The Network Origins of Aggregate Fluctuations
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Motivation (1)

Question:
Do microeconomic sector specific shocks lead to aggregate fluctuations in presence of heterogeneous intersectoral input–output linkages?

What this paper do:

- Develops a multisector model that captures input–output linkages.
- Propose four key theorems to describe how idiosyncratic shocks propagate and average out.
- Provide an evidence based application of the theory developed.

Motivation (2)

Why is this important:
Independent shocks to specific industries do not vanish as quickly as the previous literature proposed, generating more persistent effects that initially thought.

Key inside:
Microeconomic shocks effects may not remain confined to where they originate.
$\Longrightarrow$ They migth propagate throughout the economy, affect the output of other sectors, and generate sizable aggregate effects.

Main contribution:
Provide a mathematical framework for the analysis of shocks propagation and to characterize that their role in aggregate fluctuations depend on the structure of interactions between different sectors.

Previous argument

Idiosyncratic shocks generate that aggregate output concentrates around its mean at a very rapid rate.

$\Longrightarrow$ In an economy consisting of n sectors hit by independent shocks, aggregate fluctuations would have a magnitude proportional to $1/\sqrt{n}$

$\Longrightarrow$ Negligible effect at high levels of disaggregation.

This argument ignores the presence of interconnections between different firms and sectors, functioning as a potential propagation mechanism of idiosyncratic shocks throughout the economy.

Real life statement

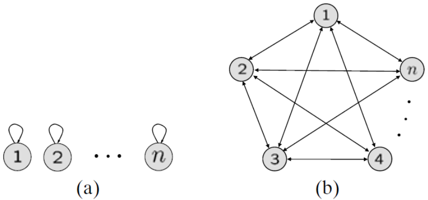
Ford's CEO requested emergency government support for General Motors and Chrysler in nov 2008 (Why in the world he will do that?)

He argued that, given the significant overlap in the suppliers and dealers of the three automakers, the collapse of either GM or Chrysler would have a ripple effect across the industry, leading to severe disruption of Ford's production operations within days, if not hours.

Graph examples: Previous argument works

As n increases and the economy becomes more disaggregated, the diversification argument based on the LLN implies that independent sectoral shocks average out rapidly at the rate $\sqrt{n}$.

Figure: Symmetric economies

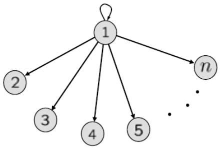


The symmetric structure of this economy ensures that aggregate output is a symmetric function of the shocks to each sector, implying that the diversification argument remains applicable.

Graph examples: Previous argument NOT works

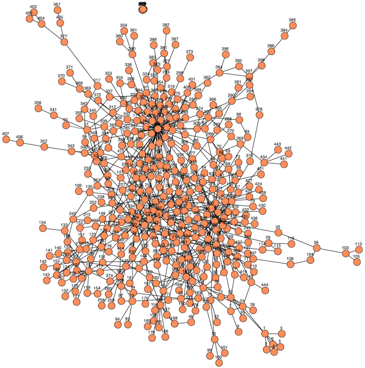
Consider an economy with small number of sectors playing a disproportionately important role as input suppliers to others. Consequently, the interplay of sectoral shocks and the intersectoral network structure may generate sizable aggregate fluctuations.

Figure: One sector is the only supplier of all other sectors



How real world graphs look

Figure: Network corresponding to the U.S. input–output matrix in 1997



$\Longrightarrow$ Small number of sectors playing a disproportionately important role as input suppliers to others

Road map

Investigate whether aggregate volatility, defined as the standard deviation of log output, vanishes as $n \to \infty$.

- In certain cases, such as the star network, the LLN fails and aggregate output does not concentrate around a constant value.
- The main focus, however, is on the more interesting cases in which the LLN holds, yet the structure of the intersectoral network still has a defining effect on aggregate fluctuations.
- Sectoral interconnections may imply that aggregate output concentrates around its mean at a rate significantly slower than $\sqrt{n}$.

Model

Household preferences

$$u(c_1, c_2, \ldots, c_n) = A \prod_{i=1}^{n} c_i^{\frac{1}{n}}$$

Sector production functions: each good produced by competitive sector; can be either consumed or used by other sectors as an input

$$x_i = z_i^{\alpha} l_i^{\alpha} \prod_{j=1}^{n} x_{ij}^{(1-\alpha) w_{ij}}$$

- x_{ij}: Amount of commodity j used in the production of good i
- w_{ij}: Share of good j in total input use of firms in sector i
  - Correspond to the entries in input-output tables
- z_i: Idiosyncratic productivity shock to sector i. Independent across sectors and $\varepsilon \equiv \log(z_i) \sim F_i$.

Model

Assumption (Assumption 1)

Input shares of all sectors add up to 1: $\sum_{j=1}^{n} w_{ij} = 1 \;\; \forall \, i$.

- Can summarize the structure of intersectoral trade with the input-output matrix W, which has entries w_{ij}.
- Economy is completely specified by the tuple

$$\mathcal{E} = (\mathcal{I}, W, \{F_i\}_{i \in \mathcal{I}}), \quad \mathcal{I} \text{ is the number of sectors}$$

- Can equivalently represent the economy as a weighted directed graph on n vertices,
  - each vertex corresponds to a sector
  - A directed edge (j, i) with weight $w_{ij} > 0$ is present from vertex j to vertex i if sector j is an input supplier to sector i.

Model

Definition (Weighted Outdegree of sector i)

Share of sector i's output in the input supply of the entire economy, normalized by $1 - \alpha$,

$$d_i \equiv \sum_{j=1}^{n} w_{ji}.$$

- When all nonzero edge weights are identical, the outdegree of vertex i is proportional to the number of sectors it is a supplier for.

Competitive Equilibrium

The competitive equilibrium of the economy can be represented by value added:

$$y = \log(GDP) = \nu' \varepsilon \tag{1}$$

Definition (Influence Vector)

$$\nu = \frac{\alpha}{n}[I - (1-\alpha)W']^{-1}$$

- Aggregate output is a linear combination of log sectoral shocks with coefficients determined by the influence vector.
- Aggregate output depends on the intersectoral network of the economy through the Leontief inverse $[I - (1-\alpha)W']^{-1}$.
- Influence vector also captures how sectoral productivity shocks propagate downstream to other sectors through the input–output matrix.

Model—Influence Vector

- The influence vector can also be interpreted as a centrality measure.

- Central sectors in the network representation of the economy play a more important role in determining aggregate output.

- ν is also the sales vector of the economy in the sense that the ith element of the influence vector is equal to the equilibrium share of sales of sector i:

$$\nu_i = \frac{p_i x_i}{\sum_{j=1}^{n} p_j x_j} \tag{2}$$

Adding Network Structure

- Focus on a sequence of economics where the number of sectors increases
- Characterize how the structure of the intersectoral network affects aggregate fluctuations
- Sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$; economy n is

$$\mathcal{E} = (\mathcal{I}_n, W_n, \{F_{in}\}_{i\in\mathcal{I}_n})$$

- Since the total supply of labor is normalized to 1, increasing n (number of sectors) means disaggregating the structure of the economy

Adding Network Structure—Notation and Assumptions

- $\{y_n\}_{n\in\mathbb{N}}$ and $\{\nu_n\}_{n\in\mathbb{N}}$ are aggregate outputs and influence vectors
- w_{ij}^n and d_i^n are elements of the intersectoral matrix W_n and the degree of sector i
- $\{\varepsilon_n\}_{n\in\mathbb{N}}$ is the sequence of vectors of (log) sectoral shocks

Assumption (Assumption 2)

Given a sequence of economies $\mathcal{E}_{n\in\mathbb{N}}$, for any sector $i \in \mathcal{I}_n$ and all $n \in \mathbb{N}$,

(a) $\mathbb{E}\varepsilon_{in} = 0$

(b) $\mathrm{Var}(\varepsilon_{in}) = \sigma_{in}^2 \in (\underline{\sigma}^2, \overline{\sigma}^2)$, *where $0 < \underline{\sigma} < \overline{\sigma}$ are independent of n.*

Aggregate Volatility

- Assumption 2(a) and independent sectoral shocks imply that we can write aggregate volatility as

$$(\operatorname{Var} y_n)^{1/2} = \sqrt{\sum_{i=1}^{n} \sigma_{in}^2 \nu_{in}^2}.$$

- For any sequence of economies satisfying Assumption 2(b),

$$(\operatorname{Var} y_n)^{1/2} = \Theta(\|\nu_n\|_2).$$

- Aggregate volatility scales with the Euclidian norm of the influence vector as the economy becomes disaggregated.
- The rate of decay of aggregate volatility may not be equal to $\sqrt{n}$ (the standard prediction from the diversification argument).
- If $\|\nu\|_2$ is bounded away from zero for all n, then aggregate volatility does not disappear as $n \to \infty$.

Asymptotic Distributions

Theorem (Theorem 1)

Consider a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ and assume that $\mathbb{E}\varepsilon_{in}^2 = \sigma^2$ for all $i \in \mathcal{I}_n$ and all $n \in \mathbb{N}$

(a) *If $\{\varepsilon_{in}\}$ are normally distributed for all i and all n, then $\frac{1}{\|v_n\|_2} y_n \xrightarrow{d} \mathcal{N}\left(0, \sigma^2\right)$*

(b) *Suppose that there exist constant $a > 0$ and random variable $\bar{\varepsilon}$ with bounded variance and cumulative distribution function $\bar{F}$, such that $F_{in}(x) < \bar{F}(x)$ for all $x < -a$, and $F_{in}(x) > \bar{F}(x)$ for all $x > a$. Also suppose that $\frac{\|v_n\|_\infty}{\|v_n\|_2} \longrightarrow 0$. Then $\frac{1}{\|v_n\|_2} y_n \xrightarrow{d} \mathcal{N}\left(0, \sigma^2\right)$*

(c) *Suppose that $\{\varepsilon_{in}\}$ are identically, but not normally distributed for all $i \in \mathcal{I}_n$ and all n. If $\frac{\|v_n\|_\infty}{\|v_n\|_2} > 0$, then the asymptotic distribution of $\frac{1}{\|v_n\|_2} y_n$, when it exists, is nonnormal and has finite variance σ^2.*

First-Order Interconnections

- Characterize the rate of decay of aggregate volatility in terms of the structural properties of the intersectoral network.
- First result: The extent of asymmetry between sectors shapes the relationship between sectoral shocks and aggregate volatility.

Definition (Coefficient of Variation)

Given an economy $\mathcal{E}_n$ with sectoral degrees $\{d_1^n, d_2^n, \ldots, d_n^n\}$, the coefficient of variation is

$$\mathrm{CV}_n \equiv \frac{1}{\bar{d}_n}\left[\frac{1}{n-1}\sum_{i=1}^{n}\left(d_i^n - \bar{d}_n\right)^2\right]^{1/2}$$

where $\bar{d}_n = \left(\sum_{i=1}^{n} d_i^n\right)/n$ is the average degree.

First-Order Interconnections

Theorem (Theorem 2)

Given a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$, aggregate volatility satisfies

$$(\operatorname{var} y_n)^{1/2} = \Omega\left(\frac{1}{n}\sqrt{\sum_{i=1}^{n}(d_i^n)^2}\right)$$

and

$$(\operatorname{var} y_n)^{1/2} = \Omega\left(\frac{1+\mathrm{CV}_n}{\sqrt{n}}\right).$$

- High variability in degree sequence of intersectoral network $\implies$ high variability in effect of shocks on aggregate output.
- High CV $\implies$ few sectors are responsible for most inputs.
- Low productivity $\implies$ low productivity in downstream sectors.
- Aggregate volatility decays slower than $\sqrt{n}$.

Interpreting Theorem 2

Definition (Power Law Degree Sequence)

A sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ has a power law degree sequence if there exist a constant $\beta > 1$, a slowly varying function $L(\cdot)$ satisfying $\lim_{t\to\infty} L(t)t^{\delta} = \infty$ and $\lim_{t\to\infty} L(t)t^{-\delta} = 0$ for all $\delta > 0$, and a sequence of positive numbers $c_n = \Theta(1)$ such that, for all $n \in \mathbb{N}$ and all $k < d^n_{\max} = \Theta\left(n^{1/\beta}\right)$, we have

$$P_n(k) = c_n k^{-\beta} L(k)$$

where $P_n(k) \equiv \frac{1}{n}\left|\{i \in \mathcal{I}_n : d^n_i > k\}\right|$ is the empirical counter-cumulative distribution function and $d^n_{\max}$ is the maximum degree of $\mathcal{E}_n$.

- Look at the special case where the intersectoral networks have power law degree sequences.
- The first part of Theorem 2 says that aggregate volatility is higher in economies whose degree sequences have "heavier tails".

Interpreting Theorem 2

Corollary (Corollary 1)

Consider a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ with a power law degree sequence and the corresponding shape parameter $\beta \in (1,2)$. Then, aggregate volatility satisfies

$$(\operatorname{var} y_n)^{1/2} = \Omega\left(n^{-(\beta-1)/\beta-\delta}\right)$$

where $\delta > 0$ is arbitrary.

- If the degree sequence of the intersectoral network exhibits heavy tails, aggregate volatility decreases at a much slower rate than predicted by the diversification argument.
- Note that so far the authors have only provided a lower bound on the rate at which aggregate volatility vanishes.
- Higher-order structural properties of the intersectoral network can still prevent output volatility from decaying at rate $\sqrt{n}$.

Second-Order Interconnections and Cascades

- First-order interconnections provide little information about how shocks to a sector affect the downstream customers of downstream customers of the affected sector, etc.

- The next theorem provides a lower bound on the decay rate of aggregate volatility in terms of second-order interconnections in the intersectoral network.

Definition (Definition 3—2nd-Order Interconnectivity Coefficient)

The second-order interconnectivity coefficient of economy $\mathcal{E}_n$ is

$$\tau_2(W_n) \equiv \sum_{i=1}^{n} \sum_{j \neq i} \sum_{k \neq i,j} w_{ji}^n w_{ki}^n d_j^n d_k^n.$$

- Measures extent to which high degree sectors are connected to each other via common suppliers

Second-Order Interactions and Cascades

Theorem (Theorem 3)

Given a sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$, *aggregate volatility satisfies*

$$(\mathrm{var}\, y_n)^{1/2} = \Omega\left(\frac{1}{\sqrt{n}} + \frac{\mathrm{CV}_n}{\sqrt{n}} + \frac{\sqrt{\tau_2\left(W_n\right)}}{n}\right)$$

- Shows how second-order interactions, captured by τ_2, affect aggregate volatility.
- Even if the empirical degree distributions of two sequences of economies are identical for all n, their aggregate volatilities may exhibit considerably different behaviors.
- This is a refinement of Theorem 2; it captures the notion that there is a clustering of significant sectors because they have common suppliers.

Interpreting Theorem 3

Corollary (Corollary 2)

Suppose that $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ *is a sequence of economies whose second-order degree sequences have power law tails with shape parameter* $\zeta \in (1,2)$*(cf. Definition 2). Then, aggregate volatility satisfies*

$$(\operatorname{var} y_n)^{1/2} = \Omega\left(n^{-(\zeta-1)/\zeta-\delta}\right)$$

for any $\delta > 0$.

- If the distributions of second-order degrees have heavy tails, aggregate volatility decreases much more slowly than predicted by diversification.
- Second-order effects may dominate first-order effects.
- If a sequence of economies has power law tails for both first- and second-order degrees, with exponents β and ζ, then the tighter bound for the decay rate of aggregate volatility is determined by $\min\{\beta, \zeta\}$.

Balanced Structures

- With limited variations in the degrees of different sectors, aggregate volatility decays at rate $\sqrt{n}$.

Definition (Definition 4—Balanced Sequence of Economics)

A sequence of economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$ is balanced if $\max_{i\in\mathcal{I}_n} d_i^n = \Theta(1)$.

- When the intersectoral network is balanced and the role of intermediate inputs is not too large, volatility decays at rate $\sqrt{n}$.
- Other structural properties of the network cannot contribute to aggregate volatility.

Theorem (Theorem 4)

Consider a sequence of balanced economies $\{\mathcal{E}_n\}_{n\in\mathbb{N}}$. Then there exists $\bar{\alpha}\in(0,1)$ such that, for $\alpha\geq\bar{\alpha}$, (var $y_n)^{1/2} = \Theta(1/\sqrt{n})$.

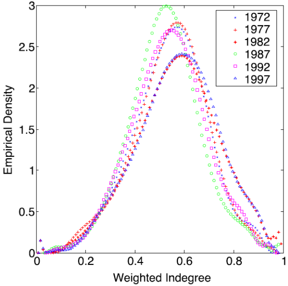
Interpreting Theorem 4

- Theorem 4 is both an aggregation and an irrelevance result for balanced economies.
- As an aggregation result, it suggests observational equivalence between the one-sector economy and any balanced multi-sector economy.
- As an irrelevance result, it shows that different input-output matrices generate roughly the same volatility for balanced economies.

Application: Setup

Data: Detailed benchmark input–output accounts spanning the 1972–2002 period, compiled every five years by the Bureau of Economic Analysis.

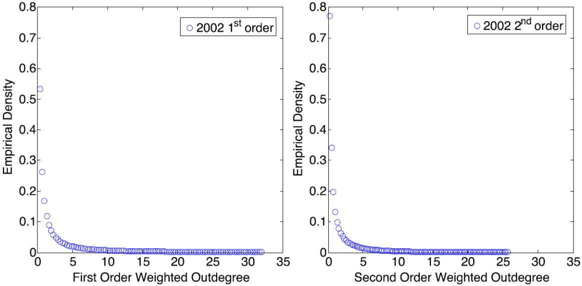
Figure: Empirical densities of intermediate input shares (indegrees).



$\Longrightarrow$ Most sectors are concentrated around the mean (0.55): on average, 71% of the sectors are within one standard deviation of the mean input share.

First- and second-order OUT-degrees densities

Figure: Empirical densities of first- and second-order degrees



Heavy right tails, meaning that some commodities are (1) General purpose inputs used by many other sectors. and (2) major suppliers to sectors that produce the general purpose inputs.

$\Longrightarrow$ The tail of the distributions is well-approximated by a power law distribution (Pareto distribution).

Distribution parameters estimation (1)

OLS regression of the empirical log-CCDF on the log-outdegree sequence are downward biased in small samples (Gabaix and Ibragimov, 2011). Thus implement the modified log rank– log size regression. Take the tail of 1 minus the empirical cumulative distribution functions to correspond to the top 20% largest sectors in terms of in an out degrees.

$$CDF_{\mathsf{Pareto}} = 1 - \left(\frac{x_m}{x}\right)^{\alpha}$$

Where x_m is the scale parameter and α is the shape parameter.

Distribution parameters estimation (2)

$\hat{\beta}$ and $\hat{\zeta}$ are the shape parameters for the first and second-order degree distribution. (As higher is the shape parameter, more skewed is the distribution.)

OLS Estimates of β and ζ[a]

	1972	1977	1982	1987	1992	1997	2002
$\widehat{\beta}$	1.38 (0.20; 97)	1.38 (0.19; 105)	1.35 (0.18; 106)	1.37 (0.19; 102)	1.32 (0.19; 95)	1.43 (0.21; 95)	1.46 (0.23; 83)
$\widehat{\zeta}$	1.14 (0.16; 97)	1.15 (0.16; 105)	1.10 (0.15; 106)	1.14 (0.16; 102)	1.15 (0.17; 95)	1.27 (0.18; 95)	1.30 (0.20; 83)
n	483	524	529	510	476	474	417

[a]The numbers in parentheses denote the associated standard errors (using Gabaix and Ibragimov (2011) correction) and the number of observations used in the estimation of the shape parameter (corresponding to the top 20% of sectors). The last row shows the total number of sectors for that year.

High degree of asymmetry in the U.S. economy in terms of the roles that different sectors play as direct or indirect suppliers to others.
$\Longrightarrow$ The interplay of sectoral shocks and network effects leads to sizable aggregate fluctuations

Quantitative extent of the network effects (1)

Aggregate effects of sectoral shocks: Compute $||\nu_n||_2$ for the U.S. input–output matrix at different levels of aggregation and for different years.

ESTIMATES FOR $\|v_n\|_2$[a]

	1972	1977	1982	1987	1992	1997	2002
$\|v_{n_d}\|_2$	0.098 ($n_d = 483$)	0.091 ($n_d = 524$)	0.088 ($n_d = 529$)	0.088 ($n_d = 510$)	0.093 ($n_d = 476$)	0.090 ($n_d = 474$)	0.094 ($n_d = 417$)
$\|v_{n_s}\|_2$	0.139 ($n_s = 84$)	0.137 ($n_s = 84$)	0.149 ($n_s = 80$)	0.133 ($n_s = 89$)	0.137 ($n_s = 89$)	0.115 ($n_s = 127$)	0.119 ($n_s = 128$)
$\frac{\|v_{n_d}\|_2}{\|v_{n_s}\|_2}$	0.705	0.664	0.591	0.662	0.679	0.783	0.790
$\frac{1/\sqrt{n_d}}{1/\sqrt{n_s}}$	0.417	0.400	0.399	0.418	0.432	0.518	0.554

[a] $\|v_{n_d}\|_2$ denotes estimates obtained from the detailed level input–output BEA data. $\|v_{n_s}\|_2$ denotes estimates obtained from the summary input–output BEA data. The numbers in parentheses denote the total number of sectors implied by each level of disaggregation.

$||\nu_{n_d}||_2$ at different years are roughly twice as large as $1/\sqrt{n}$ (First row).

$\implies$ intersectoral linkages increase the impact of sectoral shocks by at least 2 times.

The third row captures the change in the aggregate effect of sectoral shocks from more to less aggregated data.

$\implies$ Taking intersectoral linkages into account simply doubles the impact of sectoral shocks at all levels of disaggregation.

Quantitative extent of the network effects (2)

If indeed taking intersectoral linkages into account simply doubles the impact of sectoral shocks at all levels of disaggregation the ratio will be $\frac{1/\sqrt{n_d}}{1/\sqrt{n_s}}$.

If, on the other hand, network effects are more important at higher levels of disaggregation, then we would expect that:

$$\frac{||\nu_{n_d}||_2}{||\nu_{n_s}||_2} > \frac{1/\sqrt{n_d}}{1/\sqrt{n_s}}$$

Quantitative extent of the network effects (3)

The table shows that the latter is indeed the case for all years. For example, in 1972, as we move from the more aggregated measurement at the level of 84 sectors (at two-digit SIC) to an economy comprising 483 sectors (roughly at four-digit SIC), the standard diversification argument would imply a decline of 58% in the role of sectoral shocks, whereas the actual decline observed in the data, measured by $\frac{||\nu_{n_d}||_2}{||\nu_{n_s}||_2}$, is about 29%.

Back-of-the-envelope calculation (1)

Using TFP estimations across 459 four-digit (SIC) manufacturing industries from the NBER productivity database between 1958 and 2005, compute its average standard deviation to be 0.058.

Average of the U.S. GDP accounted for by manufacturing is around 20% for the same time frame.

Assuming that the manufacturing industries correspond to one-fifth of the GDP, the economy comprises $5 * 459 = 2295$ sectors. With a sector volatility of 0.06, if aggregate volatility decayed at the rate $\sqrt{n}$, it expects it to be $0.058/\sqrt{2295} = 0.001$.
$\implies$ 0.1% standard deviation of the U.S.GDP

Back-of-the-envelope calculation (2)

The shape parameter for the second order degrees $\hat{\zeta} = 1.18$ implies that aggregate volatility decays no faster than $n^{(\zeta-1)/\zeta} = n^{0.15}$.

Using the second-order degree distribution, aggregate volatility decays at the rate $n^{0.15}$, the same number would be $0.058/2295^{0.15} = 0.018$.

$\implies$ This corresponds to sizable aggregate fluctuations, in the ballpark of the approximately 2% standard deviation of the U.S. GDP (x18!).