

The Welfare Effects of Nonlinear Prices in Supply Chains^{*}

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^{*}The views expressed are those of the authors and do not necessarily represent the views of the Central Bank of Chile or its board members.

Motivation

- Policy makers have raised concerns about rising market power and its effects on efficiency
- Renewed concerns over price discrimination harming small firms in B2B markets (FTC, 2025)
- Market power per se does not generate inefficiencies, its effects rely on **pricing conduct**
- **Linear pricing** is commonly assumed

Motivation

- Policy makers have raised concerns about rising market power and its effects on efficiency
- Renewed concerns over price discrimination harming small firms in B2B markets (FTC, 2025)
- Market power per se does not generate inefficiencies, its effects rely on **pricing conduct**
- **Linear pricing** is commonly assumed
- However, we find suggestive evidence of **nonlinear pricing prevalence** (B2B for Chile)
- B2B transactions in 2016 about 15 times B2C value on e-commerce in the US (OECD, 2019)

This Paper: What are the market power costs in nonlinear price setups?

Model

- Quantitative model of nonlinear prices in supply chains
- **Average unit price is not allocative**
- Allocations and rents distribution have **different distortions relative to linear prices**

Quantification

- Using Chilean IRS VAT data, we calibrate the model
- We find that **market power welfare costs are overestimated in linear price setups**
- Identify what distortions drive welfare differences across assumed pricing regimes

Selected Related Literature

Distorted Economies and Firm Entry

- Mankiw & Whinston 1986, Hopenhayn 1992, Baqaee & Farhi 2020, Edmond, Midrigan & Xu (2023)
- **We include distortions due to price discrimination**

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Distorted Economies and Firm Entry

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- **We include distortions due to price discrimination**

Price Discrimination

- Mussa & Rosen (1978), Borenstein (1985), Wilson (1993), Goldberg (1996), Stole (2007), Bornstein & Peter 20025, Burstein, Cravino, & Rojas (2025)
- **We focus on supply chains with two-sided price discrimination**

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3. Data and Calibration

4. Model Quantification

5. Conclusion

Optimal Nonlinear Price: Basic Construct

- One seller with **constant marginal cost** c faces a **continuous of buyers** indexed by z
- Buyer **types Pareto distributed** with PDF and CDF ($z_{min} = 1$):

$$f(z) = \kappa z^{-\kappa-1} \quad F(z) = 1 - z^{-\kappa}$$

- Buyers revenue functions given by:

$$R(z, q) = \frac{z^{\frac{\sigma-1}{\sigma}} q^{\frac{\sigma-1}{\sigma}}}{(\sigma-1)/\sigma}$$

- Revenue function is increasing in buyer type z

Seller profit maximization problem

- Seller knows the distribution of buyers types but doesn't know which one is which
- **charges a nonlinear price**

Mechanism design problem

- Chooses a **transfer T and quantities q** for each type z (N_z : total buyer Mass)

$$\max_{\{T(z)q(z)\}} \Pi = \mathbb{E}_z [T(z) - cq(z)] N_z$$

Subject to

- (IR) Buyers receive non-negative surplus: $\Pi(z, q_z) \geq 0, \quad \forall z$
- (IC) Buyers self-select into their tailored menu: $q_z \in \operatorname{argmax}_z \Pi(z, q_z), \quad \forall z$

Optimal Nonlinear Price

The solution to the MD problem given a nonlinear price schedule total payment $T(q)$

- Gives total payment $T(q)$ for total quantity q
- $p(q) = T'(q)$ interpreted as the price for the q^{th} unit
- Buyer z chooses q to maximize:

$$\underbrace{\frac{z^{\frac{\sigma-1}{\sigma}} q^{\frac{\sigma-1}{\sigma}}}{(\sigma-1)/\sigma}}_{R(z,q)} - P(q)$$

- FOC will yield the **Demand for the q^{th} unit**

$$z^{\frac{\sigma-1}{\sigma}} q^{-\frac{1}{\sigma}} = T'(q) = p(q) \tag{1}$$

Optimal Nonlinear Price Result: Two-Part Tariff

- We solve the mechanism design using the revelation principle [► Details](#)
- Two-part tariff (Total Payments $T(q) = p(q) \cdot q(z)$):

$$T(q) = \mathbf{F}_{z_0} + \mathbf{p} q_z, \quad \forall z$$

- **Flat fee:** Extract surplus of lowest type $F_{z_0} = \Pi_{z_0}$
- **Linear allocative price:** $p = \frac{\rho}{\rho-1} c, \quad \rho = \frac{\sigma \kappa}{\sigma-1}, \quad \rho > \sigma$ (Result from Pareto distribution)

Is price deviation profitable for any $z > z_a$?

- Heuristic argument (Wilson 1993) to derive the optimal price $p(q)$
- Define marginal buyer $z(q, p)$ by inverting demand for the q^{th} unit (equation 1)

$$z(q, p) = q^{\frac{1}{\sigma-1}} p^{\frac{\sigma}{\sigma-1}}$$

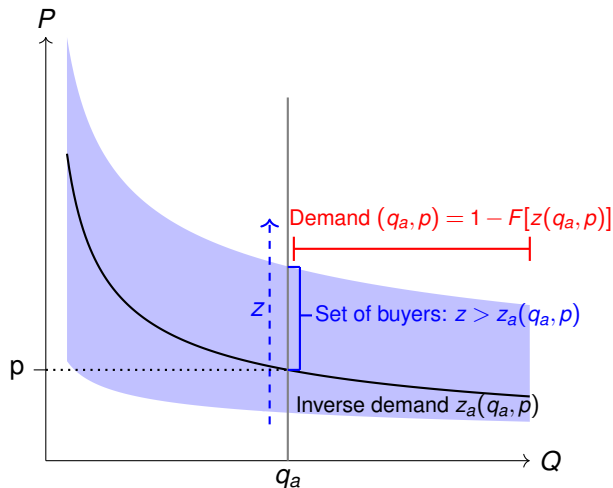
- Demand for q^{th} unit:

$$D(q, p) = 1 - F(z(q, p))$$

- Seller chooses a price for unit “q” to solve:

$$\max_p [1 - F(z_a(q_a, p))] (p - c)$$

Implication 1: No profitable deviation in price



$$\max_{\{P\}} [1 - F(z_a(q_a, p))](P - c)$$

FOC :

$$\frac{P}{c} = \frac{\frac{\kappa\sigma}{\sigma-1}}{\frac{\kappa\sigma}{\sigma-1} - 1} = \frac{\rho}{\rho - 1}$$

- The optimal price is equal to the **allocative price of the two-part tariff**
- Seller has no incentive to charge different prices for different quantities

Implication 2: Should the Seller Exclude Low Types? No

In nonlinear pricing with private information, the seller always faces a choice:

- 1 Exclude low types to better extract surplus from high types
- 2 Serve all types, but give up some rent from high types
 - However, Pareto distribution has heavy mass for low types
 - Even though low types buy little q , because of their large density the seller maximizes profits by serving them [▶ Details](#)

Reproducible argument

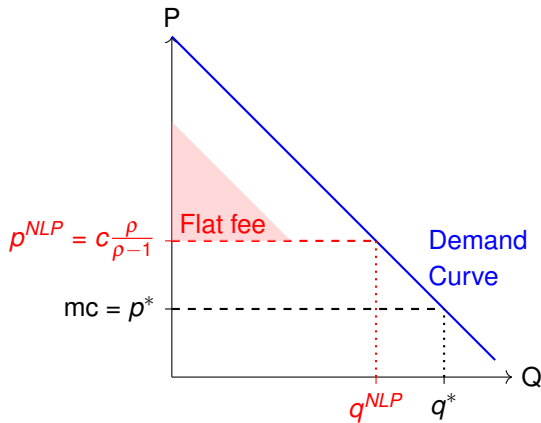
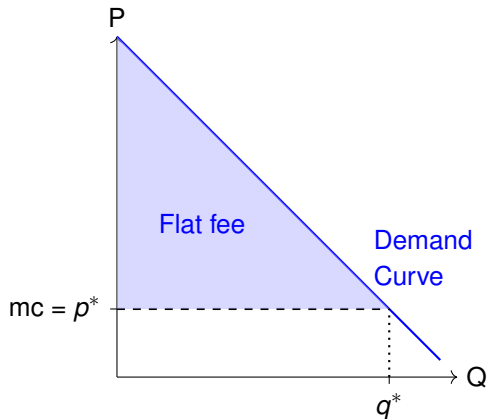
- Under a set of assumptions setup is reproducible to a supply chain with an arbitrary number of sellers, buyers and layers
- Producers of intermediate goods can price discriminate and be price discriminated against with nonlinear prices

Needed assumptions

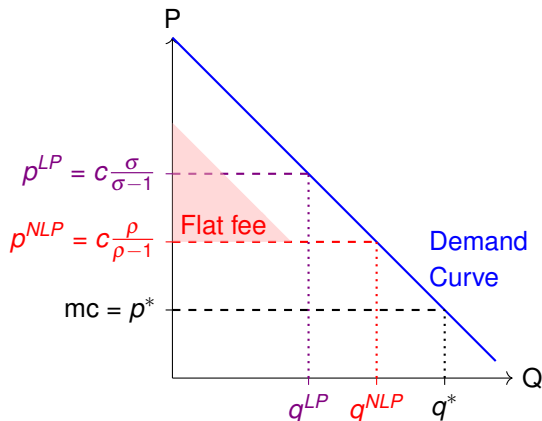
- ① Homothetic revenue functions in the shape $R(q, z) = A(z)q^\theta$, for any arbitrary θ
- ② Pareto distributed firm types
- ③ Firms have constant marginal cost (can be a result)

Optimal nonlinear prices take the form of a single Two-part tariff

Two-Part Tariff: First Degree vs. Nonlinear Pricing



Two-Part Tariff: NLP vs LP CES markup ($\rho > \sigma$)



- Allocations in NLP are less distorted relative to LP

$$q^* > q^{NLP} > q^{LP}$$

- Because of the flat fee, rents are subject to different distortions in NLP vs. LP

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1. The Optimal Nonlinear Price: Basic Construct

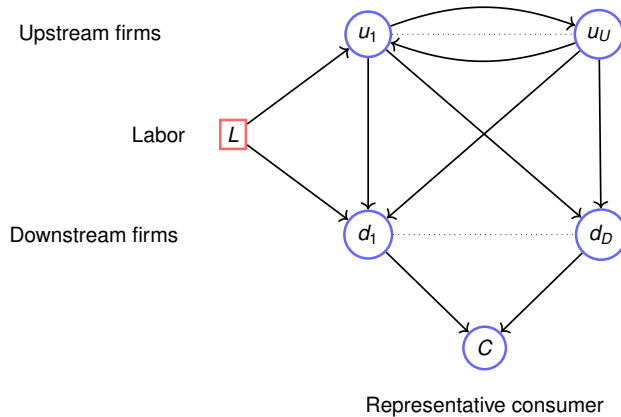
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Supply Chain Structure



Representative Consumer

- CES aggregator on downstream varieties d

$$Y = \left(\int_{d_0} q_d^{\frac{\sigma-1}{\sigma}} N_D \mu_d dd \right)^{\frac{\sigma}{\sigma-1}}$$

- N_D is the mass of downstream firms and $\mu(z)$ is the density of type z firms
- Offer labor and owns firms and receive their profits
- Budget constraint: $Y = wL^P + \Pi^U + \Pi^D$
- Exogenous aggregate labor supply; labor market clearing: $L^T = L^P + L^E$

Production

- Firm partition on upstream $u \in U$ (sells to other firms only) and downstream $d \in D$ firms (sell to final consumers only)
- Cobb-Douglas production function for all firms $i \in D + U$

$$q_i = A_i L_i^\alpha M_i^{1-\alpha}$$

- M is a CES bundle of upstream varieties:

$$M_i = \left[\int_{u_0} m_{iu}^{\frac{\sigma-1}{\sigma}} N_U \mu(\zeta) du \right]^{\frac{\sigma}{\sigma-1}}$$

- N_u is the mass of upstream firms and μ_u the density of upstream firms type u

Firm Entry

- Static model, where firms decide to enter and conditional on entry, produce
- Unbounded pool of prospective entrants that are ex-ante identical
- Pay a sunk cost c_e in units of labor
- Upon entry, firms draw a type from a layer-specific Pareto distribution G_U, G_D with tail κ_U, κ_D ,
- Free entry into both firms groups ($\mathbb{E}[\pi_\gamma] = c_e w$); M_U, M_D are endogenous
- Equilibrium mass of firms

$$\underbrace{M_E}_{\text{Entrant}} = \underbrace{(M_U + M_D)}_{\text{Mass of Firms}}$$

Linear Price Equilibrium

- Firms demand input to minimize expenditure given linear prices
- Firms choose a linear price, Upstream and Downstream

$$\max_{p_i} (p_i - MC_i) \underbrace{D_i}_{\text{Demand}}, \quad i \in \{\text{Upstream, Downstream}\}$$

- markup:

$$\frac{p_i}{MC_i} = \frac{\sigma}{\sigma - 1}$$

- Firm production is distorted and smaller relative to the planner's solution

Planner's Price Equilibrium

- Firms must charge markups to incentivize the optimal entry level
- But markup distorts input choices by acting as a uniform tax on production
- An output subsidy can restore undistorted marginal cost, conditional on entry
- The subsidy is paid via a lump sum tax to the representative consumer
- With linear prices, the **efficient allocation** is achieved with an output subsidy:

$$\tau_i = \frac{\sigma - 1}{\sigma}$$

Nonlinear Price Equilibrium Setup

- ① **Upstream firms charge nonlinear prices** to other upstream firms and downstream firms
- ② Downstream firms charge linear prices to the representative consumer, normalized to 1 price
- ③ **Firms are infinitesimal** and do not internalize other firms' outcomes nor their actions' effects on other firms

Nonlinear Price Equilibrium Reproducible Assumptions

Using the reproducible argument requires the following assumptions:

- 1 Homothetic revenue functions in the shape $R(q, z) = A(z)q^\theta$, for any arbitrary θ
Achieved through Cobb-Douglas PF and CES input demands

- 2 Pareto distributed firm types
Assumption

- 3 Firms have constant marginal cost (Result in our model)
Given CES structure give a fully connected network, under our guesses:

$$c_i(w, p_m) = \frac{1}{A_i} \left[\alpha^\eta w^{1-\eta} + (1 - \alpha)^\eta p_m^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad \text{where} \quad p_m = \left(\int_{u_0} p_u^{1-\sigma} N_u \mu_u du \right)^{\frac{1}{1-\sigma}}$$

Upstream profit maximization problem

Guesses:

- 1 Seller chooses a two-part tariff: $T = F + p(q)m$
- 2 Revenue shape: $R_i = q_i^\theta A_i, \quad \forall i$

Mechanism design problem

- o Chose a transfer (T) and quantities (m), separately for upstream (u) and downstream (d) firms

$$\max_{\substack{\{T_{ud}, m_{ud}\}, \\ \{T_{uu'}, m_{uu'}\}}} \Pi_u = \underbrace{\mathbb{E}_d [T_{ud} - c_u m_{ud}] N_D}_{\text{Downstream firms}} + \underbrace{\mathbb{E}_u [T_{uu'} - c_u m_{uu'}] N_U}_{\text{Other upstream firms}}$$

Subject to

- o Individual Rationality (IR): Buyers receive non-negative surplus from buying
- o Incentive Compatibility (IC): Buyers self-select into their tailored menu

Upstream price scheme

A flat fee and a linear component describe the solution to the mechanism design problem of the seller:

$$T_{ui} = F_{ui} + p_u m_{ui}, \quad i \in \{U, D\}$$

Flat fee: Extract surplus of lower (upstream or downstream) type

Linear allocative markup: $p_u = \frac{\rho}{\rho - 1} c_u, \quad \rho = \frac{\sigma \kappa}{\sigma - 1}, \quad \rho > \sigma$

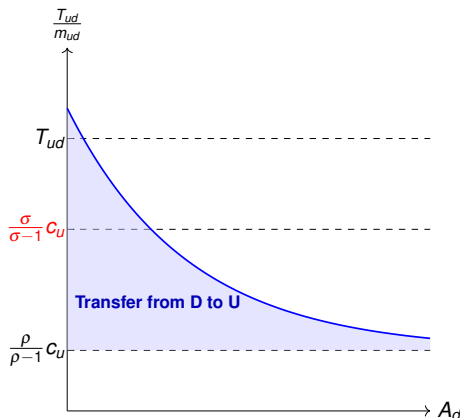
- Two different flat fees for upstream and downstream firms
- Allocative markup is smaller than linear price markup ($\rho > \sigma$)
- Upstream has no incentives to deviate on price or exclude any type

Upstream unit price scheme to downstream firms

Unit price paid by downstream firms:

$$\frac{T_{ud}}{m_{ud}} = \frac{F_{ud}}{m_{ud}} + p_u$$

- The flat fee unit price share is decreasing in firm productivity A_d :
- Allocative markup: $\frac{\rho}{\rho-1} < \frac{\sigma}{\sigma-1}$
- Transfer from downstream to upstream firms: $\frac{F_{ud}}{m_{ud}}$



Welfare: Nonlinear Pricing (NLP) vs Linear Pricing (LP)

- Weighted mean output per firm:

$$\hat{q}^r = \left[\int q_d^r \cdot \mu_d N_D^r(z) dd \right]^{\sigma/\sigma-1} / N_D^r$$

- Welfare in price regime $r \in \{LP, NLP\}$:

$$Y^r = \underbrace{(N_D^r)^{\sigma/\sigma-1}}_{\text{Extensive Margin}} \cdot \underbrace{\hat{q}^r}_{\text{Intensive Margin}}$$

- Higher output per firm in NLP because of lower “output tax”: $\hat{q}^{NLP} > \hat{q}^{LP}$
- Firms substitute m with L ; fewer L usage in entry: $N_D^{NLP} < N_D^{LP}$
- Effects on welfare will depend on what margin dominates

Welfare: Intensive Margin Wedges For Price Regime r

$$\frac{\widehat{q}^r}{\widehat{q}^{EFF}} = \left(\frac{A_d^r}{A_d^{EFF}} \right)^\tau \cdot \left(\frac{I/m^r}{I/m^{EFF}} \right)^{\alpha\tau} \cdot \left(\frac{m^r}{m^{EFF}} \right)^\tau \cdot \left(\frac{N_U^r}{N_U^{EFF}} \right)^{\tau/(\sigma-1)}$$

- 1 **Productivity Selection:** Captures productivity composition
- 2 **Input Mix:** How far firms deviate from their efficient cost-minimizing input mix
- 3 **Material Usage:** Material scale usage affected by markups (N-marginalization problem)
- 4 **Input Variety:** Fewer upstream varieties increase the price index for materials, lowering downstream demand

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Data sources

[Invoice example](#)

Invoice transactions for the universe of Chilean formal firms for 2018

- Around 1.3 billion transactions
- More than 10 million different products. Assume seller-specific products
- Data on prices and quantities for every product transacted

Merged with firms' accounting balance sheet data

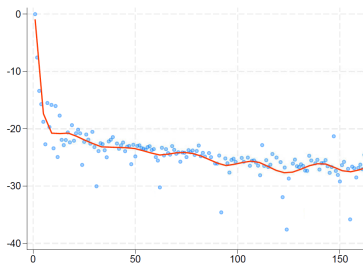
- Sales, materials, investment, 6-digit industry
- Employer-employee: Wages, headcount of employees
- Capital stock and investment

[Data Cleaning](#)

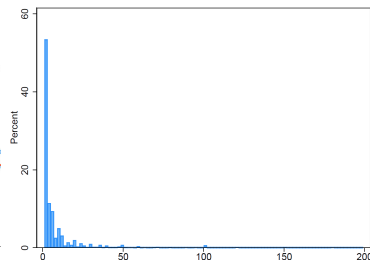
Suggestive Evidence of NLP (June 2018)

$$\ln p_{ijg} = \beta_0 + \sum_{r=1}^{200} \beta_r \ln Q \text{ bin}_r + \ln \psi_{ijg} + \ln \varepsilon_{ig}$$

Panel A. Quantity discounts fixed effects



Panel B. Quantity traded histogram



- Marginal discount decreases with quantity
- Navigation buttons: Daily, Average effect, By industry, Price-quantity menus, Firm sales Partition

Parameter Calibration

Model Parameters

	Value	Source
Labor share in production (α)	0.48	Calibrated from data
Material bundle elasticity (σ)	3.2	Assumed
Entry cost (c_e)	0.2	Assumed
Productivity Pareto tail (κ)	3.96	Calibrated from data

Productivity Pareto tail

- Firm labor demand scale up with productivity, hence labor is also Pareto distributed
- Labor Pareto Tail MLE estimate from data $\xi = \frac{\kappa}{\sigma-1} = 1.8$
- Can recover $\kappa = 3.96$

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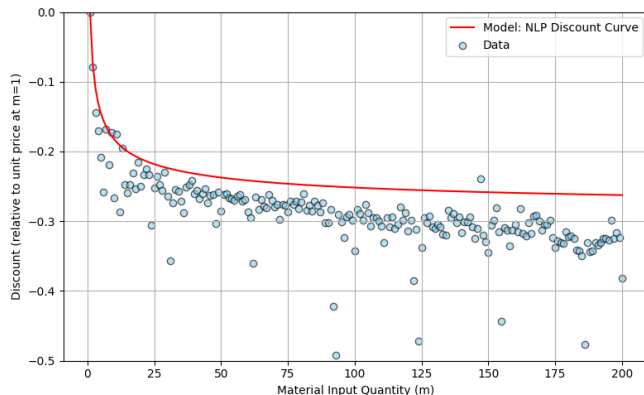
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Quantity discounts: Data vs model



- For the average upstream firm price schedule to the downstream firm, normalizing the continuous input quantity to be in the bounds of 1 to 200

Welfare Decomposition Relative to Efficiency

Define the welfare ratio: $\frac{Y^r}{Y^{\text{EFF}}} = \underbrace{\left(\frac{\hat{q}^r}{\hat{q}^{\text{EFF}}} \right)}_{\text{Intensive margin}} \cdot \underbrace{\left(\frac{M_z^r}{M_z^{\text{EFF}}} \right)^{\sigma/\sigma-1}}_{\text{Extensive margin}}$

	Linear Pricing (LP)	Nonlinear Pricing(NLP)
Welfare	0.59	0.66
Intensive Margin (Ratio)	0.42	0.50
Extensive Margin (Ratio)	$1.65^{\sigma/\sigma-1}$	$1.50^{\sigma/\sigma-1}$
Intensive Margin (log deviation)	63%	37%
Extensive Margin (log deviation)	62%	38 %

- Welfare losses under NLP are 82% of of LP $((1 - 66)/(1 - 59))$
- In **log differences** intensive margin accounts for 63% of welfare changes

Intensive and Extensive margin: Shares relative to planner pricing

$$\underbrace{\hat{q}^r}_{\text{Intensive Margin}} = \underbrace{(A_d^r)^{\sigma/\sigma-1}}_{\text{Productivity selection}} \cdot \underbrace{(l/m^r)^{\alpha\sigma/\sigma-1}}_{\text{Input mix}} \cdot \underbrace{(m^r)^{\sigma/\sigma-1}}_{\text{Material usage}} \cdot \underbrace{(N_U^r)^{1/\sigma}}_{\text{Input variety}}$$

Intensive Margin	Linear Pricing	Nonlinear Pricing
Productivity selection	1	1
Input mix	1.36	1.29
Material usage	0.41	0.48
Input variety	0.94	0.87

Extensive Margin	Linear Pricing	Price discrimination
Total firm mass	1.23	1.07
Upstream firms mass	0.85	0.66
Downtown firms mass	1.65	1.51

Intensive margin decomposition ratio: Which component matters the most

$$\underbrace{\widehat{q}^r}_{\text{Intensive Margin}} = \underbrace{(A_d^r)^{\sigma/\sigma-1}}_{\text{Productivity selection}} \cdot \underbrace{(l/m^r)^{\alpha\sigma/\sigma-1}}_{\text{Input mix}} \cdot \underbrace{(m^r)^{\sigma/\sigma-1}}_{\text{Material usage}} \cdot \underbrace{(N_U^r)^{1/\sigma}}_{\text{Input variety}}$$

Log deviations from planners pricing

	Linear Pricing	Nonlinear Pricing
Productivity selection	0%	0%
Input mix	25%	23%
Material usage	71%	65%
Input variety	4%	12%

- No selection as all firm types participate
- Input mix and material distortions explain the bulk of the intensive margin

Fixing Entry $N^{EFF} = N^{NLP}$: Shares relative to planner pricing

Intensive Margin	NLP planner Mass of Firms	NLP
Productivity selection	1	1
Input mix	1.27	1.29
Material usage	0.49	0.48
Input variety	0.86	0.87
Welfare	0.64	0.66

- Total mass of firms fixed but, masses per layer can move in general equilibrium
- With a fixed number of firms, NLP remain to distort welfare through a) inefficient input scaling, b) misallocation across inputs, c) and reduced effective supplier coverage

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Conclusion

- We develop a tractable strategy to solve for optimal linear prices
- Under Pareto distributed firm types, a two-part tariff fits the prices observed in the data
- Accounting for observed price discrimination, markups welfare costs are around 80% of linear price setup costs
- Intensive margin explains around 60% of welfare changes
- Failing to incorporate nonlinear prices overestimate markups welfare costs

Binding Constraints and IC via Envelope Theorem [▶ Return](#)

- (IR) binds only for the lowest type ($z_0 = 1$)
- For all other buyers, the only binding constraint is the downward local (IC)
- Buyer utility:

$$U(z) = \frac{z^{\sigma-1}}{\alpha} q(z)^{1-1/\sigma} - T(z), \quad \alpha = \frac{\sigma}{\sigma-1}$$

- Envelope theorem:

$$U'(z) = \frac{(\sigma-1)z^{\sigma-2}}{\alpha} q(z)^{1-1/\sigma}$$

- Integrating from lowest type:

$$U(z) = \int_1^z \frac{(\sigma-1)\tilde{z}^{\sigma-2}}{\alpha} q(\tilde{z})^{1-1/\sigma} d\tilde{z}$$

Transformed Seller Profit Maximization problem [Return](#)

$$\max_{q(z)} \int [T(z) - cq(z)] f(z) dz$$

- o **Virtual valuation, ϕ** : marginal value of a buyer type z after adjusting for (IC)
- o Using $T(z) = \frac{z^{\sigma-1}}{\alpha} q(z)^{1-1/\sigma} - U(z)$ and plugging in $U(z)$, the virtual valuation is

$$\phi(z) = \frac{z^{\sigma-1}}{\alpha} - \frac{1 - F(z)}{f(z)} \cdot \frac{d}{dz} \left(\frac{z^{\sigma-1}}{\alpha} \right)$$

- o The problem becomes a **pointwise profit maximization problem**

$$\max_{q(z)} \left\{ \phi(z) q(z)^{1-1/\sigma} - cq(z) \right\}$$

Solve For Optimal Quantities [▶ Return](#)

- From FOC:

$$q(z) = \left[\frac{(1 - 1/\sigma) \cdot \phi(z)}{c} \right]^\sigma$$

- For Pareto-distributed z with parameter κ , we have:

$$\phi(z) = \frac{z^{\sigma-1}}{\alpha} \left(1 - \frac{\sigma-1}{\kappa} \right) = \frac{z^{\sigma-1}}{\alpha} \cdot \frac{\rho - \sigma}{\rho}, \quad \text{with } \rho = \frac{\sigma\kappa}{\sigma-1}$$

- The optimal quantity scales up with type z :

$$q(z) = Bz^\rho, \quad \text{where } B = \left(\frac{(1 - 1/\sigma)}{c\alpha} \cdot \frac{\rho - \sigma}{\rho} \right)^\sigma$$

Recovering the Transfer Function

- Transfer:

$$T(z) = \frac{z^{\sigma-1}}{\alpha} q(z)^{1-1/\sigma} - U(z)$$

- With envelope theorem:

$$T(z) = \underbrace{\frac{z^{\sigma-1}}{\alpha} q(z)^{1-1/\sigma}}_{\text{Gross Surplus}} - \underbrace{\int_1^z \frac{(\sigma-1)\tilde{z}^{\sigma-2}}{\alpha} q(\tilde{z})^{1-1/\sigma} d\tilde{z}}_{\text{Information Rents}}$$

Flat Fee and Per Unit Price [▶ Return](#)

- Suppose the seller implements a two-part tariff: $T(z) = F + pq(z)$
- Match marginal utility to price:

$$p = \frac{d}{dq} \left(\frac{z^{\sigma-1}}{\alpha} q^{1-1/\sigma} \right) = \frac{z^{\sigma-1}}{\alpha} (1 - 1/\sigma) q(z)^{-1/\sigma} = \frac{\rho}{\rho-1} c$$

- From (IR) at $z = 1$: $U(1) = 0$, $F = T(1) - pq(1)$:

$$T(1) = \frac{1}{\alpha} q(1)^{1-1/\sigma}, \quad F = \frac{1}{\alpha} q(1)^{1-1/\sigma} - pq(1)$$

- 1 Flat fee extracts all surplus from the lowest type
- 2 Constant per unit price as a function of elasticity and Pareto tail

Implication 2: All types are served [▶ Return](#)

Virtual surplus: profit from serving a buyer type, net of the informational rents that must be left to higher types to preserve IC

- Virtual surplus for type z ($\alpha = \frac{\sigma}{\sigma-1}$):

$$VS(z) = \underbrace{\left(\frac{z^{\sigma-1}}{\alpha} \right) q(z)^{1-1/\sigma}}_{\text{Gains from serving type } z} - \underbrace{\left(\frac{1-F(z)}{f(z)} \cdot \frac{d}{dz} \left(\frac{z^{\sigma-1}}{\alpha} \right) \right) q(z)^{1-1/\sigma}}_{\text{Informational rents left to ensure IC}}$$

- For the lowest type $z_0 = 1$, the virtual surplus simplifies:

$$VS(1) = \left[\frac{1}{\alpha} \left(1 - \frac{\sigma-1}{\kappa} \right) \right] q(1)^{1-1/\sigma}$$

- This is strictly positive whenever $\kappa > \sigma - 1$ (necessary condition for finite output)
- If its profitable to serve the lowest type, the seller will not exclude any buyer

General Equilibrium NLP [▶ Return](#)

A decentralized nonlinear pricing equilibrium is a collection of firm level productivity A_i , linear prices $\{p_u, p_d\}$, flat fees $\{F_u, F_d\}$, wage w , and quantities $\{y_i, q_i, l_i, m_{iu}, N_U, N_D\}$ such that:

- Each producer minimizes its costs and charges a linear price that equals marginal cost times the markup
- Each producer pays a transfer, such that the lowest types have zero surplus from transacting with upstream sellers
- Entrants earn zero expected profit
- The representative consumer maximizes its consumption
- Markets clear for all goods and factors

Invoice Example [▶ Return](#)

KITCHEN CENTER SPA
IMPORTACIÓN Y DISTRIBUCIÓN DE ELECTRODOMÉSTICOS

FDU SIMPLE COOK Cuisinart CUBELL (Bellini) Jasmeg SUPER/COOK Arista LOPRA

Casa Matriz:
Sociedad:
Casa Comodoro:
Mall Parque Aracua:
Mall Plaza Los Dominicos:
Mall Buenaventura:
Mall Plaza La Sonora:
Mall Marina Aracua:
Outlet Park Wika:
Mall Plaza Masden:
Conceptos:
Temuco:
Mall Fashion Temuco:
Servicio Técnico:
Centro de Distribución:
Vila del Mar:
Alto Las Condes:
Outlet El Soltero

Av. Nueva Contadora 3000, Santiago
 An. Kennedy 5443 Local 572, Las Cañitas - Teléfono: (06-2) 24131777 - Fax: (06-2) 24131731
 Padre Marín 287 879, Local A20066274, Las Cañitas - Teléfono: (+54) 24137760
 Copalera 550 188, Local 188, Las Cañitas - Teléfono: (+54) 24137760
 Av. Libertad 1386, Local PG-0100, Vía del Mar - Teléfono: (04-2) 24337760/68
 Callema Interoceánico 1055, Local 226277, Vía del Mar - Teléfono: (04-2) 24337768
 Chiriquí 2708, Local 2, Teléfono: (06-2) 2413 7756 / 17
 Avda. Alameda 0513, Local 0513, Las Cañitas - (+56) 21343774 R
 Rubén Darío 2170, Local 1188-78, Las Cañitas - Teléfono: (+56) 21343774
 Llanada 2308, Quilicura - Teléfono: 000117767 / 797 / 804
 Marinos to Plaza #887, Las Cañitas
 R Wbo. 51a, Vía del Mar
 An. Kennedy 5401, Local 5401, Las Cañitas
 Av. El Salto 3445, Recoleta, Santiago



R.U.T. 96.999.930-7

BOLETA ELECTRÓNICA

N° 0015959119

S.I.I. - SANTIAGO NORTE

FECHA EMISIÓN : 01/08/2022

SEÑORES Antonio Martner

DIRECCIÓN Los Misioneros 1923

COMUNA Providencia

R.U.T. 16.211.960-5

TELEFONO+569995703551

CIUDAD : Santiago

EMAIL : amarthner@gmail.com

Dirección Origen: Camino lo Boza 8887

Comuna : Pudahuel

Ciudad : Santiago

Dirección Destino: Los Misioneros 1923

Ciudad : Santiago

FECHA EMISIÓN :

FECHA VENCIMIENTO : 03/08/2022

TIPO DESPACHO :

FORMA DE PAGO : Contado

COD. VENDEDOR :

Orden de Venta: 793325

Número de OC:

CÓDIGO	DETALLE	CANTIDAD	PRECIO UNITARIO	PRECIO ÍTEM
13452	Lavaplatos FDU Small Acqua bajo cubierta	1	92.429,57	92.429
14761	Encimera FDU Design 4T GLTX 65 SUT 2.0	1	142.849,74	142.849
14265	Campana Kubli Neu Slider	1	100.831,93	100.832
19110	Horno FDU Design	1	201.672,27	201.672
13377	Lavavajillas FS FDU Element 14C	1	243.689,07	243.689
14917	Grifería FDU COMICA FLEX	1	84.025,21	84.025
10232	Transporte - Providencia	1	15.529,41	15.529

Data cleaning [Return](#)

Goal: Keep all plausible transactions

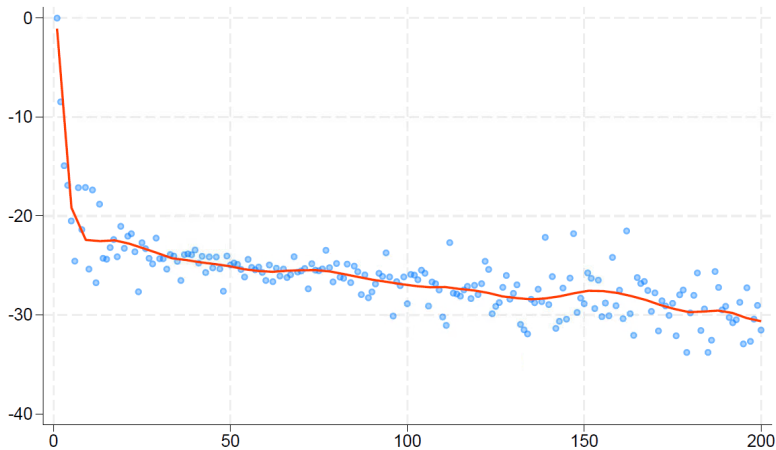
- Prices are net of discounts and recharges
- Drop if a transaction has missing or zero price or quantity
- Drop if product description is missing
- Drop prices above 10 times the mean price by seller-product-day
- Under this cleaning we keep around 99% of transactions

Quantity discounts regressions [Return](#)

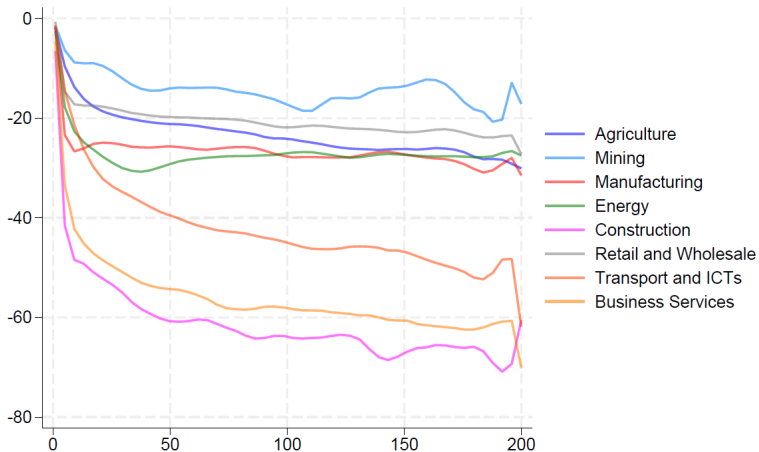
$$\ln p_{ijg} = \beta_0 + \beta_1 \ln q_{ijg} + \ln \psi + \ln \varepsilon_{ijg}$$

	(1)	(2)	(3)	(4)	(5)
$\ln q$	-0.083 (0.00006)	-0.051 (0.0001)	-0.045 (0.001)	-0.087 (0.0001)	-0.071 (0.00007)
FE Seller \times product	✓				
FE Seller \times product \times buyer		✓			
High price products (>p95)			✓		
Manufacturing				✓	
Retail and wholesale					✓
Observations (millions)	92	52	3.5	25	58
Adjusted R ²	0.928	0.956	0.756	0.911	0.945

Quantity discounts evidence June 29, 2018 [▶ Return](#)



Quantity discounts evidence by seller industry [Return](#)



Price dispersion on price-quantity menus

► Return

- Under pure second-degree: fix quantity, price variance should be zero
- Group $X: \{\text{Seller-Product-Month} + \text{Quantity}, \text{Buyer}\}$

$$C_X = \frac{\text{Standard deviation } p_X}{\text{median } p_X}; \quad X \in \{\text{Seller-Product} + \text{Quantity}, \text{Buyer}\}$$

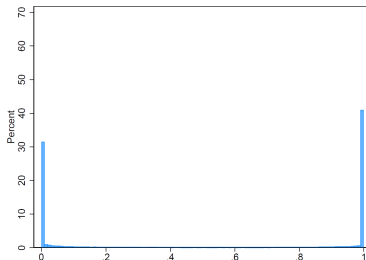
	C_X Moments				
	p10	p25	Median	p75	p90
Seller - product	0.00	0.01	0.06	0.17	2.5
Seller - product - quantity	0.00	0.00	0.04	0.14	1.41
Seller - product - quantity - buyer	0.00	0.00	0.00	0.04	0.24

- Evidence of buyer-specific quantity-menus

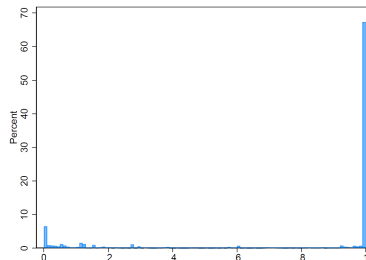
Sales partition [Return](#)

$$X_i = \begin{cases} 0 & \text{if all sales go to final consumers} \\ 1 & \text{if all sales go to other firms} \end{cases}$$

Panel A. Number of Firms



Panel B. Sales weighted



- More than 70% of firms sell only to final consumers or to other firms [By sector](#)

Sales partition: Sales shares (excluding exports)

Sector (sales)	All to final consumer	All to other firms
Firm population	0.08	0.67
Agriculture (2%)	0.04	0.60
Mining (1%)	0.27	0.08
Manufacturing (15%)	0.05	0.68
Utilities (3%)	0.20	0.51
Construction (8%)	0.02	0.89
Retail and Wholesale (32%)	0.09	0.68
Transport (10%)	0.16	0.68
Financial Services (18%)	0.18	0.67
Real Estate Services (1%)	0.24	0.37
Business Services (7%)	0.08	0.81
Personal Services (2%)	0.68	0.10