

$$= p\sum_{n=1}^{\infty}(1-p)^{n-1} = \frac{1}{p} \Rightarrow \frac{1}{.0039} = \boxed{256.4}$$

5) $P[W|SS] = .75$

$P[W|SS^c] = .4$

$P[SS] = .65$

$P[W_3|SS^c] \Rightarrow P[X=3] = \binom{5}{3}(.4)^3(.6)^2 = .23$

$P[W_3|SS] = P[X=3] = \binom{5}{3}(.75)^3(.25)^2 = .264$

$$P[SS|W_3] = \frac{P[W_3|SS]\cdot P[SS]}{P[W_3]} \Rightarrow P[W_3] = P[W_3|SS]\cdot P[SS] + P[W_3|SS^c]\cdot P[SS^c]$$

$= .264(.65) + .23(.35)$

$= .2521$

$$\Rightarrow \frac{.264(.65)}{.2521} = \boxed{68\%}$$