

The Anatomy of Aggregate Productivity^{*}

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^{*}The views expressed are those of the authors and do not necessarily represent the views of the Central Bank of Chile or its board members.

Aggregate Productivity With Distortions

- Distortions like market power, taxes, and financial frictions are ubiquitous
- They prevent resources from being used where they are most valuable

In the presence of distortions:

- Aggregate productivity can be unpacked, to a first order, into the change in technology and the change in allocative efficiency

Aggregate Productivity With Distortions

- Distortions like market power, taxes, and financial frictions are ubiquitous
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In the presence of distortions:

- Aggregate productivity can be unpacked, to a first order, into the change in technology and the change in allocative efficiency
- Still, technology and allocative efficiency remain a black box

We present an aggregation result that dissects technology and allocative efficiency

This Paper: Theory

Framework

- Arbitrary neoclassical production functions within supply chains
- Distortions in all productions inputs
- Open economy

Main structural result

- Decompose technology and allocative efficiency into arbitrary partitions
- Structurally consistent Domar weights dissecting final demand

This Paper: Measurement and Application

Measurement

- Wedges: Production function approach controlling for input and output prices
- Firm-to-firm flows: Captures direct and indirect role of distortions

Application: Revisit the aggregate TFP stagnation of Chile since 2010

- **Allocative efficiency** explain 60% of aggregate TFP stagnation
- Driven by domestic manufacturing large firms in Santiago
- **Technology** explain 40% of aggregate TFP stagnation
- Driven by construction, and business services.

Selected Related Literature

Aggregation in presence of distortions and growth accounting

Restuccia and Rogerson (2008), Hsieh and Klenow (2009, 2010), Baqaee and Farhi (2019, 2020)

- o We structurally identify which parts of the economy drive allocative efficiency and technology
- o We advance on structurally microfounding aggregate productivity growth

International trade and aggregate TFP growth

Baqaee and Farhi (2024), Burstein and Cravino (2015), Blaum et al. (2018), and Kehoe and Ruhl (2008)

- o We present a growth accounting formula for open economies with distortions
- o Adaptable to any disaggregation

Agenda

- 1 Theory
- 2 Data
- 3 Measurement and Estimation
- 4 Results
- 5 Conclusion

Production

- There are \mathcal{G} groups of households, \mathcal{F} primary factors, and \mathcal{N} firms
- Each firm has access to its own arbitrary, but CRS, production function

$$q_i = A_i F_i \left(\{q_{ij}\}_{j \in \mathcal{N}}, \{L_{if}\}_{f \in \mathcal{F}} \right)$$

- q_i is firm i output, A_i is Hicks-neutral productivity
- q_{ij} are intermediates from $j \in \mathcal{N}$ firms
- L_{if} are firms use of primary production factors
- Factors $f \in \mathcal{F}$ are inelastically supplied

Households

Each group of households g , has access to its own arbitrary, but homothetic, utility that consumes potentially all goods

$$U_g = D_g(y_{g1}, \dots, y_{gN})$$

y_{gi} is group g 's final demand of good i . Each group faces the budget constraint

$$\sum_{i \in \mathcal{N}} p_i y_{gi} = \sum_{f \in \mathcal{F}} w_f L_{gf} + \sum_{i \in \mathcal{N}} \pi_{gi}$$

- p_i and y_{gi} are the price and quantity of good i consumed by group g
- w_f and L_{gf} are the price and quantity of the factor f owned by group g
- π_{gi} is profits of firm i belonging to group g

Distortions

Input wedges τ_{ij}

- Firm i buys an input from supplier j or factor f at price p_j or w_f
- For input j , the shadow price of buyer i is $\tau_{ij}p_j$, rather than p_j
- Supplier's and buyer's marginal incentives are not aligned because of input distortions [More examples](#)

Output wedge μ_i

- Firms minimize cost given distorted input prices and charge a price $p_i = \mu_i c_i$

[Market clearing](#)

[General Equilibrium](#)

National Accounts

- Nominal expenditure of group g of households:

$$E_g = \sum_{i \in \mathcal{N}} p_i y_{gi}$$

- Nominal GDP is the sum of final demand:

$$GDP = \sum_{i \in \mathcal{N}} p_i y_i = \sum_{g \in \mathcal{G}} E_g$$

- GDP shares

$$b_i = \frac{p_i y_i}{GDP} \quad \text{if } i \in \mathcal{N}$$

$$\Lambda_f = \frac{w_f L_f}{GDP} \quad \text{if } f \in \mathcal{F}$$

- Consumption shares for each final output and household group

$$b_{gi} = \frac{p_i y_{gi}}{GDP}, \quad \text{so that} \quad b_i = \sum_{g \in \mathcal{G}} b_{gi}$$

Real indices

- o GDP deflator is defined as a chained index:

$$d \log P = \sum_{i \in \mathcal{N}} \frac{p_i y_i}{GDP} d \log p_i = \sum_{g \in \mathcal{G}} \sum_{i \in \mathcal{N}} \frac{p_i y_{gi}}{GDP} d \log p_i$$

- o Real final demand index for group g :

$$d \log Y_g = d \log E_g - \sum_{i \in \mathcal{N}} \frac{p_i y_{gi}}{E_g} d \log p_i$$

- o Real GDP growth is computed by chaining absolute indices (GDP is the numeraire):

$$\begin{aligned} d \log Y &= d \log GDP - d \log P \\ &= \sum_{g \in \mathcal{G}} \frac{E_g}{GDP} d \log E_g - \sum_{g \in \mathcal{G}} \sum_{i \in \mathcal{N}} \frac{p_i y_{gi}}{GDP} d \log p_i \end{aligned}$$

Input-output Objects

- o Direct exposure of firm i to firm j

Cost-based input-output matrix ($\tilde{\Omega}$) of dimensions $(\mathcal{N} + \mathcal{F}) \times (\mathcal{N} + \mathcal{F})$

$$\tilde{\Omega}_{ij} = \frac{\text{Value of input } j \text{ used by firm } i}{\text{Firm } i \text{ total cost}}$$

- o Direct + Indirect exposure of firm i to firm j

Cost-based Leontief inverse matrix ($\tilde{\Psi}$) accounts for firms' direct and indirect cost exposures through supply chains

$$\tilde{\Psi} \equiv (I - \tilde{\Omega})^{-1} = I + \tilde{\Omega} + \tilde{\Omega}^2 + \dots$$

- o Direct exposure of households to $S \in \{\text{good, factor}\}$

Cost-based Domar weights $\tilde{\lambda}$ ($\tilde{\Lambda}$ for factors)

$$\tilde{\lambda}' \equiv b' \tilde{\Psi}$$

Disaggregated Cost-based Domar Weights

Given a partition of final demand into $g \in G$ groups, we define the disaggregated cost-based Domar weights as :

$$\begin{aligned}\tilde{\lambda}_{gi} &\equiv \sum_{k \in \mathcal{N}} b_{gk} \tilde{\psi}_{ki} \text{ if } i \in \mathcal{N} \\ \tilde{\Lambda}_{gf} &\equiv \sum_{i \in \mathcal{N}} b_{gk} \tilde{\psi}_{kf} \text{ if } f \in \mathcal{F}\end{aligned}$$

So that cost-based Domar weights for firms and factors are:

$$\begin{aligned}\tilde{\lambda}_i &= \sum_{g \in \mathcal{G}} \tilde{\lambda}_{gi} \\ \tilde{\Lambda}_f &= \sum_{g \in \mathcal{G}} \tilde{\Lambda}_{gf}\end{aligned}$$

Change in Real GDP

The effect on real GDP of each group g from a productivity shock ($d \log A$) and a allocation shock ($d \log \mathcal{X}$) can be decomposed into: Feasible Allocations

- A pure change in technology ($d \log A$) for a given fixed allocation matrix \mathcal{X}
- The change in the distribution of resources allocation matrix ($d\mathcal{X}$) holding technology constant

$$d \log Y = \sum_{g \in \mathcal{G}} \underbrace{\frac{\partial \log Y}{\partial \log Y^g} \frac{\partial \log Y^g}{\partial \log A}}_{\Delta \text{ Technology of Group } g} d \log A + \sum_{g \in \mathcal{G}} \underbrace{\frac{\partial \log Y}{\partial \log Y^g} \frac{\partial \log Y^g}{\partial \mathcal{X}}}_{\Delta \text{ Allocative Efficiency of Group } g} d \mathcal{X}$$

Change in Real GDP: Details

$$d \log Y = \sum_{g \in \mathcal{G}} \underbrace{\sum_{i \in \mathcal{N}} \tilde{\lambda}_{gi} d \log A_i}_{\Delta \text{ Technology of Group } g} + \sum_{g \in \mathcal{G}} \underbrace{\left(- \sum_{f \in \mathcal{F}} \tilde{\Lambda}_{gf} d \log \Lambda_f - \sum_{i \in \mathcal{N}} \tilde{\lambda}_{gi} \sum_{j \in \mathcal{N}, \mathcal{F}} \tilde{\Omega}_{ij} d \log \tau_{ij} \mu_j + dE_g \right)}_{\Delta \text{ Allocative Efficiency of Group } g}$$

- ① $d \log A_i$: Cost-based Domar weighted productivity changes
- ② $d \log \Lambda_f$: Weighted average of the reductions in aggregate factor shares
- ③ $d \log \tau_{ij} \mu_j$: To isolate the changes in AE, need net out distortion changes
- ④ dE_g : Changes in group g 's nominal expenditure share alter final demand distribution across groups

2 and 3: Downstream propagation; 4: Upstream propagation

Aggregate Productivity Growth

$$\underbrace{\Delta \log Y_t - \sum_{f \in \mathcal{F}} \tilde{\Lambda}_{f,t-1} \Delta \log L_{f,t}}_{\Delta \text{ Aggregate TFP}} = \underbrace{\sum_{g \in \mathcal{G}} \sum_{i \in \mathcal{N}} \tilde{\lambda}_{ig,t-1} d \log A_i}_{\Delta \text{ Technology of Group } g}$$

$$+ \underbrace{\sum_{g \in \mathcal{G}} \left(- \sum_{f \in \mathcal{F}} \tilde{\Lambda}_{fg,t-1} d \log \Lambda_f - \sum_{i \in \mathcal{N}} \tilde{\lambda}_{ig,t-1} \sum_{j \in \mathcal{F} + \mathcal{N}} \hat{\Omega}_{ij} d \log \mu_i \tau_{ij} + dE_g \right)}_{\Delta \text{ Allocative Efficiency of Group } g}$$

- ① Cost-based Domar weighted productivity changes
- ② Resource reallocation to underproduced parts of the economy
 $\Rightarrow \Delta^-$ Factor shares: $\Rightarrow \Delta^+$ allocative efficiency $\Rightarrow \Delta^+$ TFP
- ③ Discount factor changes due to wedge changes
- ④ Changes of final demand across groups

Aggregate Productivity Growth in Open Economies

$$\underbrace{\Delta \log Y_t - \sum_{f \in \mathcal{F}^D} \tilde{\Lambda}_{f,t-1} \Delta \log L_{f,t}}_{\Delta \text{ Aggregate TFP}} = \sum_{g \in \mathcal{G}} \underbrace{\left(\sum_{i \in \mathcal{N}} \tilde{\lambda}_{gi,t-1} d \log A_i + \sum_{f \in \mathcal{F}^M} \left(\tilde{\Lambda}_{gf,t-1} - \Lambda_{gf,t-1} \right) d \log L_f \right)}_{\Delta \text{ Technology of Group } g}$$

$$+ \underbrace{\sum_{g \in \mathcal{G}} \left(- \sum_{i \in \mathcal{N}} \tilde{\lambda}_{gi,t-1} \sum_{j \in \mathcal{N}, \mathcal{F}} \hat{\Omega}_{ij} d \log \tau_{ij} \mu_j - \sum_{f \in \mathcal{F}^D} \tilde{\Lambda}_{gf,t-1} d \log \Lambda_f + dE_g - \sum_{f \in \mathcal{F}^M} \left(\tilde{\Lambda}_{gf,t-1} - \Lambda_{gf,t-1} \right) d \log \Lambda_f \right)}_{\Delta \text{ Allocative Efficiency of Group } g}$$

- We model imports as additional factors
- Imports cost and marginal revenue product gap is captured in technology
- Allocative efficiency include an homologous reallocation term form imports
- Wedges changes are not needed; accrued to foreign households

Theory: Summary

- Structural interpretation of how to disaggregate technology and allocative efficiency across different parts of the economy
- Allows to identify which parts of the economy account for technological and allocative efficiency changes
- And which parts of the economy are more relevant in terms of shocks forward and backward propagation
- We focus on ex-post measurement, ex-ante is challenging, needs firm ownership structure and elasticity of substitution across inputs

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Data Sources: Administrative Tax Data From Chilean IRS

- ① Firm accounting balance sheets: Sales, materials, investment, main industry
- ② Employer-employee: Wages, headcount employees
- ③ Capital stock: Perpetual inventory using initial fixed assets and investment
- ④ Firm-to-firm transactions: Prices, quantities, products, supplier, buyer

Data Cleaning

- Firm: Tax ID with positive sales, materials, wage bill, and capital
- Firms with less than two employees or capital below US\$10 are dropped
- Data is anonymized for the confidentiality of the firm's and workers' identities

► Sample stats

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Measurement: Distortions

Following Hall, 1998 and DLW, 2012, input and output level distortions can be derived from the firm i cost minimization problem:

$$\min_{\{X_{ij}\}} \sum_j p_j \tau_{ij} X_{ij} \quad \text{subject to} \quad Y = F(X)$$

Where X_{ij} is firm i input j usage, p_j is its price, and τ_{ij} is its price distortion. Each input j FOCs can be expressed as:

$$\mu_i \tau_{ij} = \frac{\sigma_{ij}}{\Omega_{ij}}$$

Where μ_i is the output wedge, τ_{ij} is the input wedge, σ_{ij} is the input-output elasticity and Ω_{ij} is the revenue share expenditure of input j

Estimation: Production Function

Production function estimation using control function approach (ACF)

- Control for firm-level input and output prices
- 3 factors Cobb-Douglas production function
- 6-digit sector (626) different time invariant production functions
- Time-invariant input-output elasticities σ_{ij}

Wedges time evolution

By industry

Elasticities

Factor Shares

Between-within decomposition

B-W by factor

Final Demand Groups

Group cost-based Domar weights

$$\tilde{\lambda}_{gi} = \sum_{k \in \mathcal{N}} b_{gk} \tilde{\psi}_{ki} = \sum_{k \in \mathcal{N}} b_{gk} (1 + \tilde{\Omega}_{ki} + \tilde{\Omega}_{ki}^2 + \dots)$$

Group definition

- 16 official regions of Chile and 11 industries (1 digit classification)
- Firm export status, selling domestically or exporting. For firms that perform both activities, we split them into separate units
- 3 Firm size (according to legal total sales) and inactive firms
- We measure firm export status and firm size in 4 different moments in time (2005, 2009, 2016, 2022)

Potentially, $16 \times 11 \times (2^4) \times (4^4) = 720,896$ groups (non zero around 10%)

Final Expenditures Shares b_{gk}

Final expenditure shares b is a vector of dimension $(N^D + N^X + F) \times 1$

- N^D and N^X are the number of domestic and exporting firms
- F is the number of factors (including imported inputs)

Measurement

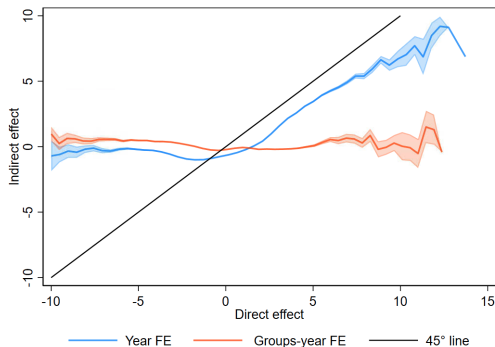
- N^D entries are the residual between firms' total sales (no exports) and firms' intermediate sales to other firms (IO data)
- N^X entries are firms exports (considered as final sales)
- Final F entries are zero (HH do not use factors directly)
- All entries are then divided by GDP

Cost-based Input-output Matrix $\tilde{\Omega}$

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_{\mathcal{D}\mathcal{D}} & \tilde{\Omega}_{\mathcal{D}\mathcal{X}} & \tilde{\Omega}_{\mathcal{D}\mathcal{F}} \\ \tilde{\Omega}_{\mathcal{X}\mathcal{D}} & \tilde{\Omega}_{\mathcal{X}\mathcal{X}} & \tilde{\Omega}_{\mathcal{X}\mathcal{F}} \\ \tilde{\Omega}_{\mathcal{F}\mathcal{D}} & \tilde{\Omega}_{\mathcal{F}\mathcal{X}} & \tilde{\Omega}_{\mathcal{F}\mathcal{F}} \end{bmatrix} = \begin{bmatrix} \tilde{\Omega}_{\mathcal{D}\mathcal{D}} & 0 & \tilde{\Omega}_{\mathcal{D}\mathcal{F}} \\ \tilde{\Omega}_{\mathcal{X}\mathcal{D}} & 0 & \tilde{\Omega}_{\mathcal{X}\mathcal{F}} \\ 0 & 0 & 0 \end{bmatrix}$$

- Row i is buying from column j , with $\tilde{\Omega}_{ij} = \tau_{ij}\mu_j\Omega_{ij}$
- Factors do not require inputs $\Rightarrow \tilde{\Omega}_{\mathcal{F}j} = 0$ for all $j = \{\mathcal{D}, \mathcal{X}, \mathcal{F}\}$
- Exports sold only internationally $\Rightarrow \tilde{\Omega}_{\mathcal{D}\mathcal{X}} = 0$ and $\tilde{\Omega}_{\mathcal{X}\mathcal{X}} = 0$
- Numerator of $\tilde{\Omega}_{\mathcal{D}\mathcal{D}}$ and $\tilde{\Omega}_{\mathcal{X}\mathcal{D}}$: Firm-to-firm trade flows
- Numerator of $\tilde{\Omega}_{\mathcal{D}\mathcal{F}}$ and $\tilde{\Omega}_{\mathcal{X}\mathcal{F}}$: Factor expenditures

Direct vs. Indirect cb Domar Weights: b_i vs. $\tilde{\lambda}'_i = \tilde{\lambda}_i - b_i$



- The direct exposure b_{gi} is a poor measure of relevance for aggregate output
- Controlling by group FEs the direct exposure is uninformative

Firm Level Productivity

Relaying on the production function estimation assumptions

$$A_{it} = \ln q_{it} - \sigma_{iL} \ln L_{it} - \sigma_{iK} \ln K_{it} - \sigma_{iM} \ln M_{it}$$

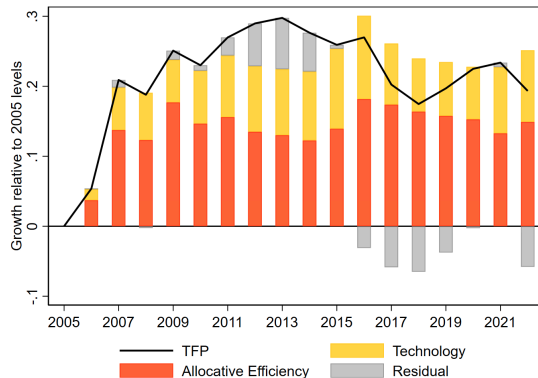
Aggregate TFP technology term:

$$\sum_{g \in \mathcal{G}} \sum_{i \in \mathcal{N}} \tilde{\lambda}_{ig,t-1} d \log A_{it}$$

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Growth Accounting With Distortions (cumulative growth, 2005=0)



- Residual is measurement error + higher order terms (2022=6%)
- Allocative efficiency stagnated since 2009 at level of 18% (2022=15%)
- Technology stagnated in 2016 at 12% (2022=10%) CB Chile TFP

Dissecting Aggregate Productivity Growth

- In which parts of the economy resource reallocation is happening? [Go](#)
- In which parts of the economy technology is moving? [Go](#)
- Does firm size explains resource reallocation and technological changes? [Go](#)
- What specific groups drive allocative efficiency and technology changes [Go](#)

Agenda

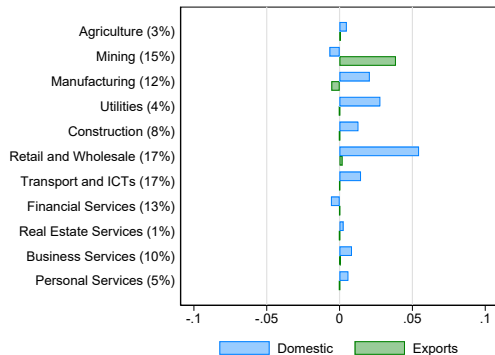
- 1 Theory
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Conclusion

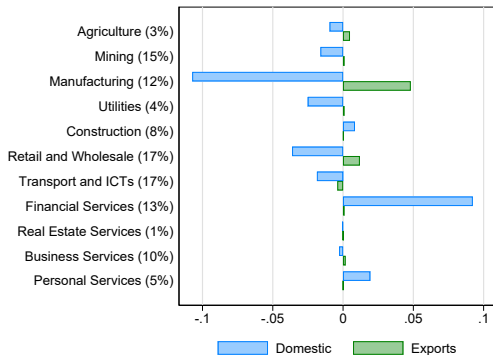
- We present an interpretable structural decomposition of allocative efficiency (AE) and technology
- For implementation, firm-level distortions and input-output flows are needed
- We find that AE explains 60% of aggregate TFP growth in Chile for 2005-2022, technology 40%
- We identify which granular groups of final demand explain aggregate TFP stagnation

Cumulative Δ A. Efficiency: Dom-Exp by Industry (GDP shares)

Expansion Period (2005-2009)



Stagnation Period (2010-2022)



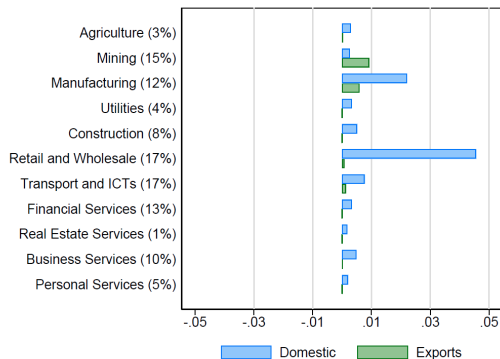
- Expansion: Dom manufacturing, utilities, retail/wholesale, and Exp mining
- Stagnation: Same Dom but negative, compensated by Exp Manufacturing and the financial sector

[Complete period](#)

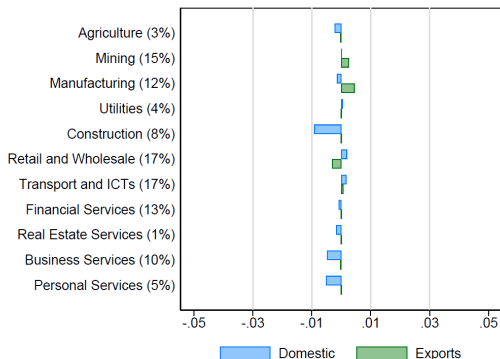
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Cumulative Δ Technology: Dom-Exp by Industry (GDP shares)

Expansion Period (2005-2016)



Stagnation Period (2017-2022)

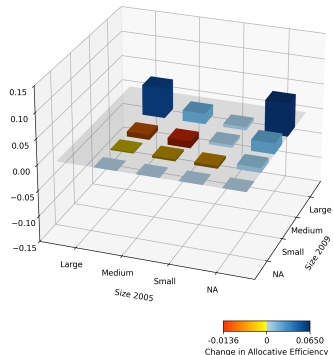


- Expansion: Dom manufacturing and retail/wholesale
- Stagnation: Construction, and non-financial services

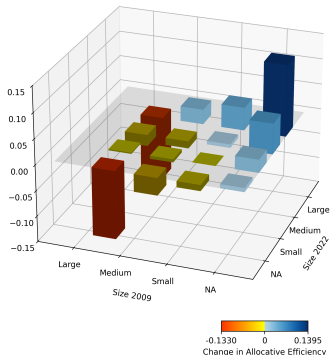
[Complete period](#)[Return](#)

Cumulative Δ Allocative Efficiency: Firm size

Expansion Period (2005-2009)



Stagnation Period (2010-2022)

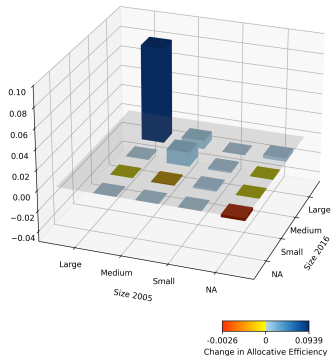


- Expansion: Large firms: + incumbents and entry
- Stagnation: Large firms: - incumbent and exit; + entrants

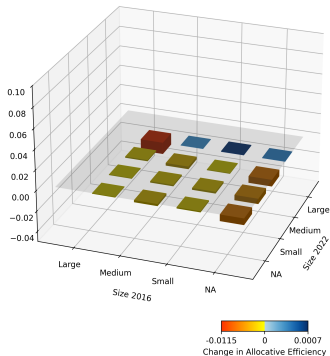
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Cumulative Δ Technology: Firm size

Expansion Period (2005-2016)



Stagnation Period (2017-2022)



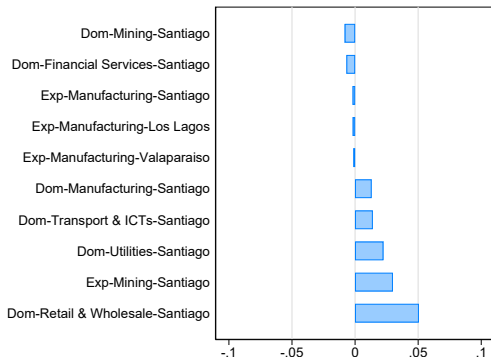
- Expansion: + Large incumbent firms
- Stagnation: - Large incumbent firms

[Return](#)

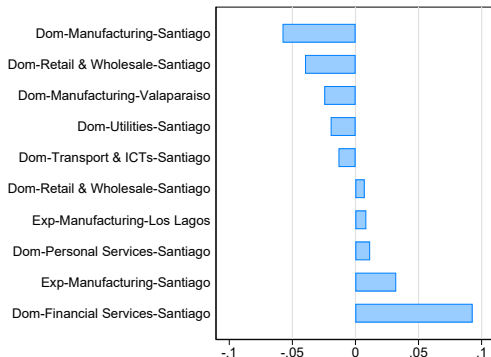
Cumulative Δ A. Efficiency: Dom-Exp by Region and Industry

(Top 5 vs, Bottom 5)

Expansion Period (2005-2009)



Stagnation Period (2010-2022)



Complete period

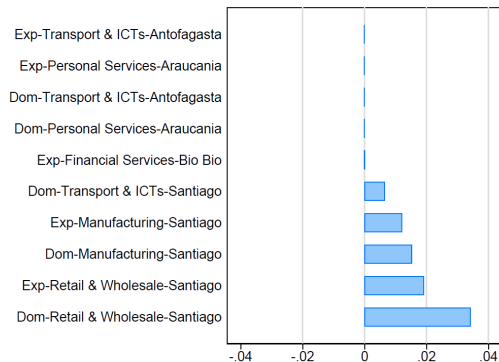
Region trade only

Return

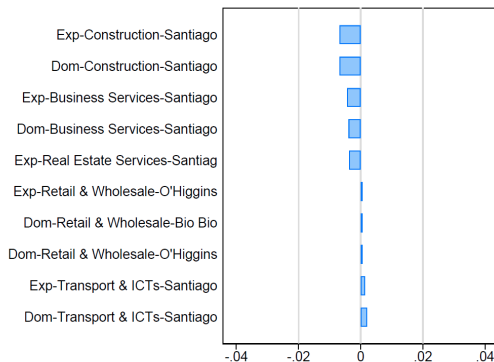
Cumulative Δ Technology: Dom-Exp by Region and Industry

(Top 5 vs, Bottom 5)

Expansion Period (2005-2016)



Stagnation Period (2017-2022)



[Complete period](#)

[Return](#)

Appendix

Theory: Preferences [Return](#)

- Domestic representative household with homothetic preferences

$$\mathcal{W} = \mathcal{W}(\{y_i\}_{i \in \mathcal{D}})$$

- Budget constraint

$$\underbrace{\sum_{i \in \mathcal{D}} p_i y_i}_{\text{Final Goods Expenditure}} = \underbrace{\sum_{f \in \{L, K\}} w_f L_f}_{\text{Factor Income}} + \underbrace{\sum_{i \in \mathcal{D} + \mathcal{E}} (1 - 1/\mu_i) p_i q_i}_{\text{Non-Factor Income: Profits}}$$

- All imports are channeled through intermediaries

Examples of $\tau_{ij} \neq 1$

Return

$\tau_{ij} > 1$ (Effective cost is higher)

- Payment or financing frictions: Buyer needs to borrow or pre-pay
- Inventory or storage constraints: Limited capacity increases input cost
- Monitoring or quality enforcement: Costs related to inspection or compliance
- Inflexible production: Poor complementarity of input

$\tau_{ij} < 1$ (Effective cost is lower)

- Rebates: Buyer receives post-hoc benefits not reflected in invoice price
- Tax credits or subsidies: reduces buyer's effective purchase cost
- Intra-group pricing: Internal transfers with hidden cross-subsidies
- Learning or IT complementarities: More efficient use lowers perceived cost

Theory: Market Clearing [Return](#)

- Goods

$$q_i = \sum_{g \in \mathcal{G}} y_{gi} + \sum_{j \in \mathcal{N}} q_{ji}$$

- Factors

$$L_f = \sum_{i \in \mathcal{N}} L_{if} = \sum_{g \in \mathcal{G}} L_{gf}$$

Theory: General equilibrium [Return](#)

Given productivity A_i , output distortions μ_i , input distortions τ_{ij} , the general equilibrium is:

- The set of prices p_i
- intermediate input choices q_{ij}
- factor input choices $\{L_{if}\}$
- consumption choices y_{gi}

such that:

- 1 Each producer minimizes its costs and charges a price of each good that is equal to its output distortion multiplied by its marginal cost
- 2 Each household group maximizes utility subject to its budget constraint taking prices as given
- 3 markets clear for all goods and factors

Real GDP Growth [Return](#)

- o GDP deflator is defined as a chained index:

$$d \log P = \sum_{i \in \mathcal{N}} \frac{p_i y_i}{GDP} d \log p_i$$

- o Real GDP growth is computed by chaining absolute indices, to a first order:

$$\begin{aligned} d \log Y &= d \log GDP - d \log P \\ &= \sum_{g \in \mathcal{G}} \frac{E_g}{GDP} d \log E_g - \sum_{g \in \mathcal{G}} \sum_{i \in \mathcal{N}} \frac{p_i y_{gi}}{GDP} d \log p_i \\ &= \sum_{g \in \mathcal{G}} \frac{E_g}{GDP} \left(d \log E_g - \sum_{i \in \mathcal{N}} \frac{p_i y_{gi}}{E_g} d \log p_i \right) \end{aligned}$$

- o GDP is the numeraire

Feasible Allocations

[Return](#)

All feasible allocations are defined by:

- An allocation matrix \mathcal{X} ($\mathcal{X}_{ij} = q_{ij}/y_j$)
- A vector of productivities A
- A vector of distortions μ_T
- a vector of factor supplies, $\mathcal{F} = [L, K, M]$

The equilibrium allocation yields:

- An allocation matrix $\mathcal{X}(A, \mathcal{F}, \mu_T)$
- Which in turn generates an output level of $\mathcal{Y}(A, \mathcal{X}(A, \mathcal{F}, \mu_T))$

Example: Direct and indirect exposure to imported inputs [Return](#)

$$\tilde{\lambda}_i \equiv b' \tilde{\Psi} = b'(I - \tilde{\Omega})^{-1}$$

Example: Decompose Leontief inverse factors' role on domestic production

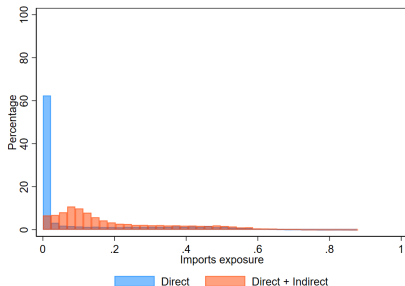
$$\tilde{\Psi}_{\mathcal{DF}} = \begin{bmatrix} \tilde{\Psi}_{DL} & \tilde{\Psi}_{DK} & \tilde{\Psi}_{DM} \end{bmatrix}$$

- $\tilde{\Psi}_{DM}$: Relevance of imports in domestic production, directly and indirectly
- Homologous for $\tilde{\Psi}_{EM}$ on exports production

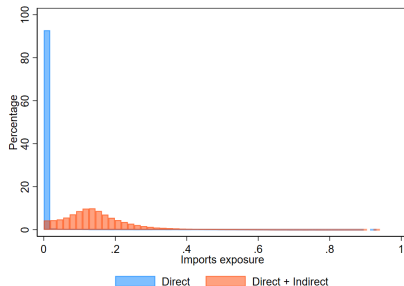
Example: Direct and indirect exposure to imported inputs

[Return](#)

(a) Domestic: $\tilde{\Omega}_{DM}$ vs $\tilde{\Psi}_{DM}$



(b) Exports: $\tilde{\Omega}_{EM}$ vs $\tilde{\Psi}_{EM}$



- 11% (99%) of firms are directly (indirectly) exposed to imports
- 38% of exporters use imports directly \Rightarrow More engaged in international trade
- Still, the majority of exporters (62%) use imports only indirectly

Aggregated objects

[▶ Return](#)

We measure the following aggregate objects directly from the data:

- Aggregate value added: Y
- Aggregate employment: L_L
- Aggregate capital: L_K
- Aggregate imports: L_{IM}
- Import Share: Λ_{IM}

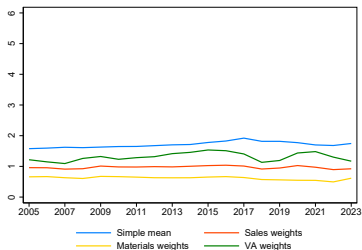
Sample statistics by industry [▶ Return](#)

	# of Tax IDs			Value Added (2022 million pesos)		
	Raw	Sample	Share sample	Raw	Sample	Share sample
Agriculture	105,010	27,821	0.26	5,884,980	4,394,102	0.75
Mining	5,042	1,570	0.31	12,603,017	12,439,233	0.99
Manufacturing	120,655	36,545	0.30	27,529,125	26,283,199	0.95
Utilities	7,085	2,047	0.29	5,489,654	4,125,247	0.75
Construction	120,981	30,589	0.25	16,053,036	10,985,617	0.68
Retail and Wholesale	487,486	124,238	0.25	28,774,760	23,163,400	0.80
Transport	148,385	33,428	0.23	29,541,936	24,449,458	0.83
Financial Services	42,222	8,513	0.20	47,436,467	45,477,105	0.96
Real Estate Services	23,034	7,041	0.31	3,732,899	1,986,035	0.53
Business Services	147,152	33,950	0.23	18,616,468	15,031,352	0.81
Personal Services	203,389	21,682	0.11	11,961,647	8,795,058	0.74
Total	1,410,441	327,424	0.23	207,623,989	177,129,806	0.85

Wedges evolution in time

[▶ Return](#)

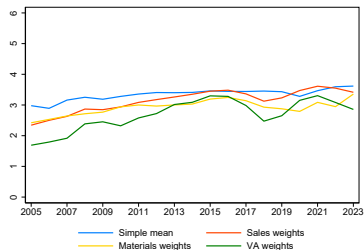
Panel A: Materials wedge



Panel B: Labor wedge



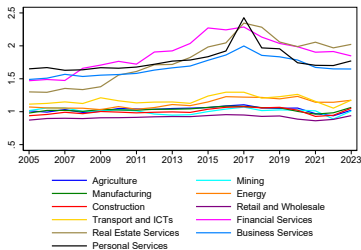
Panel C: Capital wedge



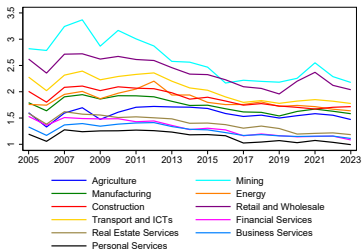
Median Wedges evolution by industry

[Return](#)

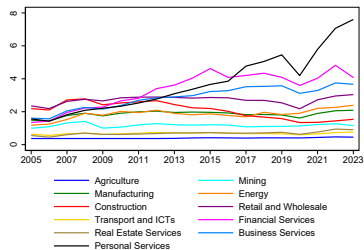
Panel A: Materials wedge



Panel B: Labor wedge



Panel C: Capital wedge



Elasticities and Returns to Scale by Industry (median)

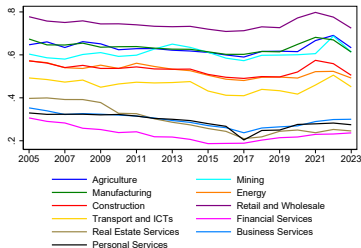
[▶ Return](#)

Sector	Materials	Labor	Capital	RTS
Agriculture	0.673	0.304	0.031	1.008
Mining	0.634	0.373	0.048	1.055
Manufacturing	0.648	0.393	0.030	1.071
Utilities	0.544	0.448	0.067	1.059
Construction	0.544	0.471	0.032	1.047
Retail and Wholesale	0.572	0.565	0.030	1.167
Transport and ICTs	0.530	0.432	0.030	0.992
Financial Services	0.425	0.571	0.031	1.027
Real Estate Services	0.523	0.469	0.024	1.016
Business Services	0.485	0.482	0.026	0.993
Personal Services	0.515	0.469	0.026	1.010

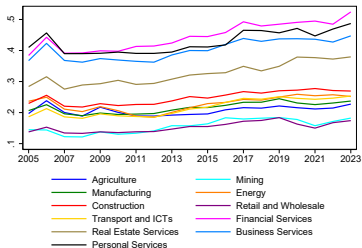
Median factor revenue shares evolution by sector

[Return](#)

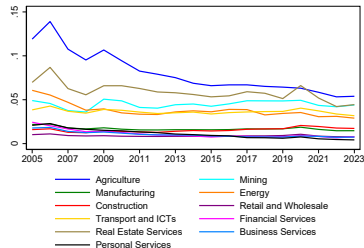
Panel A: Materials share



Panel B: Labor share



Panel C: Capital share



Evolution of distortions and within-between firm decomposition

► Return

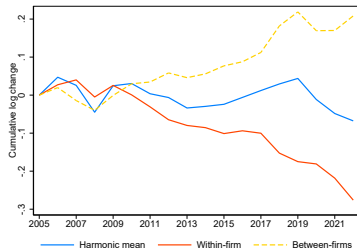
- $\gamma_i = \sum_{j \in L, K, M} \tilde{\Omega}_{ij} d \log \mu_i \tau_{ij}$
- Within: firm-level change in distortion keeping firm size constant
- Between: firm-level size change, keeping distortions constant (including extensive margin)



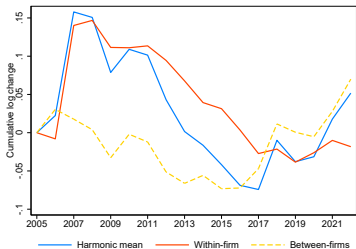
Distortions are decreasing due to within margin

Within-between firm distortions decomposition by factor [▶ Return](#)

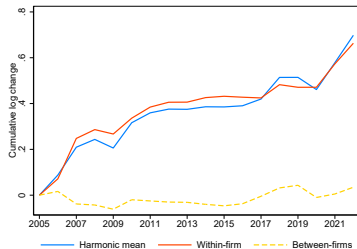
Panel A: Materials



Panel B: Labor



Panel C: Capital



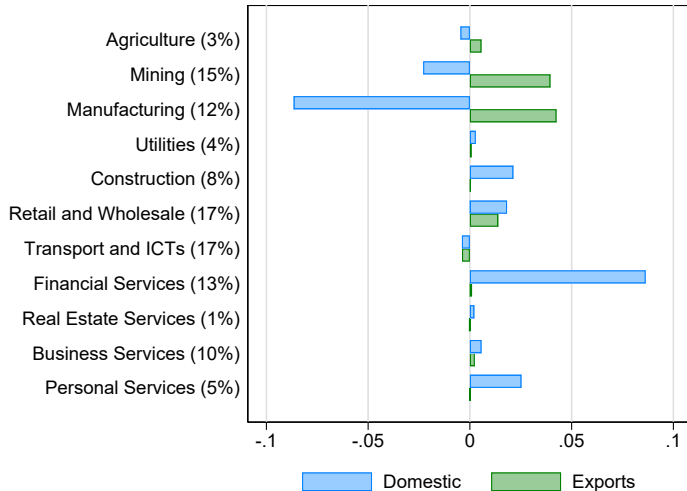
Domar weights fixed effects decomposition

[▶ Return](#)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Adjusted R2	0.0006	0.0038	0.0123	0.0297	0.0322	0.0351	0.0350	0.2265
FE region×year	✓						✓	
FE industry×year		✓				✓	✓	
FE export status×year			✓		✓	✓	✓	
FE size-group×year				✓	✓	✓	✓	
All FE interacted								✓

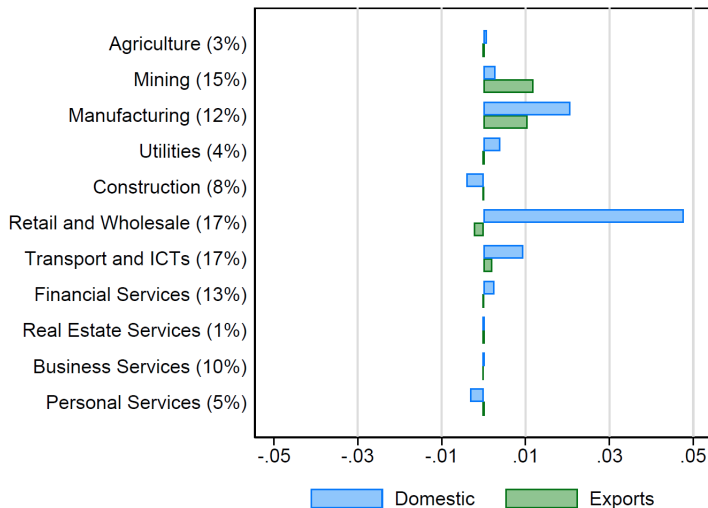
Cumulative Δ Allocative Efficiency: Dom-Exp by Industry [Return](#)

Complete sample 2005-2022



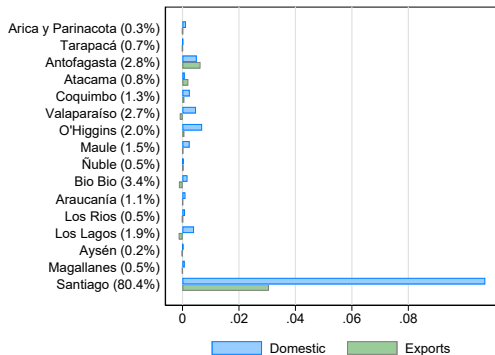
Cumulative Δ Technology: Dom-Exp by Industry [Return](#)

Complete sample 2005-2022

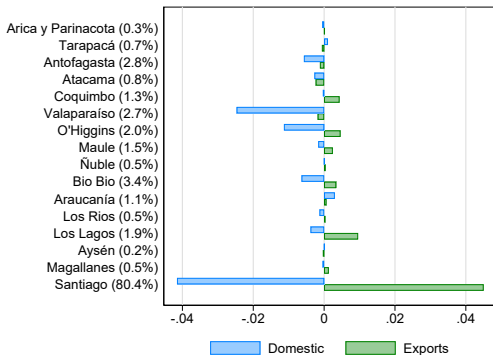


Cumulative Δ A. Efficiency: Dom-Exp by Region (GDP shares) [Return](#)

Expansion Period (2005-2009)



Stagnation Period (2010-2022)



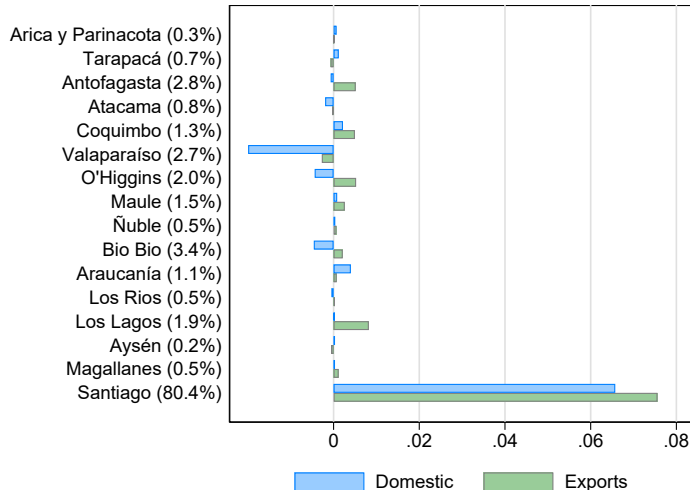
- Expansion: Santiago region dominates
- Stagnation: Valparaíso, O'Higgins and Bio Bio matter relatively more, through dom. (Manufacturing intensive regions)

[Complete period](#)

Cumulative Δ Allocative Efficiency: Dom-Exp by Region

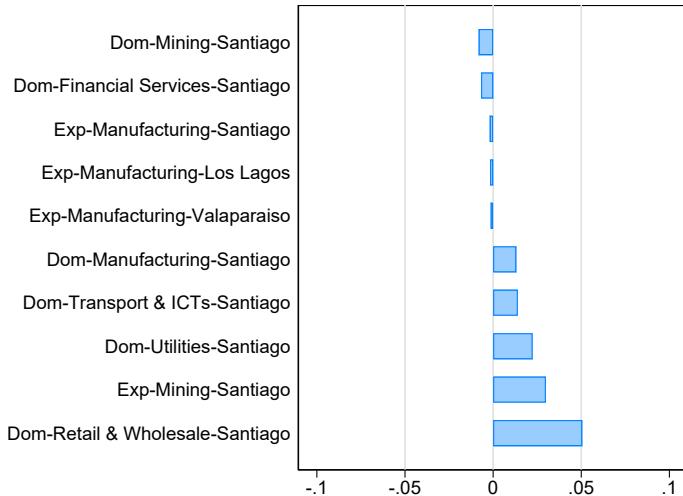
[Return](#)

Complete sample 2005-2022



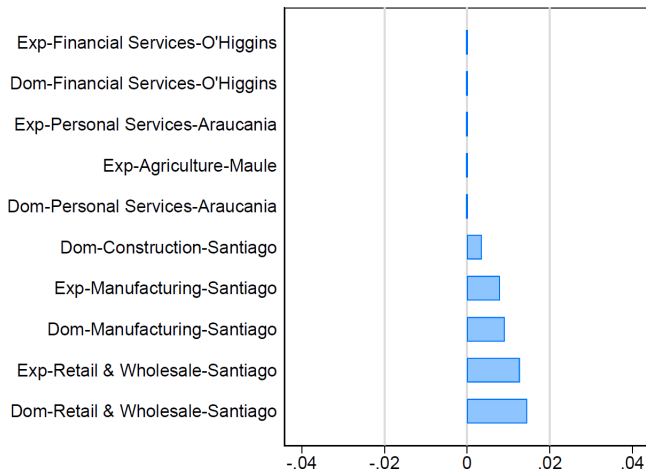
Cumulative Δ Allocative Efficiency: Dom-Exp by Region [Return](#)

Complete sample 2005-2022



Cumulative Δ Technology: Dom-Exp by Region [Return](#)

Complete sample 2005-2022



Central Bank of Chile TFP

[▶ Return](#)

