Proyecto Final para la obtención del título: Ingeniero en Informática

Future Virtual Particle Method for Pedestrian Navigation

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Abstract

This paper presents an avoidance collision method based in pedestrian self governed decisions, by calculating the position of every pedestrian in the future, a given pedestrian can adjust his velocity vector to avoid collisions instead of being affected by a repulsive force.
The model was tested using three different scenarios and compared against the Social Force Model obtaining more natural navigation, reduced number of collisions, increased average speed and reduced average deviation angle of navigating pedestrians. **keywords:** pedestrian, collision avoidance, future virtual particle, force model.

1 Introduction

1.1 Motivation and previous work

Navigation of biological, synthetic or virtual agents is a relevant problem in several fields such as pedestrian dynamics, moving robots and animation of characters for video games and motion pictures.

Modelling and simulating the displacement of agents through arbitrarily complex environments may be stated in an hierarchical structure of mechanisms depending mainly on the distance from the agent. This level has been named, from closer to further, as operational (walking, lowest level physical-computational model for displacement), tactical (way-finding, route choice) and strategic (general activity planning) [1]. These levels are not independent, factors affecting one level may impact in the following and vice-versa, for example, the route choice may vary due to congestion of agents produced from previous route choice and walking behavior. Also, obstacles can impact on the operational level or tactical level depending on the particular geometry of the environment. The particular mechanism we want to address is the avoidance of obstacles being fixed or moving (another agent) which involves operational and tactical aspects of the navigation.

A general approach is to take an existing operational model and equip it with a higher level model which allows better and smoother collision avoidance behavior. Existing low level models can be taken from pedestrian dynamics field and in general this models can be classified into rule based and force based, discrete and continuous space description, etc. [2].

A famous example of continuous and force based model is the Social Force Model [4, 5]. In this model the dynamic for virtual pedestrians is derived from the Newton equation's considering the total force exerted over each agent is the result of three forces: Contact, Social and Driving Force. While the driving force points towards the final objective of each pedestrian, the social force is repulsive and acts as a kind of collision avoidance force. However this social force term introduces several artifices in some configurations. See for example Lakoba [6], Parisi [7].

Cellular automaton models make use of a spatial grid, which can be occupied or empty, along with a set of rules determining the evolution and conflict resolution of virtual pedestrians moving over the cells of the grid. An emblematic cellular automaton model is the one proposed by Kirchner and Schadschneider [8].

Hybrid models have also been proposed such as the Contractile Particle Model [9] in which a continuous description of the space is combined with a set of simple rules governing the dynamics of the system.

The basic operational model -as the ones described above- can be improved if higher level mechanisms were added to manage more complex issues as efficient avoidance. Some recent examples can be found in the literature.

Karamouzas [10] proposed a method for collision avoidance modifying the social force model, basically, replacing the social force term by a new "evasive" force which tends to avoid future collisions. The magnitude and direction of this force is calculated considering the predictions of these possible collisions.

Kretz [11] have arrised the point that the key ingredient in social force model is the driving force instead of interaction force, so in this work the authors propose a method for dynamically adjusting the desired velocity following the gradient of a field given by a time map, in other words, the desired velocity is chosen as the quickest path to the objective taking into account the geometry and other agents (collision, congestion, jams, etc.). Also mounted on the SFM, Moussaad [12] presented a model using "cognitive heuristics" to determine the norm and direction of the desired velocity for each agent dynamically during the evolution of the system.

This paper proposes that the navigation capacity of virtual agents is concentrated in the pedestrian's decision of his desired velocity, its calculation is the key difference with the SFM.

The method proposed could be mounted on different basic displacement models like the SFM or the CPM, in the present work we have chosen the first one.

1.2 Social Force Model

The Social Force is a model presented by Helbing [4, 5] in several publications. This paper will focus on the latest version of the model [5]. In this model, each pedestrian i occupies a circular area of radius r_i and is governed by thee forces.

$$\vec{F_i} = \vec{F_{D_i}} + \vec{F_{S_i}} + \vec{F_{G_i}}$$

This forces are a measure for the internal motivations of the individual to perform certain actions.

The first term is known as the "Driving Force". It's calculated as follows:

$$\vec{F_D}(i) = m_i \frac{v_{di}\vec{e_i} - \vec{v_i}}{\tau} \tag{1}$$

$$\vec{e_i} = \frac{\vec{x}_{i0} - \vec{x}_i(t)}{||\vec{x}_{i0} - \vec{x}_i(t)||} \tag{2}$$

$\vec{F}_{D_i}$ represents the force that a pedestrian i keeps towards his desired velocity of motion.

v_{di} is the desired speed for the pedestrian i.

$\vec{e}_i$ is the desired direction of motion of the pedestrian i.

$\vec{v}_i$ is the current velocity of the pedestrian i.

$\vec{x}_i(t)$ is the actual position of the pedestrian i at the time t.

x_{i0} is the closest point from the goal (represented as an area) to pedestrian i.

Fig. 1 shows a pedestrian moving with $\vec{v}$ velocity but adjusting its trajectory towards X.

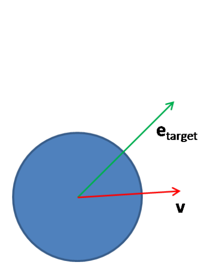

Figure 1 Driving force

The second term is known as the "Social force". It's calculated as follows:

$$\vec{F}_{S_i} = \sum_{j=1, j\neq i}^{N_P} Aexp(-\frac{\epsilon_{ij}}{B})\, \vec{e}^{\,n}_{ij} \tag{3}$$

$\vec{F}_{S_i}$ represents the fact that a pedestrian keeps a certain distance to other pedestrians and borders.

N_p is the number of existing pedestrians.

A and B are constants determined by simulations.

ϵ_{ij} is the distance from x_i towards x_j.

$\vec{e}^{\,n}_{ij}$ is the unit vector from x_i towards x_j.

Fig. 2 shows equation 3 graphically.

The third term is known as the "Contact force". It's calculated as follows:

$$\vec{F}_{G_i} = \sum_{j=1, j\neq i}^{N_P} [-\epsilon_{ij} k_n \vec{e}^{\,n}_{ij} + v^t_{ij} \epsilon_{ij} k_t \vec{e}^{\,t}_{ij}]\, g(\epsilon_{ij}) \tag{4}$$

$\vec{F}_{G_i}$ represents the physical force that a pedestrian suffers when colliding with another object (pedestrian or wall).

k_n and k_t are the normal and tangential friction coeficient respectively.

g is 0 if $\epsilon_{ij} \leq 0$ or ϵ_{ij} otherwise

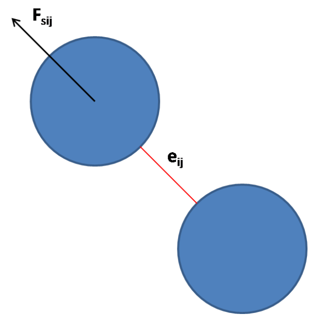

Figure 2 Social Force

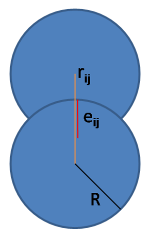

Figure 3 Colliding pedestrians

Fig. 3 shows 4 graphically.

Afterwards $\vec{F_i}$ is calculated on each simulation step for each of the pedestrians and applied until all of them had reached their goal.

Fixed parameter values:

Name	Value
A	$2000\,[N]$
B	$0.08\,[m]$
k_n	$1.2\,10^5\,[\frac{N}{m}]$
k_t	$2.4\,10^5\,[\frac{kg}{m/s}]$
τ	$0.5\,[s]$

1.3 Future Virtual Particle Model

Given that the SFM adds a fictional force on pedestrians, navigation is unnatural and doesn't resemble reality. SFM isn't validated using well known metrics for real-case scenarios such as the flow of pedestrians going out a door and the fundamental diagram [7].

In the FVPM, the social force in equation 1 is eliminated and replaced with a dynamic driving force towards a short term objective, which is equal to equation 2 replacing $\vec{x}_i^0$ with $\vec{x_{i'}}$, where $\vec{x_{i'}}$ is an estimation of the location of pedestrian i in the near future. i' will be called Future Virtual Particle.

The presented work demanded a deep understanding of the problem in question, for which a great study of related work was done. After that, we proposed various different models which were validated and reformed until it converged to this model, which validate against real world scenarios and metrics.

2 The Model

2.1 Hipotesis

The basic concepts below the model of pedestrian movement are the same as Helbin's [4, 5]:

1. The pedestrian wants to reach his goal in the shortest possible path.
2. The pedestrian's movement is influenced by other pedestrians. Depending on the distance between the two of them and the predicted trajectory, the pedestrian needs to change his route to be able to avoid obstacles.
3. Movement speed will be influenced by the presence of other pedestrians and obstacles.

2.2 Geometrical definition

A pedestrian is defined as follows:

- Circular shape

  Represents the personal space of a pedestrian. The value of the radio is generated randomly for each pedestrian. The range of values is distributed uniformly in $[0.25, 0.29]$ $[cm]$ between pedestrians.

- Long term target

  Represented by a static area. It is considered as accomplished when the pedestrian touches this area. Multiple objectives could be defined in a list, in this case, each of them must be reached in order.

- Short term target

  Called future virtual particle (FVP), it represents a point at a relative distance from the pedestrian's center. It's a dynamic objective.

  It is defined as a $1\,[kg]$ mass. Not collisionable.

- Desired speed

  The speed the pedestrian would walk if he/she was alone. This represents the constant v_{di} in ecuation 1. It takes a different value for each pedestrian. Varies with a uniform distribution between $[1.2, 1.4]\,[m/s]$.

- Reaction distance (RD)

  Maximum distance between a pedestrian and his FVP, it represents the distance at which a real pedestrian would react from an obstacle.

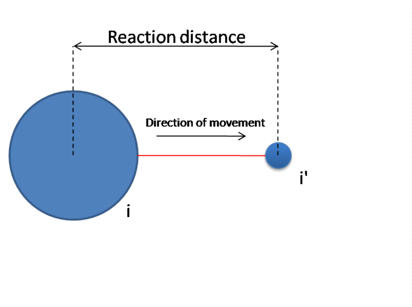

Figure 4 Pedestrian i and its FVP (i').

The naming convention used for all vectors related to a pedestrian is shown in fig. 5.

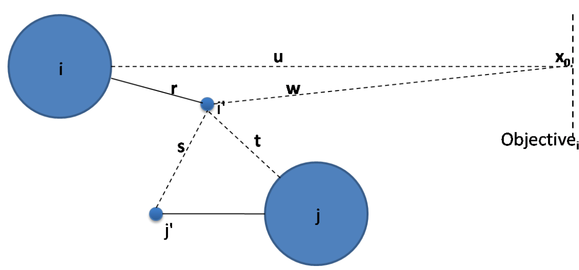

Figure 5 Vectors definitions

2.3 Dynamics

2.3.1 Force calculation

- Dynamic of the FVP

  Each pedestrian has to reach the long term objective at some point, to ensure this, the FVP needs to be aligned with the shortest path to the long term objective $\mathbf{x_o}$. On the other hand, there are sometimes obstacles in the way, which will make this impossible, in this cases, the route will have to change depending on the situation.

  To model this situations, two types of forces act over the FVP:

- Internal force

  For any pedestrian i, its internal force is

$$F_{i'}^{internal} = F_{i'}^{S1} + F_{i'}^{S2} \tag{5}$$

where

$$F_{i'}^{S1} = K_1(\theta_i) * (|\vec{r}| - RD)\hat{r}$$
$$F_{i'}^{S2} = K_2 * (\vec{x_{i'}} - (\vec{x_i} + \hat{x_{io}}RD))$$

This force aligns the future on the path $\mathbf{x_{io}}$ and is modeled using two springs S_1 and S_2:

S_1 starts on $\mathbf{x_i}$, ends on $\mathbf{x_{i'}}$ and has a stationary distance of RD with a spring constant $K_1(\theta)$ where θ is the angle between $\mathbf{u}$ and $\mathbf{r}$. With this spring the FVP will tend to always be at RD distance from the pedestrian. Because a pedestrian always tries to reach his goal in the shortest possible path (hypothesis 1), if it has to take a big detour of his ideal path, it will try to reduce his velocity drastically in order to avoid making a long travel. To recreate this, the spring constant has to be dependant of the deviation angle.

$$K_1(\theta) = \frac{K_{1c}}{\theta}$$

where K_{1c} is a constant value.
S_2 starts on $\mathbf{x_{i'}}$ and ends on $\mathbf{x_i} + RD\mathbf{\hat{x}_{io}}$ with a spring constant K_{2c}. Setting a greater K_{2c} spring constant will force a straighter path but may result in more collisions. In order to avoid an oscillatory movement, a damping γ is added to the spring S_2.
Fig. 6 shows the internal forces that a FVP i' suffers because it has to be aligned with the pedestrian's position $\mathbf{x_i}$ and the long term objective $\mathbf{x_o}$ but also at distance RD from $\mathbf{x_i}$.

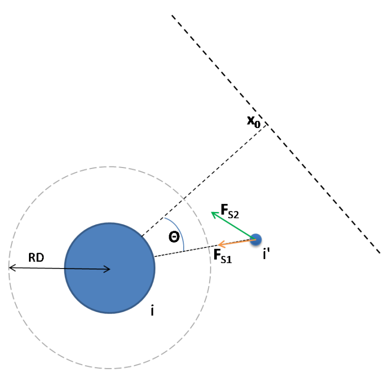

Figure 6 Internal forces

– External forces

This force will produce avoidance movements when obstacles are detected on the path. The equation for the external force that affects the FVP i' is

$$F_{i'}^{external} = FF_{i'} + FP_{i'} + FW_{i'} \tag{6}$$

where

$$FF_{i'} = \sum_{j}^{N} (\alpha_{ff} e^{-s/\beta_{ff}})$$

$$FP_{i'} = \sum_{j}^{N} (\alpha_{fp} e^{-t/\beta_{fp}})$$

$$FW_{i'} = \sum_{k}^{walls} (\alpha_{fw} e^{-\zeta_k/\beta_{fw}})$$

having $\zeta_k = |x_{i'} - x_k|$; x_k being the closest point from wall k to $x_{i'}$. α and β are constants.

The term $FF_{i'}$ in equation 6 acts as a repulsion force between i' and j', resulting in the avoidance of a future collision. The term $FP_{i'}$ in equation 6 uses this same principle but calculates the repulsion force between i' and j. The term $FW_{i'}$ in equation 6 adds the repulsion against walls, using the closest point between i' and the wall.

This force is only applied to each the pedestrian j ($j \neq i$) who is in the range of sight of pedestrian i. The restriction is verified using the following condition:

$$\mathbf{r_{ii'}} \bullet \mathbf{r_{ij}} > 0$$

The condition represents the fact that pedestrians make decisions based only on obstacles in his range of vision.

Fig. 7 shows the external forces that a FVP i' suffers because of another pedestrian (j) and also the direction in which i desires to move.

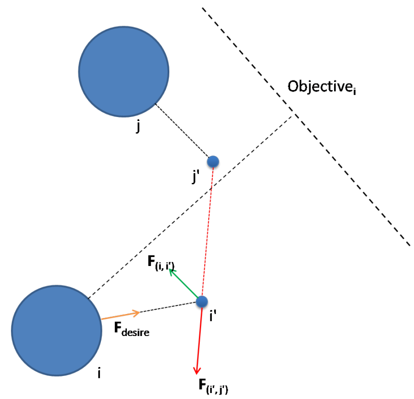

Figure 7 External forces

After this, $F_{i'}$ is computed by adding all the terms.

$$F_{i'} = F_{i'}^{external} + F_{i'}^{internal} \tag{7}$$

To avoid high symmetry situations, a small noise is added to F. This noise is calculated by taking a random value s with a random distribution from $\{-1, 1\}$ and applying:

$$M = \begin{pmatrix} 0 & -s \\ s & 0 \end{pmatrix}$$

in

$$F'_{i'} = F_{i'} + F_{i'} * M$$

Finally, movement equations are applied using $F'_{i'}$.

- Dynamic of the pedestrian

  The driving force $\vec{F}_{d_i}$ makes the pedestrian tend to move towards his FVP at velocity v_{di}

  $$\vec{F}_{d_i} = m_i \frac{|v_{di}| \frac{\vec{r}}{RD} - \vec{v}_i}{\tau}$$

  where $\tau = 0.5$

It is important to note that the only deviation from the SFM in this equation is the $k\vec{e}_i$ term. As described above, this model focuses on improving the SFM through dynamically adjusting the desired velocity.

2.3.2 Algorithm

The pedestrian movement is calculated in four steps:

1. Calculate forces for each FVP.
2. Calculate forces for each pedestrian.
3. Update positions for each FVP.
4. Update positions for each pedestrian.

3 Calibration

3.1 Scenarios

The test scenarios where crossing and hallway for they present the main types of symmetry (90 degrees and 180 degrees respectively).

Hallway scenario: A 15[m] by 4[m] hallway. Pedestrians are generated from each end at a pace of 1 pedestrian per second with the other end as target.
This scenario is shown in fig. 8.

Figure 8 Hallway Scenario

Crossing scenario: Two hallways of 25[m] length by 5[m] width put together in the center forming a cross. Pedestrians are generated from the top, targeting the bottom, and from the right end, targeting the left end, at a pace of 1 pedestrian per second.
This scenario is shown in fig. 9.

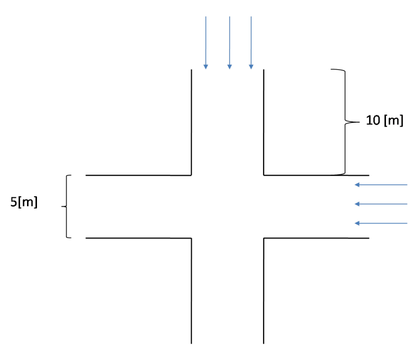

Figure 9 Cross Scenario

Room evacuation: A $20x20[m]$ room with a single exit door centered at the bottom. At the beginning, a fixed number of pedestrians are distributed across the room. Each of them targeting the exit door. When the first pedestrian exits the room, a counter starts, and when the last one crosses, it stops. This way we can measure the flow of pedestrians leaving a room, which is known to be between 1 and 4 [p/m/s] for pedestrians without emergencies [7].
The door size will be tested using the following measures: $1.2[m]$, $1.5[m]$ and $1.8[m]$.
This scenario is shown in fig. 10.

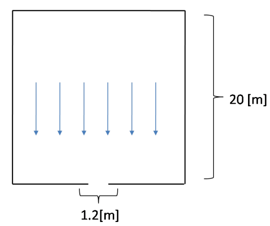

Figure 10 Room Scenario

3.2 Metrics

The metrics used to validate and verify the model against the hallway and crossing scenarios are defined as follows:

1. Number of collisions:

$$\sum_t \sum_i \sum_j collides(i,j)/2$$

where

$$collides(i,j,t) = touching(i,j,t)(1 - touching(i,j,t-1))$$

$$touching(i,j,t) = \begin{cases} 0 & \text{if } t <= 0 \\ 1 & \text{if } x_i - x_j < R_i + R_j \\ 0 & \text{if not} \end{cases}$$

2. Amount of collisions per instant: The equation for this metric is defined as:

$$\sum_t \sum_i \sum_j touching(i,j)/2$$

3. Average walking speed:

$$\sum_{t=0}^{T}((\sum_{i=0}^{N} v_i)/N)/T$$

4. Average travel time: The average time that a pedestrian takes to reach the goal. This is calculated only for pedestrians who reach the goal before the simulation ends.

5. Average travel distance: The average distance that pedestrians traveled until it reached the goal. This is calculated only for pedestrians who reach the goal before the simulation ends.

6. Average turn angle: The average angle turned by a pedestrian until it reached the goal. This is calculated only for pedestrians who reach the goal before the simulation ends.
   where the turn angle is:

$$\arccos(v_{t_n} \bullet v_{t_{n-1}} / |v_{t_n} \bullet v_{t_{n-1}}|)$$

The metric used for the room evacuation scenario is:

1. Escape flow: The amount of pedestrians coming out of the room by second.

$$\frac{N}{\Delta T}$$

   where ΔT is the time when the last pedestrian exits minus the time when the first pedestrian exits.

All results were compared to the SFM model [5].

3.3 Values

To calibrate the model, runs with varying parameters were made.

All runs used Euler method with a step of $\frac{1}{1000}$ $[s]$. For each scenario and parameter combination, 10 runs were made and their metric results averaged and compared against each set of parameters.
After numerous iterations of this process, this are the parameters which best fit all scenarios:

- Pedestrian
  - $K_1 = 80$ [N/m],
  - $\gamma = 10$

- $K_2 = 100$ [N/m]

- FVP-FVP interaction
  - $\alpha = 1000$
  - $\beta = [0.4, 0.6]$ (Uniform distribution)
- Pedestrian-FVP interaction
  - $\alpha = 1000$
  - $\beta = 0.1$
- FVP-Wall interaction
  - $\alpha = 10000$
  - $\beta = 0.1$

4 Results

All results for each metric for the crossing scenario are presented on table 1:

Metric	FVPM	SFM ($\alpha = 2000$, $\beta = 0.08$)
1	15.0 ± 4.3	23.2 ± 6.4
2	158.2 ± 60.6	57.2 ± 18.8
3	1.26 ± 0.02 $[m/s]$	1.28 ± 0.01 $[m/s]$
4	17.61 ± 0.27 $[s]$	17.33 ± 0.06 $[s]$
5	22.61 ± 0.07 $[m]$	22.52 ± 0.07 $[m]$
6	103.35 ± 32.81 $[rad]$	193.06 ± 20.44 $[rad]$

Table 1 Results for crossing scenario

Metrics for the crossing scenario don't present noticeable improvements over the SFM. The one improvement is the average turn angle, which can be recognized by looking and comparing the two models. The SFM presents a much more unnatural navigation than the FVPM.

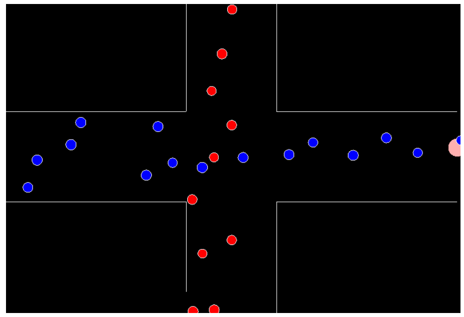

Figure 11 Crossing visualization

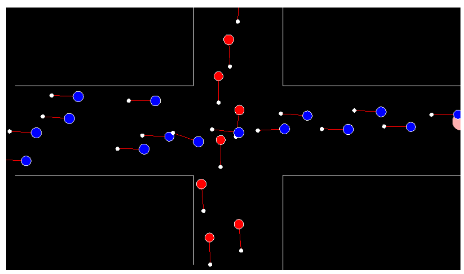

Figure 12 Crossing visualization with visible future

All results for each metric for the hallway scenario are presented on table 2:

Metric	FVPM	SFM ($\alpha = 2000$, $\beta = 0.08$)
1	7.8 ± 5.3	20.8 ± 8.5
2	53.8 ± 29.5	85.2 ± 32.3
3	1.27 ± 0.01 $[m/s]$	1.04 ± 0.08 $[m/s]$
4	13.15 ± 0.06 $[s]$	18.17 ± 2.12 $[s]$
5	17.13 ± 0.05 $[m]$	19.16 ± 0.83 $[m]$
6	109.88 ± 6.22 $[rad]$	1431.94 ± 436.44 $[rad]$

Table 2 Results for hallway scenario

In the hallway scenario, great improvements in behaviour and metrics were achieved. Every metric was beaten by the new model with considerable changes. The FVPM removes the bouncing of pedestrians which is typical of the SFM in this scenario and gives way to the forming of natural pathways for pedestrians.

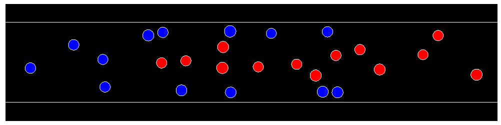

Figure 13 Hallway visualization

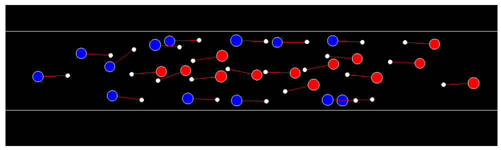

Figure 14 Hallway visualization with visible future

The number of collisions is decreased compared to the SFM. The fact that the time of collision (metric #2) is bigger than in the SFM, resembles reality, pedestrians don't shoot out at great velocities when colliding. The average travel distance is similar in the crossing scenario but much smaller in the hallway, this shows the problem of SFM in high symmetry scenarios (inverse velocities).
Each pedestrian turns (in average) 10% less with the FVPM than with the SFM in both scenarios.

For the escape room scenario, all 9 combinations for the number of pedestrians in [100, 150, 200] and the door sizes in [1.2, 1.5, 1.8] where run. Before presenting the results, two definitions need to be defined:

- Pedestrian flow: This is the amount of pedestrians that left the room per unit of time. It is defined as:

$$Q = \frac{\triangle N}{\triangle T} \; [Pedestrian/s]$$

- Pedestrian specific flow: This is the amount of pedestrians that left the room per unit of time per unit of distance. It is defined as:

$$Q_s = \frac{\triangle N}{\triangle T * door\_width} \; [Pedestrian/s/m]$$

The average pedestrian flow (Q_avg) was taken by averaging the results of 5 simulations for each configuration.
The results are shown on fig. 15, fig. 16 and fig. 17.

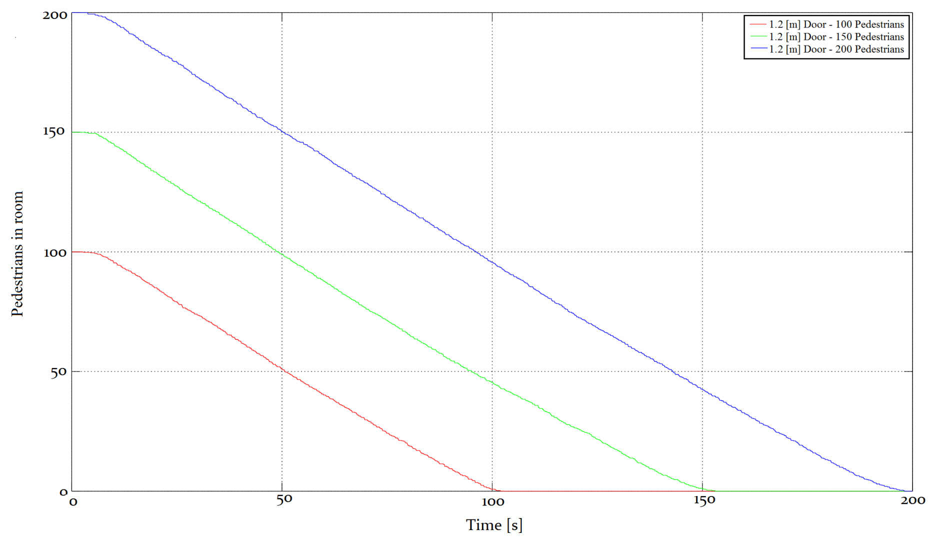

Figure 15 Room evacuation with 1.2 [m] door

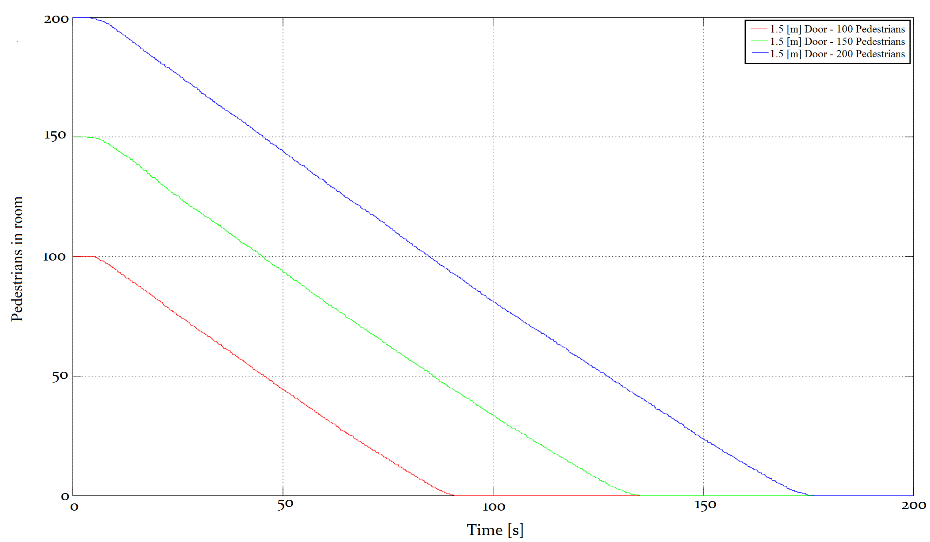

Figure 16 Room evacuation with 1.5 [m] door

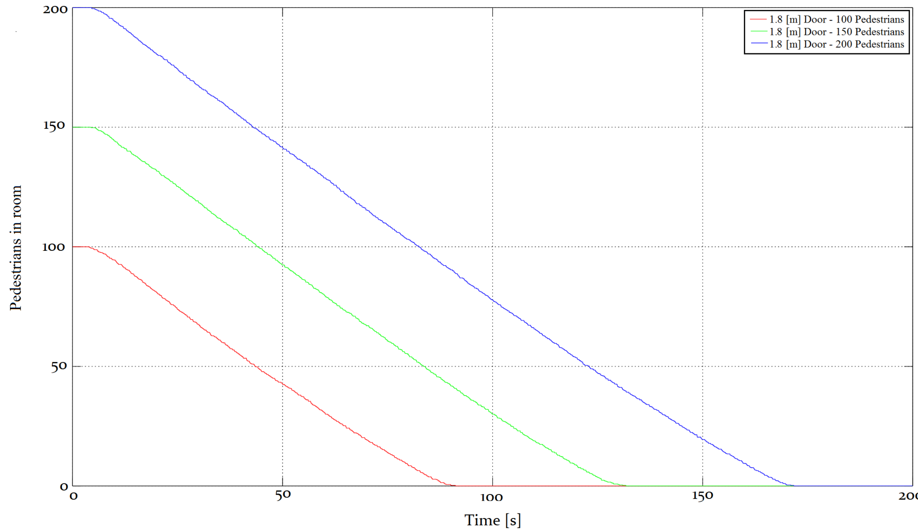

Figure 17 Room evacuation with 1.8 $[m]$ door

The escape flow for each configuration was calculated using octave's *polyfit* using a grade 1 polygon and the slopes for each door size are as follows:

N \ Door size	1.2	1.5	1.8
100	1.02	1.05	1.05
150	1.17	1.18	1.18
200	1.20	1.22	1.23

Table 3 Escape room configurations

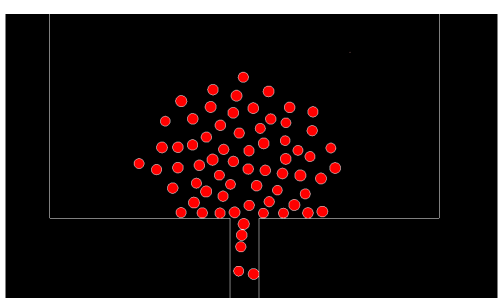

Figure 18 Room visualization

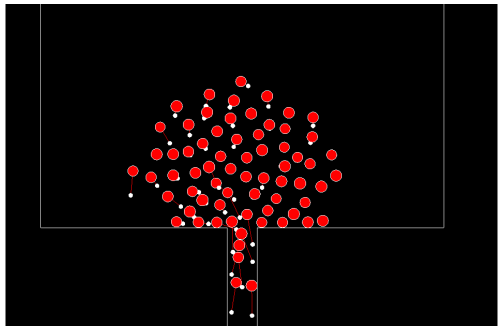

Figure 19 Room visualization with visible future

The escape flow of pedestrians does not change drastically depending on the number of pedestrians, unlike the SFM. This happens because of the removal of the social force, which increased as pedestrians increased.

5 Conclusion

Pedestrians can adjust their desired velocity at will, based on the data they perceive. We proposed a model that calculates the desired velocity of the social force model by conceptually moving the social force into the near future (FVP). The results look more natural than the original SFM and have been validated with experimental data. We used real-life metrics to validate our model, the results show that our model resembles reality with high fidelity. The proposed model produces a more natural navigation, but, at high densities, this isn't true, we still need further analysis of this scenario to adjust our model for both cases. In future work, the paths our virtual pedestrians may need to be compared and validated with real-life pedestrians paths to complement the use of metrics for the mean of pedestrians. Parameters should be re validated with more metrics and analyzed in even more detail.

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