Aggregating Distortions in Networks with Multi-Product Firms

Yasutaka Koike-Mori
UCLA

Antonio Martner
UCLA

UCLA IO proseminar
May 29, 2024

The views expressed are those of the authors and do not necessarily represent the views of the Central Bank of Chile or its board members

Motivation

- Aggregate TFP is the main source of income differences between countries
  - It is also a measure of our ignorance, ie, it is a residual
- In distorted economies:

  Δ Aggregate TFP = Δ Technology + Δ Allocative Efficiency

- Assumption: Single good firms $\implies$ no inefficiency within firms
- What if firms are multi-product objects that engage in joint production?

Motivation: Aggregage TFP as a Residual

- Central Banks around the world have been estimating aggregate TFP growth for at least 60 years
  - With better data now, but same theory
- What are the leasons derived from a residual?
  - It is informative but hard to interpret
- We make an effort to shrink the TFP residual component using product level data
  - By unpacking between vs. within firm forces shaping aggregate TFP growth

This Paper: Theory

- Growth accounting in economies with distortions
  - Baqaee & Farhi (2020) + Multi-product joint production firms
  - Heterogenous wedges (markups) shape allocative efficiency
- Multi-product joint production firms influence aggregate TFP growth
  - Non-parametric sufficient statistic to include its TFP growth effect

This Paper: Measurement and Application

- Measurement for Chile: Product-level markups + firm-to-firm product level IO matrix
  - Markups: Off-the-shelf estimation + Control for input and output prices
  - IO matrix: Captures direct and indirect role product-level distortions: “Within firm Allocative Efficiency"
- Application: Aggregate TFP growth decomposition of one decade for Chile
  - During Covid and successive high inflation years, betwen firm Allocative Efficiency is decreasing while within firm Allocative Eficciency is increasing.

Literature

- Aggregation in (in)efficient production networks
  - Liu (2019); Bigio and La'O (2020); Baqaee and Farhi (2020); Baqaee et al. (2023); Kikkawa (2022); Osotimehin and Popov (2023); Davila and Schaab (2023); Hulten (1978)
- Joint production
  - Hall (1973, 1988); Powell and Gruen (1968); Diewert (1971); Lau (1972); Ding (2023); Carrillo et al. (2023); Argente et al. (2021); Boehm and Oberfield (2023)
  - Estimation: Dhyne et al. (2017, 2022); Valmari (2023); Cairncross and Morrow (2023)
- Multi-product firms in General Equilibrium
  - Klette and Kortum (2004); Bernard et al. (2010); Mayer et al. (2014)

Testing non-joint production

- Common view of multi-product firms: A collection of independent products (Klette and Kortum (2004); Bernard et al. (2010))
- Test for non-joint production in the spirit of Ding (2023)
    - **Null hypothesis:** sales of a given product are unaffected by demand shocks to the firm's other products
    - B2B transactions data + Covid lockdown as a negative demand shock

Testing non-joint production: Covid lockdowns

Data

- Monthly B2B product level transactions for Chilean firms (Jan-Apr 2020)
  - value, product id, quantity, price, buyer and seller location

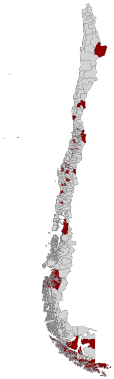
Demand shocks

- Main product: Highest sales in Jan-Feb 2020
- Shock to non-main products of firm *i*:

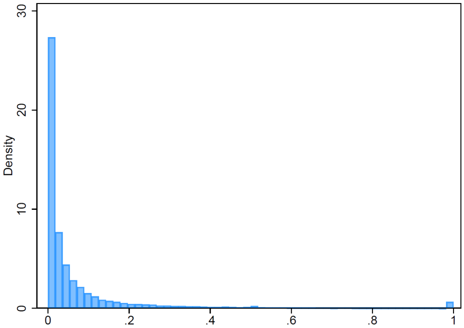
$$\phi_{it}^{-m} = \sum_{c} \left(\text{Share}_{ic,\text{Jan-Feb 2020}}^{-m} \right) \times \left(\text{Lockdown}_{c,t} \right)$$

- Lockdown by destination county *c*: $\text{Lockdown}_{c,t} = \{0, 1\}$
- $\text{Share}_{ic,\text{Jan-Feb}}^{-m}$: Pre-Covid sales share among non-main products of firm *i* to county *c*

Lockdown area and shock to non-main products



(a) Lockdown counties April 2020



(b) Non-main product lockdown share: $\phi_{i\text{April}}^{-m}$

Testing non-joint production: Estimation Strategy

$$\log Y_{it}^{m} = \alpha + \beta_1 \phi_{it}^{-m} + \beta_X X_{it} + \varepsilon_{it}$$

where $Y \in$ [Sales, Quantity] conditioning i to be a non-lockdown area

- X: input price index for firm i, and fixed effects
- If ϕ_{it}^{-m} is significant, the null hypothesis of non-joint production is rejected

Testing non-joint production: Results

	$\ln S_{imt}$				$\ln Q_{imt}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_1	-0.147***	-0.267***	-0.263***	-0.172***	-0.151***	-0.224***	-0.277***	-0.196***
Product FE	✓	✓	✓	✓	✓	✓	✓	✓
Firm FE	X	✓	✓	✓	X	✓	✓	✓
Destination county FE	X	X	✓	✓	X	X	✓	✓
Month-290 product code FE	X	X	X	✓	X	X	X	✓
Observations	1,555,795	1,518,448	1,518,448	1,518,448	1,556,544	1,519,198	1,519,198	1,519,198
R^2	0.506	0.591	0.591	0.613	0.682	0.759	0.759	0.771

*** p<0.01, ** p<0.05, * p<0.1

- Rejecting the null hypothesis motivates the need for joint production setups

Joint Production

Let $t(q,x)$ be a transformation function:

$$t(\mathbf{q},\mathbf{x}) = 0,$$

$\mathbf{x}$:input vector, $\mathbf{q}$; output vector

Assumptions:

(a) CRS: $t(\mathbf{q},\mathbf{x}) = 0$ imples $t(\lambda\mathbf{q},\lambda\mathbf{x}) = 0$

(b) Separability between Input and output bundles: $t(\mathbf{q},\mathbf{x}) = -g(\mathbf{q}) + f(\mathbf{x})$

Example Constant-Elasticity of Transformation and CES Input (CET-CES):

$$\underbrace{\left(\sum_g q_{ig}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}}_{\text{Output bundle}} = \underbrace{A\left(\omega_L L^{\frac{\sigma-1}{\sigma}} + \omega_K K^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}_{\text{Input Bundle}}$$

Setup for Joint Production within Supply Chains

- Firm $i \in N$ produces product $g \in G$ grouped in vector q_i, g
- Use intermediate inputs $g' \in G$ form firm $j \in N$ grouped in vector $x_{i,g'}$ and factors as inputs
- Joint production with CRS and separability:

$$F_i^Q\left(\left\{\underbrace{q_{ig}}_{\text{outputs}}\right\}_{i\in N, g\in G}\right) = A_i F_i^X\left(\left\{\underbrace{x_{i,jg'}}_{\text{Intermediate product } g' \text{ from j}}\right\}_{j\in N, g'\in G}, L_i, K_i\right),$$

- Cost minimization charging product-level markups

$$p_{ig} = mc_{ig} \cdot \mu_{ig}$$

Households

- Representative household:

$$U = U\left(q_{ig}, \ldots, q_{NG}\right)$$

- Budget constraint

$$\sum_{i \in N} \sum_{g \in G} p_{ig} q_{ig} = \sum_{f \in \{L,K\}} w_f L_f + \sum_{i \in N} \sum_{g \in G} \left(1 - 1/\mu_{ig}\right) p_{ig} q_{ig} + T$$

- Resource Constraint:

$$q_{ig} = q_{ig}^H + \sum_{j \in N} x_{jig}, \quad \sum_{i \in N} L_i = L, \quad \sum_{i \in N} K_i = K$$

GDP by Explicitly Aggregating Firm-Level Data

- GDP definition

$$GDP = \sum_{i \in N} \sum_{g \in g} p_{ig} q_{ig}$$

- Firm GDP shares

$$b_i = \begin{cases} \frac{p_{ig} q_{ig}}{GDP} & \text{if } i \in N, g \in G \\ 0 & \text{otherwise} \end{cases}$$

- Factor GDP shares

$$\Lambda_L = \frac{w_L L}{GDP}, \quad \Lambda_K = \frac{w_k K}{GDP}$$

Input-Output Objects (1)

- Cost-based input-output matrix ($\tilde{\Omega}$) of dimensions $(NG+F)^2$.

$$\tilde{\Omega}_{ig,jg'} = \frac{\text{Value of product g' from firm j used by firm i}}{\text{Firm i total cost}} = \frac{p_{jg'}x_{i,jg'}}{\sum_{j,g'} p_{jg'}x_{i,jg'} + \sum_f w_f L_{if}}$$

- Separability implies that shares are common for all g within i
- Cost-based Leontief inverse matrix ($\tilde{\Psi}$) accounts for products' direct and indirect cost exposures through supply chains.

$$\tilde{\Psi} \equiv (I - \tilde{\Omega})^{-1} = I + \tilde{\Omega} + \tilde{\Omega}^2 + \dots$$

Input Output Objects (2)

- Cost-based Domar weights $\tilde{\lambda}$ ($\tilde{\Lambda}$ for factors)

$$\tilde{\lambda}' = b'\tilde{\Psi}$$

- Cost-based Domar weights capture the impact of product-level cost shocks on GDP.
- Firm-level cost-based Domar weight and within-firm product share

$$\tilde{\lambda}_i = \sum_g \tilde{\lambda}_{ig}, \qquad s_{ig} = \frac{\tilde{\lambda}_{ig}}{\sum_g \tilde{\lambda}_{ig}}$$

Network product distortion

Network product distortion

$$\Xi_{ig} = \frac{sales_{ig}/\mu_{ig}}{\tilde{\lambda}_{ig}}$$

- Summarizes the cumulative downstream distortion of product g from firm i
- In the presence of markups, the impact of product-level cost shocks on GDP ($\tilde{\lambda}_{ig}$) exceeds markup-adjusted product sales (costs) $\implies \Xi_{ig} < 1$

Relative network product distortion

Relative network product distortion

$$\xi_{ig} = \frac{\Xi_{ig}}{\Xi_i}$$

- $\Xi_i = \sum_g (sales_{ig}/\mu_{ig}) / \sum_g \tilde{\lambda}_{ig}$ is firm i average distortion
- Ranks product distortions within firms
- As smaller ξ_{ig} is, good g is relatively more distorted within firm i because good g is relatively more underproduced than the average firm i good

Within-firm AE

$$\text{Within Firm AE} = \sum_{i \in N} \tilde{\lambda}_i Cov_{s_i}\left(d\log \boldsymbol{p}_i, \underbrace{\boldsymbol{\xi}_i}_{\text{Product Distortion}} \right)$$

where $d\log \boldsymbol{p}_i = (d\log p_{i1}, \ldots, d\log p_{iG})$ and $\boldsymbol{\xi}_i = (\xi_{i1}, \ldots, \xi_{iG})$

- $Cov_{s_i}(d\log \boldsymbol{p}_i, \boldsymbol{\xi}_i) > 0$: within-firm Allocative Efficiency increases
- $\downarrow$ price of more distorted goods $\Rightarrow \downarrow$ accumulated wedge in the downstream supply chain
- Sum up firm-level covariances using cost-based Domar weights, $\tilde{\lambda}_i$

Between-firm Allocative Efficiency

$$\Delta \text{ Between-firm Allocative Efficiency} = \underbrace{-\sum_{f\in\{L,K\}} \tilde{\Lambda}_{f,t-1} \frac{d\Lambda_f^D}{\Lambda_f}}_{(a)} \underbrace{-\sum_{i\in D} \tilde{\lambda}_{i,t-1}\, d\log \mu_i}_{(b)}$$

- Baqaee and Farhi (2020)
  - (a) If $(a) < 0$ resources are reallocated to the more monopolized (underproduced) part of the economy $\Rightarrow \Delta^-$ Factor shares $\Rightarrow \Delta^+ TFP$
  - (b) Factor reallocation due to markup changes must be discounted to capture the pure factor changes effect

Growth Accounting complete formula

Combining between firm and within-firm Allocative Efficiency:

$$\Delta TFP = \underbrace{\sum_i \tilde{\lambda}_i d\log A_i}_{\text{Technology}} \underbrace{- \sum_i \tilde{\lambda}_i d\log \mu_i - \sum_f \tilde{\Lambda}_{(f)} d\log \Lambda_f}_{\text{Between-Firm AE}} + \underbrace{\sum_i \tilde{\lambda}_i Cov_{s_i}\left(d\log \boldsymbol{p}_i, \xi_i\right)}_{\text{Within-Firm AE}}$$

- If firms are single-product, within-firm AE converges to zero; Baqaee and Farhi (2020).
- Without markups, between-firm AE also converges to zero, TFP changes are due to technology; Hulten (1978).

Data and measurement: overview

- Product-Level and Domar weights, $\tilde{\lambda}_i$: from B2B transaction details
- Product Network Distortion ξ_{ig}, markup: joint production from Dhyne, Petrin, Smeets & Warzynski (2022) markup estimation
- Factor Shares, $d \log p$ and TFP: observable

Product level aggregation

- Products are aggregated from around 15 million products to 290 product codes
  - Product-level output and material usage price indices by firms are built using standard Tornqvist indices

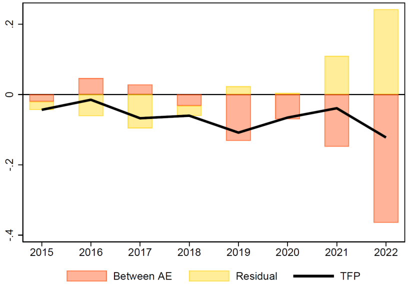
- We allow firms to produce at most 5 of the 290 available product codes
  - Product codes are restricted to account for at least 20% of the firm's total sales
  - All other goods that represent less than 20% are grouped into a composite good that combines all the remaining products.

Product distribution

TFP decomposition after Covid: Only Between firm AE

$$d\log TFP = \underbrace{\sum_i \tilde{\lambda}_i d\log A_i}_{\text{Residual}} \underbrace{- \sum_f \tilde{\Lambda}_f d\log \Lambda_f - \sum_i \tilde{\lambda}_i d\log \mu_i}_{\text{Between-Firm AE}} + \underbrace{\sum_i \tilde{\lambda}_i Cov_{s_i}\left(d\log \boldsymbol{p}_i, \xi_i\right)}_{\text{Within-Firm AE}}$$

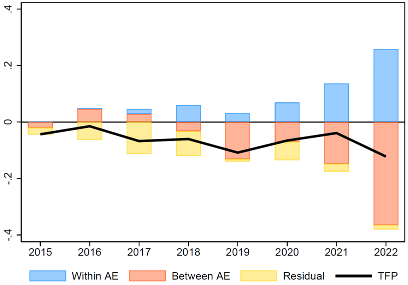
TFP cumulative change (2014=0)



TFP decomposition after Covid: Full decomposition

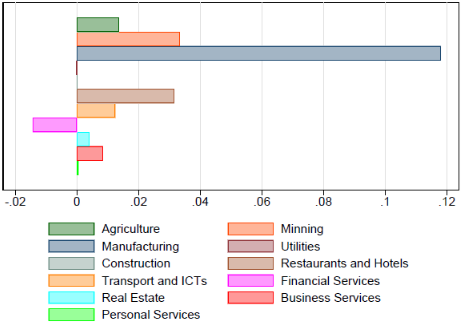
$$d\log TFP = \underbrace{\sum_i \tilde{\lambda}_i d\log A_i}_{\text{Residual}} \underbrace{- \sum_f \tilde{\Lambda}_f d\log \Lambda_f - \sum_i \tilde{\lambda}_i d\log \mu_i}_{\text{Between-Firm AE}} + \underbrace{\sum_i \tilde{\lambda}_i Cov_{s_i}\left(d\log \boldsymbol{p}_i, \xi_i\right)}_{\text{Within-Firm AE}}$$

TFP cumulative change (2014=0)



Within firm AE: Covid + high inflation (2019-2022)

Aggregate cumulative contribution by product categories of firm's main product (2019=0)



$$\text{Contribution of firm with product } c = \sum_{i \in N_C} \tilde{\lambda}_i Cov_{s_i}\left(d \log \boldsymbol{p}_i, \xi_i\right)$$

Conclusion

- TFP growth being a residual is informative but hard to interpret
  - Reducing the residual by adding within firm wedges might inform about forces driving TFP growth
- Allocative efficiency explains the bulk of aggregate TFP growth in chile after 2019
  - While resources are increaingly missalocated between firms, within firm resources allocation improoved in stressed parts of the cycle

Data

(a) Sales, materials, investment: F29 (2015-2022)

(b) Wage bill, employment: DJ1887 (2015-2022)

(c) Capital: F22 (2015-2022)
- Capital stock using perpetual inventory methods combining capital stock with investment
- UCC: interest rate - inflation expectation + depreciation rate from LA-Klems database + external financing premium (5 percent)

(d) Product, I-O matrices and output and input prices: F2F electronic receipts (2015-2022)

(e) Official deflectors for aggregate real variables back

Data cleaning

- The final sample does not include firms with a missing variable of sales, capital, wage bill, or materials
- Winzorized labor, capital and materials shares over sales at 1% of both tails of the distribution
- Firms with negative value added (sales minus materials), less than two workers or capital less than 10.000 CLP (USD 15) are excluded

Markup estimation

- Following Dhyne, Petrin, Smeets & Warzynski (2022)
- Cobb-Douglas production function using three inputs (K, L, M) and (aggregated) other outout (y^{-g}) (lower case variables denote logs)

$$y_t^g = \beta_0^g + \beta_K^g k_t + \beta_L^g l_t + \beta_M^g m_t + \gamma_{-g}^g \, y_t^{-g} + \omega_{gt}$$

- GMM Estimation was performed separately by 290 product codes
- Time invariant output elasticities recover product level markup

$$\mu_g = \beta_M^g \, \frac{p_g Y_g^*}{p_m M_n^*}$$

back

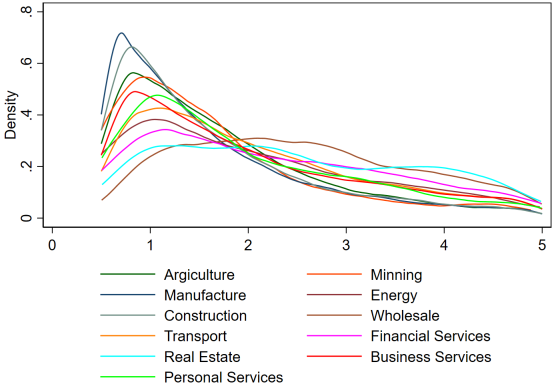
Markup estimation

- Mean coefficients by 11 aggregate product categories

	β_m	β_l	β_k	γ_{-g}
Construction	0.82	0.62	0.02	-0.07
Energy	0.82	0.56	0.16	-0.04
Manufacturing	0.93	0.41	0.03	-0.09
Agriculture	0.91	0.27	0.04	-0.10
Mining	0.86	0.40	0.06	-0.20
Wholesale	1.84	0.48	0.05	-0.13
Business Services	0.89	0.65	0.04	-0.06
Financial Services	0.87	0.49	0.01	0.09
Real Estate	1.55	0.39	0.12	-0.06
Personal Services	0.90	0.56	0.01	-0.06
Transport and ICTs	0.84	0.56	0.02	-0.03

Markup distribution

- Markup distribution by product category excluding 1 and 99 percentiles



Number of Products

Number of products distribution

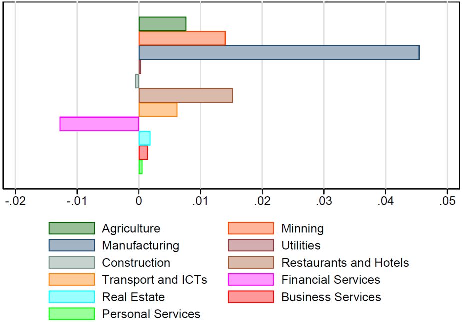
	290 product code	20% share rule
Mean	12	2
Median	6	2
Sd	16	1
p1	1	1
p25	2	1
p75	16	5
p90	33	5
p95	47	5
p99	73	5
Max	176	5

back

Within firm AE: Covid periods (2019~2021)

- Aggregate contribution by product categories of firm's main product

$$\text{Contribution of firm with product } c = \sum_{i \in N_C} \tilde{\lambda}_i Cov_{s_i}(-d \log \boldsymbol{p}_i, \xi_i)$$



Within firm AE: High-inflation periods (2021~2022)

- Aggregate contribution by product categories of firm's main product

$$\text{Contribution of firm with product } c = \sum_{i \in N_C} \tilde{\lambda}_i Cov_{s_i}(-d\log \boldsymbol{p}_i, \xi_i)$$