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Aggregating Distortions in Networks with Multi-Product Firms

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Motivation

- Aggregate TFP is the main source of income differences between countries
  - It is also a measure of our ignorance, ie, it is a residual

- In distorted economies:

  Δ Aggregate TFP = Δ Technology + Δ Allocative Efficiency

- Assumption: Single good firms $\implies$ no inefficiency within firms

- What if firms are multi-product objects that engage in joint production?

Motivation: Aggregate TFP as a Residual

- Central Banks around the world have been estimating aggregate TFP growth for at least 60 years
  - With better data now, but same theory

- What are the lessons derived from a residual?
  - It is informative but hard to interpret

- We make an effort to shrink the TFP residual component using product level data
  - By unpacking between vs. within firm forces shaping aggregate TFP growth

This Paper: Theory

- Growth accounting in economies with distortions
  - Baqaee & Farhi (2020) + Multi-product joint production firms
  - Heterogeneous wedges (markups) shape allocative efficiency

- Multi-product joint production firms influence aggregate TFP growth
  - Non-parametric sufficient statistic to include its TFP growth effect

This Paper: Measurement and Application

- Measurement for Chile: Product-level markups + firm-to-firm product level IO matrix
    - Markups: Off-the-shelf estimation + Control for input and output prices
    - IO matrix: Captures direct and indirect role product-level distortions: "Within firm Allocative Efficiency"

- Application: Aggregate TFP growth decomposition of one decade for Chile
    - During Covid and successive high inflation years, between firm Allocative Efficiency is decreasing while within firm Allocative Efficiency is increasing.

Literature

- Aggregation in (in)efficient production networks
  - Liu (2019); Bigio and La'O (2020); Baqaee and Farhi (2020); Baqaee et al. (2023); Kikkawa (2022); Osotimehin and Popov (2023); Davila and Schaab (2023); Hulten (1978)

- Joint production
  - Hall (1973, 1988); Powell and Gruen (1968); Diewert (1971); Lau (1972); Ding (2023); Carrillo et al. (2023); Argente et al. (2021); Boehm and Oberfield (2023)
  - Estimation: Dhyne et al. (2017, 2022); Valmari (2023); Cairncross and Morrow (2023)

- Multi-product firms in General Equilibrium
  - Klette and Kortum (2004); Bernard et al. (2010); Mayer et al. (2014)

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Testing non-joint production

- Common view of multi-product firms: A collection of independent products (Klette and Kortum (2004); Bernard et al. (2010))

- Test for non-joint production in the spirit of Ding (2023)
  - **Null hypothesis:** sales of a given product are unaffected by demand shocks to the firm's other products
  - B2B transactions data + Covid lockdown as a negative demand shock

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Testing non-joint production: Covid lock-downs

Data

- Monthly B2B product level transactions for Chilean firms (Jan-Apr 2020)
  - value, product id, quantity, price, buyer and seller location

Demand shocks

- Main product: Highest sales in Jan-Feb 2020
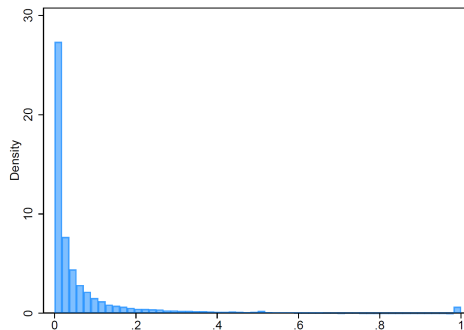- Shock to non-main products of firm i:

$$\phi_{it}^{-m} = \sum_c \left(\text{Share}_{ic,\text{Jan-Feb 2020}}^{-m} \right) \times (\text{Lockdown}_{c,t})$$

- Lockdown by destination county c: $\text{Lockdown}_{c,t} = \{0, 1\}$
- $\text{Share}_{ic,\text{Jan-Feb}}^{-m}$: Pre-Covid sales share among non-main products of firm i to county c

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Lockdown area and shock to non-main products

(a) Lockdown counties April 2020

(b) Non-main product lockdown share: $\phi_{i\text{April}}^{-m}$

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Testing non-joint production: Estimation Strategy

$$\log Y_{it}^m = \alpha + \beta_1 \phi_{it}^{-m} + \beta_X X_{it} + \varepsilon_{it}$$

where $Y \in$ [Sales, Quantity] conditioning i to be a non-lockdown area

- X: input price index for firm i, and fixed effects
- If ϕ_{it}^{-m} is significant, the null hypothesis of non-joint production is rejected

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Testing non-joint production: Results

	ln S_{imt}				ln Q_{imt}			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_1	-0.147***	-0.267***	-0.263***	-0.172***	-0.151***	-0.224***	-0.277***	-0.196***
Product FE	✓	✓	✓	✓	✓	✓	✓	✓
Firm FE	X	✓	✓	✓	X	✓	✓	✓
Destination county FE	X	X	✓	✓	X	X	✓	✓
Month-290 product code FE	X	X	X	✓	X	X	X	✓
Observations	1,555,795	1,518,448	1,518,448	1,518,448	1,556,544	1,519,198	1,519,198	1,519,198
R^2	0.506	0.591	0.591	0.613	0.682	0.759	0.759	0.771
*** p<0.01, ** p<0.05, * p<0.1								

• Rejecting the null hypothesis motivates the need for joint production setups

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Joint Production

Let $t(q,x)$ be a transformation function:

$$t(\mathbf{q},\mathbf{x}) = 0,$$

$\mathbf{x}$: input vector, $\mathbf{q}$; output vector

Assumptions:

(a) CRS: $t(\mathbf{q},\mathbf{x}) = 0$ imples $t(\lambda\mathbf{q}, \lambda\mathbf{x}) = 0$

(b) Separability between Input and output bundles: $t(\mathbf{q},\mathbf{x}) = -g(\mathbf{q}) + f(\mathbf{x})$

   Example Constant-Elasticity of Transformation and CES Input (CET-CES):

$$\underbrace{\left(\sum_g q_{ig}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}}_{\text{Output bundle}} = A \underbrace{\left(\omega_L L^{\frac{\sigma-1}{\sigma}} + \omega_K K^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}_{\text{Input Bundle}}$$

Setup for Joint Production within Supply Chains

- Firm $i \in N$ produces product $g \in G$ grouped in vector q_i, g
- Use intermediate inputs $g' \in G$ form firm $j \in N$ grouped in vector $x_{i,g'}$ and factors as inputs
- Joint production with CRS and separability:

$$
F_i^Q \left(\left\{ \underbrace{q_{ig}}_{\text{outputs}} \right\}_{i \in N, g \in G} \right) = A_i F_i^X \left(\left\{ \underbrace{x_{i,jg'}}_{\text{Intermediate product } g' \text{ from j}} \right\}_{j \in N, g' \in G}, L_i, K_i \right),
$$

- Cost minimization charging product-level markups

$$
p_{ig} = mc_{ig} \cdot \mu_{ig}
$$

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Households

- Representative household:

$$U = U(q_{ig}, ..., q_{NG})$$

- Budget constraint

$$\sum_{i \in N} \sum_{g \in G} p_{ig} q_{ig} = \sum_{f \in \{L,K\}} w_f L_f + \sum_{i \in N} \sum_{g \in G} (1 - 1/\mu_{ig}) p_{ig} q_{ig} + T$$

- Resource Constraint:

$$q_{ig} = q_{ig}^H + \sum_{j \in N} x_{jig}, \quad \sum_{i \in N} L_i = L, \quad \sum_{i \in N} K_i = K$$

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GDP by Explicitly Aggregating Firm-Level Data

- GDP definition

$$GDP = \sum_{i \in N} \sum_{g \in g} p_{ig} q_{ig}$$

- Firm GDP shares

$$b_i = \begin{cases} \frac{p_{ig} q_{ig}}{GDP} & \text{if } i \in N, g \in G \\ 0 & \text{otherwise} \end{cases}$$

- Factor GDP shares

$$\Lambda_L = \frac{w_L L}{GDP}, \quad \Lambda_K = \frac{w_k K}{GDP}$$

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Input-Output Objects (1)

- Cost-based input-output matrix ($\tilde{\Omega}$) of dimensions $(NG+F)^2$.

$$\tilde{\Omega}_{ig,jg'} = \frac{\text{Value of product g' from firm j used by firm i}}{\text{Firm i total cost}} = \frac{p_{jg'} x_{i,jg'}}{\sum_{j,g'} p_{jg'} x_{i,jg'} + \sum_f w_f L_{if}}$$

- Separability implies that shares are common for all g within i
- Cost-based Leontief inverse matrix ($\tilde{\Psi}$) accounts for products' direct and indirect cost exposures through supply chains.

$$\tilde{\Psi} \equiv (I - \tilde{\Omega})^{-1} = I + \tilde{\Omega} + \tilde{\Omega}^2 + \dots$$

Input Output Objects (2)

- Cost-based Domar weights $\tilde{\lambda}$ ($\tilde{\Lambda}$ for factors)

$$\tilde{\lambda}' = b'\tilde{\Psi}$$

- Cost-based Domar weights capture the impact of product-level cost shocks on GDP.
- Firm-level cost-based Domar weight and within-firm product share

$$\tilde{\lambda}_i = \sum_g \tilde{\lambda}_{ig}, \qquad\qquad s_{ig} = \frac{\tilde{\lambda}_{ig}}{\sum_g \tilde{\lambda}_{ig}}$$

Network product distortion

Network product distortion

$$\Xi_{ig} = \frac{sales_{ig}/\mu_{ig}}{\tilde{\lambda}_{ig}}$$

- Summarizes the cumulative downstream distortion of product g from firm i
- In the presence of markups, the impact of product-level cost shocks on GDP ($\tilde{\lambda}_{ig}$) exceeds markup-adjusted product sales (costs) $\implies \Xi_{ig} < 1$

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Relative network product distortion

Relative network product distortion

$$\xi_{ig} = \frac{\Xi_{ig}}{\Xi_i}$$

- $\Xi_i = \sum_g (sales_{ig}/\mu_{ig}) / \sum_g \tilde{\lambda}_{ig}$ is firm i average distortion

- Ranks product distortions within firms

- As smaller ξ_{ig} is, good g is relatively more distorted within firm i because good g is relatively more under-produced than the average firm i good

Within-firm AE

$$\text{Within Firm AE} = \sum_{i \in N} \tilde{\lambda}_i Cov_{s_i} \left(d\log \boldsymbol{p}_i, \underbrace{\boldsymbol{\xi}_i}_{\text{Product Distortion}} \right)$$

where $d\log \boldsymbol{p}_i = (d\log p_{i1}, ..., d\log p_{iG})$ and $\boldsymbol{\xi}_i = (\xi_{i1}, ..., \xi_{iG})$

- $Cov_{s_i}(d\log \boldsymbol{p}_i, \boldsymbol{\xi}_i) > 0$: within-firm Allocative Efficiency increases
- ↓ price of more distorted goods $\Rightarrow$ ↓ accumulated wedge in the downstream supply chain
- Sum up firm-level covariances using cost-based Domar weights, $\tilde{\lambda}_i$

Between-firm Allocative Efficiency

$$\Delta \text{ Between-Firm Allocative Efficiency} = \underbrace{-\sum_f \tilde{\Lambda}_{(f)} d\log \Lambda_f}_{(a)} \underbrace{-\sum_i \tilde{\lambda}_i d\log \mu_i}_{(b)}$$

- Baqaee and Farhi (2020)

  (a) If $(a) < 0$ resources are reallocated to the more monopolized (underproduced) part of the economy $\Rightarrow \Delta^-$ Factor shares $\Rightarrow \Delta^+$ *TFP*

  (b) Factor reallocation due to markup changes must be discounted to capture the pure factor changes effect

Growth Accounting complete formula

Combining between firm and within-firm Allocative Efficiency:

$$\Delta TFP = \underbrace{\sum_i \tilde{\lambda}_i d\log A_i}_{\text{Technology}} - \underbrace{\sum_i \tilde{\lambda}_i d\log \mu_i - \sum_f \tilde{\Lambda}_{(f)} d\log \Lambda_f}_{\text{Between-Firm AE}} + \underbrace{\sum_i \tilde{\lambda}_i Cov_{s_i}(d\log \boldsymbol{p}_i, \boldsymbol{\xi}_i)}_{\text{Within-Firm AE}}$$

- If firms are single-product, within-firm AE converges to zero; Baqaee and Farhi (2020).
- Without markups, between-firm AE also converges to zero, TFP changes are due to technology; Hulten (1978).

Data and measurement: overview

- Product-Level and Domar weights, $\tilde{\lambda}_i$: from B2B transaction (details)

- Product Network Distortion ξ_{ig}, markup: joint production from Dhyne, Petrin, Smeets & Warzynski (2022) (markup estimation)

- Factor Shares, $d \log p$ and TFP: observable

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Product level aggregation

- Products are aggregated from around 15 million products to 290 product codes
  - Product-level output and material usage price indices by firms are built using standard Tornqvist indices

- We allow firms to produce at most 5 of the 290 available product codes
  - Product codes are restricted to account for at least 20% of the firm's total sales
  - All other goods that represent less than 20% are grouped into a composite good that combines all the remaining products.

Product distribution

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TFP decomposition after Covid: Only Between firm AE

$$d \log TFP = \underbrace{\sum_i \tilde{\lambda}_i d \log A_i}_{\text{Residual}} - \underbrace{\sum_f \tilde{\Lambda}_f d \log \Lambda_f - \sum_i \tilde{\lambda}_i d \log \mu_i}_{\text{Between-Firm AE}} + \underbrace{\sum_i \tilde{\lambda}_i Cov_{s_i} (d \log p_i, \xi_i)}_{\text{Within-Firm AE}}$$
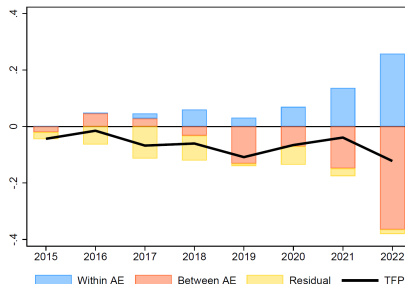
TFP cumulative change (2014=0)

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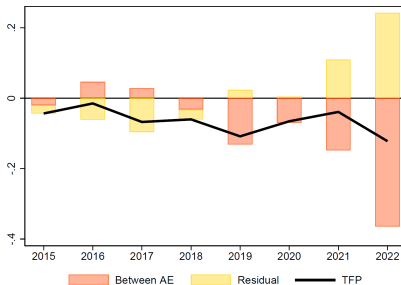
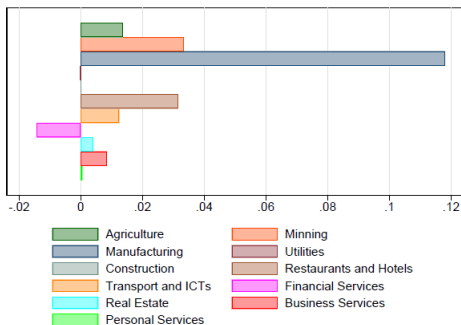# TFP decomposition after Covid: Full decomposition

$$d \log TFP = \underbrace{\sum_i \tilde{\lambda}_i d \log A_i}_{\text{Residual}} - \underbrace{\sum_f \tilde{\Lambda}_f d \log \Lambda_f - \sum_i \tilde{\lambda}_i d \log \mu_i}_{\text{Between-Firm AE}} + \underbrace{\sum_i \tilde{\lambda}_i Cov_{s_i}(d \log \boldsymbol{p}_i, \boldsymbol{\xi}_i)}_{\text{Within-Firm AE}}$$

TFP cumulative change (2014=0)

Within AE   Between AE   Residual   ▬▬ TFP

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Within firm AE: Covid + high inflation (2019-2022)

Aggregate cumulative contribution by product categories of firm's main product (2019=0)

Legend:
- Agriculture
- Manufacturing
- Construction
- Transport and ICTs
- Real Estate
- Personal Services
- Minning
- Utilities
- Restaurants and Hotels
- Financial Services
- Business Services

Contribution of firm with product $c = \sum_{i \in N_c} \tilde{\lambda}_i Cov_{s_i} \left(d \log \boldsymbol{p}_i, \boldsymbol{\xi}_i \right)$

Conclusion

- TFP growth being a residual is informative but hard to interpret
  - Reducing the residual by adding within firm wedges might inform about forces driving TFP growth

- Allocative efficiency explains the bulk of aggregate TFP growth in chile after 2019
  - While resources are increasingly missalocated between firms, within firm resources allocation improved in stressed parts of the cycle

Data

(a) Sales, materials, investment: F29 (2015-2022)

(b) Wage bill, employment: DJ1887 (2015-2022)

(c) Capital: F22 (2015-2022)
- Capital stock using perpetual inventory methods combining capital stock with investment
- UCC: interest rate - inflation expectation + depreciation rate from LA-Klems database + external financing premium (5 percent)

(d) Product, I-O matrices and output and input prices: F2F electronic receipts (2015-2022)

(e) Official deflectors for aggregate real variables (back)

Data cleaning

- The final sample does not include firms with a missing variable of sales, capital, wage bill, or materials

- Winzorized labor, capital and materials shares over sales at 1% of both tails of the distribution

- Firms with negative value added (sales minus materials), less than two workers or capital less than 10.000 CLP (USD 15) are excluded

Markup estimation

- Following Dhyne, Petrin, Smeets & Warzynski (2022)

- Cobb-Douglas production function using three inputs (K, L, M) and (aggregated) other outout (y^{-g}) (lower case variables denote logs)

$$y_t^g = \beta_0^g + \beta_K^g k_t + \beta_L^g l_t + \beta_M^g m_t + \gamma_{-g}^g \, y_t^{-g} + \omega_{gt}$$

- GMM Estimation was performed separately by 290 product codes

- Time invariant output elasticities recover product level markup

$$\mu_g = \beta_M^g \, \frac{p_g Y_g^*}{p_m M_n^*}$$

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Markup estimation

- Mean coefficients by 11 aggregate product categories

	β_m	β_l	β_k	γ_{-g}
Construction	0.82	0.62	0.02	-0.07
Energy	0.82	0.56	0.16	-0.04
Manufacturing	0.93	0.41	0.03	-0.09
Agriculture	0.91	0.27	0.04	-0.10
Mining	0.86	0.40	0.06	-0.20
Wholesale	1.84	0.48	0.05	-0.13
Business Services	0.89	0.65	0.04	-0.06
Financial Services	0.87	0.49	0.01	0.09
Real Estate	1.55	0.39	0.12	-0.06
Personal Services	0.90	0.56	0.01	-0.06
Transport and ICTs	0.84	0.56	0.02	-0.03

Markup distribution

- Markup distribution by product category excluding 1 and 99 percentiles
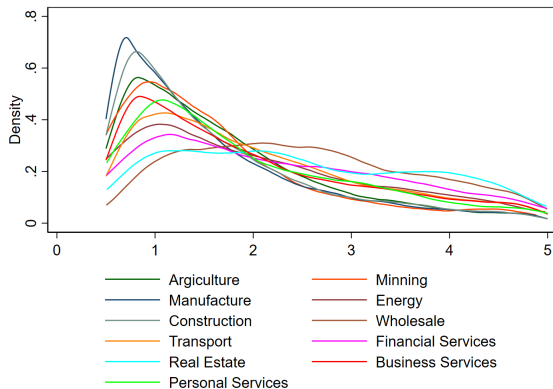
Number of Products

Number of products distribution

	290 product code	20% share rule
Mean	12	2
Median	6	2
Sd	16	1
p1	1	1
p25	2	1
p75	16	5
p90	33	5
p95	47	5
p99	73	5
Max	176	5

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Within firm AE: Covid periods (2019~2021)

- Aggregate contribution by product categories of firm's main product

$$\text{Contribution of firm with product } c = \sum_{i \in N_C} \tilde{\lambda}_i Cov_{s_i}\left(-d\log \boldsymbol{p}_i, \boldsymbol{\xi}_i\right)$$

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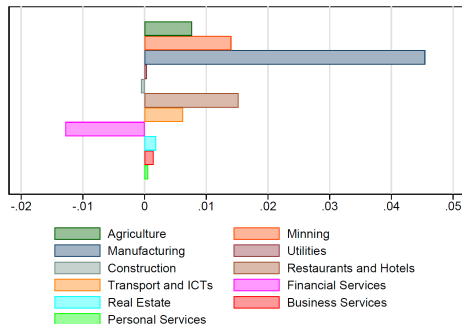
Within firm AE: High-inflation periods (2021~2022)

- Aggregate contribution by product categories of firm's main product

$$\text{Contribution of firm with product } c = \sum_{i \in N_C} \tilde{\lambda}_i Cov_{s_i} \left(-d \log \boldsymbol{p}_i, \boldsymbol{\xi}_i \right)$$

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