Aggregating Distortions in Networks with Multi-Product Firms

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Motivation: Aggregation in efficient Economies

TFP is usually measured as a residual:

$$d \log Y = \underbrace{d \log TFP}_{\text{Technology?}} + \underbrace{\Lambda_L}_{\text{Labor Share}} d \log L + \underbrace{\Lambda_K}_{\text{Capital Share}} d \log K$$

Given arbitrarily efficient supply chains, according to Hulten 1978, aggregate productivity changes are equal to the Domar-weighted (firm's sales over GDP, λ_i) sum of microeconomic changes in technology:

$$d \log TFP = \sum_i \lambda_i d \log A_i$$

Motivation: Aggregation in inefficient Economies

Restuccia and Rogerson (2008), Hsieh and Klenow (2009) in horizontal economies, and Baqaee and Farhi (2020) with firm linkages showed that distortions go beyond technological progress and lead to changes in aggregate TFP due to shifts in resource allocation between firms; Allocative Efficiency (AE).

But they abstract from potential within-firm AE. If firms jointly produce more than one product with product-specific distortions, how firms allocate productive resources within their product portfolio also shapes aggregate TFP.

Preview

What this paper does:

1. Tests if firms engage in joint production.
2. Builds a theory to include within-firm Allocative Efficiency and evaluate its contribution to aggregate TFP growth.
3. Estimates product level markups using transactional data for the universe of formal firms in Chile.
4. Performs a growth accounting in the presence of product-level markups.

Main takeaway : Within-firm AE moves opposite to between-firm AE in Chile, especially after Covid; resources are increasingly misallocated between firms, but firms product portfolio choices favor total AE.

Agenda

1. Testing joint production
2. A theory to aggregate product-level inefficiencies
3. Data
4. Growth accounting in the presence of multi-product markups
5. Conclusion

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Joint Production

Joint production implies that shocks to one product might affect the output of others as they share common production inputs.

In the absence of joint production with multi-product firms, every product can be considered a firm. If every product is treated as a firm, then there is no within-firm misallocation, and we are back into the Baqaee & Farhi, 2020 framework.

Joint production test

A firm's production technology is non-joint if the cost function for g different products can be written as (Hall, 1973 or Diewert, 1973) :

$$C(q_1, ...q_g) = \sum_{g=1}^{G} C_g(q_g)$$

If a firm's production technology is non-joint, a demand shock to one product has no impact on the sales of any other product within that firm .

Joint Production test

Under the null hypothesis, sales of a given product would be unaffected by demand shocks to the firm's other products.

Testing joint production: Demand shock Covid April 2020

Data on Covid-driven lockdowns was gathered for the 346 Chilean counties between March 2020 and December 2021.

(a) Using firm-to-firm invoices, a lockdown status is assigned for each origin and destination county.

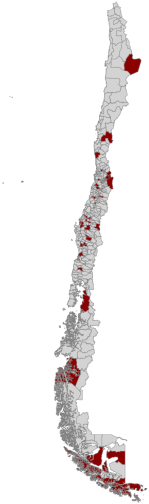
(b) A county is in lockdown if it has an active restriction for at least 15 days per month.

(c) A firm main product sold is defined as the one with the largest sales from January to February 2020.

(d) For all non-main products, its sales share is computed from January to February 2020.

(e) The latter is multiplied with monthly active lockdowns dummies, and its sum by firm is the sales share from buyers under lockdown for non-main products: $\phi_{it}^{-m} \in [0, 1]$

Testing joint production: Demand shock Covid April 2020



ϕ_{it}^{-m} :

- 24% are non-zero
- mean=0.071 ; sd=0.182

Testing joint production: Estimation Strategy

$$\log Y_{imt} = \alpha + \beta_1 \phi_{it}^{-m} + \beta_X X + \epsilon$$

Where $Y \in$ [Sales, Quantity] and X are controls, including the input price index for firm i in period t.

If ϕ_{it}^{-m} is non-zero and significant, the null hypothesis of non-join production is rejected. In this work, we take no stances on the expected relationship of ϕ_{it}^{-m} and the shock direction.

Testing joint production: Reduced form evidence

	$\ln S_{imt}$				$\ln Q_{imt}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ϕ_{it}^{-m}	-0.147***	-0.267***	-0.263***	-0.172***	-0.151***	-0.224***	-0.277***	-0.196***
Product FE	✓	✓	✓	✓	✓	✓	✓	✓
Firm FE	X	✓	✓	✓	X	✓	✓	✓
Destination county FE	X	X	✓	✓	X	X	✓	✓
Month-290 product code FE	X	X	X	✓	X	X	X	✓
Observations	1,555,795	1,518,448	1,518,448	1,518,448	1,556,544	1,519,198	1,519,198	1,519,198
R^2	0.506	0.591	0.591	0.613	0.682	0.759	0.759	0.771
*** p<0.01, ** p<0.05, * p<0.1								

ϕ_{it}^{-m} is non-zero and significant for all the specifications, which suggests it's possible to reject the null hypothesis of non-joint production.

The positive relationship of the shock with ϕ_{it}^{-m} is consistent with Ding, 2023. A negative shock on the demand for one good negatively affects the production of other goods.

Theory: Joint Production

The production set $V(q)$ is described by the transformation function $t(q,x)$:

$$V(q) = \{x | t(q,x) \geq 0\}$$

where q is the vector of outputs and x is the vector of inputs. The cost function based on cost minimization is:

$$C(q,p) \equiv \min_{x \in V(q)} p \cdot x$$

Production sets main assumptions:

(a) constant return to scale: $t(q,x) = 0 \implies t(\lambda q, \lambda x) = 0$.

(b) Separability between input and output function (Hall 1973): $t(q,x) = -g(q) + f(x)$ has the joint cost function: $C(q,p) = H(q)\varphi(p)$

Theory: Multi-Product Firms

Firm $i \in N$ produces product $g \in G$ by using products $g' \in G$ from other firms $j \in N$ and factors (L, K):

$$F_i^Q\left(\left\{\underbrace{q_{ig}}_{\text{outputs}}\right\}_{i \in N, g \in G}\right) = A_i F_i^X\left(\left\{\underbrace{x_{i,jg'}}_{\text{Intermediate product } p \text{ from j}}\right\}_{j \in N, p \in G}, L_i, K_i\right)$$

Firms charge prices with a product-specific markup μ_{ig} over its marginal cost:

$$p_{ig} = mc_{ig} \cdot \mu_{ig}$$

Theory: Final Demand

There is a representative household with constant-return to scale utility function $U(c_{ig}, ..., c_{NG})$ that receives firm profits through an income transfer T and faces the following budget constraint:

$$\sum_{i \in N} \sum_{g \in G} p_{ig} c_{ig} = \sum_{f \in \{L,K\}} w_f L_f + \sum_{i \in N} \sum_{g \in G} (1 - 1/\mu_{ig}) \, p_{ig} q_{ig} + T$$

Each product can either be consumed by the final consumers (c_{ig}) or used as an input in production by other firms ($x_{ji,g}$), facing the following resource constraints:

$$q_{ig} = c_{ig} + \sum_{j \in N} x_{jig} \qquad \sum_{i \in N} L_i = L \qquad \sum_{i \in N} K_i = K$$

Theory: National Accounts

GDP is defined as the sum of all product values consumed by the final consumers: $GDP = \sum_{i \in N} \sum_{g \in G} p_{ig} c_{ig}$

Real GDP (Y) changes can be computed as:

$$d \log Y = d \log GDP - \sum_{i \in N} \sum_{g \in G} \frac{p_{ig} c_{ig}}{GDP} d \log p_{ig}$$

Factor Shares are defined as

$$\Lambda_L = \frac{wL}{GDP}, \quad \Lambda_K = \frac{rK}{GDP}$$

Theory: Product-Input-Output Networks

The product-input-output matrix $\tilde{\Omega}$ is a $(N \times G + F)$ square matrix where N is the number of firms, G is the number of products, and F is the number of factors.

$\tilde{\Omega}$ has at its ig, jg'^{th} element the expenditure share of product g' from firm j and factor $f \in F$ used by firm i in production over firm i total costs (of producing all its goods).

From the separability assumption, the same expenditure share applies to all products, g that i makes. Thus, $\tilde{\Omega}_{ig,jg'}$ and $\tilde{\Omega}_{ig,f}$:

$$\tilde{\Omega}_{ig,jg'} = \frac{p_{jg'} x_{i,jg'}}{\sum_{j,g'} p_{jg'} x_{i,jg'} + \sum_f w_f L_{if}}, \quad \tilde{\Omega}_{ig,f} = \frac{w_f L_{if}}{\sum_{j,g'} p_{jg'} x_{i,jg'} + \sum_f w_f L_{if}}$$

Theory: Product-Input-Output Networks

The product cost-based Leontief inverse $\tilde{\Psi}$ captures every firm-product pair's direct and indirect cost exposures through production networks. Each element of $\tilde{\Psi}$ measures the weighted sums of all paths (steps) between any two non-zero firm-product pairs.

$$\tilde{\Psi} \equiv (I - \tilde{\Omega})^{-1} = I + \tilde{\Omega} + \tilde{\Omega}^2 + \ldots$$

Theory: Product-Input-Output Networks

To measure the relative importance of each firm-product pair on GDP, GDP shares in vector form are defined:

$$b_{ig} = \begin{cases} \frac{p_{ig} c_{ig}}{GDP} & \text{if } i \in N, g \in G \\ 0 & \text{otherwise} \end{cases}$$

GDP is set to be the numeraire, and the product level cost-based Domar weight vector is $\tilde{\lambda}_{ig}$ ($\tilde{\Lambda}_f$ for factors). It measures the importance of product g from firm i in final demand in two dimensions: directly when it is sold to final consumers and indirectly through the production network when it is sold to other firms that, eventually, downstream production networks will reach final consumers.

$$\tilde{\lambda}' \equiv b'\tilde{\Psi} = b' + b'\tilde{\Omega} + b'\tilde{\Omega}^2 + \dots$$

Theory: Firm Level Aggregation

Summing over products by firms, the firm-level cost-based Domar weight $\tilde{\lambda}_i$ is computed, which is used to compute the within-firm product-level Domar weight share s_{ig}:

$$\tilde{\lambda}_i = \sum_{g \in G} \tilde{\lambda}_{ig}, \qquad s_{ig}^{g} = \frac{\tilde{\lambda}_{ig}}{\tilde{\lambda}_i}$$

Theory: Product Network Distortion

The product network distortion Ξ_{ig} measures each firm-product pair distortion through production networks. Is the product-level ratio of product i total cost (or $sales_{ig}/\mu_{ig}$) to product level cost-based Domar weight, $\tilde{\lambda}_{ig}$.

$$\Xi_{ig} \equiv \frac{tc_{ig}}{\tilde{\lambda}_{ig}} \equiv \frac{sales_{ig}/\mu_{ig}}{\tilde{\lambda}_{ig}}$$

In efficient economies, $tc_{ig} = \tilde{\lambda}_{ig}$; there is not product network distortions.

Theory: Product Network Distortion

$\Xi_{ig'}$ measures the size of the inefficiency by firms using product ig' as an intermediate input, which in turn is trespassed from product ig' buyers downstream the supply chain.

If $\Xi_{ig'} < \tilde{\lambda}_{ig}$, good ig' is relatively underproduced with respect to the optimal outcome; to few productive resources are allocated to its production.

Theory: Product Network Distortion

The relative product network distortion ξ_{ig} ranks product distortions within a given firm.

$$\xi_{ig} \equiv \frac{\Xi_{ig}}{\Xi_i}$$

Where $\Xi_i \equiv \sum_g tc_{ig} / \sum_g \tilde{\lambda}_{ig}$ is firm i average distortion.

ξ_{ig} describes relative distortion differences within a firm and will be the object that will determine within-firm allocative efficiency.

Value of ξ_{ig}	relative distortion within firm i
> 1	less distorted
$= 1$	average distortion
< 1	more distorted

Theory: Growth Accounting

$\mathcal{X}^{IN}$ is a $(N+F)\times(N\times G+F)$ admissible input allocation matrix, where the columns are buyer firms and the rows are seller-product pairs. Each of its elements $\mathcal{X}^{IN}_{ijg}=\frac{q_{ijg}}{q_{jg}}$ is the share of the output of good g produced by firm j that firm i uses as an input.

The output correspondence $\mathcal{X}^{OUT}_i$ maps the rows of the matrix $\mathcal{X}^{IN}_i$, corresponding to firm i, to a vector of outputs for each good produced by firm i, where $\mathcal{X}^{OUT}=\{\mathcal{X}^{OUT}_1,\mathcal{X}^{OUT}_2,\dots,\mathcal{X}^{OUT}_N\}$.

The set of all input allocation matrices and their corresponding output correspondences for each firm is defined as a collection:

$$\mathcal{X}=\left(\mathcal{X}^{IN},\mathcal{X}^{OUT}\right)$$

Theory: Growth Accounting

A productivity shock ($\mathrm{d}\log A$) and a markup shock ($\mathrm{d}\log\mu$) effect in real GDP ($\mathcal{Y}$) can be decomposed into the change in technology ($\mathrm{d}\log A$) for a given fixed $\mathcal{X}$ term and, the change in the distribution of $\mathcal{X}$ ($\mathrm{d}\mathcal{X}$) holding technology constant:

$$\mathrm{d}\log\mathcal{Y} = \underbrace{\frac{\partial\log\mathcal{Y}}{\partial\log A}\mathrm{d}\log A}_{\Delta\text{ Technology}} + \underbrace{\frac{\partial\log\mathcal{Y}}{\partial\mathcal{X}}\mathrm{d}\log\mathcal{X}}_{\Delta\text{ Allocative Efficiency}}$$

Theory: Growth Accounting

For single-good firms that charge one markup, the Baqaee & Farhi, 2020 distorted Solow residual is:

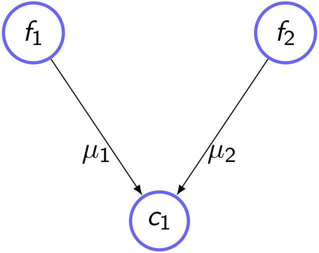
$$d \log TFP = \underbrace{\sum_{i \in N} \tilde{\lambda}_i d \log A_i}_{\Delta \text{ Technology}} - \underbrace{\sum_{i \in N} \tilde{\lambda}_i d \log \mu_i - \sum_{f \in F} \tilde{\Lambda}_f d \log \Lambda_f}_{\Delta \text{(Between firm) Allocative Efficiency}}$$

- Allocative efficiency changes depend on a) markups and b) factor shares.
- Markups harm efficiency because w and r do not equal MRPL and MRPK; hence, fewer units are produced relative to the optimal allocation.
- With markups, aggregate factor usage also declines relative to the optimal allocation.

Theory: Growth Accounting Example Between AE

- A representative household has Cobb-Douglas utility: $U = c_1^{\alpha_1} c_2^{\alpha_2}$
- There are two single-product firms that produce using linear technology, and labor is inelastically supplied: $q_i = L_i$, for $i = 1, 2$
- Factor allocations are: $(L_1^*, L_2^*) \propto \left(\frac{\alpha_1}{\mu_1}, \frac{\alpha_2}{\mu_2}\right)$

Between firm misalocation



Theory: Growth Accounting Example Between AE

- Assume $\mu_1 > \mu_2$ and $\alpha_1 = \alpha_2 = 0.5$. Both firms are underproducing relative to a competitive allocation and firm 1 employs relatively fewer labor units than firm 2.
- If there is a negative shock to the markup of firm 1 ($d \log \mu_1 = -\epsilon$), there is a reallocation of labor to firm 1, improving AE and hence boosting TFP growth.

$$
\begin{aligned}
d \log TFP &= \underbrace{-\alpha_1 d \log \mu_1 - d \log \Lambda_f}_{\text{Between Firm AE}} \\
&= \alpha_1 \epsilon - \frac{1}{1 + \frac{\alpha_2 \mu_1}{\alpha_1 \mu_2}} \epsilon \\
&= \epsilon \alpha_1 \left[1 - \frac{1}{\alpha_1 + \alpha_2 \frac{\mu_1}{\mu_2}} \right] > 0
\end{aligned}
$$

Theory: Growth Accounting within firm AE

Under the single-product firm assumption, misallocations arise between firms, but if firms are multiproduct and engage in joint production within firm AE arises:

$$\begin{aligned} \Delta\text{Within-Firm AE} &= -\sum_{i \in N} \tilde{\lambda}_i \underbrace{Cov_{s_i}\left(d\log p_i, \xi_i\right)}_{\text{Within-Firm AE of firm } i} \\ &= -\sum_{i \in N} \tilde{\lambda}_i \mathbb{E}\left[\mathbb{E}[d\log p_{ig} - d\log p_i]\ \mathbb{E}[\xi_{ig} - \Xi_i]\right] \end{aligned}$$

where $\xi_i = (\xi_{i1}, ..., \xi_{iG})$ and $d\log p_i = (d\log p_{i1}, ..., d\log p_{iG})$.

Theory: Growth Accounting within firm AE

Each product contribution to within-firm AE depends on:

- The change in the price of the product g relative to the firm's average price change.
- How distorted is product g relative to the average product distortion of firm i.

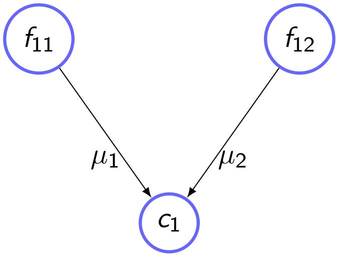
For example, if the price change for a given product g in firm i is smaller than the average price change in firm i, and product g is more distorted relative to the average product within firm i, its contribution to the covariance will be negative.

If markups cumulate in downstream supply chains, relatively lower price changes for a relatively distorted upstream input will reduce distortions downstream and improve TFP growth.

Theory: Growth Accounting Example within firm AE

Now, instead of two firms, only one firm produces two products. It is the same economy as the prior example but relabeled.

Within firm misalocation



Theory: Growth Accounting Example within firm AE

With$\mu_1 > \mu_2$ and $\alpha_1 = \alpha_2 = 0.5$:

- Product 1 is more distorted than product 2:
  $\Xi_1 < \Xi_2 \implies \frac{\Xi_1}{\Xi} = \xi_1 < 1$

If there is reduction in product 1 markup ($d \log \mu_{i1} = -\epsilon$)

- $Cov_\alpha \left([\epsilon, 0], [\mu_1, \mu_2] \right) < 0$

Hence,

$$\begin{aligned} d \log TFP &= - \left[Cov_\alpha \left([\epsilon, 0], [\mu_1, \mu_2] \right) \right] \\ &= \epsilon \alpha_1 \left[1 - \frac{1}{\alpha_1 + \alpha_2 \frac{\mu_1}{\mu_2}} \right] > 0 \end{aligned}$$

Theory: Growth Accounting

Combining between firm and within-firm AE, TFP growth can be decomposed as:

$$d\log TFP = \underbrace{\sum_i \tilde{\lambda}_i d\log A_i}_{\text{Technology}} \underbrace{- \sum_i \tilde{\lambda}_i d\log \mu_i - \sum_f \tilde{\Lambda}_{(f)} d\log \Lambda_f}_{\text{Between-Firm AE}} - \underbrace{\sum_i \tilde{\lambda}_i Cov_{s_i}\left(d\log p_i, \xi_i\right)}_{\text{Within-Firm AE}}$$

- If firms are single-product firms, they charge a unique markup over the same marginal costs, so the within-firm AE converges to zero, and we are back to Baqaee and Farhi. (2020).
- If prices equal marginal cost, there are no markups, and then the Between firm AE also converges to zero, where all TFP changes are due to technology converging to Hulten (1978).

Data Sources

From the Chilean IRS (SII):

(a) F29, firm accounting variables: Sales, Materials, Investment.
(b) DJ1878, DJ1879, labor information: Wages and headcount employees.
(c) F22. Capital stock. The level of capital is computed using the perpetual inventory method.
(d) DTE. Firm-to-Firm transactions, including prices, quantities, products, origins, and destinations.

Data on multiproduct firms

The universe of formal firm-to-firm transactions is observed as the firm's total sales (which include the latter and firm-to-consumer sales). Firm-specif product shares are computed for the firm-to-firm universe, and it is assumed that its distributions are equivalent to firm-to-consumer transactions to recover the complete distribution of firm sales by product.

Every firm-to-firm transaction reports a "detail" column that records each product name transacted. "Details" are often firm idiosyncratic and differ from firm to firm for the same product. There are around 15 million firm "detail" pairs.

Data treatment

The data is anonymized to ensure confidentiality regarding the firm's and workers' identities. A set of filters is applied over the raw data to obtain the final data set for the empirical analysis:

(a) A firm is defined as a taxpayer with a tax ID, positive sales, positive materials, positive wage bill, and capital for any given year.

(b) Firms that hire less than two employees or capital valued below US$20 a year are dropped.

(c) All variables are winterized at 1% and 99% levels to avoid as much measurement error as possible.

Data: Product level aggregation

For the empirical part of the paper, products are aggregated from around 15 million products to 290 product codes, hence, it is necessary to create aggregated product-level output and material usage price indices by firms.

Each firm's product code-level output and intermediate goods input price indices are built using standard Tornqvist indices. This method is widely used for estimating aggregate production functions at the firm or plant level when price data is accessible at the product level following Dhyne et al. (2022) and De Roux et al. (2021)).

Data implementation

To bring the theory to the data to test the importance of within-firm AE, information on the ratio of product-level sales to cost-based Domar weights and markups is needed.

While product-level sales to cost-based Domar weights are straightforward to build with the data available, product-level markups represent a challenge.

The literature workhorse markup estimation strategy, De Loecker and Warzynski (2012), or De Loecker et al. (2016) are based on non-joint production setups.

To recover product level cost information and product level markups, this work follows Dhyne et al. (2022), which developed further on multi-product production functions assuming joint production.

Data implementation

This work estimation strategy allows firms to produce at most 5 of the 290 available product codes. Product codes are restricted to account for at least 20% of the firm's total sales. All other goods that represent less than 20% of the firm's total sales are grouped into a composite good that combines all the remaining products.

The record of product-specific prices and quantities to build price indices of the composite good is kept.

Data implementation

Number of products distribution

	290 product code	20% share rule
Mean	12	2
Median	6	2
Sd	16	1
p1	1	1
p25	2	1
p75	16	5
p90	33	5
p95	47	5
p99	73	5
Max	176	5

Multi-product Production Functions

A firm has a production possibilities set, V, that consists of a set of feasible inputs $x = (x_1, ..., x_M)$ and outputs $q = (q_1, ..., q_G)$. For any (q_g, x) the transformation function is defined as

$$q_g^* = f_g(q_g, x) \equiv \max\{q_g | (q_g, q_{-g}, x) \in V\}$$

To identify the unobserved marginal cost for each firm's product, from the (variable) cost minimization problem, firms have $N - 1$ freely variable inputs and one fixed input, capital (K).

Every firm minimizes its variables cost to produce its output vector q_i^* given the input prices vector $p_x = (p_{x1}, ..., p_{xM})$ and unobserved productivity for products, $\omega = (\omega_1, ..., \omega_G)$.

Multi-product Production Functions

Defining the Lagrangian multiplier of the cost minimization problem, λ_g, as the marginal cost of product g, the first order condition for every optimal input demand yield:

$$p_m = \lambda_g \frac{\partial f(q^*_{-g}, x, K, \omega)}{\partial x_m} \quad \forall m = 1, .., M,$$

It is possible to solve for product g marginal cost as the expenditure on production input m divided by its output elasticity (β^g_n) times product g production:

$$\lambda_g = \frac{p_m}{\frac{\partial f(q^*_{-g}, x, K, \omega)}{\partial x_m}} = \frac{p_m x^*_m}{\beta^g_m q^*_g},$$

As prices are observed, markups for every product can be computed as $\mu_g = \frac{p_g}{\lambda_g}$

Multi-product Production Functions

Firms are assumed to use a Cobb-Douglas production function with three factors: (in logs, Capital k, Labor l, and Materials m). A multi-product firm will produce physical units of product g using the following production function:

$$\log q_{gt} = \beta_0^g + \beta_k^g \log k_t + \beta_l^g \log l_t + \beta_m^g \log m_t^j + \gamma_{-g}^g \log q_{-gt} + \omega_{gt}$$

A GMM estimation is performed using control functions for the unobserved productivity terms to account for unobserved productivity, homologous to ACF with the difference of the need for instruments for q_{-g}; following Dhyne, 2022 lagged values of q_{-g} are used.

Multi-product Production Functions

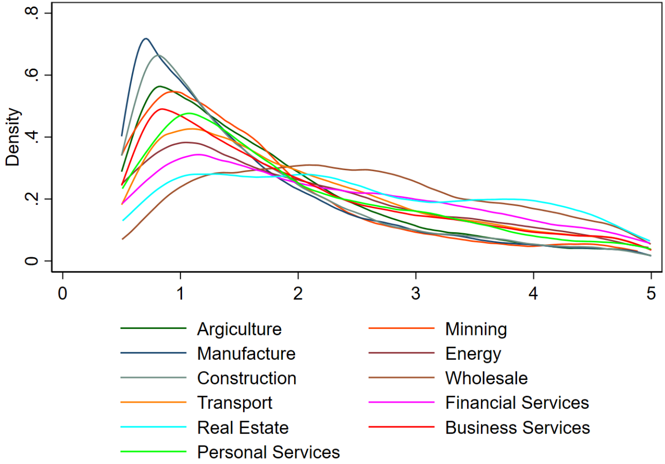
The production function is performed separately for each 290 product category; the table below shows the mean coefficients by 11 aggregate product categories

Production function estimation coefficients by product category

	β_m	β_l	β_k	γ_{-g}
Construction	0.82	0.62	0.02	-0.07
Energy	0.82	0.56	0.16	-0.04
Manufacturing	0.93	0.41	0.03	-0.09
Agriculture	0.91	0.27	0.04	-0.10
Mining	0.86	0.40	0.06	-0.20
Wholesale	1.84	0.48	0.05	-0.13
Business Services	0.89	0.65	0.04	-0.06
Financial Services	0.87	0.49	0.01	0.09
Real Estate	1.55	0.39	0.12	-0.06
Personal Services	0.90	0.56	0.01	-0.06
Transport and ICTs	0.84	0.56	0.02	-0.03

Multi-product Production Functions

Markups distribution by product category



Note: values of the bottom 5 and top 5% from the full markup distribution are excluded from the graph.

Product Level Network Distortions

The estimation procedure for Product Network Distortions, Ξ, can be summarized in 5 steps:

(a) Year by year build each element of the cost-based input-output matrix by product denoted as $\tilde{\Omega}$
- Denominator: Firm i total costs, $\sum_{j,g'} p_{jg'} x_{i,jg'} + \sum_f w_f L_{if}$
- Numerator: Expenditure on product g' from firm j used by firm i in production, $p_{jg'} x_{i,jg'}$

(b) Cost-based Leontief inverse, $\tilde{\Psi} = (I - \tilde{\Omega})^{-1}$

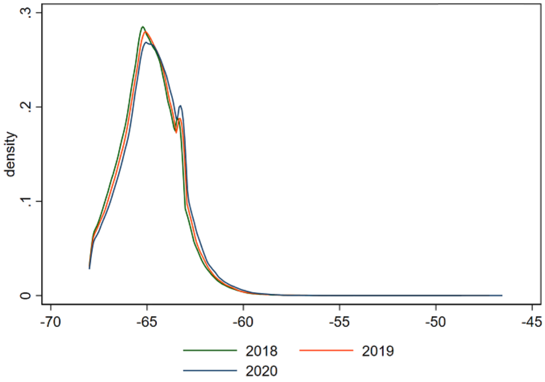
(c) b-vector: $b_{ig} = \frac{p_{ig} c_{ig}}{GDP}$

(d) Cost-based Domar weight vector: $\tilde{\lambda}' = b' \tilde{\Psi}$

(e) Recovering marginal cost from the product level production function estimation:
$\Xi_{ig} = \frac{tc_{ig}}{\tilde{\lambda}_{ig}}$

Product Level Network Distortions

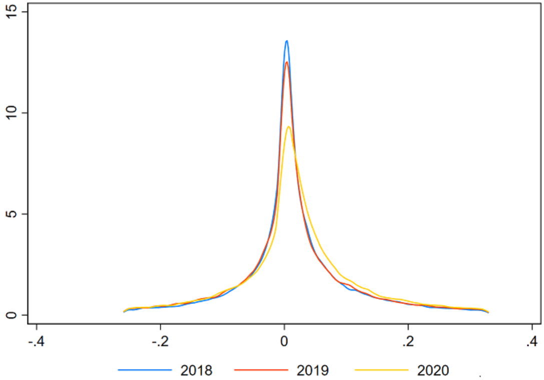
Distribution of Ξ (in logs)



- While Ξ distribution does not vary in time, product level distortions are highly heterogeneous; some products can potentially have high aggregate distortionary effects.

Product Level Network Distortions

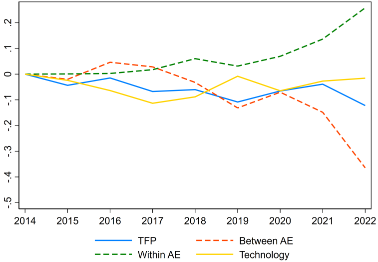
$Cov_{s_i}(\xi_i, \mathrm{dlog} p_i)$ distribution



- The dispersion of the covariance implies misallocation within firms and heterogeneity across those firms. The fact that the distribution varies yearly also suggests a possible contribution to the business cycle.

Aggregate TFP growth

$$d\log TFP = \underbrace{\sum_i \tilde{\lambda}_i d\log A_i}_{\text{Technology}} \underbrace{- \sum_i \tilde{\lambda}_i d\log \mu_i - \sum_f \tilde{\Lambda}_{(f)} d\log \Lambda_f}_{\text{Between-Firm AE}} \underbrace{- \sum_i \tilde{\lambda}_i Cov_{s_i}(d\log p_i, \xi_i)}_{\text{Within-Firm AE}}$$



- While the main component explaining TFP growth is the allocation of resources between firms (red dashed line), within-firm AE or how firms choose their product portfolio (green dashed line) plays a relevant role in explaining TFP changes.
- Within-firm AE moves opposite to between-firm AE, especially after Covid; resources are increasingly misallocated between firms, but the firm's product portfolio choices favor total AE.

Conclusion: Unpacking within and between AE is Key for Development

In a context of distortions, aggregate productivity growth combines technology and within and between AE.

Understanding the distortions behind aggregate productivity growth can shed some light on how to target policy to enhance development.

- Especially in the context of productivity stagnation, and after COVID.

$\Longrightarrow$ Allocative efficiency explains the bulk of productivity growth in Chile since the Pandemic.

Two additional steps we are still working on:

- Micro-patterns decomposition explaining within and between AE.
- Counterfactual to the frontier.