Aggregate Outcomes of Nonlinear Prices in Supply Chains*

Luca Lorenzini
UCLA Anderson

Antonio Martner
UCLA & Central Bank of Chile

October 1, 2025

*The views expressed are those of the authors and do not necessarily represent the views of the Central Bank of Chile or its board members.

Motivation and Research Question

- Understanding the aggregate costs of market power is central in research and policy debates
- But market power aggregate analysis often **omits price discrimination**

  "There can be no doubt that firms are well aware of the **benefits of price discrimination.**"

  —Varian, *Handbook of IO* (1989)

Motivation and Research Question

- Understanding the aggregate costs of market power is central in research and policy debates

- But market power aggregate analysis often **omits price discrimination**

  "There can be no doubt that firms are well aware of the **benefits of price discrimination.**"

  —Varian, *Handbook of IO* (1989)

- Policy revival: first Robinson-Patman lawsuits in 20+ years; FTC *v.* Southern Glazer's (2024):

  "Southern engages in **discriminatory pricing** . . . offering **quantity discounts and rebates** to large buyers that are **inaccessible to smaller rivals** and not justified by cost." —FTC Press Release, Dec. 2024

  "When local businesses get squeezed . . . Americans face **fewer choices**, **higher prices**, and **communities suffer.**"

  —*FTC Chair Lina M. Khan*, Dec. 2024

Motivation and Research Question

- Understanding the aggregate costs of market power is central in research and policy debates
- But market power aggregate analysis often **omits price discrimination**

  "There can be no doubt that firms are well aware of the **benefits of price discrimination.**"

  —Varian, *Handbook of IO* (1989)
- Policy revival: first Robinson-Patman lawsuits in 20+ years; FTC *v.* Southern Glazer's (2024):

  "Southern engages in **discriminatory pricing** . . . offering **quantity discounts and rebates** to large buyers that are **inaccessible to smaller rivals** and not justified by cost." —FTC Press Release, Dec. 2024

  "When local businesses get squeezed . . . Americans face **fewer choices**, **higher prices**, and **communities suffer.**"

  —*FTC Chair Lina M. Khan*, Dec. 2024

Research Question

What are the aggregate outcomes of price discrimination in supply chains?

Market Power and Efficiency

- Market power per se does not generate aggregate inefficiencies
- **What matters is how market power is exercised through pricing mechanisms**
- In linear pricing, average prices determine quantities (allocative); sufficient statistic for welfare
- In supply chains in Chile, we find indicative evidence of widespread **nonlinear prices**
- **Average prices are not fully allocative** and can misstate aggregate welfare costs evaluations
- **Specially problematic in supply chains** where markup distortions can accumulate

This Paper: Main Mechanism

- Under standard assumptions, the optimal nonlinear price is a two-part tariff:

$$\underbrace{pq}_{\text{Total Payment}} = \underbrace{F}_{\text{Flat fee}} + \underbrace{p_{\text{marg}}}_{\text{Marginal price}} q$$

Flat Fee Distorts Entry

- The flat fee does not affect input choices; it reallocate rents from buyer to seller
- Affects firms profits distribution and can distort entry decisions

Marginal Price Improve Allocation Relative to Linear Prices

- The marginal price determines quantity allocations (it is allocative)
- In our setting, it's unambiguously smaller relative to the allocative linear price

This Paper: Roadmap

Theory

- Optimal nonlinear price characterization in partial equilibrium and testable footprint
- Multi-sector supply chain model where firms charge and pay nonlinear prices

This Paper: Roadmap

Theory

- Optimal nonlinear price characterization in partial equilibrium and testable footprint
- Multi-sector supply chain model where firms charge and pay nonlinear prices

Indicative Evidence and Measurement

- Administrative Data: P and Q at the seller-product-buyer level for the universe of B2B in Chile
- Pricing Diagnosis: **Nonlinear prices by buyer group** (hybrid $2^{nd} + 3^{rd}$ degree PD)
- Measurement: Estimate **all** model parameters

This Paper: Roadmap

Theory

- Optimal nonlinear price characterization in partial equilibrium and testable footprint
- Multi-sector supply chain model where firms charge and pay nonlinear prices

Indicative Evidence and Measurement

- Administrative Data: P and Q at the seller-product-buyer level for the universe of B2B in Chile
- Pricing Diagnosis: **Nonlinear prices by buyer group** (hybrid $2^{nd} + 3^{rd}$ degree PD)
- Measurement: Estimate **all** model parameters

Quantification

- **Nonlinear prices improve welfare relative to linear prices** 75% v. 50% rel. to efficiency
- Sufficient statistics to unpack welfare drivers across pricing regimes

Selected Related Literature

Distorted Economies, Misallocation and Firm Entry

- Harberger (1956) & Mankiw & Whinston (1986), Hopenhayn (1992), Baqaee & Farhi (2020), Edmond, Midrigan & Xu (2023)
- **We incorporate nonlinear pricing distortions**

Selected Related Literature

Distorted Economies, Misallocation and Firm Entry

- Harberger (1956) & Mankiw & Whinston (1986), Hopenhayn (1992), Baqaee & Farhi (2020), Edmond, Midrigan & Xu (2023)
- **We incorporate nonlinear pricing distortions**

Price Discrimination

- Mussa & Rosen (1978), Borenstein (1985), Wilson (1993), Goldberg (1996), Stole (2007), Bornstein & Peter (2025), Burstein, Cravino, & Rojas (2025)
- **We focus on supply chains with two-sided $2^{nd} + 3^{rd}$ degree price discrimination**

Table of Contents

Primitives and Behavior

- One seller with **constant marginal cost** c faces a **continuum of buyers** indexed by z
- Seller knows the distribution of buyer types, but type information is private
- Chooses a **nonlinear transfer $T(z)$ and quantity $q(z)$** for each type z

$$\max_{\{T(z),q(z)\}} \Pi_{\text{seller}} = \int_{\underline{z}}^{\infty} \left[T(z) - c\,q(z)\right] f(z)\,dz$$

Subject to

- (IR) Buyers receive non-negative surplus:

$$\Pi(z, q(z)) = zq(z) - T(z) \geq 0, \quad \forall z$$

- (IC) No buyer prefers the contract designed for another buyer:

$$zq(z) - T(z) \geq zq(\tilde{z}) - T(\tilde{z}), \quad \forall z, \tilde{z}$$

Mirrlees Reduction and Virtual Surplus (Standard)

- Using the virtual surplus ϕ , the problem can be written as a pointwise optimization problem

$$\max_{\{q(z)\}} \Pi_{\text{seller}} = \int_{\underline{z}}^{\infty} \left[\phi(\mathbf{z}, \mathbf{q}(\mathbf{z})) - cq(z)\right] f(z)\, dz,$$

$$\text{with} \quad \phi(\mathbf{z}, \mathbf{q}) = \underbrace{R(z,q)}_{1} - \underbrace{\frac{1}{h(z)} \frac{\partial R(z,q)}{\partial z}}_{2}$$

- Inverse hazard rate, $h(z)^{-1} = (1 - F(z))/f(z)$ is the weight on the remaining higher types
- The virtual surplus represents the seller's effective revenue from serving type z:
  1. Buyer z total revenue from the transaction (seller wants to extract it)
  2. Rents the seller must leave to higher types to prevent them from mimicking type z

Functional Forms and Optimal Nonlinear Price

- So far, standard screening problem, now **we impose two additional new assumptions**:
  1. Buyer **types are Pareto distributed** with tail parameter κ
  2. Buyers' **revenue functions are homogenous** on the quantity transacted with the seller: $R(z,q) = z^{\frac{\sigma-1}{\sigma}} q^{\frac{\sigma-1}{\sigma}}$. $\sigma > 1$ is the demand elasticity faced by the seller
- Buyer type shifts demand without altering curvature

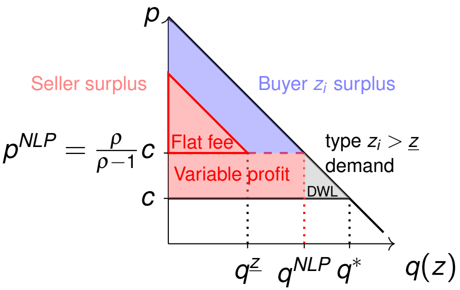
Lemma 1: Optimal Nonlinear Price

Under (i) constant marginal cost, (ii) Pareto distributed types, and (iii) homogenous revenue, the optimal nonlinear price schedule is isomorphic to a **two-part tariff** *when* $\kappa > \sigma - 1$*:*

$$T(z) = F + p^{\mathrm{NLP}} q(z), \qquad p^{\mathrm{NLP}} = \frac{\rho}{\rho - 1} c, \qquad \rho \equiv \frac{\kappa\sigma}{\sigma - 1} > \sigma, \qquad F \text{ is set so that}: \ \Pi(\underline{z}) = 0.$$

Optimal Nonlinear Price: Allocations

- Virtual surplus for lower type is strictly positive: **All types are served** Details
- **Quantities are pinned down by marginal price p^{NLP}**
- **Flat fee only redistributes surplus**; is not allocative



Optimal Nonlinear Price: Testable Prediction

- If pricing in the data is equivalent to a two-part tariff: $T(z) = F + p^{\text{NLP}} q(z)$
- **Average unit price is**: $\frac{T(z)}{q(z)} = \frac{F}{q(z)} + p^{\text{NLP}}$
- **Decreasing and convex in** q
- Has a **horizontal asymptote at** p^{NLP}

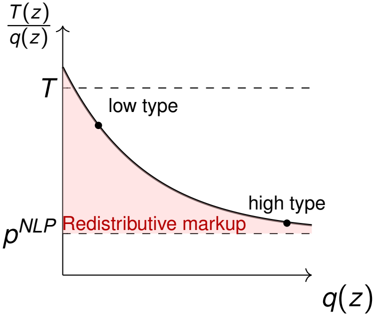


Table of Contents

Data Sources

Invoice example

Invoice transactions for the universe of Chilean formal firms for 2024

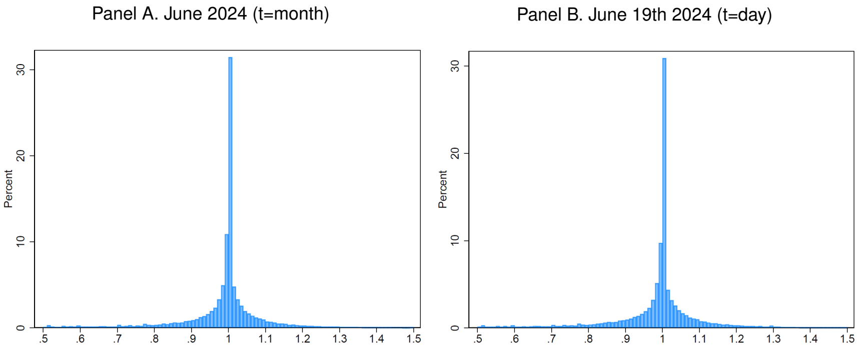
- 1.4 billion transactions
- More than 10 million different products. We assume products are seller-specific
- Data on **prices and quantities for every product transacted**

Merged with firms' accounting balance sheet data

- Sales, materials, investment, 6-digit industry
- Employer-employee: Wages, headcount of employees
- Capital stock and investment

Data Cleaning

Price Dispersion



- $\theta_{jgt} = \frac{p_{jgit}}{\bar{p}_{jgt}}$; seller j, product g, buyer i, time t (excluding products with one transaction)
- $\text{Var}(\log \theta_{jgd}) = 0.65$, **price discrimination cannot be rejected in 70% of transactions**

Residual Drivers

Indicative Evidence of Nonlinear Prices (assume equilibrium $\{p, q\}$)

Seller j, product g, buyer i, time t, day d, quantity q, buyer group B (11 sectors $\times$ 3 sizes $\times$ 16 regions)

$$\ln p_{jgit} = \beta_0 + \beta_1 \ln q_{jgit} + \Psi_S + \varepsilon_{jgit}$$

- **Unconditional average discount is 4% per unit when doubling quantity purchased**
- Conditioning on buyers (and groups of buyers), the average discount increases
- Even within buyer groups, the average discount is 90% of the unconditional average

By sector

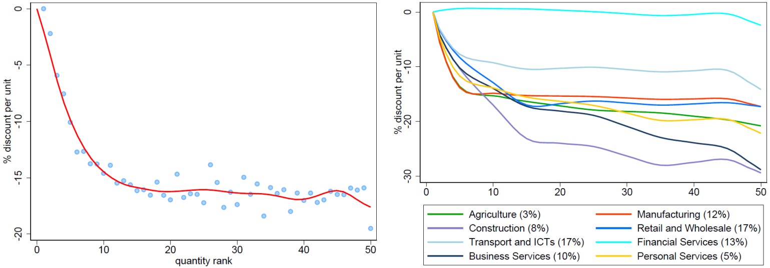
	(1)	(2)	(3)	(4)
$\ln q_{jgit}$	-0.042	-0.084	-0.065	-0.037
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
$S_{Base} = j \times g \times d$	✓			
$S = \text{Base} + i$		✓		
$S = \text{Base} + B$			✓	
$S = \text{Base} \times B$				✓
N	430M	430M	430M	430M
R^2	0.9646	0.9678	0.9659	0.9790

Nonlinear Prices by Quantity Bins

Bins construction Bins histogram

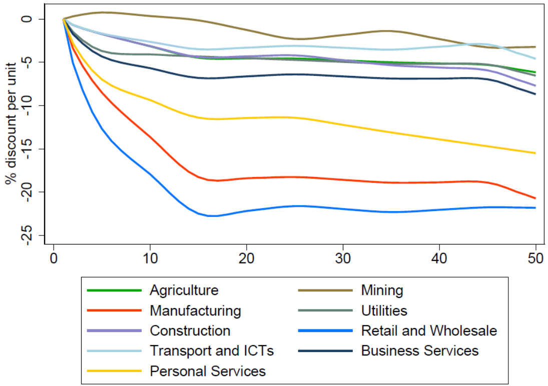
$$\ln p_{jgit} = \beta_0 + \sum_{b=2}^{50} \beta_b \mathbb{1}_{\{B_{jgit}=b\}} + \Psi_{jgd} + \varepsilon_{jgit}$$

(A) All sectors (B) Fix the seller industry



- **Unit prices fall steeply at small q and flatten as q grows** (consistent with Lemma 1)
- Pronounced between-sector heterogeneity in both steepness and curvature

Retail & Wholesale Seller Sector: Pricing to Buyer Sectors



- **Within a seller sector, nonlinear price schedules differ by buyer sector**

Taking Stock

- Within seller×product×day, **unit prices decline with quantity and flatten at higher ranks**
- Curvature, levels, and steepness are different across seller industries
- Within a seller industry, curvature shifts with buyer sector
- **Reject linear pricing**: pricing consistent with hybrid of $2^{nd} + 3^{rd}$ degree price discrimination:
  - 2^{nd} **degree screening drives curvature**
  - 3^{rd} **degree shifts levels and steepness across buyer industries**

Table of Contents

Environment

- There are two firm types, $\ell \in \{u, r\}$, defined by their position relative to final demand ▸ Evidence
- Upstream firms u sell both to other u and to retailers firms r, and buy from other u
- r purchase inputs from u and sell exclusively to the representative final consumer
- Within each type $\ell \in \{u, r\}$ sectors are indexed by $s \in S$
- Firms as buyers are denote by i and by j as sellers, buyer sectors as s and seller sectors as s'
- In each (ℓ, s) there is a continuum of firms with productivity z_i distributed Pareto with tail κ_s^ℓ
- A firm i is thus characterized by the triple (ℓ, s, z_i), denoting type, sector, and productivity

Market Structure

- Retail firms sell to the representative consumer at uniform per-unit prices
- Upstream firms set nonlinear prices to other upstream firms and retailers
- Firms can price discriminate across types and sectors (ℓ, s) but no z_i within a (ℓ, s) ($2^{nd} + 3^{rd}$)
- Firms are atomistic in input markets as buyers and take the wage as given

Preferences

- The representative consumer owns all firms and inelastically supplies one unit of labor ($L = 1$)
- Final demand is Cobb–Douglas across retail sectors with within–sector CES over reatil varieties:

$$Y = \prod_{s \in S} Y_s^{\theta_s}, \qquad \sum_{s \in S} \theta_s = 1, \qquad Y_s = \left(\int_{j \in R_s} y_j^{\frac{\varphi_s - 1}{\varphi_s}} \, d\nu_s(j) \right)^{\frac{\varphi_s}{\varphi_s - 1}}$$

- $\theta_s \in (0, 1)$ are Cobb–Douglas output elasticities
- $\varphi_s > 1$ is the within-sector elasticity, and $d\nu_s(j)$ denotes number active retail varieties R_s in s
- The total number of active varieties in (r, s) is $N_s^r := \nu_s(R_s)$, an endogenous equilibrium object

Technology

- Firm $i \in (\ell, s)$ output (Q_i) use CD in labor (l_i) and a CD aggregator of sectoral materials (M_i)

$$Q_i = z_i \, l_i^{\alpha_s^\ell} \, M_i^{1-\alpha_s^\ell}, \qquad 0 < \alpha_s^\ell < 1,$$

$$M_i = \prod_{s' \in S} M_{is'}^{\theta_{ss'}^\ell}, \qquad \sum_{s' \in S} \theta_{ss'}^\ell = 1, \qquad M_{is'} = \left(\int_{j \in U_{s'}} m_{ij}^{\frac{\sigma_{s'}-1}{\sigma_{s'}}} \, d\nu_{s'}(j) \right)^{\frac{\sigma_{s'}}{\sigma_{s'}-1}}$$

- α_s^ℓ is the labor output elasticity, $M_{is'}$ is firm i material bundle from upstream sector s'
- $\theta_{ss'}^\ell \geq 0$ are sector–s firm type ℓ input elasticities
- m_{ij} is firm i's input of upstream variety j in sector s'
- $\sigma_{s'} > 1$ is the CES across varieties inside upstream sector s'
- $N_{s'} := \nu_{s'}(U_{s'})$ is the endogenous measure over the active upstream firms in sector s'

Firm Entry

- In each (ℓ, s) there is an unbounded pool of identical potential entrants
- Entryants pay a sunk cost $c_s^{E\,\ell} > 0$ in units of labor, and then observe their productivity z
- Active firms exit exogenously at the end of the period with probability $\delta_s^\ell \in [0, 1)$
- Free entry requires that the expected discounted value of profits $(\pi_i^{\ell s}(z))$ equals entry cost $(c_s^{E\,\ell})$

$$\frac{1}{1-\delta_s^\ell}\mathbb{E}_z\left[\pi_i^{\ell s}(z)\right] = c_s^{E\,\ell} w, \quad \forall(\ell, s)$$

Model Recap

1. In every period potential entrants in each (ℓ, s) pay $c_s^{E\,\ell}$ and then draw productivity z
2. Each upstream seller j observes only the buyer's pair (ℓ, s) (not z_i) and offers a pair–specific nonlinear contract menu $\{m_j^{\ell,s}, T_j^{\ell,s}\}$
3. Retail sellers j post linear prices to final consumers
4. Buyers $i = (\ell, s, z_i)$ observe menus and w and choose l_i and $\{m_{ij}\}_j$ to max. profits
5. Production and trade occur, transfers $\{T_{ij}\}_j$ are realized, and final demand $\{y_j\}$ is met
6. Firms exit with probability δ_s^ℓ
7. Contracts are enforceable, resale/arbitrage is ruled out
8. We consider a steady state: all aggregates are time-invariant

Working Guesses Motivated by Lemma 1

Guess 1: Optimal contracts are isomorphic to a two-part tariff specific to (ℓ, s)

$$T_{ij} = p^{\ell}_{js} m_{ij} + F^{\ell}_{js'} = \mu^{\ell}_{ss'} c_j m_{ij} + F^{\ell}_{js'}$$

- Transfer T_{ij} depends on allocation m_{ij}, markup $\mu^{\ell}_{ss'}$, and flat fee $F^{\ell}_{js'}$
- The marginal (allocative) price is $p^{\ell}_{js} = \mu^{\ell}_{ss'} c_j$
- Flat fees are inframarginal and do not affect marginal input choices;

Guess 2: Revenue functions are homogeneous in quantity

$$R^{\ell}_{i,s} = A^{\ell}_{s} \left(Q^{\ell}_{i,s}\right)^{\psi^{\ell}_{s}}$$

- For parameters A^{ℓ}_{s} and ψ^{ℓ}_{s} that are constants at buyer type and sector (ℓ, s)

Costs and Price Indices Under Guess 1

- Marginal prices are quantity–invariant within a buyer type–sector (i, s) and seller sector s'

CES Sectoral Price Index

$$P^{\ell}_{ss'} = \left(\int_{j \in U_{s'}} \left(p^{\ell}_{js'} \right)^{1-\sigma_{s'}} d\nu_{ss'}(j) \right)^{\frac{1}{1-\sigma_{s'}}}$$

Cobb–Douglas Materials Cost Index

$$P^M_i = \prod_{s' \in S} \left(P^{\ell}_{ss'} \right)^{\theta^{\ell}_{ss'}}, \qquad \sum_{s' \in S} \theta^{\ell}_{ss'} = 1, \;\; \theta^{\ell}_{ss'} \geq 0$$

Firm–Level Marginal Cost

$$c_i = \frac{\Theta^{\ell}_s}{z_i} \, w^{\alpha^{\ell}_s} \left(P^M_i \right)^{1-\alpha^{\ell}_s}, \qquad \Theta^{\ell}_s \equiv \left(\alpha^{\ell}_s \right)^{-\alpha^{\ell}_s} \left(1 - \alpha^{\ell}_s \right)^{-(1-\alpha^{\ell}_s)} \prod_{s' \in S} \left(\theta^{\ell}_{ss'} \right)^{-(1-\alpha^{\ell}_s)\theta^{\ell}_{ss'}}$$

Type Re-parametrization and Distribution for Screening

- For buyer i, the total surplus from transacting with seller $j \in S'$ can be expressed as:

$$\left.\frac{d\Pi_i}{d\left(v_{s'}(z_j)\right)}\right|_{\arg\max \Pi_i} \equiv TS_{is'}(m_{ij}) = \frac{\sigma_{s'}}{\sigma_{s'}-1} P^{\ell}_{js'} M_{is'}^{\frac{1}{\sigma_{s'}}} m_{ij}^{\frac{\sigma_{s'}-1}{\sigma_{s'}}} - T.$$

- For a seller in sector s', the buyer i matters only through valuation index: $\tau_{is'} \equiv P^{\ell}_{ss'} M_{is'}^{1/\sigma_{s'}}$
- Depends on sector s' price level $P^{\ell}_{ss'}$ with the buyer's scale $M_{is'}$
- Productivity Pareto with tail parameter κ^{ℓ}_s implies buyer types $\tau_{is'}$ are distributed according to:

$$\rho^{\ell}_{ss'} = \xi^{\ell}_s \sigma_{s'} > 1, \qquad \xi^r_s \equiv \frac{\kappa^r_s}{\varphi - 1}, \text{ for retailers} \quad \xi^u_s \equiv \frac{\kappa^r_s}{\sigma_s - 1}, \text{ for upstream}$$

Upstream Seller Profit Maximization Problem

- The seller chooses a menu of total transfer and a allocation $\{T_{ij}^{\ell}, m_{ij}^{\ell}\}$ for each (ℓ, s')

$$\max_{\{T,m\}} \sum_{\ell \in \{u,r\}} \sum_{s \in S} N_s^{\ell} \mathbb{E}_{\tau_{is'}} \left[T(\tau) - c_j m_{ij}(\tau) \right], \quad \text{s.t. for each } (\ell, b)\text{: IC, IR}$$

- The problem is additively separable across (ℓ, s, s') and can be solved partition-by-partition.
- Following Lemma 1, the virtual–surplus reduction yields the pointwise optimization problem:

$$\max_{\{m(\tau)\}} N_s^{\ell} \mathbb{E}_{\tau_{is'}} \left[\left(\tau - \frac{\tau}{\rho_{ss'}^{\ell}} \right) \frac{\sigma_{s'}}{\sigma_{s'} - 1} m(\tau)^{\frac{\sigma_{s'}-1}{\sigma_{s'}}} - c_j m(\tau) \right].$$

- which is strictly concave in m since $(\sigma_{s'} - 1)/\sigma_{s'} \in (0, 1)$

Optimal Nonlinear Price

Proposition 1: Optimal Nonlinear Price in Supply Chains

In equilibrium, the optimal contract offered by an upstream seller $j \in U_{s'}$ to any buyer $i = (\ell, s, z_i)$ is a two–part tariff:

$$T_{ij} = p^{\ell}_{js} m_{ij} + F^{\ell}_{js},$$

with a marginal price p that is constant across types and equals

$$p^{\ell}_{js} = \mu^{\ell}_{ss'} c_j, \qquad \mu^{\ell}_{ss'} = \frac{\rho^{\ell}_{ss'}}{\rho^{\ell}_{ss'} - 1}, \qquad \rho^{\ell}_{ss'} = \xi^{\ell}_{s} \sigma_{s'},$$

and a flat fee F chosen so that the lowest type's participation constraint binds,

$$\Pi(\underline{z}^{\ell}_{s}) = 0 \quad \Longleftrightarrow \quad F^{\ell}_{js} = \frac{1}{\sigma_{s'}} R^{\ell}_{ss'}\left(\underline{z}^{\ell}_{i}, m^{*}(\underline{z}^{\ell}_{i})\right).$$

For all partitions on firm types $\ell \in \{u, r\}$ and buyer sectors s, each with its sector-specific two-part tariffs.

Guesses Verification · Firms Profits · General Equilibrium Definition

Two Upstream Pricing Counterfactuals For Welfare Comparisons

Planer Efficient Pricing (as in Baqaee and Farhi, 2021)

- Firms must charge markups to incentivize the optimal entry level
- But markup distorts input choices by acting as a uniform tax on production
- An output subsidy can restore undistorted marginal-cost, conditional on entry
- The subsidy is paid via a lump sum tax to the representative consumer

Linear prices (e.g, as in Edmond, Midrigan & Xu, 2023)

- Linear markup over marginal cost from monopolistic competition
- CES markups $\mu^{LP} = \frac{\sigma}{\sigma-1}$, strictly higher than $\mu^{NLP} = \frac{\rho}{\rho-1}$
- Because unambiguously $\sigma < \rho$

Upstream Firm Profits Under Nonlinear Prices:

$$\mathbb{E}\left[\Pi_j^u\right] = \underbrace{\sum_{\ell}\sum_{s}\int_{i\in B_{\ell s}} \left(p_{js}^{\ell} - c_j\right) m_{ij}\, dv_{\ell s}^{i}}_{\text{allocative margin}} + \underbrace{\sum_{\ell}\sum_{s}\int_{i\in B_{\ell s}} F_{jsi}^{\ell}\, dv_{\ell s}^{i}}_{\text{flat–fee revenue}} - \underbrace{\sum_{s'}\int_{j'\in D_j} F_{j',s'}^{u}\, dv_{s'}^{j}}_{\text{flat–fee payments}}$$

- B denotes seller firm j client set, D denote seller firm j suppliers set (exogenous sets)
- **NLP Marginal Prices.** Charge and pay smaller marginal prices (p_{js}^{ℓ}, c_j) relative to Lin. P.
- **Rents.** Through flat fees seller j, extracts rents, but it's also rent extracted
- **GE incidence.** Cheaper c_j lift downstream demand; double marginizalzation attenuation
- **Entry.** Depends on m_{ij} expansion and net rent extraction (firm entry is misallocated)

Retailer Firm Profits Under Nonlinear Prices

$$\mathbb{E}\left[\Pi_i^r\right] = \underbrace{\left(\frac{1}{\varphi_s}\right) R_i}_{\text{allocative margin}} - \underbrace{\sum_{s'} \int_{j \in D_j} F_{ijs'}^r \, d\nu_{s'}^j}_{\text{Flat fees payments to upstream}} .$$

- **Constant markup** Allocative margin has a constant markup and a fixed share of revenue
- **NLP Marginal Prices.** NLP lowers input costs, retail prices fall with constant markup and revenue expands; the allocative term scales proportionally with R_i
- **Rents.** Extracted via fee payments to upstream
- **Entry.** Depends on change in profits: revenue expansion versus rent extraction

Welfare Decomposition: Intensive vs. Extensive Margins

- If wage is the numeraire, welfare is the inverse final price index: $W \equiv \frac{1}{P_Y}$ derivation

$$\Delta \log W = \underbrace{-\sum_{s\in S}\theta_s\,\Delta\log\mu_s^{rc} - \sum_{s\in S}\tilde{\lambda}_s^{ru}\,\Delta\log\mu_s^{ur} - \sum_{s\in S}\tilde{\lambda}_s^{uu}\,\Delta\log\mu_s^{uu}}_{\textbf{Intensive Margin } (r\to c),\ (u\to r),\ (u\to u)}$$

$$+ \underbrace{\sum_{s\in S}\theta_s\,\frac{1}{\varphi_s-1}\,\Delta\log N_s^r - \sum_{s\in S}\tilde{\lambda}_s^{uu}\left(\frac{1}{1-\sigma_s}\,\Delta\log N_s^u\right)}_{\textbf{Extensive Margin: } \textit{variety}(\textit{masses})}$$

- $\tilde{\lambda}$ are final consumption direct and indirect costs exposures (direct $\times$ network exposure)
- Three markup layers, upstream wedges are amplified by network propagation
- Variety (N) improve welfare with elasticities $\theta_s/(\varphi_s-1)$ downstream and $\tilde{\lambda}_u/(\sigma_u-1)$ upstream
- Flat fees affect welfare through profit function and extensive channels, not through price indices

Welfare Ratios Across Price Regimes: Nonlinear vs. Linear

$$\frac{W^{\mathrm{NLP}}}{W^{\mathrm{Lin}}} = \underbrace{\prod_{s\in S}\left(\frac{\mu_s^{ur,\mathrm{NLP}}}{\mu_s^{ur,\mathrm{Lin}}}\right)^{-\tilde{\lambda}_s^{ru}} \prod_{s\in S}\left(\frac{\mu_s^{uu,\mathrm{NLP}}}{\mu_s^{uu,\mathrm{Lin}}}\right)^{-\tilde{\lambda}_s^{uu}}}_{\textbf{Intensive Margin}} \times \underbrace{\prod_{s\in S}\left(\frac{N_s^{r,\mathrm{NLP}}}{N_s^{r,\mathrm{Lin}}}\right)^{\frac{\theta_s}{\varphi_s-1}} \prod_{s\in S}\left(\frac{N_s^{u,\mathrm{NLP}}}{N_s^{u,\mathrm{Lin}}}\right)^{-\frac{\tilde{\lambda}_s^{uu}}{1-\sigma_s}}}_{\textbf{Extensive Margin}}.$$

- Intensive margin (unambiguous gain):
  $\mu^{\mathrm{NLP}} < \mu_u^{\mathrm{Lin}}$, attenuating double marginalization
- Extensive margin (ambiguous):
  Flat fees shift profits with ambiguous sign, firm entry could go either way
- Welfare comparison? Need to quantify the model fully

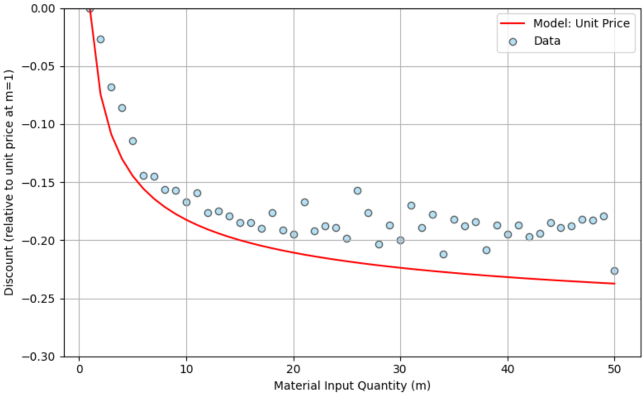
Table of Contents

Measurement

Parameter	Strategy	Granularity
Labor output elasticity (α_s^ℓ)	Measured from data	626 sectors $\times$ firm type
Final demand elasticity (θ_r)	Measured from data	626 sectors
Input-Output elasticity ($\theta_{ss'}^\ell$)	Measured from data	626 sectors $\times$ firm type
Final demand bundle elasticity (φ_s)	Pin down by CES results and data	11 sectors
Material bundle elasticity ($\sigma_{s'}$)	Covid shock for Chile estimation	11 sectors
Exit rate(δ^ℓ)	Measured from data	626 sectors $\times$ firm type
Entry cost (c_e^ℓ)	Pin down by free entry and data	626 sectors $\times$ firm type
Productivity Pareto tail (κ^ℓ)	MLE estimation	11 sectors $\times$ firm type

Estimation Details

Nonlinear Prices: Data vs model (untargeted)



- For the average upstream firm price schedule to retailers, normalizing the continuous input quantity to be in the bounds of 1 to 50

Aggregate Welfare Ratios Relative to Efficiency (Reg $\in \{$NLP, Lin$\}$)

$$\frac{\mathbf{W}^{\mathrm{Reg}}}{\mathbf{W}^{\mathrm{Eff}}} = \underbrace{\prod_{s\in S}\left(\frac{\mu_s^{ur,\mathrm{Reg}}}{\mu_s^{ur,\mathrm{Eff}}}\right)^{-\tilde{\lambda}_s^{ru}} \prod_{s\in S}\left(\frac{\mu_s^{uu,\mathrm{Reg}}}{\mu_s^{uu,\mathrm{Eff}}}\right)^{-\tilde{\lambda}_s^{uu}}}_{\text{Intensive Margin}} \times \underbrace{\prod_{s\in S}\left(\frac{N_s^{r,\mathrm{Reg}}}{N_s^{r,\mathrm{Eff}}}\right)^{\frac{\theta_s}{\varphi_s-1}} \prod_{s\in S}\left(\frac{N_s^{u,\mathrm{Reg}}}{N_s^{u,\mathrm{Eff}}}\right)^{-\frac{\tilde{\lambda}_s^{uu}}{1-\sigma_s}}}_{\text{Extensive Margin}}.$$

Price Regime	W^R / W^{Eff}	$W^{\mathrm{Reg}} / W^{\mathrm{Lin}}$
Nonlinear (NLP)	0.745	1.534
Linear pricing (Lin)	0.486	

- NLP attains $\approx 75\%$ of efficient welfare vs. $\approx 50\%$ under linear pricing
- Moving from Lin to NLP closes about half of the efficiency gap:

$$\frac{W^{\mathrm{NLP}} - W^{\mathrm{Lin}}}{W^{\mathrm{Eff}} - W^{\mathrm{Lin}}} = \frac{0.745 - 0.486}{1 - 0.486} \approx 0.50.$$

Aggregate Welfare Decomposition and Masses (v. efficient)

$$\frac{\mathbf{W}^{\text{Reg}}}{\mathbf{W}^{\text{Eff}}} = \underbrace{\prod_{s\in S}\left(\frac{\mu_s^{ur,\text{Reg}}}{\mu_s^{ur,\text{Eff}}}\right)^{-\tilde{\lambda}_s^{ru}} \prod_{s\in S}\left(\frac{\mu_s^{uu,\text{Reg}}}{\mu_s^{uu,\text{Eff}}}\right)^{-\tilde{\lambda}_s^{uu}}}_{\text{Intensive Margin}} \times \underbrace{\prod_{s\in S}\left(\frac{N_s^{r,\text{Reg}}}{N_s^{r,\text{Eff}}}\right)^{\frac{\theta_s}{\varphi_s-1}} \prod_{s\in S}\left(\frac{N_s^{u,\text{Reg}}}{N_s^{u,\text{Eff}}}\right)^{-\frac{\tilde{\lambda}_s^{uu}}{1-\sigma_s}}}_{\text{Extensive Margin}}.$$

Regime	Intensive	Extensive	$\text{Share}_{\text{int}}$	$\text{Share}_{\text{ext}}$	N^u	N^r
Nonlinear	0.67	1.12	0.79	0.21	1.18	1.17
Linear	0.46	1.06	0.93	0.07	1.03	1.44

- **Result**
  Higher markups create higher expected profits, and thus more entry: More firms at a smaller scale
- **Intensive Margin Dominates (as a share of total log deviation relative to Eff.)**
  NLP: 79%, Lin: 93%. Extensive is pro-competitive (factors >1) but modest
- **Entry Responses**
  NLP and Lin increase firm masses in both firm types, large retail entry in Lin

Nonlinear vs. Linear Pricing: Opening Welfare Ratios by Sector

$$\frac{W^{\mathrm{NLP}}}{W^{\mathrm{Lin}}} = \underbrace{\prod_{s\in S}\left(\frac{\mu_s^{ur,\mathrm{NLP}}}{\mu_s^{ur,\mathrm{Lin}}}\right)^{-\tilde{\lambda}_s^{ru}} \prod_{s\in S}\left(\frac{\mu_s^{uu,\mathrm{NLP}}}{\mu_s^{uu,\mathrm{Lin}}}\right)^{-\tilde{\lambda}_s^{uu}}}_{\textbf{Intensive Margin}} \times \underbrace{\prod_{s\in S}\left(\frac{N_s^{r,\mathrm{NLP}}}{N_s^{r,\mathrm{Lin}}}\right)^{\frac{\theta_s}{\varphi_s-1}} \prod_{s\in S}\left(\frac{N_s^{u,\mathrm{NLP}}}{N_s^{u,\mathrm{Lin}}}\right)^{-\frac{\tilde{\lambda}_s^{uu}}{1-\sigma_s}}}_{\textbf{Extensive Margin}}.$$

Sector	Intensive (allocative)		Extensive (variety)		Net NLP/Lin
	Retailers	Upstream	Retailers	Upstream	
Agriculture	1.010	1.010	0.997	1.005	1.022
Mining	1.003	1.003	0.999	1.014	1.019
Manufacturing	1.024	1.029	0.991	1.002	1.047
Utilities	1.016	1.006	0.996	1.033	1.051
Construction	1.061	1.022	0.980	1.119	1.189
Retail and Wholesale	1.037	1.070	0.992	1.005	1.106
Transport and ICTs	1.007	1.023	0.981	1.000	1.011
Financial Services	1.012	1.008	0.943	0.998	0.960
Real Estate Services	1.009	1.004	0.996	1.023	1.033
Business Services	1.005	1.006	0.989	0.999	0.999
Personal Services	1.001	1.001	0.998	1.000	1.000
Product over sectors	1.197	1.198	0.870	1.207	1.507

Quantification: Main Takeaways

1. Intensive Margin Dominates Aggregate Markups

- Nonlinear pricing's welfare edge comes primarily from relieving allocative wedges
- In the final-demand–exposed upstream sectors (Construction and Retail & Wholesale)

2. Extensive Margin is Reinforcing but Modest

- GE participation expands upstream and retail entry
- Yet these variety effects are quantitatively small relative to intensive gains

Table of Contents

Conclusion

- We reject uniform pricing and find evidence of hybrid $2^{nd} + 3^{rd}$ degree price discrimination
- NL. P. improves allocations relative to Lin. P., but shifts rents via flat fees: distorts entry
- Net: NL. P. beats Lin. P. on welfare 75% vs. 50% of efficient pricing
- Average prices can mislead. Policy should monitor marginal prices and rent extractions
- Don't ban quantity discounts. Target markup accumulation along the supply chain
- The method is plug-and-play with standard microdata on transactions:
  - Ex-ante evaluations of Robinson–Patman–style enforcement
  - Ex-post sector-specific contracting rules evaluations

Is it profitable to exclude any buyer type? Return

In nonlinear pricing with private information, the seller always faces a choice:

1. Exclude low types to better extract surplus from high types
2. Serve all types, but give up some rent from high types

- However, Pareto distribution has heavy mass for low types
- Even though low types buy little q, because of their large density the seller maximizes profits by serving them

No, Positive Virtual Surplus for all types ▸ Return

Virtual surplus: profit from serving a buyer type, net of the informational rents that must be left to higher types to preserve IC

- Virtual surplus for type z (with $\alpha = \frac{\sigma}{\sigma-1}$):

$$\mathrm{VS}(z) = \underbrace{\left(\frac{z^{\sigma-1}}{\alpha}\right) q(z)^{1-1/\sigma}}_{\text{Gains from serving type } z} - \underbrace{\left(\frac{1-F(z)}{f(z)} \cdot \frac{d}{dz}\left(\frac{z^{\sigma-1}}{\alpha}\right)\right) q(z)^{1-1/\sigma}}_{\text{Informational rents left to ensure IC}}$$

- For the lowest type $z_0 = 1$, the virtual surplus simplifies:

$$\mathrm{VS}(1) = \left[\frac{1}{\alpha}\left(1 - \frac{\sigma-1}{\kappa}\right)\right] q(1)^{1-1/\sigma}$$

- This is strictly positive whenever $\kappa > \sigma - 1$ (necessary condition for finite output)
- If its profitable to serve the lowest type, the seller will not exclude any buyer

Is price deviation profitable for any $z > z_a$? ▸ Return

- Heuristic argument (Wilson 1993) to derive the optimal price $p(q)$
- Define marginal buyer $z(q,p)$ by inverting demand for the q^{th} unit (equation 1)

$$z(q,p) = q^{\frac{1}{\sigma-1}} p^{\frac{\sigma}{\sigma-1}}$$

- Demand for q^{th} unit:

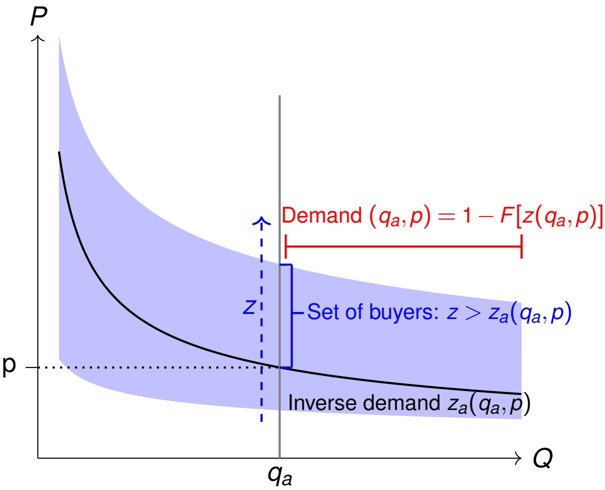
$$D(q,p) = 1 - F(z(q,p))$$

- Seller chooses a price for unit "q" to solve:

$$\max_p [1 - F(z_a(q_a,p))](p-c)$$

No profitable deviation in price

▸ Return



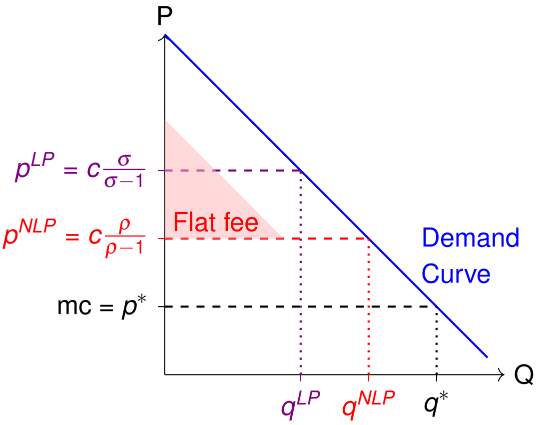
$$\max_{\{P\}} [1 - F(z_a(q_a, p))](P - c)$$

FOC :

$$\frac{P}{c} = \frac{\frac{\kappa\sigma}{\sigma-1}}{\frac{\kappa\sigma}{\sigma-1} - 1} = \frac{\rho}{\rho - 1}$$

- The optimal price is equal to the **allocative price of the two-part tariff**
- Seller has no incentive to charge different prices for different quantities

Two-Part Tariff: NLP vs LP CES markup ($\rho > \sigma$) Return



- Allocations in NLP are less distorted relative to LP

$$q^* > q^{NLP} > q^{LP}$$

- Because of the flat fee, rents are subject to different distortions in NLP vs. LP

General Equilibrium NLP

Return

A decentralized nonlinear pricing equilibrium is a collection of firm level productivity z_i, linear prices $\{p_u, p_r\}$, flat fees $\{F_u, F_r\}$, wage w, and quantities $\{y_i, q_i, l_i, m_{iu}, N_U, N_R\}$ such that:

- Each producer minimizes its costs and charges a linear price that equals marginal cost times the markup
- Each producer pays a transfer, such that the lowest types have zero surplus from transacting with upstream sellers
- Entrants earn zero expected profit
- The representative consumer maximizes its consumption
- Markets clear for all goods and factors

Invoice Example

Return

KITCHEN CENTER SPA
IMPORTACIÓN Y DISTRIBUCIÓN DE ELECTRODOMÉSTICOS

Casa Matriz:	Av. El Salto 3485, Recoleta, Santiago
Sucursales:	
Casa Costanera:	Av. Nueva Costanera 3900, Vitacura
Mall Parque Arauco:	Av. Kennedy 5413 Local 572, Las Condes - Teléfono: (56-2) 24117777 - Fax: (56-2) 24117711
Mall Plaza Los Dominicos:	Padre Hurtado Sur 875, Local A2080/2076, Las Condes - Teléfono: (+56 2) 24117738
Mall Buenaventura:	San Ignacio 500 Local 12, Quilicura - Teléfono: (+56 2) 24117769
Mall Plaza La Serena:	Av. Huanhualí 105 BA 190, La Serena - Teléfono: (56-2) 24117732
Mall Marina Arauco:	Av. Libertad 1348, Local PD-01/02, Viña del Mar - Teléfono: (56-2) 24117767/68
Outlet Park Viña:	Camino Internacional 2440 Local 72, Viña del Mar
Mall Plaza Maule:	Circunvalación 1055, Local 226/227, Talca - Teléfono: (56-2) 24117748
Concepción:	Paicaví 2367, Local 2, Teléfono: (56-2) 2411 7716 / 17
Temuco:	Avda. Alemania 0611, Temuco - Teléfono: (+56 2) 24117774 R
Mall Easton Temuco:	Rudencio Ortega 01780, Local L168-170, Temuco - Teléfono: (+56 2) 24117714
Servicio Técnico:	Lautaro #260, Quilicura - Teléfono: 6004117709 / 737 / 504
Centro de Distribución:	Camino lo Boza #8887, Pudahuel, Santiago
Viña del Mar:	8 Nte. 514, Viña del Mar.
Alto Las Condes:	Av. Kennedy 9001 Local 1017, Las Condes.
Outlet El Salto:	Av. El Salto 3400, Recoleta, Santiago.

R.U.T. 96.999.930-7

BOLETA ELECTRÓNICA

Nº 0015959119

S.I.I. - SANTIAGO NORTE

FECHA EMISIÓN : 01/08/2022

SEÑORES Antonio Martner
DIRECCIÓN Los Misioneros 1923 TELEFONO +569995703551
COMUNA Providencia CIUDAD : Santiago
R.U.T. 16.211.960-5 EMAIL : amartner@gmail.com

FECHA EMISIÓN :
FECHA VENCIMIENTO : 03/08/2022
TIPO DESPACHO :
FORMA DE PAGO : Contado
COD. VENDEDOR :

Dirección Origen: Camino lo Boza 8887
Comuna : Pudahuel Ciudad : Santiago
Dirección Destino: Los Misioneros 1923
Comuna : Providencia Ciudad : Santiago

Orden de Venta: 793325
Número de OC:

CÓDIGO	DETALLE	CANTIDAD	PRECIO UNITARIO	PRECIO ÍTEM
13452	Lavaplatos FDV Small Acqua bajo cubierta	1	92.428,57	92.429
14761	Encimera FDV Design 4T GLTX 65 BUT 2.0	1	142.848,74	142.849
14265	Campana Kubli Neu Slider	1	100.831,93	100.832
19110	Horno FDV Design	1	201.672,27	201.672
13377	Lavavajillas FS FDV Element 14C	1	243.689,07	243.689
14917	Griferia FDV CONICA FLEX	1	84.025,21	84.025
10232	Transporte - Providencia	1	15.529,41	15.529

Data cleaning Return

Goal: Keep all plausible transactions

- Prices are net of discounts and recharges
- Drop if a transaction has missing or zero price or quantity
- Drop if product description is missing
- Drop transactions where seller-product-day maxmin price ratio is above the 99^{th} percentile
- Under this cleaning we keep around 95% of transactions

Price Variance Determinants for 2024: Strategy Return

Step 1

- Make goods comparable and eliminate possible demand and supply shocks
- Store residuals from:

$$\ln p_{jgit} = \beta_0 + \Psi_{jgd} + \varepsilon_{jgit}$$

p_{jgit} is the price for seller j, product g, buyer i in time t, Ψ is a fixed effect including day d

Step 2

- Project residuals on different observables (quantity transacted and buyers' observables)
- Compare R^2

Price Determinants for 2024: Results Return

Seller j, product g, buyer i, time t, day d, quantity q, buyer group B (11 sectors$\times$3 sizes$\times$16 regions)

$$\varepsilon_{jgit} = \beta_0 + \Psi_{jgdS} + \varepsilon_{jgit}$$

	(1)	(2)	(2)
R^2	0.34	0.28	0.53
$S =$ Quantity	✓		
$S =$ Buyer Group		✓	
$S =$ Quantity $\times$ Buyer group			✓
N	147M	147M	147M

- Consistent whit hybrid second + thrid dregree price discrimination schemes

Monthly Fixed Effects Selected sectors Buyer Power

Price Determinants for 2024: Monthly Fixed Effects

Return

Seller j, product g, buyer i, time t, month m, quantity q, buyer group B (11 sectors$\times$3 sizes$\times$16 regions)

$$\ln p_{jgit} = \beta_0 + \Psi_{jgdS} + \varepsilon_{jgit}$$

	(1)	(2)	(3)	(4)
R^2	0.34	0.51	0.41	0.62
$S =$ Quantity	✓			
$S =$ Buyer		✓		
$S =$ Buyer Group			✓	
$S =$ Quantity $\times$ Buyer group				✓
N	363M	363M	363M	363M

Price Determinants for 2024: Manufacturing

Return

Seller j, product g, buyer i, time t, month m, quantity q, buyer group B (11 sectors$\times$3 sizes$\times$16 regions)

$$\ln p_{jgit} = \beta_0 + \Psi_{jgmS} + \varepsilon_{jgit}$$

	(1)	(2)	(3)	(4)
R^2	0.45	0.54	0.46	0.81
$S =$ Quantity	✓			
$S =$ Buyer		✓		
$S =$ Buyer Group			✓	
$S =$ Quantity $\times$ Buyer group				✓
N	136M	136M	136M	136M

Price Determinants for 2024: Retail and Wholesale Return

Seller j, product g, buyer i, time t, month m, quantity q, buyer group B (11 sectors$\times$3 sizes$\times$16 regions)

$$\ln p_{jgit} = \beta_0 + \Psi_{jgmS} + \varepsilon_{jgit}$$

	(1)	(2)	(3)	(4)
R^2	0.38	0.65	0.49	0.68
$S =$ Quantity	✓			
$S =$ Buyer		✓		
$S =$ Buyer Group			✓	
$S =$ Quantity $\times$ Buyer group				✓
N	180M	180M	180M	180M

Buyer Market Power? Return

- Exploit cross-sectional variation in the number of suppliers each buyer transacts with
- A larger number of providers may indicate stronger outside options; better pricing terms

$$\ln p_{jgim} = \beta_0 + \beta_1 \ln q_{jgim} + \beta_2 (\log q_{jgim} \times \log \text{NumProviders}_i) + \Psi_{jgm} + \varepsilon_{jgit},$$

- $\beta_2 > 0$ would suggest that quantity discounts become flatter as buyer power increases
- We find that $\beta_1 = -0.0462\ (0.0001)$ and $\beta_2 = -0.0098\ (0.0001)$
- Buyer power does not appear to be the primary mechanism generating quantity discounts

Nonlinear Prices by Sector

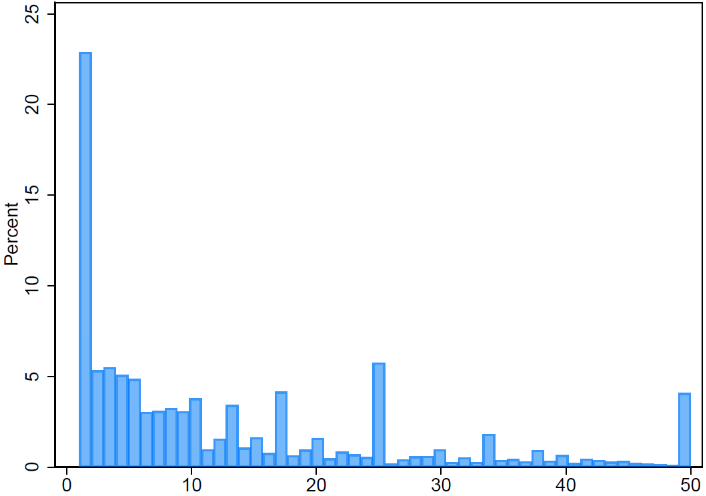
Return

Sector	Mean Q discount	N transactions
All sectors	-0.042	430M
Agriculture	-0.042	2M
Mining	-0.016	1M
Manufacturing	-0.036	118M
Utilities	0.000	6M
Construction	-0.129	1M
Retail and Wholesale	-0.048	270M
Transport & ICTs	-0.032	12M
Financial Services	-0.002	49M
Real Estate Services	-0.052	1M
Business Services	-0.089	5M
Personal Services	-0.053	1M

Quantity Quantiles Bins Return

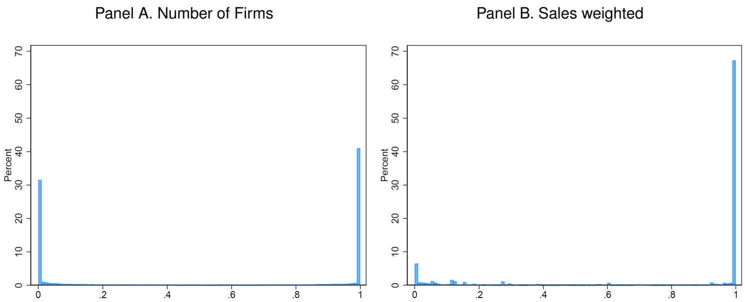
- Products have different scales, we compare prices across each product's q rank distribution
- For each product g, $F_g(\cdot)$: empirical CDF of transacted quantities q_{jgit}
- Define the within-product rank: $r_{jgit} \equiv F_g\left(q_{jgit}\right)$.
- Partition $[0,1]$ into 50 equal-probability intervals $I_b \equiv \left((b-1)/50,\ b/50\right]$ for $b = 1, \dots, 50$
- Assign each transaction to a bin $B_{jgit} = b$ whenever $r_{jgit} \in I_b$
- With discrete quantities and mass points, we assign observations to the smallest b

Quantity Quantiles Bins Histogram Return



Sales partition ▸ Return

$$X_i = \begin{cases} 0 & \text{if all sales go to final consumers} \\ 1 & \text{if all sales go to other firms} \end{cases}$$



- More than 70% of firms sell only to final consumers or to other firms By sector

Sales partition: Sales shares (excluding exports) ▸ Return

Sector (sales)	All to final consumer	All to other firms
Firm population	0.08	0.67
Agriculture (2%)	0.04	0.60
Mining (1%)	0.27	0.08
Manufacturing (15%)	0.05	0.68
Utilities (3%)	0.20	0.51
Construction (8%)	0.02	0.89
Retail and Wholesale (32%)	0.09	0.68
Transport & ICTs (10%)	0.16	0.68
Financial Services (18%)	0.18	0.67
Real Estate Services (1%)	0.24	0.37
Business Services (7%)	0.08	0.81
Personal Services (2%)	0.68	0.10

Guesses Verification ▸ Return

Guess 1 (two–part tariffs with quantity–invariant marginal price within each (ℓ, b, u))

- follows immediately from the two-part tariff and constant markup in Proposition 1

Guess 2 (homogenous link revenue)

- Is verified by aggregating optimal link choices across partitions
- The seller's total revenue is isoelastic in own quantity with exponent $(\sigma_u - 1)/\sigma_u$
- Admits a closed–form scale A_{su} that explicitly includes a flat–fee component driven by the seller's CES share in buyers' materials bundles

General Equilibrium Under Nonlinear Prices

Return

A general equilibrium under nonlinear pricing is a collection

$$\left\{ (m^{\ell}_{ubi}(\cdot), T^{\ell}_{ubi}(\cdot), B_{\ell bi})_{(\ell,b),i},\ (p^{\ell,*}_{bi})_{(\ell,b),i},\ (P^{\ell}_{ub})_{u,b,\ell},\ (N^{\ell}_{s})_{s,\ell},\ (Q_j, I_j)_j \right\}$$

such that: (i) mechanisms $(m^{\ell}_{ubi}(\cdot), T^{\ell}_{ubi}(\cdot))$ implement the two–part–tariff optimum with $p^{\ell,*}_{bi}$ and $F^{\ell,*}_{ubi}$ in Proposition 1; (ii) buyers' choices satisfy the best–response condition above; (iii) price and cost indices satisfy ; (iv) materials and labor markets clear with $L = 1$; and (v) free entry holds in each (ℓ, s). A detailed proof of existence and uniqueness is provided in the paper.

Welfare Decomposition: Intensive vs. Extensive Margins ▸ Return

Welfare is the inverse final price index:

$$W \equiv \frac{1}{P_Y}, \qquad \log P_Y = \sum_{s \in \mathscr{S}} \theta_s \log P_s.$$

With wage normalization and free entry, the representative household's income equals the wage, so $W = 1/P_Y$

- θ_s: final-expenditure share on retail sector s.
- P_s: sectoral retail price index.

Sectoral Price Indices (Retail Interface)

Within each retail sector s:

$$P_s = \mu_s^r \, \Theta_s^r \, w^{\alpha_s^r} \left(\prod_{s' \in \mathscr{S}} \left(P^r_{s's}\right)^{(1-\alpha_s^r)\,\theta^r_{ss'}} \right) \left(N_s^r\right)^{-\frac{1}{\varphi_s - 1}} \mathscr{V}_s, \qquad P^r_{s's} = \mu^r_{s's} \, C_{s'}. \tag{1}$$

- μ_s^r: retail-to-consumer markup (allocative wedge).
- $\mu^r_{s's}$: buyer-specific markup charged by upstream s' to retail s.
- $C_{s'}$: upstream sector-s' marginal cost index.
- N_s^r: mass of active retail varieties; $\mathscr{V}_s$: CES selection term.

Upstream Marginal Cost Recursion

For each upstream seller sector s':

$$C_{s'} = \Theta^u_{s'} \, w^{\alpha^u_{s'}} \left(\prod_{v \in \mathscr{S}} \left(P^u_{vs'}\right)^{(1-\alpha^u_{s'})\,\theta^u_{s'v}} \right) \left(N^u_{s'}\right)^{-\frac{1}{\sigma_{s'}-1}} \mathscr{V}^u_{s'}, \qquad P^u_{vs'} = \mu^u_{vs'} \, C_v. \tag{2}$$

Taking logs and substituting $P^u_{vs'} = \mu^u_{vs'} C_v$:

$$\log C_{s'} = \sum_{v \in \mathscr{S}} (1 - \alpha^u_{s'}) \, \theta^u_{s'v} \left(\log \mu^u_{vs'} + \log C_v \right) + \alpha^u_{s'} \log w + \log \Theta^u_{s'} - \frac{1}{\sigma_{s'} - 1} \log N^u_{s'} + \log \mathscr{V}^u_{s'}. \tag{3}$$

Upstream System and Response Across Regimes

Stacking (3) with $A^{uu}_{s'v} := (1-\alpha^u_{s'})\,\theta^u_{s'v}$ gives

$$\log C^u = A^{uu}\log C^u + \log\mu^{uu} + \alpha^u \log w + \log\Theta^u - \frac{\log N^u}{\sigma-1} + \log\mathscr{V}^u, \tag{4}$$

where division by $(\sigma-1)$ is elementwise and $\log\mu^{uu}$ stacks upstream→upstream wedges. In changes across regimes (technology and w drop out):

$$\Delta\log C^u = (I-A^{uu})^{-1}\left(\Delta\log\mu^{uu} - \frac{\Delta\log N^u}{\sigma-1} + \Delta\log\mathscr{V}^u\right). \tag{5}$$

Final Demand Exposures

Define upstream–upstream and retail–upstream cost-share matrices:

$$A^{uu}_{s'v} := (1-\alpha^{u}_{s'})\,\theta^{u}_{s'v}, \qquad B^{ru}_{ss'} := (1-\alpha^{r}_{s})\,\theta^{r}_{ss'}.$$

Final-demand exposures that load upstream objects into $\log P_Y$:

$$\tilde{\lambda}_{ru} := \theta^{\top} B^{ru} \in \mathbb{R}^{1\times|\mathscr{S}|}, \qquad \tilde{\lambda}_{u} := \tilde{\lambda}_{ru}(I-A^{uu})^{-1} \in \mathbb{R}^{1\times|\mathscr{S}|}. \tag{6}$$

- $\tilde{\lambda}_{ru}$: exposure at the retail interface (no upstream propagation).
- $\tilde{\lambda}_{u}$: full upstream propagation via the Leontief inverse.

Exact Welfare Decomposition

Starting from (1)–(6) and the upstream recursion, the change in welfare satisfies

$$\Delta \log W = \underbrace{-\sum_s \theta_s \, \Delta \log \mu_s^r \; - \; \tilde{\lambda}_{ru} \, \Delta \log \mu^r \; - \; \tilde{\lambda}_u \, \Delta \log \mu^{uu}}_{\textbf{Intensive (allocative) markups: retail}\to\textbf{consumer, retail}\leftrightarrow\textbf{upstream, upstream}\leftrightarrow\textbf{upstream}}$$

$$+ \underbrace{\sum_s \frac{\theta_s}{\varphi_s - 1} \, \Delta \log N_s^r \; + \; \tilde{\lambda}_u \left(\frac{\Delta \log N^u}{\sigma - 1} \right)}_{\textbf{Extensive (variety/masses)}}$$

$$\underbrace{-\sum_s \theta_s \, \Delta \log \mathscr{V}_s \; - \; \tilde{\lambda}_u \, \Delta \log \mathscr{V}^u}_{\textbf{Selection (composition)}} .$$

Labor Output Elasticity $\boldsymbol{\alpha_s}$ ▸ Return

- **What.** Cobb–Douglas weight on *non-materials* (labor $+$ user cost of capital).
- **Identify.** Cost-share mapping under cost minimization:

$$\alpha_i = 1 - \frac{\sum_j p_{ji} m_{ji}}{w_i L_i + r_i K_i + \sum_j p_{ji} m_{ji}}.$$

  Flat fees: $TC_i = F_i + VC_i$; for large buyers F_i / TC_i is small $\Rightarrow$ variable share $\approx$ total share.
- **Sample.** Keep firms above 75th pctl. revenue; winsorize α_i at 1–99; aggregate to (s, ℓ) at 6-digit; average 2005–2022.
- **Why.** Governs response to wage vs. input-price shocks: higher α_s amplifies wage relevance, dampens price conduct action from materials prices.

Labor Shares by Sector (Results)

Labor Shares by Sector (mean)

Sector	Retailers	Upstream	Sector mean
Agriculture	0.43	0.41	0.42
Mining	0.25	0.32	0.29
Manufacturing	0.39	0.42	0.41
Utilities	0.37	0.38	0.38
Construction	0.48	0.42	0.45
Retail and Wholesale	0.37	0.31	0.34
Transport and ICTs	0.55	0.47	0.51
Financial Services	0.58	0.62	0.60
Real Estate Services	0.66	0.53	0.59
Business Services	0.72	0.65	0.69
Personal Services	0.71	0.57	0.64
Type mean	0.50	0.46	0.48

Final-Demand Output Elasticity $\boldsymbol{\theta_s}$

- **What.** Cobb–Douglas weights across *retail sectors* in final demand.
- **Identify.** With linear pricing to consumers, retail revenues identify expenditure shares:

$$\theta_s \approx \frac{\text{retail revenue in } s}{\sum_{s'} \text{retail revenue in } s'}.$$

- **Sample.** Large retailers (>75th pctl.), compute annual sector shares, average 2005–2022; check revenue-weighted robustness.
- **Why.** Anchors final-demand system and welfare accounting in counterfactuals.

Final-Demand Shares $\boldsymbol{\theta_s}$ (Results)

Cobb–Douglas Output Elasticities by Retail Sector

Sector	θ_s
Agriculture	0.0446
Mining	0.0085
Manufacturing	0.1318
Utilities	0.0505
Construction	0.1521
Retail and Wholesale	0.2768
Transport and ICTs	0.0979
Financial Services	0.1132
Real Estate Services	0.0152
Business Services	0.0911
Personal Services	0.0183

Materials Input–Output Shares $\boldsymbol{\theta}^{\ell}_{iss'}$

- **What.** Buyer-facing expenditure shares over upstream seller sectors s':

$$\theta^{\ell}_{iss'} = \frac{\sum_{j \in \mathscr{U}_{s'}} p_{ij} m_{ij}}{\sum_{s''} \sum_{j \in \mathscr{U}_{s''}} p_{ij} m_{ij}}, \quad \sum_{s'} \theta^{\ell}_{iss'} = 1.$$

- **Identify.** From transaction-level variable payments (VC); $TC = F + VC$, large buyers $\Rightarrow F/TC$ small.
- **Sample.** Compute firm-level θ for $\ell \in \{r, u\}$; keep >75th pctl. revenue; aggregate to 6-digit, then to 1-digit by year; average 2005–2022.
- **Why.** Micro foundation for the IO network; pins exposures and intensive-margin substitution scope.

Materials IO Shares: Retailers as Buyers (Results)

Input-output Elasticities by Retailers as Buyers

Buyer \ Seller	Agr.	Min.	Man.	Uti.	Cons.	R. & W.	T. & ICTs	F. Serv.	RE. Serv.	B. Serv.	P. Serv.
Agriculture	0.25	0.00	0.21	0.02	0.03	0.32	0.05	0.07	0.00	0.04	0.00
Mining	0.00	0.04	0.19	0.06	0.15	0.30	0.07	0.02	0.00	0.17	0.00
Manufacturing	0.13	0.02	0.35	0.02	0.03	0.25	0.11	0.03	0.00	0.06	0.00
Utilities	0.07	0.01	0.18	0.03	0.03	0.26	0.17	0.05	0.00	0.20	0.00
Construction	0.10	0.00	0.10	0.02	0.22	0.24	0.15	0.03	0.00	0.14	0.00
Retail and Wholesale	0.16	0.01	0.24	0.01	0.02	0.34	0.08	0.05	0.00	0.09	0.00
Transport and ICTs	0.07	0.01	0.14	0.02	0.03	0.24	0.19	0.04	0.00	0.26	0.00
Financial Services	0.08	0.00	0.12	0.01	0.01	0.22	0.06	0.15	0.01	0.33	0.00
Real Estate Services	0.03	0.00	0.12	0.01	0.02	0.30	0.04	0.06	0.05	0.37	0.00
Business Services	0.07	0.00	0.13	0.01	0.01	0.22	0.09	0.06	0.00	0.41	0.00
Personal Services	0.07	0.00	0.17	0.02	0.02	0.25	0.07	0.08	0.00	0.33	0.01

Materials IO Shares: Upstream as Buyers (Results)

Input-output Elasticities by Upstream Firms as Buyers

Buyer \ Seller	Agr.	Min.	Man.	Uti.	Cons.	R. & W.	T. & ICTs	F. Serv.	RE. Serv.	B. Serv.	P. Serv.
Agriculture	0.26	0.00	0.12	0.02	0.04	0.29	0.10	0.06	0.00	0.10	0.00
Mining	0.01	0.07	0.39	0.05	0.06	0.13	0.11	0.03	0.00	0.15	0.00
Manufacturing	0.08	0.02	0.49	0.03	0.02	0.15	0.09	0.02	0.00	0.10	0.00
Utilities	0.06	0.02	0.18	0.07	0.03	0.18	0.15	0.04	0.00	0.27	0.00
Construction	0.07	0.00	0.14	0.03	0.30	0.18	0.12	0.03	0.00	0.13	0.00
Retail and Wholesale	0.12	0.01	0.27	0.01	0.02	0.38	0.07	0.03	0.00	0.10	0.00
Transport and ICTs	0.06	0.02	0.14	0.02	0.04	0.21	0.22	0.03	0.00	0.26	0.00
Financial Services	0.05	0.00	0.12	0.02	0.01	0.20	0.07	0.12	0.01	0.41	0.00
Real Estate Services	0.03	0.00	0.11	0.01	0.02	0.27	0.04	0.04	0.06	0.41	0.00
Business Services	0.07	0.00	0.13	0.01	0.01	0.23	0.09	0.05	0.00	0.40	0.00
Personal Services	0.06	0.00	0.15	0.03	0.02	0.21	0.07	0.11	0.00	0.33	0.01

Upstream Materials Elasticity $\boldsymbol{\sigma}_{u'}$

- **What.** Substitutability across varieties *within* an upstream seller sector u'.
- **Identify.** IV from March 2020 municipal lockdown of *main supplier* u^*:

$$\Delta_{12} \log \frac{m_{isut}}{m_{isu^*t}} = -\sigma_{u'} \, \Delta_{12} \widehat{\log \frac{p_{isut}}{p_{isu^*t}}} + \gamma_s + \varepsilon.$$

- **Design.** 2SLS by seller sector; instrument $Z_{isu} = \mathbf{1}\{u^* \text{ locked}\}$; 12m diffs; large buyers; exclude buyer/clients/other inputs under lockdown; cluster at buyer level.
- **Why.** Higher $\sigma \Rightarrow$ faster rewiring, stronger intensive reallocation, lower pass-through; feeds κ mapping. (Conservatively set $\sigma \geq 1.45$ where $\hat{\sigma} < 1$.)

Estimated Elasticities $\boldsymbol{\sigma}_{u'}$ (Results)

Estimated Elasticities of Substitution by Seller Sector

Sector	$\sigma_{u'}$	SE	1^{st} Stage F stat.	Obs.
Agriculture	2.59	(1.35)	10.24	4,387
Manufacturing	3.41	(0.84)	16.37	186,912
Construction	1.45	(0.42)	7.36	6,062
Retail and Wholesale	3.80	(0.39)	94.08	680,985
Transport and ICTs	5.07	(2.22)	25.19	24,054
Financial Services	3.09	(1.56)	9.35	3,631
Business Services	5.21	(2.02)	17.55	4,514
Personal Services	6.69	(3.37)	13.29	7,579
All sectors	3.04	(1.12)	149.87	918,124

Three sectors (Mining, Utilities, Real Estate Services) yield $\hat{\sigma}_{u'} < 1$; we set $\sigma_{u'} = 1.45$ (minimum estimate above one) for model quantification.

Final-Consumer Variety Elasticity $\boldsymbol{\varphi}_{s_r}$

- **What.** CES elasticity across retail *varieties* within sector s_r; markup $\mu = \varphi/(\varphi - 1)$.
- **Identify.** Sectoral accounting identity under linear pricing:

$$\varphi_{s_r,t} = \frac{\sum_j R_{j,t}}{w_{s_r,t} \sum_j F_{j,t} + \sum_j \Pi_{j,t}}, \qquad \Pi_j^{\text{var}} = \frac{1}{\varphi_{s_r}} R_j.$$

- **Sample.** Large retailers; $F_{j,t}$ small, pool to sector-year; average 2005–2022.
- **Why.** Higher $\varphi \Rightarrow$ keener competition, smaller wedges; also maps retailer labor tails ν into productivity tails $\kappa = (\varphi - 1)\nu$.

Final-Consumer Elasticities $\boldsymbol{\varphi}_{s_r}$ (Results)

Retailer Parameter φ_{s_r} by Sector

Sector	φ_{s_r}
Agriculture	4.54
Mining	2.68
Manufacturing	4.22
Utilities	3.94
Construction	2.59
Retail and Wholesale	8.17
Transport and ICTs	2.05
Financial Services	1.40
Real Estate Services	1.82
Business Services	2.73
Personal Services	2.56
Type mean	3.34

Notes: φ_{s_r} computed from pooled sectoral sums of revenue, fixed costs (labor units), and profits.

Exit Hazard $\boldsymbol{\delta}_{s_\ell}$

- **What.** One-year hazard that an active firm exits.
- **Measure.** For cell (s, ℓ, t):

$$\delta_{s_\ell,t} = 1 - \frac{\text{survivors}_{s_\ell,t}}{\text{active}_{s_\ell,t}}, \qquad \delta_{s_\ell} = \frac{1}{|\mathscr{T}|}\sum_{t\in\mathscr{T}} \delta_{s_\ell,t}.$$

- **Sample.** Compute at 6-digit $\times$ type; track 2005–2022; average across years.
- **Why.** Disciplines expected lifespan and shock persistence; higher δ increases payoff needed for entry, tilts adjustments toward the extensive margin.

Exit Rates δ by Sector (Results)

Exit Rates (δ) by Sector (Means)

Sector	Retailers	Upstream	Sector mean
Agriculture	0.090	0.086	0.088
Mining	0.084	0.093	0.088
Manufacturing	0.093	0.071	0.082
Utilities	0.070	0.064	0.067
Construction	0.140	0.110	0.125
Retail and Wholesale	0.103	0.076	0.089
Transport and ICTs	0.088	0.093	0.091
Financial Services	0.101	0.062	0.081
Real Estate Services	0.115	0.099	0.107
Business Services	0.099	0.077	0.088
Personal Services	0.093	0.090	0.092
Type mean	0.098	0.084	0.091

Entry Cost c_{e,s_ℓ} (in labor units)

- **What.** Sunk labor resources required to create an operating firm.
- **Identify.** Free entry with survival hazard δ:

$$\mathrm{PV}_{s_\ell} = \frac{\bar{\Pi}_{s_\ell}}{1-\beta(1-\delta_{s_\ell})}, \quad w_{s_\ell} c_{e,s_\ell} = p^{\mathrm{succ}}_{s_\ell} \cdot \mathrm{PV}_{s_\ell} \Rightarrow c_{e,s_\ell} = \frac{p^{\mathrm{succ}}_{s_\ell}}{w_{s_\ell}} \cdot \frac{\bar{\Pi}_{s_\ell}}{1-\beta(1-\delta_{s_\ell})}.$$

- **Sample.** Use observed profits Π, wages w, positive-profit share p^{succ}, and δ at 6-digit $\times$ type; report currency and wage-bill equivalents.
- **Why.** Shapes steady-state firm mass/scale; interacts with NLP's rent reallocation along the chain.

Entry Costs c_e by Sector (Results)

Entry Costs and Equivalent Yearly Wage-Bills by Sector

	Retailers		Upstream	
Sector	Entry cost c_e	Wage-bill eq.	Entry cost c_e	Wage-bill eq.
Agriculture	81.03	3.68	84.12	4.78
Mining	29212.81	43.99	177.12	7.20
Manufacturing	101.87	4.25	120.80	4.53
Utilities	700.66	14.15	306.11	5.50
Construction	109.72	7.78	109.05	4.18
Retail and Wholesale	63.92	6.06	83.61	5.13
Transport and ICTs	299.85	10.28	98.03	6.40
Financial Services	263.84	8.64	248.44	9.05
Real Estate Services	82.11	11.68	100.69	8.70
Business Services	82.91	5.76	125.21	3.11
Personal Services	127.87	4.56	94.76	4.57
Type mean	2829.69	10.98	140.72	5.74

Notes: Entry costs c_e are in the currency units used for calibration; “Wage-bill eq.” reports multiples of the annual wage bill.

Productivity Tail Exponent κ

- **What.** Thickness of the upper tail of firm productivity.
- **Identify.** Estimate labor tail by MLE above threshold:

$$\widehat{\nu} = \frac{n}{\sum_{i:\, L_i \geq L_{\min}} \ln(L_i / L_{\min})}, \quad \mathrm{SE}(\widehat{\nu}) \approx \widehat{\nu}/\sqrt{n}.$$

  Map to productivity using $l(z) \propto z^{\sigma-1}$ (or $\varphi - 1$ for retail):

$$\kappa^u = (\sigma - 1)\nu^u, \qquad \kappa^r = (\varphi - 1)\nu^r.$$

- **Sample.** Compute ν by 1-digit $\times$ type; combine with sectoral σ / φ; report implied κ.
- **Why.** Thicker tails (small κ) magnify selection/reallocation gains and shape how NLP shifts surplus across the distribution.

Labor and Implied Productivity Tails $\boldsymbol{\nu}$, $\boldsymbol{\kappa}$ (Results)

Labor and Implied Productivity Pareto Tails by Sector

	Retailers		Upstream		
Sector	ν_r	$\kappa_r = (\varphi_s - 1)\nu_r$	ν_u	$\kappa_u = (\sigma_{u'} - 1)\nu_u$	$\sigma_{u'}$
Agriculture	2.49	8.82	2.63	4.18	2.59
Mining	1.43	2.40	2.20	0.99	1.45
Manufacturing	2.66	8.58	2.15	5.18	3.41
Utilities	2.17	6.38	1.94	0.87	1.45
Construction	3.23	5.13	2.19	0.99	1.45
Retail and Wholesale	3.45	24.74	2.40	6.72	3.80
Transport and ICTs	2.20	2.32	3.04	12.37	5.07
Financial Services	2.55	1.02	2.26	4.72	3.09
Real Estate Services	4.36	3.59	3.03	1.36	1.45
Business Services	2.45	4.25	1.93	8.13	5.21
Personal Services	2.03	3.17	2.58	14.69	6.69

Notes: $\kappa = (\sigma_{u'} - 1)\nu$ uses seller-sector elasticities $\sigma_{u'}$ from the IV estimates. For Mining, Utilities, and Real Estate Services, we set $\sigma_{u'} = 1.45$ (minimum estimate above one).

Sectoral Allocative Markups Final-Demand Weighted

Return

Sector	Lin	NLP: $u \to r$	NLP: $u \to u$	Share(r→u)	Share(full up)
Agriculture	1.63	1.18	1.17	0.053	0.050
Mining	3.27	1.94	1.46	0.005	0.009
Manufacturing	1.41	1.12	1.16	0.186	0.175
Utilities	3.27	1.47	1.56	0.036	0.024
Construction	3.27	1.27	1.46	0.121	0.084
Retail & Wholesale	1.36	1.08	1.12	0.319	0.364
Transport & ICTs	1.25	1.10	1.07	0.096	0.136
Financial Services	1.48	1.15	1.17	0.087	0.071
Real Estate Services	3.27	1.19	1.30	0.018	0.013
Business Services	1.24	1.08	1.11	0.067	0.065
Personal Services	1.18	1.08	1.06	0.012	0.009
Weighted aggregate	**1.61**	**1.14**	**1.17**		