

# Aggregate Outcomes of Nonlinear Prices in Supply Chains<sup>\*</sup>

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<sup>\*</sup>The views expressed are those of the authors and do not necessarily represent the views of the Central Bank of Chile or its board members.

## Motivation and Research Question

- o Understanding the aggregate costs of market power is central in research and policy debates
- o But market power aggregate analysis often **omits price discrimination**
  - “There can be no doubt that **firms are well aware of the benefits of price discrimination.**”
  - “Price discrimination is one of the most **prevalent forms of marketing practices**”

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“When local businesses get squeezed . . . Americans face **fewer choices**, **higher prices**, and **communities suffer.**”

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**Research Question: What are the aggregate outcomes of price discrimination in supply chains?**

## Market Power and Aggregate Efficiency

- o Estimating the aggregate cost of market power hinges on **assumptions of pricing contracts**
- o **Standard assumption is uniform pricing**: a single quantity-invariant price to all buyers  
(average prices = marginal prices)
- o In supply chains in Chile, we find indicative evidence of widespread **nonlinear prices**  
(average prices  $\neq$  marginal prices)
- o Under nonlinear pricing, **average prices are not allocative, whereas marginal prices are**
- o **Relevant in supply chains**: distortions in marginal prices accumulate (double marginalization)

## This Paper: Main Mechanism

- Under standard assumptions, the optimal nonlinear price is equivalent to a two-part tariff:

$$\underbrace{pq}_{\text{Total Payment}} = \underbrace{F}_{\text{Flat fee}} + \underbrace{p_{\text{marg}}}_{\text{Marginal price}} q$$

### Allocations: Nonlinear Pricing Improves Allocations relative to Uniform Pricing (Welfare Enhancing)

- The marginal price determines quantity allocations (it is allocative)
- In our setting, the marginal price is lower than under uniform pricing

### Entry: Flat Fee Distorts Entry (Ambiguous Welfare Effect)

- The flat fee does not affect input choices; it reallocates rents from buyers to sellers
- Affects firm profit distribution and distorts entry decisions

# This Paper: What we do

## Theory

- Multi-sector general equilibrium model in supply chains with two-sided price discrimination
- Endogenous nonlinear prices by buyer sector ( $2^{nd} + 3^{rd}$  PD) and endogenous entry



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## Empirics (Population-level B2B transactions for Chile)

- Pricing Diagnosis: **Nonlinear prices by buyer sector** (combination of  $2^{nd} + 3^{rd}$  degree PD)
- Calibration: Estimate model parameters **under uniform and nonlinear pricing assumptions**

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## Results: Welfare Outcomes

- **Policy:** **Banning PD increases the costs from market power**, 75% v. 49% distance to efficiency
- **Measurement:** PD yields lower distance to eff. rel. to uniform price assumption, 75% v. 57%

## Selected Related Literature

### Aggregate Cost of Market Power (Misallocation and Firm Entry)

- Quesnay (1758), Harberger (1956), Mankiw & Whinston (1986), Hopenhayn (1992), Hsieh & Klenow (2009), Jones (2011), Baqaee & Farhi (2019, 2020), Edmond, Midrigan & Xu (2023), Bornstein & Peter (2025), Burstein, Cravino, & Rojas (2025)

### Price Discrimination and Screening

- Dupuit (1849), Mirrlees (1971), Spence (1977), Mussa & Rosen (1978), Maskin & Riley (1984), Borenstein (1985), Tirole (1988), Varian (1989), Wilson (1993), Laffont & Tirole (1993), Armstrong (1996), Stole (2007)

We embed endogenous  $2^{nd} + 3^{rd}$  PD into a GE model in supply chains with endogenous entry

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## 1. Partial Equilibrium: Nonlinear Price Characterization

## 2. General Equilibrium Model in Supply Chains

## 3. Descriptive Evidence

## 4. Model Quantification

## 5. Conclusion

## Primitives and Behavior (Standard)

- One seller with **constant marginal cost**  $c$  faces a **continuum of buyers** indexed by  $z$
- Seller has full bargaining power and makes a take-it-or-leave-it offer
- Seller knows the distribution of buyer types, but type information is private
- Chooses a **nonlinear transfer**  $T(z)$  and **quantity**  $q(z)$  for each type  $z$

$$\max_{\{T(z), q(z)\}} \Pi_{\text{seller}} = \int_{\underline{z}}^{\infty} [T(z) - cq(z)] f(z) dz$$

### Subject to

- (IR) Buyers receive non-negative surplus:  $\Pi(z, q(z)) = zq(z) - T(z) \geq 0, \quad \forall z$
- (IC) Buyers choose their tailored contract:  $zq(z) - T(z) \geq zq(\tilde{z}) - T(\tilde{z}), \quad \forall z, \tilde{z}$

## Mirrlees Reduction and Virtual Surplus (Standard)

- Using the virtual surplus  $\phi$ , the problem can be written as a pointwise optimization problem

$$\max_{\{q(z)\}} \Pi_{\text{seller}} = \int_{\underline{z}}^{\infty} [\phi(z, q(z)) - cq(z)] f(z) dz,$$
$$\text{with } \phi(z, q) = \underbrace{R(z, q)}_1 - \underbrace{\frac{1}{h(z)} \frac{\partial R(z, q)}{\partial z}}_2$$

- Inverse hazard rate,  $h(z)^{-1} = (1 - F(z))/f(z)$  is the weight on the remaining higher types
- The virtual surplus represents the seller's effective revenue from serving type  $z$ :
  - Buyer  $z$  total revenue from the transaction (seller wants to extract it)
  - Rents the seller must leave to higher types to prevent them from mimicking type  $z$

## Functional Forms and Optimal Nonlinear Price (New)

- o So far, standard screening problem, now we impose two additional assumptions:
  - 1 Buyer types are Pareto distributed with tail parameter  $\kappa$
  - 2 Buyers have isoelastic demands ( $\sigma > 1$ )
- o Buyer type shifts demand without altering curvature

### Lemma 1: Optimal Nonlinear Price

Under (i) constant marginal cost, (ii) Pareto distributed types, and (iii) isoelastic demands, the optimal nonlinear price schedule is equivalent to a *two-part tariff* when  $\kappa > \sigma - 1$ :

$$T(z) = F + p^{\text{NLP}} q(z), \quad p^{\text{NLP}} = \frac{\rho}{\rho - 1} c, \quad \rho \equiv \frac{\kappa \sigma}{\sigma - 1} > \sigma, \quad F \text{ is set so that: } \Pi(\underline{z}) = 0.$$

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## Environment

- Two **observable** firm types,  $\ell \in \{u, r\}$ , defined by their position relative to final demand ► Evidence
- Upstream firms  $u$  sell both to other  $u$  and to retail firms  $r$ , and buy from other  $u$  firms
- $r$  purchase inputs from  $u$  and sell exclusively to the representative final consumer
- Within each type  $\ell \in \{u, r\}$  **observable** sectors are indexed by  $s \in S$
- Firms as buyers are denoted by  $i$  and by  $j$  as sellers, buyer sectors as  $s$  and seller sectors as  $s'$
- Each  $(\ell, s)$  has a continuum of firms with **unobserved** productivity  $z_i$  distributed Pareto, tail  $\kappa_s^\ell$
- A firm  $i$  is thus characterized by the triple  $(\ell, s, z_i)$ , denoting type, sector, and productivity

## Market Structure: Upstream Second and Third Degree Price Discrimination

- Retail firms sell to the representative consumer at uniform per-unit prices
- Upstream firms set nonlinear prices to other upstream firms and retailers
- Firms can price discriminate:
  - ① Within types and sectors  $(\ell, s)$  with unobserved  $z_i$ , but know  $z_i$  distribution ( $2^{nd}$ )
  - ② Across observed types and sectors  $(\ell, s)$  ( $3^{rd}$ )
- Firms are atomistic in input markets as buyers and take the wage as given

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**Main Challenge: Find endogenous sufficient conditions to get  $2^{nd} + 3^{rd}$  tractable in supply chains:**

- Constant marginal costs
- Isoelastic demands

## Preferences

- The representative consumer owns all firms and inelastically supplies one unit of labor ( $L=1$ )
- Final demand is Cobb–Douglas across retail sectors with within–sector CES over retail varieties:

$$Y = \prod_{s \in S} Y_s^{\theta_s}, \quad \sum_{s \in S} \theta_s = 1, \quad Y_s = \left( \int_{j \in R_s} y_j^{\frac{\varphi_s-1}{\varphi_s}} dv_s(j) \right)^{\frac{\varphi_s}{\varphi_s-1}}$$

- $\theta_s \in (0, 1)$  are Cobb–Douglas output elasticities
- $\varphi_s > 1$  is the within-sector elasticity, and  $dv_s(j)$  denotes number active retail varieties  $R_s$  in  $s$
- The total number of active varieties in  $(r, s)$  is  $N_s^r := v_s(R_s)$ , an endogenous equilibrium object

## Technology

- Firm  $i \in (\ell, s)$  output ( $Q_i$ ) is CD in labor ( $l_i$ ) and in a CD aggregator of sectoral materials ( $M_i$ )

$$Q_i = z_i l_i^{\alpha_s^\ell} M_i^{1-\alpha_s^\ell}, \quad 0 < \alpha_s^\ell < 1,$$

$$M_i = \prod_{s' \in S} M_{is'}^{\theta_{ss'}^\ell}, \quad \sum_{s' \in S} \theta_{ss'}^\ell = 1, \quad M_{is'} = \left( \int_{j \in U_{s'}} m_{ij}^{\frac{\sigma_{s'}-1}{\sigma_{s'}}} dv_{s'}(j) \right)^{\frac{\sigma_{s'}}{\sigma_{s'}-1}}$$

- $\alpha_s^\ell$  is the labor output elasticity,  $M_{is'}$  is firm  $i$ ' material bundle from upstream sector  $s'$
- $\theta_{ss'}^\ell \geq 0$  are the sector  $s'$  input elasticities for sector  $s$  and firm type  $\ell$
- $m_{ij}$  is firm  $i$ 's input of upstream variety  $j$  in sector  $s'$
- $\sigma_{s'} > 1$  is the CES across varieties inside upstream sector  $s'$
- $N_{s'} := v_{s'}(U_{s'})$  is the endogenous measure over the active upstream firms in sector  $s'$

## Firm Entry

- In each  $(\ell, s)$  there is an unbounded pool of identical potential entrants
- Entrants pay a sunk cost  $c_s^{E\ell} > 0$  in units of labor, then observe their productivity  $z$
- Active firms exit exogenously at the end of the period with probability  $\delta_s^\ell \in [0, 1)$
- Free entry requires that the expected discounted value of profits  $(\pi_i^{\ell s}(z))$  equals entry cost  $(c_s^{E\ell})$

$$\frac{1}{1 - \delta_s^\ell} \mathbb{E}_z \left[ \pi_i^{\ell s}(z) \right] = c_s^{E\ell} w, \quad \forall (\ell, s)$$

## Model Recap

- 1 Potential entrants in each firm type and sector pair  $(\ell, s)$  pay  $c_s^{E\ell}$  and then draw productivity  $z$
- 2 Upstream sellers see  $(\ell, s)$  and  $z$  distribution; post  $(\ell, s)$  nonlinear contracts  $\{m_j^{\ell,s}, T_j^{\ell,s}\}$
- 3 Retail sellers post uniform prices to final consumers
- 4 Buyers  $i = (\ell, s, z_i)$  observe price menus and  $w$ , then choose  $l_i$  and  $\{m_{ij}\}_j$  to max. profits
- 5 Production and trade occur, transfers  $\{T_{ij}\}_j$  are realized, and final demand  $\{y_j\}$  is met
- 6 Firms exit with probability  $\delta_s^\ell$
- 7 Contracts are enforceable, resale/arbitrage is ruled out

# Towards Setting the Seller Profit Maximization Problem

## 1 Guess and Verify [▶ Details](#)

- Guess 1: For every seller, optimal contracts are two-part tariffs specific to  $(\ell, s)$
- Guess 2: Upstream sellers face isoelastic demands,  $\sigma$

## 2 Guesses imply standard CES and Cobb-Douglas costs and price indices functional forms

[▶ Details](#)

## 3 For a seller $j$ , $\tau_{is'}^\ell \equiv P_{ss'}^\ell M_{is'}^{1/\sigma_{s'}}$ is a sufficient statistic for buyer's heterogeneity [▶ Details](#)

- $\tau_{is'}^\ell$  is Pareto distributed with tail parameter:  $\rho_{ss'}^\ell = \sigma_{s'} \xi_s^\ell > 1$

$$\xi_s^r = \frac{\kappa_s^r}{\varphi - 1} \quad (\text{retail}), \quad \xi_s^u = \frac{\kappa_s^u}{\sigma_s - 1} \quad (\text{upstream})$$



## Upstream Seller Profit Maximization Problem

- A seller  $j \in s'$  chooses a menu of a total transfer and a allocation  $\{T_{ij}^\ell, m_{ij}^\ell\}$  for each  $(\ell, s)$

$$\max_{\{T, m\}} \sum_{\ell \in \{u, r\}} \sum_{s \in S} N_s^\ell \mathbb{E}_{\tau_{is'}} [T(\tau) - c_j m_{ij}(\tau)], \quad \text{s.t. for each } (\ell, s): \text{IC, IR}$$

- The problem is additively separable across  $(\ell, s)$  and can be solved partition-by-partition.
- Following Lemma 1, the virtual-surplus reduction yields the pointwise optimization problem:

$$\max_{\{m(\tau)\}} N_s^\ell \mathbb{E}_{\tau_{is'}} \left[ \left( \tau - \frac{\tau}{\rho_{ss'}^\ell} \right) \frac{\sigma_{s'}}{\sigma_{s'} - 1} m(\tau)^{\frac{\sigma_{s'} - 1}{\sigma_{s'}}} - c_j m(\tau) \right]$$

- which is strictly concave in  $m$  since  $(\sigma_{s'} - 1)/\sigma_{s'} \in (0, 1)$

# Optimal Nonlinear Price

## Proposition 1: Optimal Nonlinear Price in Supply Chains

There is an equilibrium where the optimal contract offered by an upstream seller  $j \in U_{s'}$  to any buyer  $i = (\ell, s, z_i)$  is a two-part tariff:

$$T_{ij} = p_{js}^{\ell} m_{ij} + F_{js}^{\ell},$$

with a marginal price  $p$  that is constant across types and equals:

$$p_{js}^{\ell} = \mu_{ss'}^{\ell} c_j, \quad \mu_{ss'}^{\ell} = \frac{\rho_{ss'}^{\ell}}{\rho_{ss'}^{\ell} - 1}, \quad \rho_{ss'}^{\ell} = \xi_s^{\ell} \sigma_{s'}, \quad \xi_s^r \equiv \frac{\kappa_s^r}{\varphi - 1}, \text{ for retailers } \quad \xi_s^u \equiv \frac{\kappa_s^r}{\sigma_s - 1}, \text{ for upstream}$$

and a flat fee  $F$  chosen so that the lowest type's participation constraint binds,

$$\Pi(z_s^{\ell}) = 0 \iff F_{js}^{\ell} = \frac{1}{\sigma_{s'}} R_{ss'}^{\ell}(z_i^{\ell}, m^*(z_i^{\ell})).$$

For all partitions of firm types  $\ell \in \{u, r\}$  and buyer sectors  $s$ , each with its own sector-specific two-part tariff.

## Two Upstream Pricing Counterfactuals For Welfare Comparisons

### Planer Efficient Pricing (as in Baqaee and Farhi, 2021)

- Firms must charge markups to incentivize the optimal entry level
- But markup distorts input choices by acting as a uniform tax on production
- An output subsidy can restore undistorted marginal-cost, conditional on entry
- The subsidy is paid via a lump sum tax to the representative consumer

### Uniform prices (e.g, as in Edmond, Midrigan & Xu, 2023)

- Constant markup over marginal cost from monopolistic competition
- CES markups  $\mu^{LP} = \frac{\sigma}{\sigma-1}$ , strictly higher than  $\mu^{NLP} = \frac{\rho}{\rho-1}$
- Because unambiguously  $\sigma < \rho$

# Exact Welfare Decomposition: Intensive vs. Extensive Margins

Profit Functions

- If the wage is the numeraire, welfare is the inverse final price index:  $W \equiv \frac{1}{P_Y}$  Derivation

$$\Delta \log W = \underbrace{-\sum_s \tilde{\lambda}_s^{cr} \Delta \log \mu_s^r - \sum_{s'} \bar{\lambda}_{s'}^{ru} \Delta \log \bar{\mu}_{s'}^{ru} - \sum_{s'} \bar{\lambda}_{s'}^{uu} \Delta \log \bar{\mu}_{s'}^{uu}}_{\text{Intensive margin (allocative markups)}} + \underbrace{\sum_s \frac{\tilde{\lambda}_s^{cr}}{\varphi_s - 1} \Delta \log N_s^r + \sum_{s'} \frac{\tilde{\lambda}_{s'}^{uu}}{\sigma_{s'} - 1} \Delta \log N_{s'}^u}_{\text{Extensive margin (Firm masses, entry/variety)}}$$

- $\tilde{\lambda}$  are final consumption direct and indirect costs exposures
- $\bar{\mu}$  contains seller sectors charging buyer-sector specific markups
- Allocative prices markups drive the intensive margin ( $\mu$ , extent of double marginalization)
- Flat fees drive the extensive margin through distorted profits ( $N$ , firm masses and love of variety)

## Welfare Ratios Across Price Regimes: Nonlinear vs. Uniform

$$\frac{W^{\text{NLP}}}{W^{\text{Uni}}} = \underbrace{\prod_{s' \in S} \left( \frac{\bar{\mu}_{s'}^{ru, \text{NLP}}}{\bar{\mu}_{s'}^{ru, \text{Uni}}} \right)^{-\bar{\lambda}_{s'}^{ru}} \times \prod_{s' \in S} \left( \frac{\bar{\mu}_{s'}^{uu, \text{NLP}}}{\bar{\mu}_{s'}^{uu, \text{Uni}}} \right)^{-\bar{\lambda}_{s'}^{uu}}}_{\text{Intensive Margin}} \times \underbrace{\prod_{s \in S} \left( \frac{N_s^{r, \text{NLP}}}{N_s^{r, \text{Uni}}} \right)^{\frac{\tilde{\lambda}_s^{cr}}{\varphi_s - 1}} \times \prod_{s' \in S} \left( \frac{N_{s'}^{u, \text{NLP}}}{N_{s'}^{u, \text{Uni}}} \right)^{\frac{\tilde{\lambda}_{s'}^{uu}}{\sigma_{s'} - 1}}}_{\text{Extensive Margin}}$$

- o Intensive margin (unambiguous gain):

$\mu^{\text{NLP}} < \mu^{\text{Uni}}$ , attenuating double marginalization

- o Extensive margin (ambiguous):

Flat fees shift profits with ambiguous sign, Firm entry can go either way

- o Welfare comparison? Requires full model quantification

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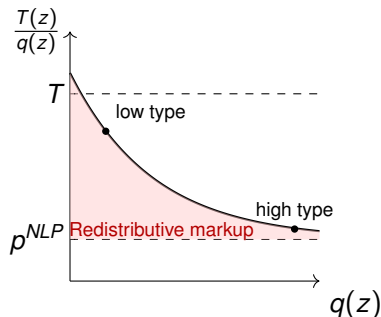
**3. Descriptive Evidence**

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## Optimal Nonlinear Price: Testable Prediction

- If pricing in the data is equivalent to a two-part tariff:  $T(z) = F + p^{\text{NLP}} q(z)$
- Average unit price is:  $\frac{T(z)}{q(z)} = \frac{F}{q(z)} + p^{\text{NLP}}$
- Decreasing and convex in  $q$
- Has a horizontal asymptote at  $p^{\text{NLP}}$



# Data Sources

Invoice example

## Invoice transactions for the universe of Chilean formal firms in 2024

- 1.4 billion transactions
- More than 10 million different products. We assume products are seller-specific
- Data on **prices and quantities for every product transacted**  
(Transport cost is usually a different product)

## Merged with firms' accounting balance-sheet data

- Sales, materials, investment, 6-digit sectors
- Employer-employee data: Wages, headcount
- Capital stock and investment

Data cleaning



## Real Data Example: Fix seller (X. inc), Product (Y), and Day

**X Inc. — Product Y** Invoice # 0001  
Buyer sector: **Manufacturing**

| Qty | Unit price (CLP) | Total (CLP) |
|-----|------------------|-------------|
| 1   | 4.990            | 4.990       |

**X Inc. — Product Y** Invoice # 0002  
Buyer sector: **Manufacturing**

| Qty | Unit price (CLP) | Total (CLP) |
|-----|------------------|-------------|
| 5   | 4.390            | 21.950      |

**X Inc. — Product Y** Invoice # 0003  
Buyer sector: **Mining**

| Qty | Unit price (CLP) | Total (CLP) |
|-----|------------------|-------------|
| 1   | 5.990            | 5.990       |

**X Inc. — Product Y** Invoice # 0004  
Buyer sector: **Mining**

| Qty | Unit price (CLP) | Total (CLP) |
|-----|------------------|-------------|
| 5   | 5.490            | 27.450      |

- Manufacturing: unit prices drop from 4.990 to 4.390 when moving from  $q=1$  to  $q=5$
- Mining: a distinct menu with higher unit prices, 5.990 at  $q=1$  and 5.490 at  $q=5$
- Consistent with buyer-sector-specific nonlinear prices

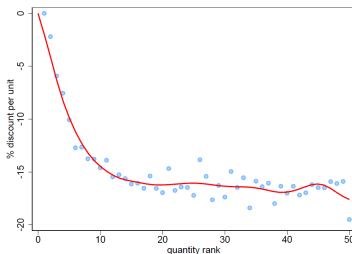
# Nonlinear Prices by Quantity Bins and Seller Sector

Bins construction

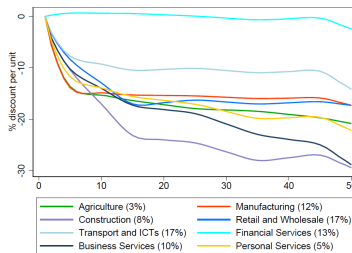
Bins histogram

$$\ln p_{jgit} = \beta_0 + \sum_{b=2}^{50} \beta_b \mathbb{1}_{\{B_{jgit}=b\}} + \psi_{jgd} + \varepsilon_{jgit}$$

(A) All sectors

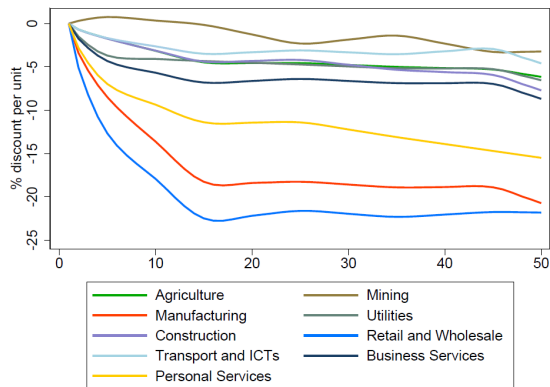


(B) By seller sector



- Unit prices fall steeply at small  $q$  and flatten as  $q$  grows (consistent with prediction)
- Between seller sector heterogeneity in both steepness and curvature

## Retail & Wholesale Seller Sector: Pricing to Different Buyer Sectors



- Within a seller sector, nonlinear price schedules differ by buyer sector
- Buyer sector shifts price schedule without much change in curvature

## Taking Stock

- Within seller×product×day, unit prices decline with quantity and flatten at higher ranks
- Curvature, levels, and steepness are different across seller sectors
- Within a seller sector, the price schedule shifts with buyer sector
- Evidence inconsistent with uniform pricing
- Pricing consistent with a combination of price-menus ( $2^{nd}$ ) specific to buyer sectors ( $3^{rd}$ ):
  - $2^{nd}$  degree screening drives curvature
  - $3^{rd}$  degree shifts levels across buyer sectors

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## Three Quantitative Exercises

- Calibration using population-level B2B transactions and firm balance sheets for Chile
- Two calibrations depending on observed prices interpretation (nonlinear and uniform)

### ① Model fit

How much of the observed pricing schemes the model can explain

### ② Policy

Welfare outcomes of banning all forms of price discrimination

### ③ Measurement

Welfare cost under two interpretations of the same data: nonlinear vs. uniform pricing

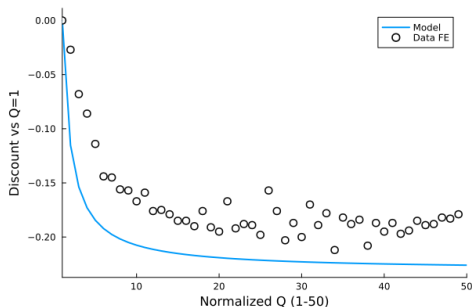
## Parameter Estimation

| Parameter                                       | Strategy                            | Granularity                    |
|---|-------------------------------------|--------------------------------|
| Labor output elasticity ( $\alpha_s^\ell$ )     | Estimated from data                 | 626 sectors $\times$ firm type |
| Final demand elasticity ( $\theta_r$ )          | Estimated from data                 | 626 sectors                    |
| Input-Output elasticity ( $\theta_{ss'}^\ell$ ) | Estimated from data                 | 626 sectors $\times$ firm type |
| Final demand bundle elasticity ( $\varphi_s$ )  | Pinned down by CES results and data | 11 sectors                     |
| Material bundle elasticity ( $\sigma_{s'}$ )    | COVID-19 shock for Chile estimation | 11 sectors                     |
| Exit rate ( $\delta^\ell$ )                     | Estimated from data                 | 626 sectors $\times$ firm type |
| Entry cost ( $c_e^\ell$ )                       | Pinned down by free entry and data  | 626 sectors $\times$ firm type |
| Productivity Pareto tail ( $\kappa^\ell$ )      | MLE estimation                      | 11 sectors $\times$ firm type  |

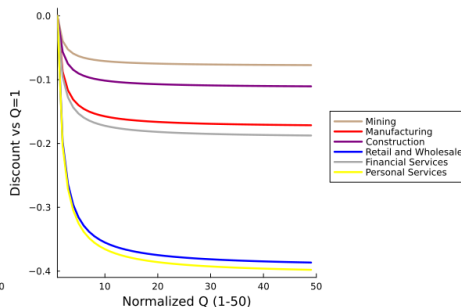
- $\sigma_{s'}$ ,  $\kappa^\ell$  jointly pin the marginal price:
  - Lower  $\sigma_{s'}$ ,  $\kappa^\ell$  (fatter tail, more dispersion) implies higher marginal marked-up prices
- Buyer surplus can be extracted by flat fees, is mainly determined by  $\kappa^\ell$ :
  - Large  $\kappa^\ell$  implies low marginal price and thus a higher flat fee

## Model Fit (untargeted): Nonlinear Prices Interpretation

A. Model unit prices vs.  
data fixed-effects regression



B. Model retail and wholesale sector  
unit prices to selected buyer sectors



- For the average upstream firm price schedule to retailers, normalizing the continuous input quantity to lie between 1 and 50



## Policy: Ban on Price Discrimination Welfare Outcome

$$\frac{W^{\text{Reg}}}{W^{\text{Eff}}} = \underbrace{\prod_{s \in S} \left( \frac{\bar{\mu}_s^{ur, \text{Reg}}}{\bar{\mu}_s^{ur, \text{Eff}}} \right)^{-\tilde{\lambda}_s^{ru}} \prod_{s \in S} \left( \frac{\bar{\mu}_s^{uu, \text{Reg}}}{\bar{\mu}_s^{uu, \text{Eff}}} \right)^{-\tilde{\lambda}_s^{uu}}}_{\text{Intensive Margin}} \times \underbrace{\prod_{s \in S} \left( \frac{N_s^{r, \text{Reg}}}{N_s^{r, \text{Eff}}} \right)^{\frac{\theta_s}{\varphi_s - 1}} \prod_{s \in S} \left( \frac{N_s^{u, \text{Reg}}}{N_s^{u, \text{Eff}}} \right)^{\frac{\tilde{\lambda}_s^{uu}}{\sigma_s - 1}}}_{\text{Extensive Margin}}$$

| Price regime          | $W^R / W^{\text{Eff}}$ |
|-----------------------|------------------------|
| Nonlinear (NLP)       | 0.745                  |
| Uniform pricing (Uni) | 0.486                  |

- Banning price discrimination reduces welfare from  $\approx 75\%$  of efficient welfare to  $\approx 50\%$
- Allowing for price discrimination closes about half of the efficiency gap:

$$\frac{W^{\text{NLP}} - W^{\text{Uni}}}{W^{\text{Eff}} - W^{\text{Uni}}} = \frac{0.745 - 0.486}{1 - 0.486} \approx 0.50.$$

## Policy: Aggregate Welfare Decomposition (rel. to the efficient BMK)

$$\frac{W^{\text{Reg}}}{W^{\text{Eff}}} = \underbrace{\prod_{s \in S} \left( \frac{\bar{\mu}_s^{ur, \text{Reg}}}{\bar{\mu}_s^{ur, \text{Eff}}} \right)^{-\tilde{\lambda}_s^{ur}} \prod_{s \in S} \left( \frac{\bar{\mu}_s^{uu, \text{Reg}}}{\bar{\mu}_s^{uu, \text{Eff}}} \right)^{-\tilde{\lambda}_s^{uu}}}_{\text{Intensive Margin}} \times \underbrace{\prod_{s \in S} \left( \frac{N_s^{r, \text{Reg}}}{N_s^{r, \text{Eff}}} \right)^{\frac{\theta_s}{\varphi_s - 1}} \prod_{s \in S} \left( \frac{N_s^{u, \text{Reg}}}{N_s^{u, \text{Eff}}} \right)^{\frac{\tilde{\lambda}_s^{uu}}{\sigma_s - 1}}}_{\text{Extensive Margin}}$$

| Regime rel. to. eff. | Intensive | Extensive | Share <sub>int</sub> | Share <sub>ext</sub> |
|----------------------|-----------|-----------|----------------------|----------------------|
| Nonlinear (NLP)      | 0.67      | 1.12      | 0.79                 | 0.21                 |
| Uniform (Uni)        | 0.46      | 1.06      | 0.93                 | 0.07                 |

### o Result

Relative to efficiency, markups create higher expected profits, and thus more entry: More firms at a smaller scale

### o Intensive Margin Dominates (as a share of total log deviation relative to Eff.)

NLP: 79%, Uni: 93%. **Banning price discrimination raises double marginalization along the supply chain**

### o Extensive Margin: Entry Responses

The extensive margin is pro-competitive (factors > 1) but modest

## Policy: Opening Welfare Ratios by Sector

$$\frac{W^{\text{NLP}}}{W^{\text{Uni}}} = \underbrace{\prod_{s \in S} \left( \frac{\bar{\mu}_s^{ur, \text{NLP}}}{\bar{\mu}_s^{ur, \text{Uni}}} \right)^{-\tilde{\lambda}_s^{ru}} \prod_{s \in S} \left( \frac{\bar{\mu}_s^{uu, \text{NLP}}}{\bar{\mu}_s^{uu, \text{Uni}}} \right)^{-\tilde{\lambda}_s^{uu}}}_{\text{Intensive Margin}} \times \underbrace{\prod_{s \in S} \left( \frac{N_s^{r, \text{NLP}}}{N_s^{r, \text{Uni}}} \right)^{\frac{\theta_s}{\varphi_s - 1}} \prod_{s \in S} \left( \frac{N_s^{u, \text{NLP}}}{N_s^{u, \text{Uni}}} \right)^{\frac{\tilde{\lambda}_s^{uu}}{\sigma_s - 1}}}_{\text{Extensive Margin}}$$

| Sector                 | Intensive (allocative) |          | Extensive (variety) |          | Net NLP/Uni |
|------------------------|------------------------|----------|---------------------|----------|-------------|
|                        | Retailers              | Upstream | Retailers           | Upstream |             |
| Agriculture            | 1.010                  | 1.010    | 0.997               | 1.005    | 1.022       |
| Mining                 | 1.003                  | 1.003    | 0.999               | 1.014    | 1.019       |
| Manufacturing          | 1.024                  | 1.029    | 0.991               | 1.002    | 1.047       |
| Utilities              | 1.016                  | 1.006    | 0.996               | 1.033    | 1.051       |
| Construction           | 1.061                  | 1.022    | 0.980               | 1.119    | 1.189       |
| Retail and Wholesale   | 1.037                  | 1.070    | 0.992               | 1.005    | 1.106       |
| Transport and ICTs     | 1.007                  | 1.023    | 0.981               | 1.000    | 1.011       |
| Financial Services     | 1.012                  | 1.008    | 0.943               | 0.998    | 0.960       |
| Real Estate Services   | 1.009                  | 1.004    | 0.996               | 1.023    | 1.033       |
| Business Services      | 1.005                  | 1.006    | 0.989               | 0.999    | 0.999       |
| Personal Services      | 1.001                  | 1.001    | 0.998               | 1.000    | 1.000       |
| Product across sectors | 1.197                  | 1.198    | 0.870               | 1.207    | 1.507       |

## Measurement: Nonlinear vs. Uniform Pricing Interpretation

| Price Lens | $W^L / W^{\text{Eff}}$ | Intensive  | Extensive  |
|------------|------------------------|------------|------------|
| Nonlinear  | 0.748                  | 0.67 (79%) | 1.12 (21%) |
| Uniform    | 0.565                  | 0.55 (97%) | 1.02 (3%)  |

- Two model quantifications, dependent on the data interpretation
- Welfare falls from 75% (NLP) to 57% (Uni) when changing pricing assumption
- Nonlinear prices interpretation closes the gap by about 18 percentage points
- Aggregate costs of market power are lower when the model allows for price discrimination

# Table of Contents

- 1. Partial Equilibrium: Nonlinear Price Characterization
- 2. General Equilibrium Model in Supply Chains
- 3. Descriptive Evidence
- 4. Model Quantification
- 5. Conclusion

## Conclusion

- We find indicative widespread evidence that sellers set nonlinear prices (NLP) in supply chains
- NLP improves allocations relative to uniform, but shifts rents via flat fees: distorts entry
- Banning price discrimination in supply chains can raise market power aggregate costs
- Average prices can mislead. Policy should monitor marginal prices and rent extraction
- Don't necessarily ban quantity discounts; target markup accumulation along the supply chain
- The method is plug-and-play with standard micro-data on transactions



Virtual surplus: profit from serving a buyer type, net of the informational rents that must be left to higher types to preserve IC

- Virtual surplus for type  $z$  (with  $\alpha = \frac{\sigma}{\sigma-1}$ ):

$$VS(z) = \underbrace{\left(\frac{z^{\sigma-1}}{\alpha}\right) q(z)^{1-1/\sigma}}_{\text{Gains from serving type } z} - \underbrace{\left(\frac{1-F(z)}{f(z)} \cdot \frac{d}{dz} \left(\frac{z^{\sigma-1}}{\alpha}\right)\right) q(z)^{1-1/\sigma}}_{\text{Informational rents left to ensure IC}}$$

- For the lowest type  $z_0 = 1$ , the virtual surplus simplifies:

$$VS(1) = \left[ \frac{1}{\alpha} \left( 1 - \frac{\sigma - 1}{\kappa} \right) \right] q(1)^{1-1/\sigma}$$

- This is strictly positive whenever  $\kappa > \sigma - 1$  (necessary condition for finite output)
- If its profitable to serve the lowest type, the seller will not exclude any buyer



## Is price deviation profitable for any $z > z_a$ ? [Return](#)

- o Heuristic argument (Wilson 1993) to derive the optimal price  $p(q)$
- o Define marginal buyer  $z(q, p)$  by inverting demand for the  $q^{th}$  unit (equation 1)

$$z(q, p) = q^{\frac{1}{\sigma-1}} p^{\frac{\sigma}{\sigma-1}}$$

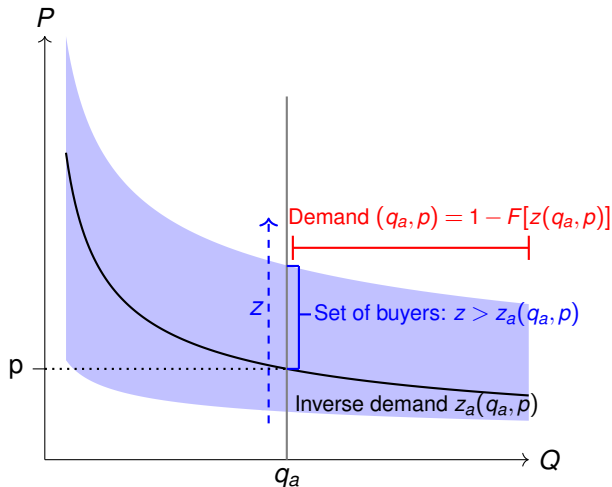
- o Demand for  $q^{th}$  unit:

$$D(q, p) = 1 - F(z(q, p))$$

- o Seller chooses a price for unit “q” to solve:

$$\max_p [1 - F(z_a(q_a, p))] (p - c)$$

## No profitable deviation in price ▶ Return



$$\max_{\{P\}} [1 - F(z_a(q_a, p))](P - c)$$

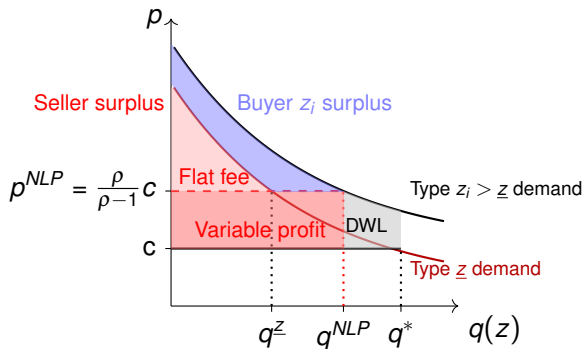
FOC :

$$\frac{P}{c} = \frac{\frac{\kappa\sigma}{\sigma-1}}{\frac{\kappa\sigma}{\sigma-1} - 1} = \frac{\rho}{\rho - 1}$$

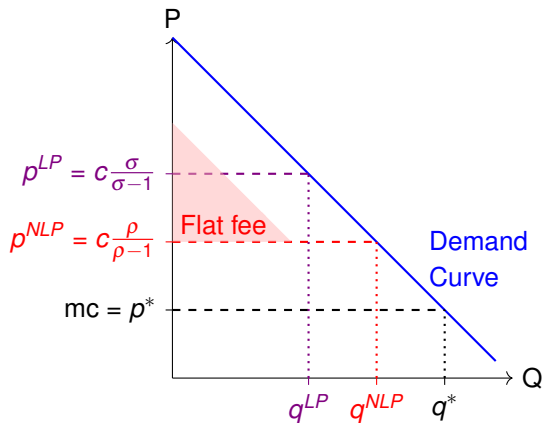
- o The optimal price is equal to the **allocative price of the two-part tariff**
- o Seller has no incentive to charge different prices for different quantities

## Optimal Nonlinear Price: Allocations [Return](#)

- Virtual surplus for lower type is strictly positive: **All types are served** [Details](#)
- Quantities are pinned down by marginal price  $p^{NLP}$
- Flat fee only redistributes surplus; is not allocative



## Two-Part Tariff: NLP vs LP CES markup ( $\rho > \sigma$ ) [Return](#)



- Allocations in NLP are less distorted relative to LP

$$q^* > q^{NLP} > q^{LP}$$

- Because of the flat fee, rents are subject to different distortions in NLP vs. LP



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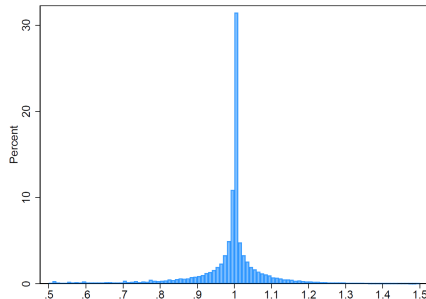
## Data cleaning [Return](#)

**Goal:** Keep all plausible transactions

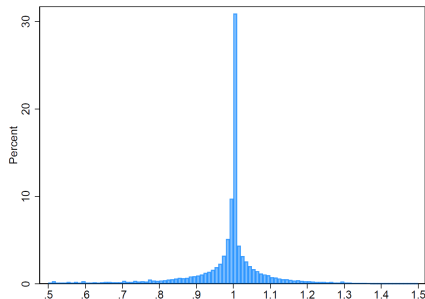
- Prices are net of discounts and recharges
- Drop if a transaction has missing or zero price or quantity
- Drop if product description is missing
- Drop transactions where seller-product-day maxmin price ratio is above the 99<sup>th</sup> percentile
- Under this cleaning we keep around 95% of transactions

# Price Dispersion Return

Panel A. June 2024 (t=month)



Panel B. June 19th 2024 (t=day)



- $\theta_{jgt} = \frac{p_{jgit}}{\bar{p}_{jgt}}$  ; seller  $j$ , product  $g$ , buyer  $i$ , time  $t$  (excluding products with one transaction)
- $\text{Var}(\log \theta_{jgd}) = 0.65$
- indicative evidence inconsistent with uniform pricing in 70% of transactions





## Price Variance Determinants for 2024: Strategy [Return](#)

### Step 1

- o Make goods comparable and eliminate possible demand and supply shocks
- o Store **residuals** from:

$$\ln p_{jgit} = \beta_0 + \Psi_{jgd} + \epsilon_{jgit}$$

$p_{jgit}$  is the price for seller  $j$ , product  $g$ , buyer  $i$  in time  $t$ ,  $\Psi$  is a fixed effect including day  $d$

### Step 2

- o Project residuals on different observables (quantity transacted and buyers' observables)
- o Compare  $R^2$

## Price Determinants for 2024: Results [Return](#)

Seller  $j$ , product  $g$ , buyer  $i$ , time  $t$ , day  $d$ , quantity  $q$ , buyer group  $B$  (11 sectors  $\times$  3 sizes  $\times$  16 regions)

$$\varepsilon_{jgit} = \beta_0 + \psi_{jgdS} + \varepsilon_{jgit}$$

|   | (1)  | (2)  | (2)  |
|---|------|------|------|
| $R^2$   | 0.34 | 0.28 | 0.53 |
| $S = \text{Quantity}$                           | ✓    |      |      |
| $S = \text{Buyer Group}$                        |      | ✓    |      |
| $S = \text{Quantity} \times \text{Buyer group}$ |      |      | ✓    |
| N   | 147M | 147M | 147M |

- o Consistent whit hybrid second + thrid dregree price discrimination schemes

## Price Determinants for 2024: Monthly Fixed Effects [Return](#)

Seller  $j$ , product  $g$ , buyer  $i$ , time  $t$ , month  $m$ , quantity  $q$ , buyer group  $B$  (11 sectors  $\times$  3 sizes  $\times$  16 regions)

$$\ln p_{jgit} = \beta_0 + \psi_{jgdS} + \varepsilon_{jgit}$$

|   | (1)  | (2)  | (3)  | (4)  |
|---|------|------|------|------|
| $R^2$   | 0.34 | 0.51 | 0.41 | 0.62 |
| $S = \text{Quantity}$                           | ✓    |      |      |      |
| $S = \text{Buyer}$                              |      | ✓    |      |      |
| $S = \text{Buyer Group}$                        |      |      | ✓    |      |
| $S = \text{Quantity} \times \text{Buyer group}$ |      |      |      | ✓    |
| N   | 363M | 363M | 363M | 363M |

## Price Determinants for 2024: Manufacturing [Return](#)

Seller  $j$ , product  $g$ , buyer  $i$ , time  $t$ , month  $m$ , quantity  $q$ , buyer group  $B$  (11 sectors  $\times$  3 sizes  $\times$  16 regions)

$$\ln p_{jgit} = \beta_0 + \psi_{jgmS} + \varepsilon_{jgit}$$

|   | (1)  | (2)  | (3)  | (4)  |
|---|------|------|------|------|
| $R^2$   | 0.45 | 0.54 | 0.46 | 0.81 |
| $S = \text{Quantity}$                           | ✓    |      |      |      |
| $S = \text{Buyer}$                              |      | ✓    |      |      |
| $S = \text{Buyer Group}$                        |      |      | ✓    |      |
| $S = \text{Quantity} \times \text{Buyer group}$ |      |      |      | ✓    |
| N   | 136M | 136M | 136M | 136M |

## Price Determinants for 2024: Retail and Wholesale [Return](#)

Seller  $j$ , product  $g$ , buyer  $i$ , time  $t$ , month  $m$ , quantity  $q$ , buyer group  $B$  (11 sectors  $\times$  3 sizes  $\times$  16 regions)

$$\ln p_{jgit} = \beta_0 + \psi_{jgmS} + \varepsilon_{jgit}$$

|   | (1)  | (2)  | (3)  | (4)  |
|---|------|------|------|------|
| $R^2$   | 0.38 | 0.65 | 0.49 | 0.68 |
| $S = \text{Quantity}$                           | ✓    |      |      |      |
| $S = \text{Buyer}$                              |      | ✓    |      |      |
| $S = \text{Buyer Group}$                        |      |      | ✓    |      |
| $S = \text{Quantity} \times \text{Buyer group}$ |      |      |      | ✓    |
| N   | 180M | 180M | 180M | 180M |

## Buyer Market Power? [Return](#)

- Exploit cross-sectional variation in the number of suppliers each buyer transacts with
- A larger number of providers may indicate stronger outside options; better pricing terms

$$\ln p_{jgim} = \beta_0 + \beta_1 \ln q_{jgim} + \beta_2 (\log q_{jgim} \times \log \text{NumProviders}_i) + \Psi_{jgm} + \varepsilon_{jgit},$$

- $\beta_2 > 0$  would suggest that quantity discounts become flatter as buyer power increases
- We find that  $\beta_1 = -0.0462$  (0.0001) and  $\beta_2 = -0.0098$  (0.0001)
- Buyer power does not appear to be the primary mechanism generating quantity discounts

## Nonlinear Prices by Sector [▶ Return](#)

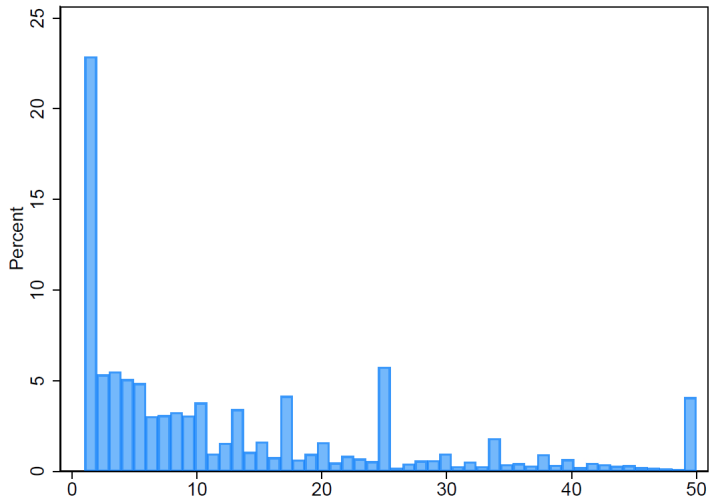
| Sector               | Mean Q discount | N transactions |
|----------------------|-----------------|----------------|
| All sectors          | -0.042          | 430M           |
| Agriculture          | -0.042          | 2M             |
| Mining               | -0.016          | 1M             |
| Manufacturing        | -0.036          | 118M           |
| Utilities            | 0.000           | 6M             |
| Construction         | -0.129          | 1M             |
| Retail and Wholesale | -0.048          | 270M           |
| Transport & ICTs     | -0.032          | 12M            |
| Financial Services   | -0.002          | 49M            |
| Real Estate Services | -0.052          | 1M             |
| Business Services    | -0.089          | 5M             |
| Personal Services    | -0.053          | 1M             |



## Quantity Quantiles Bins [Return](#)

- Products have different scales, we compare prices across each product's  $q$  rank distribution
- For each product  $g$ ,  $F_g(\cdot)$ : empirical CDF of transacted quantities  $q_{jgit}$
- Define the within-product rank:  $r_{jgit} \equiv F_g(q_{jgit})$ .
- Partition  $[0, 1]$  into 50 equal-probability intervals  $I_b \equiv ((b-1)/50, b/50]$  for  $b = 1, \dots, 50$
- Assign each transaction to a bin  $B_{jgit} = b$  whenever  $r_{jgit} \in I_b$
- With discrete quantities and mass points, we assign observations to the smallest  $b$

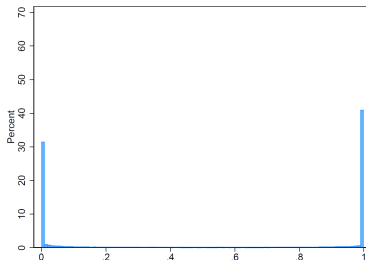
# Quantity Quantiles Bins Histogram [Return](#)



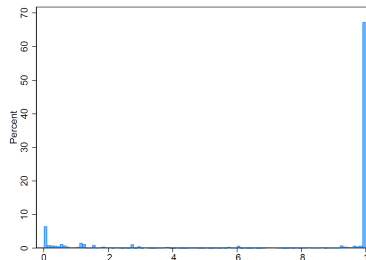
# Sales partition [Return](#)

$$X_i = \begin{cases} 0 & \text{if all sales go to final consumers} \\ 1 & \text{if all sales go to other firms} \end{cases}$$

Panel A. Number of Firms



Panel B. Sales weighted



- More than 70% of firms sell only to final consumers or to other firms [By sector](#)



## Guess and Verify [▶ Return](#)

Guess 1: Optimal contracts are isomorphic to a two-part tariff specific to  $(\ell, s)$

$$T_{ij} = p_{js}^{\ell} m_{ij} + F_{js'}^{\ell} = \mu_{ss'}^{\ell} c_j m_{ij} + F_{js'}^{\ell}$$

- Transfer  $T_{ij}$  depends on allocation  $m_{ij}$ , markup  $\mu_{ss'}^{\ell}$ , and flat fee  $F_{js'}^{\ell}$
- The marginal (allocative) price is  $p_{js}^{\ell} = \mu_{ss'}^{\ell} c_j$
- Flat fees are inframarginal and do not affect marginal input choices;

Guess 2: Revenue functions are homogeneous in quantity

$$R_{i,s}^{\ell} = A_s^{\ell} (Q_{i,s}^{\ell})^{\psi_s^{\ell}}$$

- For parameters  $A_s^{\ell}$  and  $\psi_s^{\ell}$  that are constants at buyer type and sector  $(\ell, s)$
- Imply isoelastic demands for intermediate inputs

- Marginal prices are quantity-invariant within a buyer type-sector  $(i, s)$  and seller sector  $s'$

## CES Sectoral Price Index

$$P_{ss'}^\ell = \left( \int_{j \in U_{s'}} \left( p_{js'}^\ell \right)^{1-\sigma_{s'}} dv_{ss'}(j) \right)^{\frac{1}{1-\sigma_{s'}}}$$

## Cobb–Douglas Materials Cost Index

$$P_i^M = \prod_{s' \in S} \left( P_{ss'}^\ell \right)^{\theta_{ss'}^\ell}, \quad \sum_{s' \in S} \theta_{ss'}^\ell = 1, \quad \theta_{ss'}^\ell \geq 0$$

## Firm-Level Marginal Cost

$$c_i = \frac{\Theta_s^\ell}{z_i} w_s^{\alpha_s^\ell} (P_i^M)^{1-\alpha_s^\ell}, \quad \Theta_s^\ell \equiv \left(\alpha_s^\ell\right)^{-\alpha_s^\ell} \left(1-\alpha_s^\ell\right)^{-(1-\alpha_s^\ell)} \prod_{s' \in S} \left(\theta_{ss'}^\ell\right)^{-(1-\alpha_s^\ell)\theta_{ss'}^\ell}$$

## Type Re-parametrization and Distribution for Screening [Return](#)

- For a seller  $j \in s'$ , each buyer  $i \in s$  matters only through the valuation index:  $\tau_{is'}^\ell \equiv P_{ss'}^\ell M_{is'}^{1/\sigma_{s'}}$
- For a seller  $j$ ,  $\tau_{is'}^\ell$  is a sufficient statistic for buyer's heterogeneity
- $P_{ss'}^\ell$  price level faced by  $i$  for inputs from  $s'$ , and  $M_{is'}$  is the buyer's demand shifter (scale)
- Under Pareto distributed buyer productivity,  $\tau_{is'}^\ell$  is Pareto with tail parameter:

$$\rho_{ss'}^\ell = \sigma_{s'} \xi_s^\ell > 1$$

- Type-specific heterogeneity maps to  $\xi_s^\ell$ :

$$\xi_s^r = \frac{\kappa_s^r}{\varphi - 1} \quad (\text{retail}), \quad \xi_s^u = \frac{\kappa_s^u}{\sigma_s - 1} \quad (\text{upstream})$$

## Guesses Verification [▶ Return](#)

Guess 1 (two-part tariffs with quantity-invariant marginal price within each  $(\ell, b, u)$ )

- o follows immediately from the two-part tariff and constant markup in Proposition 1

Guess 2 (homogenous link revenue)

- o Is verified by aggregating optimal link choices across partitions
- o The seller's total revenue is isoelastic in own quantity with exponent  $(\sigma_u - 1)/\sigma_u$
- o Admits a closed-form scale  $A_{su}$  that explicitly includes a flat-fee component driven by the seller's CES share in buyers' materials bundles



## General Equilibrium Under Nonlinear Prices [▶ Return](#)

A general equilibrium under nonlinear pricing is a collection

$$\left\{ (m_{ubi}^{\ell}(\cdot), T_{ubi}^{\ell}(\cdot), B_{\ell bi})_{(\ell,b),i}, (p_{bi}^{\ell,*})_{(\ell,b),i}, (P_{ub}^{\ell})_{u,b,\ell}, (N_s^{\ell})_{s,\ell}, (Q_j, l_j)_j \right\}$$

such that: (i) mechanisms  $(m_{ubi}^{\ell}(\cdot), T_{ubi}^{\ell}(\cdot))$  implement the two-part-tariff optimum with  $p_{bi}^{\ell,*}$  and  $F_{ubi}^{\ell,*}$  in Proposition 1; (ii) buyers' choices satisfy the best-response condition above; (iii) price and cost indices satisfy ; (iv) materials and labor markets clear with  $L = 1$ ; and (v) free entry holds in each  $(\ell, s)$ . A detailed proof of existence and uniqueness is provided in the paper.

## Upstream Firm Profits Under Nonlinear Prices: [Return](#)

$$\mathbb{E} [\Pi_j^u] = \underbrace{\sum_{\ell} \sum_s \int_{i \in B_{\ell s}} (p_{js}^{\ell} - c_j) m_{ij} dv_{\ell s}^i}_{\text{allocative margin}} + \underbrace{\sum_{\ell} \sum_s \int_{i \in B_{\ell s}} F_{jsi}^{\ell} dv_{\ell s}^i}_{\text{flat-fee revenue}} - \underbrace{\sum_{s'} \int_{j' \in D_j} F_{j',s'}^u dv_{s'}^{j'}}_{\text{flat-fee payments}}$$

- o  $B$  denotes seller firm  $j$  client set,  $D$  denote seller firm  $j$  suppliers set (exogenous sets)
- o **NLP Marginal Prices.** Charge and pay smaller marginal prices  $(p_{js}^{\ell}, c_j)$  relative to Lin. P.
- o **Rents.** Through flat fees seller  $j$ , extracts rents, but it's also rent extracted
- o **GE incidence.** Cheaper  $c_j$  lift downstream demand; double marginalization attenuation
- o **Entry.** Depends on  $m_{ij}$  expansion and net rent extraction (firm entry is misallocated)

## Retailer Firm Profits Under Nonlinear Prices [Return](#)

$$\mathbb{E}[\Pi'_i] = \underbrace{\left(\frac{1}{\phi_s}\right) R_i}_{\text{allocative margin}} - \underbrace{\sum_{s'} \int_{j \in D_j} F_{ijs'}^r dv_{s'}^j}_{\text{Flat fees payments to upstream}}$$

- o **Constant markup** Allocative margin has a constant markup and a fixed share of revenue
- o **NLP Marginal Prices.** NLP lowers input costs, retail prices fall with constant markup and revenue expands; the allocative term scales proportionally with  $R_i$
- o **Rents.** Extracted via fee payments to upstream
- o **Entry.** Depends on change in profits: revenue expansion versus rent extraction



$$P_s = \mu_s^r \prod_{s'} (\mu_{ss'}^{ru} C_{s'})^{(1-\alpha_s^r)\theta_{ss'}^r} (N_s^r)^{-\frac{1}{\phi_s-1}} \gamma_s.$$
$$C_{S'} = \prod_V (\mu_{VS'}^{UU} C_V)^{(1-\alpha_{S'}^U)\theta_{S'V}^U} (N_{S'}^U)^{-\frac{1}{\sigma_{S'}-1}} \gamma_{S'}^U.$$

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## Final-Demand Exposure

Buyer–seller markups load into welfare through exposure matrices:

$$\Lambda^{ru} = \text{Diag}(b)\Omega^{ru}, \quad \Lambda^{uu} = \text{Diag}(b\Omega^{ru})\Psi^{uu}.$$

For each seller  $s'$ :

$$\bar{\lambda}_{s'}^{ru} = \sum_s \Lambda_{ss'}^{ru}, \quad \bar{\lambda}_{s'}^{uu} = \sum_u \Lambda_{us'}^{uu}.$$

Define normalized buyer weights:

$$\omega_{s|s'}^{ru} := \frac{\Lambda_{ss'}^{ru}}{\bar{\lambda}_{s'}^{ru}}, \quad \omega_{u|s'}^{uu} := \frac{\Lambda_{us'}^{uu}}{\bar{\lambda}_{s'}^{uu}}, \quad \text{with } \sum_s \omega_{s|s'}^{ru} = \sum_u \omega_{u|s'}^{uu} = 1.$$

Then the exposure-weighted seller-level markup changes are

$$\Delta \log \bar{\mu}_{s'}^{ru} = \sum_s \omega_{s|s'}^{ru} \Delta \log \mu_{ss'}^{ru}, \quad \Delta \log \bar{\mu}_{s'}^{uu} = \sum_u \omega_{u|s'}^{uu} \Delta \log \mu_{us'}^{uu}.$$

- Welfare aggregates over seller-level distortions, weighted by exposure to final demand
- Pareto structure guarantees sectoral aggregation is exact—no loss of generality

## Exact Welfare Decomposition

Using exposure mappings and free entry:

$$\begin{aligned} \Delta \log W = & \underbrace{- \sum_s \tilde{\lambda}_s^{cr} \Delta \log \mu_s^r - \sum_{s'} \bar{\lambda}_{s'}^{ru} \Delta \log \bar{\mu}_{s'}^{ru} - \sum_{s'} \bar{\lambda}_{s'}^{uu} \Delta \log \bar{\mu}_{s'}^{uu}}_{\text{Intensive margin (allocative markups)}} \\ & + \underbrace{\sum_s \frac{\tilde{\lambda}_s^{cr}}{\phi_s - 1} \Delta \log N_s^r + \sum_{s'} \frac{\tilde{\lambda}_{s'}^{uu}}{\sigma_{s'} - 1} \Delta \log N_{s'}^u}_{\text{Extensive margin (entry/variety)}} \end{aligned}$$

- Each markup or entry change affects welfare in proportion to its exposure weight.
- Buyer-specific pricing (nonlinear) alters both weights and effective markups.
- The aggregate welfare impact of market power depends on pricing form—uniform vs. nonlinear.

## Labor Output Elasticity $\alpha_s$ [▶ Return](#)

- o **What.** Cobb–Douglas weight on *non-materials* (labor + user cost of capital).
- o **Identify.** Cost-share mapping under cost minimization:

$$\alpha_i = 1 - \frac{\sum_j p_{ji} m_{ji}}{w_i L_i + r_i K_i + \sum_j p_{ji} m_{ji}}.$$

Flat fees:  $TC_i = F_i + VC_i$ ; for large buyers  $F_i / TC_i$  is small  $\Rightarrow$  variable share  $\approx$  total share.

- o **Sample.** Keep firms above 75th pctl. revenue; winsorize  $\alpha_i$  at 1–99; aggregate to  $(s, \ell)$  at 6-digit; average 2005–2022.
- o **Why.** Governs response to wage vs. input-price shocks: higher  $\alpha_s$  amplifies wage relevance, dampens price conduct action from materials prices.



| Sector               | Retailers | Upstream | Sector mean |
|----------------------|-----------|----------|-------------|
| Agriculture          | 0.43      | 0.41     | 0.42        |
| Mining               | 0.25      | 0.32     | 0.29        |
| Manufacturing        | 0.39      | 0.42     | 0.41        |
| Utilities            | 0.37      | 0.38     | 0.38        |
| Construction         | 0.48      | 0.42     | 0.45        |
| Retail and Wholesale | 0.37      | 0.31     | 0.34        |
| Transport and ICTs   | 0.55      | 0.47     | 0.51        |
| Financial Services   | 0.58      | 0.62     | 0.60        |
| Real Estate Services | 0.66      | 0.53     | 0.59        |
| Business Services    | 0.72      | 0.65     | 0.69        |
| Personal Services    | 0.71      | 0.57     | 0.64        |
| Type mean            | 0.50      | 0.46     | 0.48        |

## Final-Demand Output Elasticity $\theta_s$

- o **What.** Cobb–Douglas weights across *retail sectors* in final demand.
- o **Identify.** With linear pricing to consumers, retail revenues identify expenditure shares:

$$\theta_s \approx \frac{\text{retail revenue in } s}{\sum_{s'} \text{retail revenue in } s'}.$$

- o **Sample.** Large retailers (>75th pctl.), compute annual sector shares, average 2005–2022; check revenue-weighted robustness.
- o **Why.** Anchors final-demand system and welfare accounting in counterfactuals.

## Final-Demand Shares $\theta_s$ (Results)

### Cobb–Douglas Output Elasticities by Retail Sector

| Sector               | $\theta_s$ |
|----------------------|------------|
| Agriculture          | 0.0446     |
| Mining               | 0.0085     |
| Manufacturing        | 0.1318     |
| Utilities            | 0.0505     |
| Construction         | 0.1521     |
| Retail and Wholesale | 0.2768     |
| Transport and ICTs   | 0.0979     |
| Financial Services   | 0.1132     |
| Real Estate Services | 0.0152     |
| Business Services    | 0.0911     |
| Personal Services    | 0.0183     |

## Materials Input–Output Shares $\theta_{iss'}^\ell$

- o **What.** Buyer-facing expenditure shares over upstream seller sectors  $s'$ :

$$\theta_{iss'}^\ell = \frac{\sum_{j \in \mathcal{U}_{s'}} p_{ij} m_{ij}}{\sum_{s''} \sum_{j \in \mathcal{U}_{s''}} p_{ij} m_{ij}}, \quad \sum_{s'} \theta_{iss'}^\ell = 1.$$

- o **Identify.** From transaction-level variable payments ( $VC$ );  $TC = F + VC$ , large buyers  $\Rightarrow F/TC$  small.
- o **Sample.** Compute firm-level  $\theta$  for  $\ell \in \{r, u\}$ ; keep >75th pctl. revenue; aggregate to 6-digit, then to 1-digit by year; average 2005–2022.
- o **Why.** Micro foundation for the IO network; pins exposures and intensive-margin substitution scope.

| Buyer \ Seller       | Agr. | Min. | Man. | Ut.  | Cons. | R. & W. | T. & ICTs | F. Serv. | RE. Serv. | B. Serv. | P. Serv. |
|----------------------|------|------|------|------|-------|---------|-----------|----------|-----------|----------|----------|
| Agriculture          | 0.25 | 0.00 | 0.21 | 0.02 | 0.03  | 0.32    | 0.05      | 0.07     | 0.00      | 0.04     | 0.00     |
| Mining               | 0.00 | 0.04 | 0.19 | 0.06 | 0.15  | 0.30    | 0.07      | 0.02     | 0.00      | 0.17     | 0.00     |
| Manufacturing        | 0.13 | 0.02 | 0.35 | 0.02 | 0.03  | 0.25    | 0.11      | 0.03     | 0.00      | 0.06     | 0.00     |
| Utilities            | 0.07 | 0.01 | 0.18 | 0.03 | 0.03  | 0.26    | 0.17      | 0.05     | 0.00      | 0.20     | 0.00     |
| Construction         | 0.10 | 0.00 | 0.10 | 0.02 | 0.22  | 0.24    | 0.15      | 0.03     | 0.00      | 0.14     | 0.00     |
| Retail and Wholesale | 0.16 | 0.01 | 0.24 | 0.01 | 0.02  | 0.34    | 0.08      | 0.05     | 0.00      | 0.09     | 0.00     |
| Transport and ICTs   | 0.07 | 0.01 | 0.14 | 0.02 | 0.03  | 0.24    | 0.19      | 0.04     | 0.00      | 0.26     | 0.00     |
| Financial Services   | 0.08 | 0.00 | 0.12 | 0.01 | 0.01  | 0.22    | 0.06      | 0.15     | 0.01      | 0.33     | 0.00     |
| Real Estate Services | 0.03 | 0.00 | 0.12 | 0.01 | 0.02  | 0.30    | 0.04      | 0.06     | 0.05      | 0.37     | 0.00     |
| Business Services    | 0.07 | 0.00 | 0.13 | 0.01 | 0.01  | 0.22    | 0.09      | 0.06     | 0.00      | 0.41     | 0.00     |
| Personal Services    | 0.07 | 0.00 | 0.17 | 0.02 | 0.02  | 0.25    | 0.07      | 0.08     | 0.00      | 0.33     | 0.01     |

## Input-output Elasticities by Upstream Firms as Buyers

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## Upstream Materials Elasticity $\sigma_{u'}$

- o **What.** Substitutability across varieties *within* an upstream seller sector  $u'$ .
- o **Identify.** IV from March 2020 municipal lockdown of *main supplier*  $u^*$ :

$$\Delta_{12} \log \frac{m_{isut}}{m_{isu^*t}} = -\sigma_{u'} \widehat{\Delta_{12} \log \frac{p_{isut}}{p_{isu^*t}}} + \gamma_s + \varepsilon.$$

- o **Design.** 2SLS by seller sector; instrument  $Z_{isu} = \mathbf{1}\{u^* \text{ locked}\}$ ; 12m diffs; large buyers; exclude buyer/clients/other inputs under lockdown; cluster at buyer level.
- o **Why.** Higher  $\sigma \Rightarrow$  faster rewiring, stronger intensive reallocation, lower pass-through; feeds  $\kappa$  mapping. (Conservatively set  $\sigma \geq 1.45$  where  $\hat{\sigma} < 1$ .)

| Sector               | $\sigma_{U'}$ | SE     | 1 <sup>st</sup> Stage F stat. | Obs.    |
|----------------------|---------------|--------|-------------------------------|---------|
| Agriculture          | 2.59          | (1.35) | 10.24                         | 4,387   |
| Manufacturing        | 3.41          | (0.84) | 16.37                         | 186,912 |
| Construction         | 1.45          | (0.42) | 7.36                          | 6,062   |
| Retail and Wholesale | 3.80          | (0.39) | 94.08                         | 680,985 |
| Transport and ICTs   | 5.07          | (2.22) | 25.19                         | 24,054  |
| Financial Services   | 3.09          | (1.56) | 9.35                          | 3,631   |
| Business Services    | 5.21          | (2.02) | 17.55                         | 4,514   |
| Personal Services    | 6.69          | (3.37) | 13.29                         | 7,579   |
| All sectors          | 3.04          | (1.12) | 149.87                        | 918,124 |

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## Final-Consumer Variety Elasticity $\varphi_{s_r}$

- o **What.** CES elasticity across retail *varieties* within sector  $s_r$ ; markup  $\mu = \varphi/(\varphi - 1)$ .
- o **Identify.** Sectoral accounting identity under linear pricing:

$$\varphi_{s_r,t} = \frac{\sum_j R_{j,t}}{w_{s_r,t} \sum_j F_{j,t} + \sum_j \Pi_{j,t}}, \quad \Pi_j^{\text{var}} = \frac{1}{\varphi_{s_r}} R_j.$$

- o **Sample.** Large retailers;  $F_{j,t}$  small, pool to sector-year; average 2005–2022.
- o **Why.** Higher  $\varphi \Rightarrow$  keener competition, smaller wedges; also maps retailer labor tails  $v$  into productivity tails  $\kappa = (\varphi - 1)v$ .



## Exit Hazard $\delta_{s_\ell}$

- o **What.** One-year hazard that an active firm exits.
- o **Measure.** For cell  $(s, \ell, t)$ :

$$\delta_{s_\ell, t} = 1 - \frac{\text{survivors}_{s_\ell, t}}{\text{active}_{s_\ell, t}}, \quad \delta_{s_\ell} = \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} \delta_{s_\ell, t}.$$

- o **Sample.** Compute at 6-digit  $\times$  type; track 2005–2022; average across years.
- o **Why.** Disciplines expected lifespan and shock persistence; higher  $\delta$  increases payoff needed for entry, tilts adjustments toward the extensive margin.







## Productivity Tail Exponent $\kappa$

- o **What.** Thickness of the upper tail of firm productivity.
- o **Identify.** Estimate labor tail by MLE above threshold:

$$\hat{v} = \frac{n}{\sum_{i: L_i \geq L_{\min}} \ln(L_i/L_{\min})}, \quad \text{SE}(\hat{v}) \approx \hat{v}/\sqrt{n}.$$

Map to productivity using  $l(z) \propto z^{\sigma-1}$  (or  $\varphi - 1$  for retail):

$$\kappa^u = (\sigma - 1)v^u, \quad \kappa^r = (\varphi - 1)v^r.$$

- o **Sample.** Compute  $v$  by 1-digit  $\times$  type; combine with sectoral  $\sigma / \varphi$ ; report implied  $\kappa$ .
- o **Why.** Thicker tails (small  $\kappa$ ) magnify selection/reallocation gains and shape how NLP shifts surplus across the distribution.

## Labor and Implied Productivity Pareto Tails by Sector

Notes:  $\kappa = (\sigma_{U'} - 1)v$  uses seller-sector elasticities  $\sigma_{U'}$  from the IV estimates. For Mining, Utilities, and Real Estate Services, we set  $\sigma_{U'} = 1.45$  (minimum estimate above one).



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