Aggregate Outcomes of Nonlinear Prices in Supply Chains*

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*The views expressed are those of the authors and do not necessarily represent the views of the Central Bank of Chile or its board members.

Motivation and Research Question

- Price discrimination shapes firm contracts and has been broadly studied in partial equilibrium

  "Price discrimination is one of the most prevalent forms of marketing practices" —Varian, *Handbook of IO* (1989)

- In macro, models are rich in incorporating GE forces, but usually omit price discrimination

- In this paper, we study these two together: Price discrimination in GE

- Particularly, in a setting with supply chains: a natural space for price discrimination

Research Question: What are the aggregate outcomes of price discrimination in supply chains?

This paper

Empirics (Population-level B2B transactions for Chile)

- Shed light on what price discrimination looks like for the population of firms in a country
- Indicative evidence: widespread nonlinear prices (i.e., quantity discounts)

Theory

- GE model in supply chains where firms price discriminate and are price discriminated against

Theory + Empirics: Price discrimination is quantitatively relevant for welfare

1. Improves allocative efficiency rel. to uniform pricing (single Q-invariant price): Δ^{+} welfare
2. Distorts firms' profit distribution and entry": $\Delta^{?}$ welfare

1 dominates: price discrimination reduces the aggregate cost of market power rel. to uniform prices

Selected Related Literature

Aggregate Cost of Market Power (Misallocation and Firm Entry)

- Quesnay (1758), Harberger (1956), Mankiw & Whinston (1986), Hopenhayn (1992), Hsieh & Klenow (2009), Jones (2011), Baqaee & Farhi (2019, 2020), Edmond, Midrigan & Xu (2023), Bornstein & Peter (2025), Burstein, Cravino, & Rojas (2025)

Price Discrimination and Screening

- Dupuit (1849), Mirrlees (1971), Spence (1977), Mussa & Rosen (1978), Maskin & Riley (1984), Borenstein (1985), Tirole (1988), Varian (1989), Wilson (1993), Laffont & Tirole (1993), Armstrong (1996), Stole (2007)

We embed endogenous $2^{nd} + 3^{rd}$ PD into a GE model in supply chains with endogenous entry

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1. Partial Equilibrium: Nonlinear Price Characterization
2. Descriptive Evidence
3. General Equilibrium: Model in Supply Chains
4. Model Quantification
5. Conclusion

Primitives and Behavior (Standard)

- One seller with constant marginal cost c faces a continuum of buyers indexed by z
- Seller has full bargaining power and makes a take-it-or-leave-it offer
- Seller knows the distribution of buyer types, but type information is private
- Chooses a nonlinear transfer $T(z)$ and quantity $q(z)$ for each type z

$$\max_{\{T(z),q(z)\}} \Pi_{\text{seller}} = \int_{\underline{z}}^{\infty} \left[T(z) - c\,q(z)\right] f(z)\,dz$$

Subject to

- (IR) Buyers receive non-negative surplus: $\Pi(z, q(z)) = zq(z) - T(z) \geq 0, \quad \forall z$
- (IC) Buyers choose their tailored contract: $zq(z) - T(z) \geq zq(\tilde{z}) - T(\tilde{z}), \quad \forall z, \tilde{z}$

Problem in terms of Virtual Surplus

Functional Forms and Optimal Nonlinear Price (New)

- So far, standard screening problem, now we impose two additional assumptions:
  1. Buyer types are Pareto distributed with tail parameter κ
  2. Buyers have isoelastic demands ($\sigma > 1$) (Buyer type shifts demand without altering curvature)

Lemma 1: Optimal Nonlinear Price

Under (i) constant marginal cost, (ii) Pareto distributed types, and (iii) isoelastic demands, the optimal nonlinear price schedule is equivalent to a two-part tariff when $\kappa > \sigma - 1$:

$$T(z) = F + p^{\mathrm{NLP}} q(z), \qquad p^{\mathrm{NLP}} = \frac{\rho}{\rho - 1} c, \qquad \rho \equiv \frac{\kappa \sigma}{\sigma - 1} > \sigma, \qquad F \textit{ is set so that}: \ \Pi(\underline{z}) = 0.$$

Optimal Nonlinear Price: Implications ▸ Graph

$$\underbrace{T(z)}_{\text{Total Payment}} = \underbrace{F}_{\text{Flat fee}} + \underbrace{p^{\text{NLP}}}_{\text{Marginal price}} q(z)$$

1. Selection: It is optimal for the seller to serve all types ▸ Proof

2. Allocations: Marginal Price
   - Determines quantity allocations (it is allocative)
   - Is lower relative to uniform pricing (smaller markup): $\rho > \sigma \implies \frac{\rho}{\rho-1} < \frac{\sigma}{\sigma-1}$ ▸ Graph

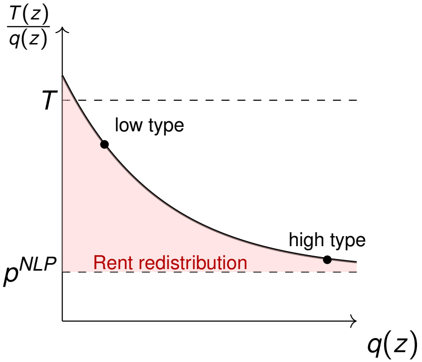
3. Rents: Flat Fee
   - Does not affect input choices; it reallocates rents from buyers to sellers
   - Affects firms' profit distribution (and entry in GE)

Optimal Nonlinear Price: Testable Prediction

- Average unit price is:

$$\frac{T(z)}{q(z)} = \frac{F}{q(z)} + p^{\text{NLP}}$$

- Decreasing and convex in q
- Has a horizontal asymptote at p^{NLP}



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Data Sources

Invoice example | Data cleaning

Invoice transactions for the universe of Chilean formal firms in 2024

- 1.4 billion transactions
- More than 10 million different products. We assume products are seller-specific
- Data on prices and quantities for every product transacted

  (Transport cost is usually a different product)

Merged with firms' accounting balance-sheet data

- Sales, materials, investment, 6-digit sectors
- Employer-employee data: Wages, headcount
- Capital stock and investment

Data Example: Fix seller (X. inc), Product (Y), and Day

X Inc. — Product **Y** Invoice # 001
Buyer sector: **Manufacturing**

Qty	Unit price (CLP)	Total (CLP)
1	4.990	4.990

X Inc. — Product **Y** Invoice # 003
Buyer sector: **Mining**

Qty	Unit price (CLP)	Total (CLP)
1	5.990	5.990

X Inc. — Product **Y** Invoice # 002
Buyer sector: **Manufacturing**

Qty	Unit price (CLP)	Total (CLP)
5	4.390	21.950

X Inc. — Product **Y** Invoice # 004
Buyer sector: **Mining**

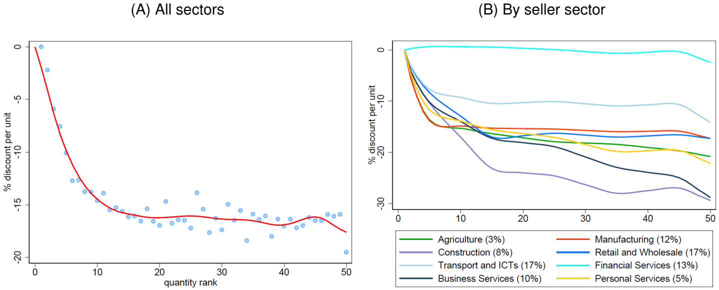
Qty	Unit price (CLP)	Total (CLP)
5	5.490	27.450

- Manufacturing: unit prices drop from 4.990 to 4.390 when moving from $q{=}1$ to $q{=}5$
- Mining: a distinct menu with higher unit prices, 5.990 at $q = 1$ and 5.490 at $q{=}5$
- Consistent with buyer-sector–specific nonlinear prices Aggregate Correlation

Nonlinear Prices by Quantity Bins and Seller Sector

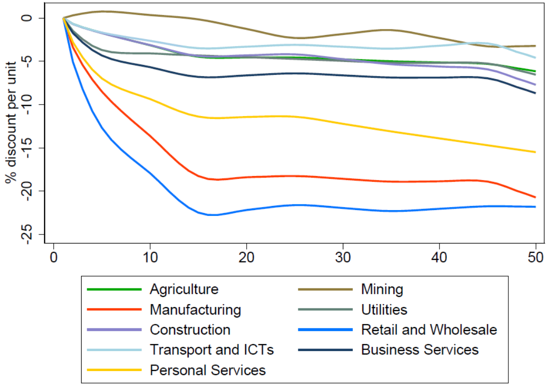
Bins construction Bins histogram

$$\ln p_{jgit} = \beta_0 + \sum_{b=2}^{50} \beta_b \mathbb{1}_{\{B_{jgit}=b\}} + \Psi_{jgd} + \varepsilon_{jgit}$$



- Unit prices fall steeply at small q and flatten as q grows (consistent with prediction)
- Heterogeneity between seller sectors in both the steepness and curvature of discounts

Retail & Wholesale Seller Sector: Pricing to Different Buyer Sectors



- Within a seller sector, nonlinear price schedules differ by buyer sector
- Buyer sector shifts price schedule without much change in curvature

Taking Stock

- Within seller$\times$product$\times$day, unit prices decline with quantity and flatten at higher quantities
- Evidence inconsistent with uniform pricing
- Pricing consistent with a combination of price-menus (2^{nd}) specific to buyer sectors (3^{rd}):
  - 2^{nd} degree screening drives curvature
  - 3^{rd} degree shifts levels across buyer sectors

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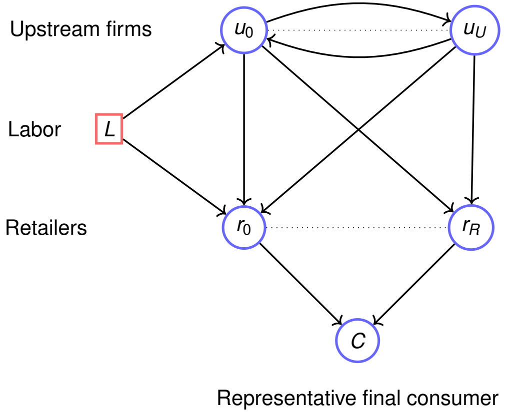
Supply Chain Structure

- Two observable firm types $\ell \in \{u, r\}$, defined by their position relative to final demand Evidence
- Upstream Roundabout
  u firms sell and buy from other u firms
- Retailers sell to final consumer only
  r purchase inputs from u and sell exclusively to the representative final consumer



Firm Heterogeneity

- Within each type $\ell \in \{u, r\}$ observable sectors are indexed by $s \in S$
- Firms as buyers are denoted by i and by j as sellers, buyer sectors as s and seller sectors as s'
- Each (ℓ, s) has a continuum of firms with unobserved productivity z_i distributed Pareto, tail κ_s^ℓ
- A firm i is thus characterized by the triple (ℓ, s, z_i), denoting type, sector, and productivity

Market Structure

- Retail firms sell to the representative consumer at uniform per-unit prices
- Upstream firms set nonlinear prices to other upstream firms and retailers
- Firms can price discriminate:
  1. Within types and sectors (ℓ, s) with unobserved z_i, but know z_i distribution (2^{nd})
  2. Across observed types and sectors (ℓ, s) (3^{rd})
- Firms are atomistic in input markets as buyers and take the wage as given

Main Challenge: Find endogenous sufficient conditions to get 2^{nd} + 3^{rd} tractable in supply chains:

- Constant marginal costs
- Isoelastic demands

Functional Forms and Firm Entry

Preferences ▸ Details

- Final demand Y is Cobb–Douglas across retail sectors aggregators Y_s
- Y_s are CES over retail sector firm varieties y_j^s with endogenous mass N_s^r

Technology ▸ Details

- Q_i is Cobb–Douglas in labor L_i and upstream sectoral material aggregator M_i
- M_i is CES over upstream varieties m_{ij}^s with endogenous mass N_s^l

Firm Entry ▸ Details

- In each (ℓ, s) there is an unbounded pool of identical potential entrants
- Entrants pay a sunk cost $c_s^{E\,\ell} > 0$ in labor units
- Free entry: expected discounted value of profits $(\pi_i^{\ell s}(z))$ equals entry cost $(c_s^{E\,\ell})$

Solution Concept ▸ Model Recap

1. Guess and Verify ▸ Details
   - Guess 1: For every seller, optimal contracts are two-part tariffs specific to (ℓ, s)
   - Guess 2: Upstream sellers face isoelastic demands, σ
   - imply standard CES and Cobb-Douglas costs and price indices functional forms ▸ Details

2. For a seller j, $\tau^{\ell}_{is'} \equiv P^{\ell}_{ss'} M^{1/\sigma_{s'}}_{is'}$ is a sufficient statistic for buyer's heterogeneity ▸ Details
   - $\tau^{\ell}_{is'}$ is Pareto distributed with tail parameter: $\rho^{\ell}_{ss'} = \sigma_{s'} \xi^{\ell}_{s} > 1$

$$\xi^{r}_{s} = \frac{\kappa^{r}_{s}}{\varphi - 1} \quad \text{(retail)}, \qquad \xi^{u}_{s} = \frac{\kappa^{u}_{s}}{\sigma_{s} - 1} \quad \text{(upstream)}$$

3. For any upstream seller, profit max. problem is additively separable acrros (ℓ, s) ▸ Details

Optimal Nonlinear Price

Proposition 1: Optimal Nonlinear Price in Supply Chains

Optimal contract offered by an upstream seller $j \in U_{s'}$ to any buyer $i = (\ell, s, z_i)$ is a two–part tariff:

$$T_{ij} \; = \; p^{\ell}_{js}\, m_{ij} \; + \; F^{\ell}_{js}$$

Marginal price p is constant across buyers within (ℓ, s): $\quad p^{\ell}_{js} = \mu^{\ell}_{ss'}\, c_j,$

$$\mu^{\ell}_{ss'} \; = \; \frac{\rho^{\ell}_{ss'}}{\rho^{\ell}_{ss'} - 1}, \qquad \rho^{r}_{ss'} \; \equiv \; \frac{\sigma_{s'}\, \kappa^{r}_{s}}{\varphi - 1}, \text{ for retailers } \quad \rho^{u}_{ss'} \; \equiv \; \frac{\sigma_{s'}\, \kappa^{u}_{s}}{\sigma_s - 1}, \text{ for upstream}$$

and a flat fee F chosen so that the lowest type's participation constraint binds,

$$\Pi(\underline{z}^{\ell}_{s}) = 0 \quad \iff \quad F^{\ell}_{js} = \frac{1}{\sigma_{s'}}\, R^{\ell}_{ss'}\left(\underline{z}^{\ell}_{i},\, m^{*}(\underline{z}^{\ell}_{i})\right).$$

For all partitions of firm types $\ell \in \{u, r\}$ and buyer sectors s, each with its own sector-specific two-part tariff.

Guesses Verification Firms Profits General Equilibrium Definition

Two Upstream Pricing Counterfactuals For Welfare Comparisons

Return

Planer Efficient Pricing (as in Baqaee and Farhi, 2021)

- Firms must charge markups to incentivize the optimal entry level
- But markup distorts input choices by acting as a uniform tax on production
- An output subsidy can restore undistorted marginal-cost, conditional on entry
- The subsidy is paid via a lump sum tax to the representative consumer

Uniform prices (e.g, as in Edmond, Midrigan & Xu, 2023)

- Constant markup over marginal cost from monopolistic competition
- CES markups $\mu^{LP} = \frac{\sigma}{\sigma-1}$, strictly higher than $\mu^{NLP} = \frac{\rho}{\rho-1}$
- Because unambiguously $\sigma < \rho$

Exact Welfare Decomposition: Intensive vs. Extensive Margins

- If the wage is the numeraire, welfare is the inverse final price index: $W \equiv \frac{1}{P_Y}$ Derivation Profit Functions

$$\Delta \log W = \underbrace{-\sum_s \tilde{\lambda}_s^{cr} \Delta \log \mu_s^r - \sum_{s'} \bar{\lambda}_{s'}^{ru} \Delta \log \bar{\mu}_{s'}^{ru} - \sum_{s'} \bar{\lambda}_{s'}^{uu} \Delta \log \bar{\mu}_{s'}^{uu}}_{\textbf{Intensive margin (allocative markups)}}$$

$$+ \underbrace{\sum_s \frac{\tilde{\lambda}_s^{cr}}{\varphi_s - 1} \Delta \log N_s^r + \sum_{s'} \frac{\tilde{\lambda}_{s'}^{uu}}{\sigma_{s'} - 1} \Delta \log N_{s'}^u}_{\textbf{Extensive margin (Firm masses, entry/variety)}}$$

- $\tilde{\lambda}$ are final consumption direct and indirect costs exposures
- $\bar{\mu}$ contains seller sectors charging buyer-sector specific markups
- Allocative prices markups drive the intensive margin (μ, extent of double marginalization)
- Flat fees drive the extensive margin trough distorted profits (N, firm masses and love of variety)

Welfare Ratios Across Price Regimes: Nonlinear vs. Uniform

$$\frac{W^{\text{NLP}}}{W^{\text{Uni}}} = \underbrace{\prod_{s' \in S} \left(\frac{\bar{\mu}_{s'}^{ru,\text{NLP}}}{\bar{\mu}_{s'}^{ru,\text{Uni}}} \right)^{-\bar{\lambda}_{s'}^{ru}} \times \prod_{s' \in S} \left(\frac{\bar{\mu}_{s'}^{uu,\text{NLP}}}{\bar{\mu}_{s'}^{uu,\text{Uni}}} \right)^{-\bar{\lambda}_{s'}^{uu}}}_{\textbf{Intensive Margin}} \times \underbrace{\prod_{s \in S} \left(\frac{N_s^{r,\text{NLP}}}{N_s^{r,\text{Uni}}} \right)^{\frac{\tilde{\lambda}_s^{cr}}{\varphi_s - 1}} \times \prod_{s' \in S} \left(\frac{N_{s'}^{u,\text{NLP}}}{N_{s'}^{u,\text{Uni}}} \right)^{\frac{\tilde{\lambda}_{s'}^{uu}}{\sigma_{s'} - 1}}}_{\textbf{Extensive Margin}}$$

- Intensive margin (unambiguous gain):
  $\mu^{\text{NLP}} < \mu^{\text{Uni}}$, attenuating double marginalization
- Extensive margin (ambiguous):
  Flat fees shift profits with ambiguous sign, Firm entry can go either way
- Welfare comparison? Requires full model quantification

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Three Quantitative Exercises

- Calibration using population-level B2B transactions and firm balance sheets for Chile
- Two calibrations based on pricing conduct (nonlinear and uniform) ▸ Pricing Counterfactuals
- Welfare decomposition: Intensive vs. Extensive margin (and by sectors) ▸ Derivation

1. Model fit
   How much of the observed pricing schemes the model can explain ▸ Details
2. Policy
   Welfare outcomes of banning all forms of price discrimination
3. Measurement
   Welfare cost under two interpretations of the same data: nonlinear vs. uniform pricing

Policy: Ban on Price Discrimination Welfare Outcome

$$\frac{W^{\text{Reg}}}{W^{\text{Eff}}} = \underbrace{\prod_{s\in S}\left(\frac{\bar{\mu}_s^{ur,\text{Reg}}}{\bar{\mu}_s^{ur,\text{Eff}}}\right)^{-\tilde{\lambda}_s^{ru}} \prod_{s\in S}\left(\frac{\bar{\mu}_s^{uu,\text{Reg}}}{\bar{\mu}_s^{uu,\text{Eff}}}\right)^{-\tilde{\lambda}_s^{uu}}}_{\text{Intensive Margin}} \times \underbrace{\prod_{s\in S}\left(\frac{N_s^{r,\text{Reg}}}{N_s^{r,\text{Eff}}}\right)^{\frac{\theta_s}{\varphi_s-1}} \prod_{s\in S}\left(\frac{N_s^{u,\text{Reg}}}{N_s^{u,\text{Eff}}}\right)^{\frac{\tilde{\lambda}_s^{uu}}{\sigma_s-1}}}_{\text{Extensive Margin}}$$

Price regime	W^R / W^{Eff}
Nonlinear (NLP)	0.745
Uniform pricing (Uni)	0.486

- Banning price discrimination reduces welfare from $\approx 75\%$ of efficient welfare to $\approx 50\%$
- Allowing for price discrimination closes about half of the efficiency gap:

$$\frac{W^{\text{NLP}} - W^{\text{Uni}}}{W^{\text{Eff}} - W^{\text{Uni}}} = \frac{0.745 - 0.486}{1 - 0.486} \approx 0.50.$$

Policy: Aggregate Welfare Decomposition (rel. to the efficient BMK)

$$\frac{W^{\text{Reg}}}{W^{\text{Eff}}} = \underbrace{\prod_{s\in S}\left(\frac{\bar{\mu}_s^{ur,\text{Reg}}}{\bar{\mu}_s^{ur,\text{Eff}}}\right)^{-\tilde{\lambda}_s^{ru}} \prod_{s\in S}\left(\frac{\bar{\mu}_s^{uu,\text{Reg}}}{\bar{\mu}_s^{uu,\text{Eff}}}\right)^{-\tilde{\lambda}_s^{uu}}}_{\text{Intensive Margin}} \times \underbrace{\prod_{s\in S}\left(\frac{N_s^{r,\text{Reg}}}{N_s^{r,\text{Eff}}}\right)^{\frac{\theta_s}{\varphi_s-1}} \prod_{s\in S}\left(\frac{N_s^{u,\text{Reg}}}{N_s^{u,\text{Eff}}}\right)^{\frac{\tilde{\lambda}_s^{uu}}{\sigma_s-1}}}_{\text{Extensive Margin}}$$

Regime rel. to. eff.	Intensive	Extensive	$\text{Share}_{\text{int}}$	$\text{Share}_{\text{ext}}$
Nonlinear (NLP)	0.67	1.12	0.79	0.21
Uniform (Uni)	0.46	1.06	0.93	0.07

- Result
  Rel. to efficiency, markups create higher expected profits, boost entry: more firms at a smaller scale
- Intensive Margin Dominates (as a share of total log deviation relative to Eff.)
  NLP: 79%, Uni: 93%. Banning price discrimination raises double marginalization in supply chains
- Extensive Margin: Entry Responses
  The extensive margin is pro-competitive (factors > 1) but modest By sector

Measurement: Nonlinear vs. Uniform Pricing Interpretation

Price Lens	W^L/W^{Eff}	Intensive	Extensive
Nonlinear	0.748	0.67 (79%)	1.12 (21%)
Uniform	0.565	0.55 (97%)	1.02 (3%)

- Two model quantifications, dependent on the data interpretation
- Welfare falls from 75% (NLP) to 57% (Uni) when changing pricing assumption
- Nonlinear prices interpretation closes the gap by about 18 percentage points
- Aggregate costs of market power are lower when the model allows for price discrimination

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Conclusion

- We find indicative widespread evidence that sellers set nonlinear prices (NLP) in supply chains
- NLP improves allocations rel. to uniform pricing but shifts rents via flat fees which distorts entry
- Banning price discrimination in supply chains can raise the aggregate costs of market power
- Average prices can mislead. Policy should monitor marginal prices and rent extraction
- Don't necessarily ban quantity discounts; target markup accumulation along the supply chain
- The method is plug-and-play with standard micro-data on transactions

Mirrlees Reduction and Virtual Surplus (Standard) ▸ Return

- Using the virtual surplus ϕ , the problem can be written as a pointwise optimization problem

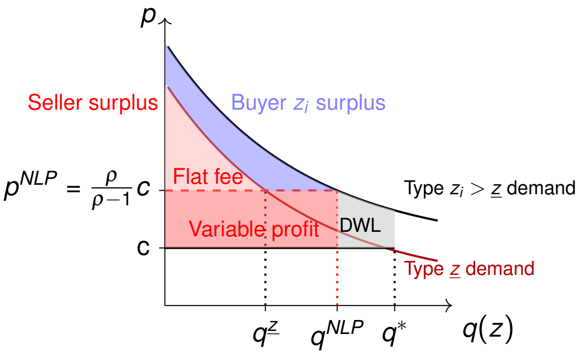
$$\max_{\{q(z)\}} \Pi_{\text{seller}} = \int_{\underline{z}}^{\infty} \left[\phi(z, q(z)) - cq(z)\right] f(z)\, dz,$$

$$\text{with} \quad \phi(z,q) = \underbrace{R(z,q)}_{1} - \underbrace{\frac{1}{h(z)} \frac{\partial R(z,q)}{\partial z}}_{2}$$

- Inverse hazard rate, $h(z)^{-1} = (1 - F(z))/f(z)$ is the weight on the remaining higher types
- The virtual surplus represents the seller's effective revenue from serving type z:
  1. Buyer z total revenue from the transaction (seller wants to extract it)
  2. Rents the seller must leave to higher types to prevent them from mimicking type z

Optimal Nonlinear Price: Allocations ▸ Return

- Virtual surplus for lower type is strictly positive: All types are served ▸ Details
- Quantities are pinned down by marginal price p^{NLP}
- Flat fee only redistributes surplus; is not allocative



Is it profitable to exclude any buyer type? ▸ Return

In nonlinear pricing with private information, the seller always faces a choice:

1. Exclude low types to better extract surplus from high types
2. Serve all types, but give up some rent from high types

- However, Pareto distribution has heavy mass for low types
- Even though low types buy little q, because of their large density the seller maximizes profits by serving them

No, Positive Virtual Surplus for all types Return

Virtual surplus: profit from serving a buyer type, net of the informational rents that must be left to higher types to preserve IC

- Virtual surplus for type z (with $\alpha = \frac{\sigma}{\sigma-1}$):

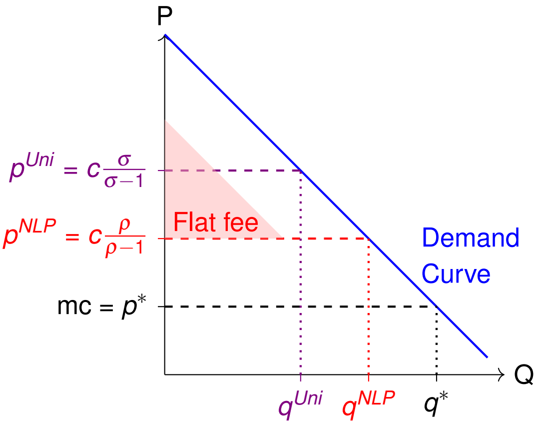
$$\mathrm{VS}(z) = \underbrace{\left(\frac{z^{\sigma-1}}{\alpha}\right) q(z)^{1-1/\sigma}}_{\text{Gains from serving type } z} - \underbrace{\left(\frac{1-F(z)}{f(z)} \cdot \frac{d}{dz}\left(\frac{z^{\sigma-1}}{\alpha}\right)\right) q(z)^{1-1/\sigma}}_{\text{Informational rents left to ensure IC}}$$

- For the lowest type $z_0 = 1$, the virtual surplus simplifies:

$$\mathrm{VS}(1) = \left[\frac{1}{\alpha}\left(1 - \frac{\sigma-1}{\kappa}\right)\right] q(1)^{1-1/\sigma}$$

- This is strictly positive whenever $\kappa > \sigma - 1$ (necessary condition for finite output)
- If its profitable to serve the lowest type, the seller will not exclude any buyer

Two-Part Tariff: NLP vs Uni CES markup ($\rho > \sigma$) Return



- Allocations in NLP are less distorted relative to Uni

$$q^* > q^{NLP} > q^{Uni}$$

- Because of the flat fee, rents are subject to different distortions in NLP vs. Uni

Is price deviation profitable for any $z > z_a$? ‣ Return

- Heuristic argument (Wilson 1993) to derive the optimal price $p(q)$
- Define marginal buyer $z(q,p)$ by inverting demand for the q^{th} unit (equation 1)

$$z(q,p) = q^{\frac{1}{\sigma-1}} p^{\frac{\sigma}{\sigma-1}}$$

- Demand for q^{th} unit:

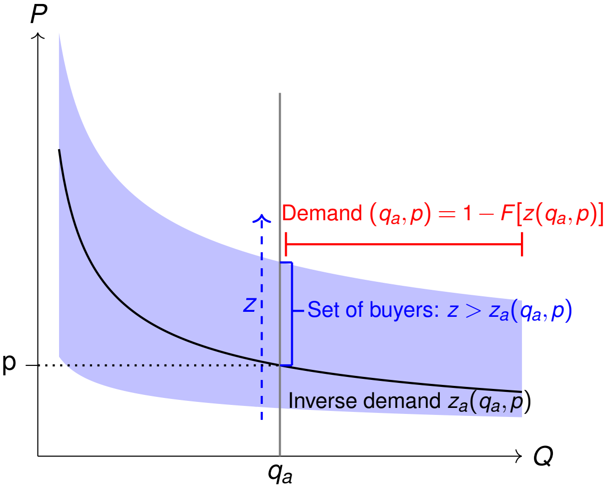
$$D(q,p) = 1 - F(z(q,p))$$

- Seller chooses a price for unit "q" to solve:

$$\max_p [1 - F(z_a(q_a,p))](p-c)$$

No profitable deviation in price

Return



$$\max_{\{P\}} [1 - F(z_a(q_a, p))](P - c)$$

FOC :

$$\frac{P}{c} = \frac{\frac{\kappa\sigma}{\sigma-1}}{\frac{\kappa\sigma}{\sigma-1} - 1} = \frac{\rho}{\rho - 1}$$

- The optimal price is equal to the allocative price of the two-part tariff
- Seller has no incentive to charge different prices for different quantities

General Equilibrium NLP

Return

A decentralized nonlinear pricing equilibrium is a collection of firm level productivity z_i, linear prices $\{p_u, p_r\}$, flat fees $\{F_u, F_r\}$, wage w, and quantities $\{y_i, q_i, l_i, m_{iu}, N_U, N_R\}$ such that:

- Each producer minimizes its costs and charges a linear price that equals marginal cost times the markup
- Each producer pays a transfer, such that the lowest types have zero surplus from transacting with upstream sellers
- Entrants earn zero expected profit
- The representative consumer maximizes its consumption
- Markets clear for all goods and factors

Invoice Example

Return

KITCHEN CENTER SPA
IMPORTACIÓN Y DISTRIBUCIÓN DE ELECTRODOMÉSTICOS

Casa Matriz:	Av. El Salto 3485, Recoleta, Santiago
Sucursales:	
Casa Costanera:	Av. Nueva Costanera 3900, Vitacura
Mall Parque Arauco:	Av. Kennedy 5413 Local 572, Las Condes - Teléfono: (56-2) 24117777 - Fax: (56-2) 24117711
Mall Plaza Los Dominicos:	Padre Hurtado Sur 875, Local A2080/2076, Las Condes - Teléfono: (+56 2) 24117738
Mall Buenaventura:	San Ignacio 500 Local 12, Quilicura - Teléfono: (+56 2) 24117769
Mall Plaza La Serena:	Av. Huanhualí 105 BA 190, La Serena - Teléfono: (56-2) 24117732
Mall Marina Arauco:	Av. Libertad 1348, Local PD-01/02, Viña del Mar - Teléfono: (56-2) 24117767/68
Outlet Park Viña:	Camino Internacional 2440 Local 72, Viña del Mar
Mall Plaza Maule:	Circunvalación 1055, Local 226/227, Talca - Teléfono: (56-2) 24117748
Concepción:	Paicaví 2567, Local 2, Teléfono: (56-2) 2411 7716 / 17
Temuco:	Avda. Alemania 0611, Temuco - Teléfono: (+56 2) 24117774 R
Mall Easton Temuco:	Rudencio Ortega 01780, Local L168-170, Temuco - Teléfono: (+56 2) 24117714
Servicio Técnico:	Lautaro #260, Quilicura - Teléfono: 6004117709 / 737 / 504
Centro de Distribución:	Camino lo Boza #8887, Pudahuel, Santiago
Viña del Mar:	8 Nte. 514, Viña del Mar.
Alto Las Condes:	Av. Kennedy 9001 Local 1017, Las Condes.
Outlet El Salto:	Av. El Salto 3400, Recoleta, Santiago.

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FECHA EMISIÓN :
FECHA VENCIMIENTO : 03/08/2022
TIPO DESPACHO :
FORMA DE PAGO : Contado
COD. VENDEDOR :

Dirección Origen: Camino lo Boza 8887
Comuna : Pudahuel Ciudad : Santiago
Dirección Destino: Los Misioneros 1923
Comuna : Providencia Ciudad : Santiago

Orden de Venta: 793325
Número de OC:

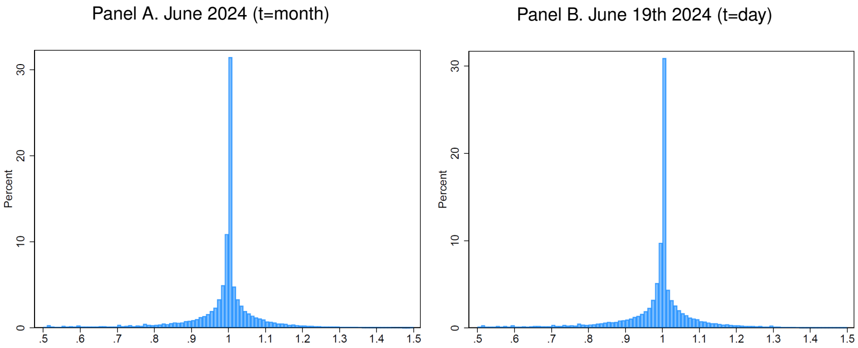
CÓDIGO	DETALLE	CANTIDAD	PRECIO UNITARIO	PRECIO ÍTEM
13452	Lavaplatos FDV Small Acqua bajo cubierta	1	92.428,57	92.429
14761	Encimera FDV Design 4T GLTX 65 BUT 2.0	1	142.848,74	142.849
14265	Campana Kubli Neu Slider	1	100.831,93	100.832
19110	Horno FDV Design	1	201.672,27	201.672
13377	Lavavajillas FS FDV Element 14C	1	243.689,07	243.689
14917	Griferia FDV CONICA FLEX	1	84.025,21	84.025
10232	Transporte - Providencia	1	15.529,41	15.529

Data cleaning Return

Goal: Keep all plausible transactions

- Prices are net of discounts and recharges
- Drop if a transaction has missing or zero price or quantity
- Drop if product description is missing
- Drop transactions where seller-product-day maxmin price ratio is above the 99^{th} percentile
- Under this cleaning we keep around 95% of transactions

Price Dispersion Return



- $\theta_{jgt} = \frac{p_{jgit}}{\bar{p}_{jgt}}$; seller j, product g, buyer i, time t (excluding products with one transaction)
- $\text{Var}(\log \theta_{jgd}) = 0.65$
- indicative evidence inconsistent with uniform pricing in 70% of transactions Residual Drivers

Prices and Quantities Correlation (assumed in equilibrium) Return

- We do not observe contract details; instead assume contracts are specific to buyer groups
- If we observe price variation within a day, we assume we are tracing an equilibrium contract

$$\ln p_{jgit} = \beta_0 + \beta_1 \ln q_{jgit} + \Psi_S + \varepsilon_{jgit}$$

Seller j, product g, buyer i, time t, day d, quantity q, buyer group B (11 sectors$\times$3 sizes$\times$16 regions)

- Unconditional average discount is 2.9% ($\ln 2 * 0.042$) per unit when doubling quantity purchased
- Conditioning on buyers (and groups of buyers), the average discount increases
- Even within buyer groups, the average discount is 90% of the unconditional average

	(1)	(2)	(3)	(4)
$\ln q_{jgit}$	-0.042	-0.084	-0.065	-0.037
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
$S_{Base} = j \times g \times d$	✓			
$S = \text{Base} + i$		✓		
$S = \text{Base} + B$			✓	
$S = \text{Base} \times B$				✓
N	430M	430M	430M	430M
R^2	0.9646	0.9678	0.9659	0.9790

Price dispersion By sector

Price Variance Determinants for 2024: Strategy

Return

Step 1

- Make goods comparable and eliminate possible demand and supply shocks
- Store residuals from:

$$\ln p_{jgit} = \beta_0 + \Psi_{jgd} + \varepsilon_{jgit}$$

p_{jgit} is the price for seller j, product g, buyer i in time t, Ψ is a fixed effect including day d

Step 2

- Project residuals on different observables (quantity transacted and buyers' observables)
- Compare R^2

Price Determinants for 2024: Results Return

Seller j, product g, buyer i, time t, day d, quantity q, buyer group B (11 sectors$\times$3 sizes$\times$16 regions)

$$\varepsilon_{jgit} = \beta_0 + \Psi_{jgdS} + \varepsilon_{jgit}$$

	(1)	(2)	(2)
R^2	0.34	0.28	0.53
$S =$ Quantity	✓		
$S =$ Buyer Group		✓	
$S =$ Quantity $\times$ Buyer group			✓
N	147M	147M	147M

- Consistent whit hybrid second + thrid dregree price discrimination schemes

Monthly Fixed Effects Selected sectors Buyer Power

Price Determinants for 2024: Monthly Fixed Effects Return

Seller j, product g, buyer i, time t, month m, quantity q, buyer group B (11 sectors$\times$3 sizes$\times$16 regions)

$$\ln p_{jgit} = \beta_0 + \Psi_{jgdS} + \varepsilon_{jgit}$$

	(1)	(2)	(3)	(4)
R^2	0.34	0.51	0.41	0.62
$S =$ Quantity	✓			
$S =$ Buyer		✓		
$S =$ Buyer Group			✓	
$S =$ Quantity $\times$ Buyer group				✓
N	363M	363M	363M	363M

Price Determinants for 2024: Manufacturing

Return

Seller j, product g, buyer i, time t, month m, quantity q, buyer group B (11 sectors$\times$3 sizes$\times$16 regions)

$$\ln p_{jgit} = \beta_0 + \Psi_{jgmS} + \varepsilon_{jgit}$$

	(1)	(2)	(3)	(4)
R^2	0.45	0.54	0.46	0.81
$S =$ Quantity	✓			
$S =$ Buyer		✓		
$S =$ Buyer Group			✓	
$S =$ Quantity $\times$ Buyer group				✓
N	136M	136M	136M	136M

Price Determinants for 2024: Retail and Wholesale Return

Seller j, product g, buyer i, time t, month m, quantity q, buyer group B (11 sectors$\times$3 sizes$\times$16 regions)

$$\ln p_{jgit} = \beta_0 + \Psi_{jgmS} + \varepsilon_{jgit}$$

	(1)	(2)	(3)	(4)
R^2	0.38	0.65	0.49	0.68
$S =$ Quantity	✓			
$S =$ Buyer		✓		
$S =$ Buyer Group			✓	
$S =$ Quantity $\times$ Buyer group				✓
N	180M	180M	180M	180M

Buyer Market Power? Return

- Exploit cross-sectional variation in the number of suppliers each buyer transacts with
- A larger number of providers may indicate stronger outside options; better pricing terms

$$\ln p_{jgim} = \beta_0 + \beta_1 \ln q_{jgim} + \beta_2 \left(\log q_{jgim} \times \log \text{NumProviders}_i\right) + \Psi_{jgm} + \varepsilon_{jgit},$$

- $\beta_2 > 0$ would suggest that quantity discounts become flatter as buyer power increases
- We find that $\beta_1 = -0.0462\ (0.0001)$ and $\beta_2 = -0.0098\ (0.0001)$
- Buyer power does not appear to be the primary mechanism generating quantity discounts

Nonlinear Prices by Sector

Return

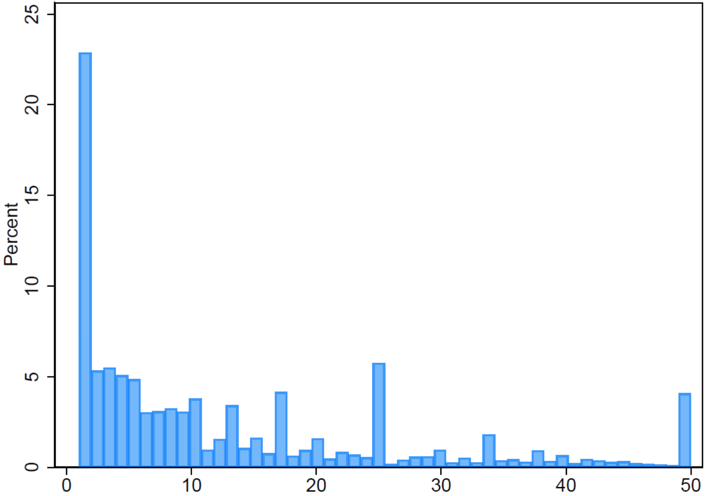
Sector	Mean Q discount	N transactions
All sectors	-0.042	430M
Agriculture	-0.042	2M
Mining	-0.016	1M
Manufacturing	-0.036	118M
Utilities	0.000	6M
Construction	-0.129	1M
Retail and Wholesale	-0.048	270M
Transport & ICTs	-0.032	12M
Financial Services	-0.002	49M
Real Estate Services	-0.052	1M
Business Services	-0.089	5M
Personal Services	-0.053	1M

Quantity Quantiles Bins Return

- Products have different scales, we compare prices across each product's q rank distribution
- For each product g, $F_g(\cdot)$: empirical CDF of transacted quantities q_{jgit}
- Define the within-product rank: $r_{jgit} \equiv F_g\left(q_{jgit}\right)$.
- Partition $[0,1]$ into 50 equal-probability intervals $I_b \equiv \left((b-1)/50,\ b/50\right]$ for $b = 1, \ldots, 50$
- Assign each transaction to a bin $B_{jgit} = b$ whenever $r_{jgit} \in I_b$
- With discrete quantities and mass points, we assign observations to the smallest b

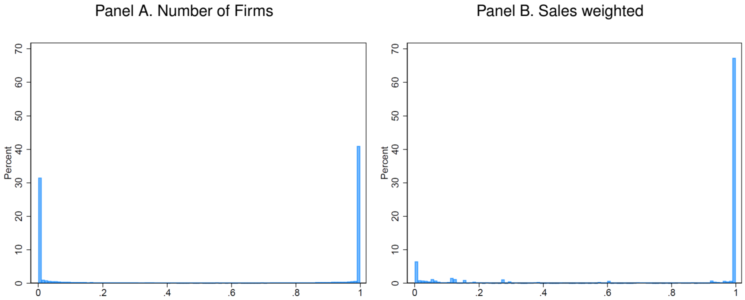
Quantity Quantiles Bins Histogram

Return



Sales partition ▸ Return

$$X_i = \begin{cases} 0 & \text{if all sales go to final consumers} \\ 1 & \text{if all sales go to other firms} \end{cases}$$



- More than 70% of firms sell only to final consumers or to other firms By sector

Sales partition: Sales shares (excluding exports)

Return

Sector (sales)	All to final consumer	All to other firms
Firm population	0.08	0.67
Agriculture (2%)	0.04	0.60
Mining (1%)	0.27	0.08
Manufacturing (15%)	0.05	0.68
Utilities (3%)	0.20	0.51
Construction (8%)	0.02	0.89
Retail and Wholesale (32%)	0.09	0.68
Transport & ICTs (10%)	0.16	0.68
Financial Services (18%)	0.18	0.67
Real Estate Services (1%)	0.24	0.37
Business Services (7%)	0.08	0.81
Personal Services (2%)	0.68	0.10

Preferences

▸ Return

- The representative consumer owns all firms and inelastically supplies one unit of labor ($L=1$)
- Final demand is Cobb–Douglas across retail sectors with within–sector CES over retail varieties:

$$Y = \prod_{s \in S} Y_s^{\theta_s}, \qquad \sum_{s \in S} \theta_s = 1, \qquad Y_s = \left(\int_{j \in R_s} y_j^{\frac{\varphi_s - 1}{\varphi_s}} \, d\nu_s(j) \right)^{\frac{\varphi_s}{\varphi_s - 1}}$$

- $\theta_s \in (0,1)$ are Cobb–Douglas output elasticities
- $\varphi_s > 1$ is the within-sector elasticity, and $d\nu_s(j)$ denotes number active retail varieties R_s in s
- The total number of active varieties in (r,s) is $N_s^r := \nu_s(R_s)$, an endogenous equilibrium object

Technology ‣ Return

- Firm $i \in (\ell, s)$ output (Q_i) is CD in labor (l_i) and in a CD aggregator of sectoral materials (M_i)

$$Q_i = z_i \, l_i^{\alpha_s^\ell} \, M_i^{1-\alpha_s^\ell}, \qquad 0 < \alpha_s^\ell < 1,$$

$$M_i = \prod_{s' \in S} M_{is'}^{\theta_{ss'}^\ell}, \qquad \sum_{s' \in S} \theta_{ss'}^\ell = 1, \qquad M_{is'} = \left(\int_{j \in U_{s'}} m_{ij}^{\frac{\sigma_{s'}-1}{\sigma_{s'}}} \, d\nu_{s'}(j) \right)^{\frac{\sigma_{s'}}{\sigma_{s'}-1}}$$

- α_s^ℓ is the labor output elasticity, $M_{is'}$ is firm i' material bundle from upstream sector s'
- $\theta_{ss'}^\ell \geq 0$ are the sector s' input elasticities for sector s and firm type ℓ
- m_{ij} is firm i's input of upstream variety j in sector s'
- $\sigma_{s'} > 1$ is the CES across varieties inside upstream sector s'
- $N_{s'} := \nu_{s'}(U_{s'})$ is the endogenous measure over the active upstream firms in sector s'

Firm Entry ▸ Return

- In each (ℓ, s) there is an unbounded pool of identical potential entrants
- Entrants pay a sunk cost $c_s^{E\,\ell} > 0$ in units of labor, then observe their productivity z
- Active firms exit exogenously at the end of the period with probability $\delta_s^\ell \in [0, 1)$
- Free entry requires that the expected discounted value of profits $(\pi_i^{\ell s}(z))$ equals entry cost $(c_s^{E\,\ell})$

$$\frac{1}{1-\delta_s^\ell}\mathbb{E}_z\left[\pi_i^{\ell s}(z)\right] = c_s^{E\,\ell} w, \quad \forall(\ell, s)$$

Model Recap ‣ Return

1. Potential entrants in each firm type and sector pair (ℓ, s) pay $c_s^{E\,\ell}$ and then draw productivity z
2. Upstream sellers see (ℓ, s) and z distribution; post (ℓ, s) nonlinear contracts $\{m_j^{\ell,s}, T_j^{\ell,s}\}$
3. Retail sellers post uniform prices to final consumers
4. Buyers $i = (\ell, s, z_i)$ observe price menus and w, then choose l_i and $\{m_{ij}\}_j$ to max. profits
5. Production and trade occur, transfers $\{T_{ij}\}_j$ are realized, and final demand $\{y_j\}$ is met
6. Firms exit with probability δ_s^ℓ
7. Contracts are enforceable, resale/arbitrage is ruled out

Upstream Seller Profit Maximization Problem ▸ Return

- A seller $j \in s'$ chooses a menu of a total transfer and a allocation $\{T_{ij}^{\ell}, m_{ij}^{\ell}\}$ for each (ℓ, s)

$$\max_{\{T,m\}} \sum_{\ell \in \{u,r\}} \sum_{s \in S} N_s^{\ell} \mathbb{E}_{\tau_{is'}} \left[T(\tau) - c_j m_{ij}(\tau) \right], \quad \text{s.t. for each } (\ell, s)\text{: IC, IR}$$

- The problem is additively separable across (ℓ, s) and can be solved partition-by-partition.
- Following Lemma 1, the virtual–surplus reduction yields the pointwise optimization problem:

$$\max_{\{m(\tau)\}} N_s^{\ell} \mathbb{E}_{\tau_{is'}} \left[\left(\tau - \frac{\tau}{\rho_{ss'}^{\ell}} \right) \frac{\sigma_{s'}}{\sigma_{s'} - 1} m(\tau)^{\frac{\sigma_{s'}-1}{\sigma_{s'}}} - c_j\, m(\tau) \right]$$

- which is strictly concave in m since $(\sigma_{s'} - 1)/\sigma_{s'} \in (0, 1)$

Guess and Verify

▸ Return

Guess 1: Optimal contracts are isomorphic to a two-part tariff specific to (ℓ, s)

$$T_{ij} = p^{\ell}_{js} m_{ij} + F^{\ell}_{js'} = \mu^{\ell}_{ss'} c_j m_{ij} + F^{\ell}_{js'}$$

- Transfer T_{ij} depends on allocation m_{ij}, markup $\mu^{\ell}_{ss'}$, and flat fee $F^{\ell}_{js'}$
- The marginal (allocative) price is $p^{\ell}_{js} = \mu^{\ell}_{ss'} c_j$
- Flat fees are inframarginal and do not affect marginal input choices;

Guess 2: Revenue functions are homogeneous in quantity

$$R^{\ell}_{i,s} = A^{\ell}_{s} \left(Q^{\ell}_{i,s}\right)^{\psi^{\ell}_{s}}$$

- For parameters A^{ℓ}_{s} and ψ^{ℓ}_{s} that are constants at buyer type and sector (ℓ, s)
- Imply isoelastic demands for intermediate inputs

Costs and Price Indices Under Guess 1 ▸ Return

- Marginal prices are quantity–invariant within a buyer type–sector (i, s) and seller sector s'

CES Sectoral Price Index

$$P^{\ell}_{ss'} = \left(\int_{j \in U_{s'}} \left(p^{\ell}_{js'} \right)^{1-\sigma_{s'}} d\nu_{ss'}(j) \right)^{\frac{1}{1-\sigma_{s'}}}$$

Cobb–Douglas Materials Cost Index

$$P^{M}_{i} = \prod_{s' \in S} \left(P^{\ell}_{ss'} \right)^{\theta^{\ell}_{ss'}}, \qquad \sum_{s' \in S} \theta^{\ell}_{ss'} = 1, \ \ \theta^{\ell}_{ss'} \geq 0$$

Firm–Level Marginal Cost

$$c_i = \frac{\Theta^{\ell}_{s}}{z_i} w^{\alpha^{\ell}_{s}} \left(P^{M}_{i} \right)^{1-\alpha^{\ell}_{s}}, \qquad \Theta^{\ell}_{s} \equiv \left(\alpha^{\ell}_{s} \right)^{-\alpha^{\ell}_{s}} \left(1 - \alpha^{\ell}_{s} \right)^{-(1-\alpha^{\ell}_{s})} \prod_{s' \in S} \left(\theta^{\ell}_{ss'} \right)^{-(1-\alpha^{\ell}_{s})\theta^{\ell}_{ss'}}$$

Type Re-parametrization and Distribution for Screening

Return

- For a seller $j \in s'$, each buyer $i \in s$ matters only through the valuation index: $\tau^{\ell}_{is'} \equiv P^{\ell}_{ss'} M^{1/\sigma_{s'}}_{is'}$
- For a seller j, $\tau^{\ell}_{is'}$ is a sufficient statistic for buyer's heterogeneity
- $P^{\ell}_{ss'}$ price level faced by i for inputs from s', and $M_{is'}$ is the buyer's demand shifter (scale)
- Under Pareto distributed buyer productivity, $\tau^{\ell}_{is'}$ is Pareto with tail parameter:

$$\rho^{\ell}_{ss'} = \sigma_{s'} \xi^{\ell}_{s} > 1$$

- Type-specific heterogeneity maps to ξ^{ℓ}_{s}:

$$\xi^{r}_{s} = \frac{\kappa^{r}_{s}}{\varphi - 1} \quad \text{(retail)}, \qquad \xi^{u}_{s} = \frac{\kappa^{u}_{s}}{\sigma_{s} - 1} \quad \text{(upstream)}$$

Guesses Verification ▸ Return

Guess 1 (two–part tariffs with quantity–invariant marginal price within each (ℓ, b, u))

- follows immediately from the two-part tariff and constant markup in Proposition 1

Guess 2 (homogenous link revenue)

- Is verified by aggregating optimal link choices across partitions
- The seller's total revenue is isoelastic in own quantity with exponent $(\sigma_u - 1)/\sigma_u$
- Admits a closed–form scale A_{su} that explicitly includes a flat–fee component driven by the seller's CES share in buyers' materials bundles

General Equilibrium Under Nonlinear Prices

Return

A general equilibrium under nonlinear pricing is a collection

$$\left\{(m^{\ell}_{ubi}(\cdot), T^{\ell}_{ubi}(\cdot), B_{\ell bi})_{(\ell,b),i},\ (p^{\ell,*}_{bi})_{(\ell,b),i},\ (P^{\ell}_{ub})_{u,b,\ell},\ (N^{\ell}_{s})_{s,\ell},\ (Q_j, I_j)_j\right\}$$

such that: (i) mechanisms $(m^{\ell}_{ubi}(\cdot), T^{\ell}_{ubi}(\cdot))$ implement the two–part–tariff optimum with $p^{\ell,*}_{bi}$ and $F^{\ell,*}_{ubi}$ in Proposition 1; (ii) buyers' choices satisfy the best–response condition above; (iii) price and cost indices satisfy ; (iv) materials and labor markets clear with $L = 1$; and (v) free entry holds in each (ℓ, s). A detailed proof of existence and uniqueness is provided in the paper.

Upstream Firm Profits Under Nonlinear Prices: ▸ Return

$$\mathbb{E}\left[\Pi_j^u\right] = \underbrace{\sum_\ell \sum_s \int_{i \in B_{\ell s}} \left(p_{js}^\ell - c_j\right) m_{ij} \, dv_{\ell s}^i}_{\text{allocative margin}} + \underbrace{\sum_\ell \sum_s \int_{i \in B_{\ell s}} F_{jsi}^\ell \, dv_{\ell s}^i}_{\text{flat–fee revenue}} - \underbrace{\sum_{s'} \int_{j' \in D_j} F_{j',s'}^u \, dv_{s'}^j}_{\text{flat–fee payments}}$$

- B denotes seller firm j client set, D denote seller firm j suppliers set (exogenous sets)
- NLP Marginal Prices. Charge and pay smaller marginal prices (p_{js}^ℓ, c_j) relative to Lin. P.
- Rents. Through flat fees seller j, extracts rents, but it's also rent extracted
- GE incidence. Cheaper c_j lift downstream demand; double marginizalzation attenuation
- Entry. Depends on m_{ij} expansion and net rent extraction (firm entry is misallocated)

Retailer Firm Profits Under Nonlinear Prices ‣ Return

$$\mathbb{E}[\Pi_i^r] = \underbrace{\left(\frac{1}{\varphi_s}\right) R_i}_{\text{allocative margin}} \quad - \quad \underbrace{\sum_{s'} \int_{j \in D_i} F_{ijs'}^r \, d\nu_{s'}^j}_{\text{Flat fees payments to upstream}}$$

- **Constant markup** Allocative margin has a constant markup and a fixed share of revenue
- NLP Marginal Prices. NLP lowers input costs, retail prices fall with constant markup and revenue expands; the allocative term scales proportionally with R_i
- Rents. Extracted via fee payments to upstream
- Entry. Depends on change in profits: revenue expansion versus rent extraction

Welfare Decomposition: Overview ▸ Return

Aggregate welfare is the inverse of the final price index:

$$W \equiv \frac{1}{P_Y}, \qquad \log P_Y = \sum_{s \in S} \theta_s \log P_s$$

With wage normalization and free entry, $W = 1/P_Y$

Idea: Under Pareto-distributed firm types and CES aggregation, firm-level heterogeneity collapses into sectoral sufficient statistics. Hence, sector-level markups, firm masses, and elasticities fully determine welfare without microdata.

Goal: Express $\Delta \log W$ as a linear map from changes in:

- Buyer–seller markups $\mu^{ru}_{ss'}$, $\mu^{uu}_{us'}$ (intensive margin)
- Firm masses N^r_s, $N^u_{s'}$ (extensive margin)

Buyer–Seller Price Structure

Retail interface (consumer $\rightarrow$ retail $\leftrightarrow$ upstream):

$$P_s = \mu_s^r \prod_{s'} \left(\mu_{ss'}^{ru} C_{s'}\right)^{(1-\alpha_s^r)\theta_{ss'}^r} \left(N_s^r\right)^{-\frac{1}{\varphi_s - 1}} \mathscr{V}_s.$$

Upstream recursion (within production network):

$$C_{s'} = \prod_{v} \left(\mu_{vs'}^{uu} C_v\right)^{(1-\alpha_{s'}^u)\theta_{s'v}^u} \left(N_{s'}^u\right)^{-\frac{1}{\sigma_{s'}-1}} \mathscr{V}_{s'}^u.$$

- Firms charge different markups to each buyer sector
- Buyer-specific markups $\mu_{ss'}^{ru}$, $\mu_{us'}^{uu}$ propagate along the input–output structure
- CES structure allows aggregation using cost-based exposure weights

Final-Demand Exposure

Buyer–seller markups load into welfare through exposure matrices:

$$\Lambda^{ru} = \mathrm{Diag}(b)\Omega^{ru}, \qquad \Lambda^{uu} = \mathrm{Diag}(b\Omega^{ru})\Psi^{uu}.$$

For each seller s':

$$\bar{\lambda}^{ru}_{s'} = \sum_s \Lambda^{ru}_{ss'}, \qquad \bar{\lambda}^{uu}_{s'} = \sum_u \Lambda^{uu}_{us'}.$$

Define normalized buyer weights:

$$\omega^{ru}_{s|s'} := \frac{\Lambda^{ru}_{ss'}}{\bar{\lambda}^{ru}_{s'}}, \qquad \omega^{uu}_{u|s'} := \frac{\Lambda^{uu}_{us'}}{\bar{\lambda}^{uu}_{s'}}, \quad \text{with } \sum_s \omega^{ru}_{s|s'} = \sum_u \omega^{uu}_{u|s'} = 1.$$

Then the exposure-weighted seller-level markup changes are

$$\Delta \log \bar{\mu}^{ru}_{s'} = \sum_s \omega^{ru}_{s|s'} \Delta \log \mu^{ru}_{ss'}, \qquad \Delta \log \bar{\mu}^{uu}_{s'} = \sum_u \omega^{uu}_{u|s'} \Delta \log \mu^{uu}_{us'}.$$

- Welfare aggregates over seller-level distortions, weighted by exposure to final demand
- Pareto structure guarantees sectoral aggregation is exact—no loss of generality

Exact Welfare Decomposition

Using exposure mappings and free entry:

$$\Delta \log W = \underbrace{-\sum_s \tilde{\lambda}_s^{cr} \Delta \log \mu_s^r - \sum_{s'} \bar{\lambda}_{s'}^{ru} \Delta \log \bar{\mu}_{s'}^{ru} - \sum_{s'} \bar{\lambda}_{s'}^{uu} \Delta \log \bar{\mu}_{s'}^{uu}}_{\textbf{Intensive margin (allocative markups)}}$$

$$+ \underbrace{\sum_s \frac{\tilde{\lambda}_s^{cr}}{\varphi_s - 1} \Delta \log N_s^r + \sum_{s'} \frac{\tilde{\lambda}_{s'}^{uu}}{\sigma_{s'} - 1} \Delta \log N_{s'}^u}_{\textbf{Extensive margin (entry/variety)}}$$

- Each markup or entry change affects welfare in proportion to its exposure weight.
- Buyer-specific pricing (nonlinear) alters both weights and effective markups.
- The aggregate welfare impact of market power depends on pricing form—uniform vs. nonlinear.

Parameter Estimation

Return

Parameter	Strategy	Granularity
Labor output elasticity (α_s^ℓ)	Estimated from data	626 sectors $\times$ firm type
Final demand elasticity (θ_r)	Estimated from data	626 sectors
Input-Output elasticity ($\theta_{ss'}^\ell$)	Estimated from data	626 sectors $\times$ firm type
Final demand bundle elasticity (φ_s)	Pinnned down by CES results and data	11 sectors
Material bundle elasticity ($\sigma_{s'}$)	COVID-19 shock for Chile estimation	11 sectors
Exit rate(δ^ℓ)	Estimated from data	626 sectors $\times$ firm type
Entry cost (c_e^ℓ)	Pinned down by free entry and data	626 sectors $\times$ firm type
Productivity Pareto tail (κ^ℓ)	MLE estimation	11 sectors $\times$ firm type

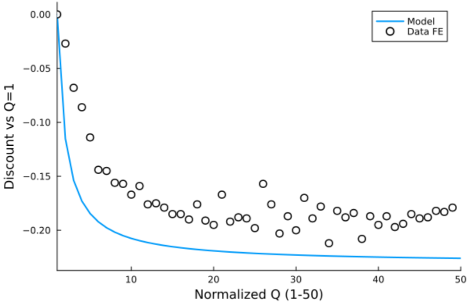
- $\sigma_{s'}, \kappa^\ell$ jointly pin the marginal price:
  Lower $\sigma_{s'}, \kappa^\ell$ (fatter tail, more dispersion) implies higher marginal marked-up prices
- Buyer surplus can be extracted by flat fees, is mainly determined by κ^ℓ:
  Large κ^ℓ implies low marginal price and thus a higher flat fee

Estimation Details

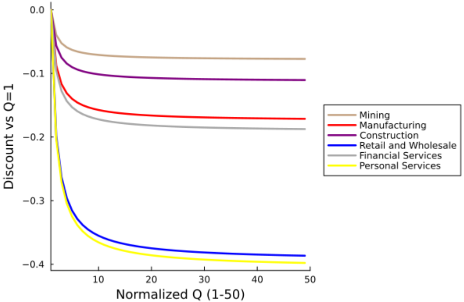
Model Fit (untargeted): Nonlinear Prices Interpretation

Return

A. Model unit prices vs.
data fixed-effects regression



B. Model retail and wholesale sector
unit prices to selected buyer sectors



- For the average upstream firm price schedule to retailers, normalizing the continuous input quantity to lie between 1 and 50

Policy: Opening Welfare Ratios by Sector Return

$$\frac{W^{\text{NLP}}}{W^{\text{Uni}}} = \underbrace{\prod_{s\in S}\left(\frac{\bar{\mu}_s^{ur,\text{NLP}}}{\bar{\mu}_s^{ur,\text{Uni}}}\right)^{-\tilde{\lambda}_s^{ru}} \prod_{s\in S}\left(\frac{\bar{\mu}_s^{uu,\text{NLP}}}{\bar{\mu}_s^{uu,\text{Uni}}}\right)^{-\tilde{\lambda}_s^{uu}}}_{\textbf{Intensive Margin}} \times \underbrace{\prod_{s\in S}\left(\frac{N_s^{r,\text{NLP}}}{N_s^{r,\text{Uni}}}\right)^{\frac{\theta_s}{\varphi_s-1}} \prod_{s\in S}\left(\frac{N_s^{u,\text{NLP}}}{N_s^{u,\text{Uni}}}\right)^{\frac{\tilde{\lambda}_s^{uu}}{\sigma_s-1}}}_{\textbf{Extensive Margin}}$$

Sector	Intensive (allocative)		Extensive (variety)		Net NLP/Uni
	Retailers	Upstream	Retailers	Upstream	
Agriculture	1.010	1.010	0.997	1.005	1.022
Mining	1.003	1.003	0.999	1.014	1.019
Manufacturing	1.024	1.029	0.991	1.002	1.047
Utilities	1.016	1.006	0.996	1.033	1.051
Construction	1.061	1.022	0.980	1.119	1.189
Retail and Wholesale	1.037	1.070	0.992	1.005	1.106
Transport and ICTs	1.007	1.023	0.981	1.000	1.011
Financial Services	1.012	1.008	0.943	0.998	0.960
Real Estate Services	1.009	1.004	0.996	1.023	1.033
Business Services	1.005	1.006	0.989	0.999	0.999
Personal Services	1.001	1.001	0.998	1.000	1.000
Product across sectors	1.197	1.198	0.870	1.207	1.507

Labor Output Elasticity α_s ▸ Return

- **What.** Cobb–Douglas weight on *non-materials* (labor $+$ user cost of capital).
- **Identify.** Cost-share mapping under cost minimization:

$$\alpha_i \;=\; 1 - \frac{\sum_j p_{ji} m_{ji}}{w_i L_i + r_i K_i + \sum_j p_{ji} m_{ji}}.$$

  Flat fees: $TC_i = F_i + VC_i$; for large buyers F_i / TC_i is small $\Rightarrow$ variable share $\approx$ total share.
- **Sample.** Keep firms above 75th pctl. revenue; winsorize α_i at 1–99; aggregate to (s, ℓ) at 6-digit; average 2005–2022.
- **Why.** Governs response to wage vs. input-price shocks: higher α_s amplifies wage relevance, dampens price conduct action from materials prices.

Labor Shares by Sector (Results)

Labor Shares by Sector (mean)

Sector	Retailers	Upstream	Sector mean
Agriculture	0.43	0.41	0.42
Mining	0.25	0.32	0.29
Manufacturing	0.39	0.42	0.41
Utilities	0.37	0.38	0.38
Construction	0.48	0.42	0.45
Retail and Wholesale	0.37	0.31	0.34
Transport and ICTs	0.55	0.47	0.51
Financial Services	0.58	0.62	0.60
Real Estate Services	0.66	0.53	0.59
Business Services	0.72	0.65	0.69
Personal Services	0.71	0.57	0.64
Type mean	0.50	0.46	0.48

Final-Demand Output Elasticity $\boldsymbol{\theta_s}$

- **What.** Cobb–Douglas weights across *retail sectors* in final demand.
- **Identify.** With linear pricing to consumers, retail revenues identify expenditure shares:

$$\theta_s \approx \frac{\text{retail revenue in } s}{\sum_{s'} \text{retail revenue in } s'}.$$

- **Sample.** Large retailers (>75th pctl.), compute annual sector shares, average 2005–2022; check revenue-weighted robustness.
- **Why.** Anchors final-demand system and welfare accounting in counterfactuals.

Final-Demand Shares θ_s (Results)

Cobb–Douglas Output Elasticities by Retail Sector

Sector	θ_s
Agriculture	0.0446
Mining	0.0085
Manufacturing	0.1318
Utilities	0.0505
Construction	0.1521
Retail and Wholesale	0.2768
Transport and ICTs	0.0979
Financial Services	0.1132
Real Estate Services	0.0152
Business Services	0.0911
Personal Services	0.0183

Materials Input–Output Shares $\boldsymbol{\theta}^{\ell}_{iss'}$

- **What.** Buyer-facing expenditure shares over upstream seller sectors s':

$$\theta^{\ell}_{iss'} = \frac{\sum_{j\in\mathscr{U}_{s'}} p_{ij} m_{ij}}{\sum_{s''}\sum_{j\in\mathscr{U}_{s''}} p_{ij} m_{ij}}, \quad \sum_{s'} \theta^{\ell}_{iss'} = 1.$$

- **Identify.** From transaction-level variable payments (VC); $TC = F + VC$, large buyers $\Rightarrow F/TC$ small.
- **Sample.** Compute firm-level θ for $\ell \in \{r, u\}$; keep >75th pctl. revenue; aggregate to 6-digit, then to 1-digit by year; average 2005–2022.
- **Why.** Micro foundation for the IO network; pins exposures and intensive-margin substitution scope.

Materials IO Shares: Retailers as Buyers (Results)

Input-output Elasticities by Retailers as Buyers

Buyer \ Seller	Agr.	Min.	Man.	Uti.	Cons.	R. & W.	T. & ICTs	F. Serv.	RE. Serv.	B. Serv.	P. Serv.
Agriculture	0.25	0.00	0.21	0.02	0.03	0.32	0.05	0.07	0.00	0.04	0.00
Mining	0.00	0.04	0.19	0.06	0.15	0.30	0.07	0.02	0.00	0.17	0.00
Manufacturing	0.13	0.02	0.35	0.02	0.03	0.25	0.11	0.03	0.00	0.06	0.00
Utilities	0.07	0.01	0.18	0.03	0.03	0.26	0.17	0.05	0.00	0.20	0.00
Construction	0.10	0.00	0.10	0.02	0.22	0.24	0.15	0.03	0.00	0.14	0.00
Retail and Wholesale	0.16	0.01	0.24	0.01	0.02	0.34	0.08	0.05	0.00	0.09	0.00
Transport and ICTs	0.07	0.01	0.14	0.02	0.03	0.24	0.19	0.04	0.00	0.26	0.00
Financial Services	0.08	0.00	0.12	0.01	0.01	0.22	0.06	0.15	0.01	0.33	0.00
Real Estate Services	0.03	0.00	0.12	0.01	0.02	0.30	0.04	0.06	0.05	0.37	0.00
Business Services	0.07	0.00	0.13	0.01	0.01	0.22	0.09	0.06	0.00	0.41	0.00
Personal Services	0.07	0.00	0.17	0.02	0.02	0.25	0.07	0.08	0.00	0.33	0.01

Materials IO Shares: Upstream as Buyers (Results)

Input-output Elasticities by Upstream Firms as Buyers

Buyer \ Seller	Agr.	Min.	Man.	Uti.	Cons.	R. & W.	T. & ICTs	F. Serv.	RE. Serv.	B. Serv.	P. Serv.
Agriculture	0.26	0.00	0.12	0.02	0.04	0.29	0.10	0.06	0.00	0.10	0.00
Mining	0.01	0.07	0.39	0.05	0.06	0.13	0.11	0.03	0.00	0.15	0.00
Manufacturing	0.08	0.02	0.49	0.03	0.02	0.15	0.09	0.02	0.00	0.10	0.00
Utilities	0.06	0.02	0.18	0.07	0.03	0.18	0.15	0.04	0.00	0.27	0.00
Construction	0.07	0.00	0.14	0.03	0.30	0.18	0.12	0.03	0.00	0.13	0.00
Retail and Wholesale	0.12	0.01	0.27	0.01	0.02	0.38	0.07	0.03	0.00	0.10	0.00
Transport and ICTs	0.06	0.02	0.14	0.02	0.04	0.21	0.22	0.03	0.00	0.26	0.00
Financial Services	0.05	0.00	0.12	0.02	0.01	0.20	0.07	0.12	0.01	0.41	0.00
Real Estate Services	0.03	0.00	0.11	0.01	0.02	0.27	0.04	0.04	0.06	0.41	0.00
Business Services	0.07	0.00	0.13	0.01	0.01	0.23	0.09	0.05	0.00	0.40	0.00
Personal Services	0.06	0.00	0.15	0.03	0.02	0.21	0.07	0.11	0.00	0.33	0.01

Upstream Materials Elasticity $\boldsymbol{\sigma}_{u'}$

- **What.** Substitutability across varieties *within* an upstream seller sector u'.
- **Identify.** IV from March 2020 municipal lockdown of *main supplier* u^*:

$$\Delta_{12} \log \frac{m_{isut}}{m_{isu^*t}} = -\sigma_{u'} \, \Delta_{12} \widehat{\log \frac{p_{isut}}{p_{isu^*t}}} + \gamma_s + \varepsilon.$$

- **Design.** 2SLS by seller sector; instrument $Z_{isu} = \mathbf{1}\{u^* \text{ locked}\}$; 12m diffs; large buyers; exclude buyer/clients/other inputs under lockdown; cluster at buyer level.
- **Why.** Higher $\sigma \Rightarrow$ faster rewiring, stronger intensive reallocation, lower pass-through; feeds κ mapping. (Conservatively set $\sigma \geq 1.45$ where $\hat{\sigma} < 1$.)

Estimated Elasticities $\sigma_{u'}$ (Results)

Estimated Elasticities of Substitution by Seller Sector

Sector	$\sigma_{u'}$	SE	1^{st} Stage F stat.	Obs.
Agriculture	2.59	(1.35)	10.24	4,387
Manufacturing	3.41	(0.84)	16.37	186,912
Construction	1.45	(0.42)	7.36	6,062
Retail and Wholesale	3.80	(0.39)	94.08	680,985
Transport and ICTs	5.07	(2.22)	25.19	24,054
Financial Services	3.09	(1.56)	9.35	3,631
Business Services	5.21	(2.02)	17.55	4,514
Personal Services	6.69	(3.37)	13.29	7,579
All sectors	3.04	(1.12)	149.87	918,124

Three sectors (Mining, Utilities, Real Estate Services) yield $\hat{\sigma}_{u'} < 1$; we set $\sigma_{u'} = 1.45$ (minimum estimate above one) for model quantification.

Final-Consumer Variety Elasticity $\boldsymbol{\varphi}_{s_r}$

- **What.** CES elasticity across retail *varieties* within sector s_r; markup $\mu = \varphi/(\varphi - 1)$.
- **Identify.** Sectoral accounting identity under linear pricing:

$$\varphi_{s_r,t} = \frac{\sum_j R_{j,t}}{w_{s_r,t}\sum_j F_{j,t} + \sum_j \Pi_{j,t}}, \qquad \Pi_j^{\mathrm{var}} = \frac{1}{\varphi_{s_r}} R_j.$$

- **Sample.** Large retailers; $F_{j,t}$ small, pool to sector-year; average 2005–2022.
- **Why.** Higher $\varphi \Rightarrow$ keener competition, smaller wedges; also maps retailer labor tails ν into productivity tails $\kappa = (\varphi - 1)\nu$.

Final-Consumer Elasticities $\boldsymbol{\varphi}_{s_r}$ (Results)

Retailer Parameter φ_{s_r} by Sector

Sector	φ_{s_r}
Agriculture	4.54
Mining	2.68
Manufacturing	4.22
Utilities	3.94
Construction	2.59
Retail and Wholesale	8.17
Transport and ICTs	2.05
Financial Services	1.40
Real Estate Services	1.82
Business Services	2.73
Personal Services	2.56
Type mean	3.34

Notes: φ_{s_r} computed from pooled sectoral sums of revenue, fixed costs (labor units), and profits.

Exit Hazard $\boldsymbol{\delta}_{s_\ell}$

- **What.** One-year hazard that an active firm exits.
- **Measure.** For cell (s, ℓ, t):

$$\delta_{s_\ell,t} = 1 - \frac{\text{survivors}_{s_\ell,t}}{\text{active}_{s_\ell,t}}, \qquad \delta_{s_\ell} = \frac{1}{|\mathscr{T}|} \sum_{t \in \mathscr{T}} \delta_{s_\ell,t}.$$

- **Sample.** Compute at 6-digit $\times$ type; track 2005–2022; average across years.
- **Why.** Disciplines expected lifespan and shock persistence; higher δ increases payoff needed for entry, tilts adjustments toward the extensive margin.

Exit Rates δ by Sector (Results)

Exit Rates (δ) by Sector (Means)

Sector	Retailers	Upstream	Sector mean
Agriculture	0.090	0.086	0.088
Mining	0.084	0.093	0.088
Manufacturing	0.093	0.071	0.082
Utilities	0.070	0.064	0.067
Construction	0.140	0.110	0.125
Retail and Wholesale	0.103	0.076	0.089
Transport and ICTs	0.088	0.093	0.091
Financial Services	0.101	0.062	0.081
Real Estate Services	0.115	0.099	0.107
Business Services	0.099	0.077	0.088
Personal Services	0.093	0.090	0.092
Type mean	0.098	0.084	0.091

Entry Cost c_{e,s_ℓ} (in labor units)

- **What.** Sunk labor resources required to create an operating firm.
- **Identify.** Free entry with survival hazard δ:

$$\mathrm{PV}_{s_\ell} = \frac{\bar{\Pi}_{s_\ell}}{1-\beta(1-\delta_{s_\ell})}, \quad w_{s_\ell} c_{e,s_\ell} = p^{\mathrm{succ}}_{s_\ell} \cdot \mathrm{PV}_{s_\ell} \Rightarrow c_{e,s_\ell} = \frac{p^{\mathrm{succ}}_{s_\ell}}{w_{s_\ell}} \cdot \frac{\bar{\Pi}_{s_\ell}}{1-\beta(1-\delta_{s_\ell})}.$$

- **Sample.** Use observed profits Π, wages w, positive-profit share p^{succ}, and δ at 6-digit $\times$ type; report currency and wage-bill equivalents.
- **Why.** Shapes steady-state firm mass/scale; interacts with NLP's rent reallocation along the chain.

Entry Costs c_e by Sector (Results)

Entry Costs and Equivalent Yearly Wage-Bills by Sector

	Retailers		Upstream	
Sector	Entry cost c_e	Wage-bill eq.	Entry cost c_e	Wage-bill eq.
Agriculture	81.03	3.68	84.12	4.78
Mining	29212.81	43.99	177.12	7.20
Manufacturing	101.87	4.25	120.80	4.53
Utilities	700.66	14.15	306.11	5.50
Construction	109.72	7.78	109.05	4.18
Retail and Wholesale	63.92	6.06	83.61	5.13
Transport and ICTs	299.85	10.28	98.03	6.40
Financial Services	263.84	8.64	248.44	9.05
Real Estate Services	82.11	11.68	100.69	8.70
Business Services	82.91	5.76	125.21	3.11
Personal Services	127.87	4.56	94.76	4.57
Type mean	2829.69	10.98	140.72	5.74

Notes: Entry costs c_e are in the currency units used for calibration; "Wage-bill eq." reports multiples of the annual wage bill.

Productivity Tail Exponent κ

- **What.** Thickness of the upper tail of firm productivity.
- **Identify.** Estimate labor tail by MLE above threshold:

$$\widehat{\nu} = \frac{n}{\sum_{i:\, L_i \geq L_{\min}} \ln(L_i/L_{\min})}, \quad \mathrm{SE}(\widehat{\nu}) \approx \widehat{\nu}/\sqrt{n}.$$

Map to productivity using $l(z) \propto z^{\sigma-1}$ (or $\varphi - 1$ for retail):

$$\kappa^u = (\sigma - 1)\nu^u, \qquad \kappa^r = (\varphi - 1)\nu^r.$$

- **Sample.** Compute ν by 1-digit $\times$ type; combine with sectoral σ / φ; report implied κ.
- **Why.** Thicker tails (small κ) magnify selection/reallocation gains and shape how NLP shifts surplus across the distribution.

Labor and Implied Productivity Tails $\boldsymbol{\nu}$, $\boldsymbol{\kappa}$ (Results)

Labor and Implied Productivity Pareto Tails by Sector

	Retailers		Upstream		
Sector	ν_r	$\kappa_r = (\varphi_s - 1)\nu_r$	ν_u	$\kappa_u = (\sigma_{u'} - 1)\nu_u$	$\sigma_{u'}$
Agriculture	2.49	8.82	2.63	4.18	2.59
Mining	1.43	2.40	2.20	0.99	1.45
Manufacturing	2.66	8.58	2.15	5.18	3.41
Utilities	2.17	6.38	1.94	0.87	1.45
Construction	3.23	5.13	2.19	0.99	1.45
Retail and Wholesale	3.45	24.74	2.40	6.72	3.80
Transport and ICTs	2.20	2.32	3.04	12.37	5.07
Financial Services	2.55	1.02	2.26	4.72	3.09
Real Estate Services	4.36	3.59	3.03	1.36	1.45
Business Services	2.45	4.25	1.93	8.13	5.21
Personal Services	2.03	3.17	2.58	14.69	6.69

Notes: $\kappa = (\sigma_{u'} - 1)\nu$ uses seller-sector elasticities $\sigma_{u'}$ from the IV estimates. For Mining, Utilities, and Real Estate Services, we set $\sigma_{u'} = 1.45$ (minimum estimate above one).

Sectoral Allocative Markups Final-Demand Weighted

Return

Sector	Lin	NLP: $u \rightarrow r$	NLP: $u \rightarrow u$	Share(r→u)	Share(full up)
Agriculture	1.63	1.18	1.17	0.053	0.050
Mining	3.27	1.94	1.46	0.005	0.009
Manufacturing	1.41	1.12	1.16	0.186	0.175
Utilities	3.27	1.47	1.56	0.036	0.024
Construction	3.27	1.27	1.46	0.121	0.084
Retail & Wholesale	1.36	1.08	1.12	0.319	0.364
Transport & ICTs	1.25	1.10	1.07	0.096	0.136
Financial Services	1.48	1.15	1.17	0.087	0.071
Real Estate Services	3.27	1.19	1.30	0.018	0.013
Business Services	1.24	1.08	1.11	0.067	0.065
Personal Services	1.18	1.08	1.06	0.012	0.009
Weighted aggregate	1.61	1.14	1.17		