

# Nonlinear Prices and Firm Participation

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# Motivation I

- Market power in vertical supply chains has been argued to generate efficiency and welfare losses
- However, most of the previous literature has not considered the effects of price discrimination on resource misallocation and firm participation

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- Market power in vertical supply chains has been argued to generate efficiency and welfare losses
- However, most of the previous literature has not considered the effects of price discrimination on resource misallocation and firm participation
- Using invoice data for Chile, we find that while linear prices are observed, non-linear prices are prevalent
  - ★ Around 30% transactions have linear prices
  - ★ Quantity discounts explain around 60% of the price variation relative to buyer dispersion

amazon business



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## Quantity Discounts

Save 5% or more with Quantity Discounts on over 60 million products starting at just two units of the same item.

# Motivation II: Price discrimination

## 1. Static allocative efficiency

- ★ First-degree: No misallocation
- ★ Second-degree: No misallocation at the top, large distortion at the bottom  
⇒ Misallocation due to heterogeneous marginal product of inputs

# Motivation II: Price discrimination

## 1. Static allocative efficiency

- ★ First-degree: No misallocation
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⇒ Misallocation due to heterogeneous marginal product of inputs

## 2. Dynamic surplus distortion

- ★ First-degree: no firm will enter with a positive entry cost; hold up problem
- ★ Second-degree: Implications about firm profits are not so obvious
- ★ Distorted entry (firm wedge)  
⇒ Trade-off between participation and surplus extracting

# This paper

## How does nonlinear pricing affect welfare through resource misallocation and firm participation?

- Reduced form evidence
  - ★ Firm-to-Firm transactions for Chile descriptive statistics
  - ★ Provide evidence on nonlinear pricing
- Theory: Tractable setting to illustrate main mechanisms
  - ★ Supply chain model with second-degree price discrimination
  - ★ Firm participation in steady state
  - ★ Welfare: compare nonlinear, linear prices and perfect competition setups

# This paper: Theory preview

- Assume firms can exert first-degree price discrimination
  - ★ No static inefficiency
  - ★ But no firm will enter a dynamic setting with a positive entry cost; hold up problem
- In the data we observe second-degree, which is a special case of first-degree when the distribution of types is degenerated
  - ★ There is a wide dispersion of productivities in the data ( $\sim$  Pareto)
  - ★ Tradeoff between participation and surplus extracting
  - ★ Implications about firm profits are not so obvious

# Literature

Heterogenous firm-level market power transmission and aggregate implications

Peter and Bornestein (2024), Burstein, Cravino, and Rojas (2024)

Firm dynamics with distortions

Hsieh and Klenow (2014), Edmond, Midrigan, and Xu (2023), De Loecker, Eeckhout, and Mongey (2022), Boehm, Oberfield, South and Waseem (2024)

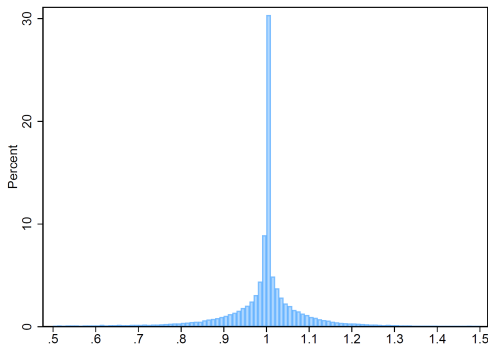


# Reduced form evidence: Data

- Invoice transactions for the universe of Chilean formal firms for 2018
  - ★ Around 1.3 billion transactions
  - ★ More than 10 million different products
  - ★ Data on prices and quantities for every (seller-specific) product transacted
- Merged with firm's accounting balance sheet data
  - ★ Sales, materials, investment, 6-digit industry
  - ★ Employer-employee: Wages, headcount employees
  - ★ Capital stock and investment

## Reduced form evidence: Price dispersion

- Seller  $i$ , product  $g$  price and mean:  $p_{ig}$ ,  $\bar{p}_{ig}$
- $\tilde{p}_{ig} = \frac{p_{ig}}{\bar{p}_{ig}}$



- Variance of  $\log \tilde{p}_{ig} = 0.47$  (excluding products with one transaction)
- No price discrimination in around 30% of transactions

## Reduced form evidence: Unpacking price variance

$$\ln p_{ijg} = \ln G + \ln \psi_{ig} + \mathbb{1}_j \ln \theta_j + \mathbb{1}_q \ln \gamma_q + \ln \omega_{ijg} \quad (1)$$

	(1)	(2)	(3)	(4)
Residual variance	4.35	0.25	0.26	0.24
FE Seller x product	Yes	Yes	Yes	Yes
FE Buyer	No	Yes	No	Yes
FE quantity	No	No	Yes	Yes
Observations	1.2 billions	1.2 billions	1.2 billions	1.2 billions
Adj R2	0.92	0.93	0.93	0.94

- What share of variance can be explained by buyer vs. quantity dispersion?

	$\frac{\text{var}(\theta_j)}{\text{var}(\theta_j + \gamma_q)}$	$\frac{\text{var}(\gamma_q)}{\text{var}(\theta_j + \gamma_q)}$	$\frac{2\text{cov}(\theta_j, \gamma_q)}{\text{var}(\theta_j + \gamma_q)}$
Main	0.39	0.77	-0.16
Lower bound	0.23	0.61	
Upper bound	0.39	0.77	

- Variation in quantities explains a higher share of the price variance

## Reduced form evidence:

Do large buyer firms purchase larger quantities?

$$\log q_{ijg} = \alpha_1 + \beta_1 \log L_j + \pi_{ig} + \epsilon_1 \quad (2)$$

	(1)	(2)	(3)
$\log L$ buyer (SE)	0.137 (0.00005)	0.136 (0.00006)	0.133 (0.00008)
FE Seller x product	Y	Y	Y
Ex-Manufacturing products	N	Y	N
Ex-retail products	N	N	Y
R2	0.67	0.66	0.71
obs	1.2 billions	900 million	800 million

- Quantity traded increases with buyer firm size

## Reduced form evidence: Quantity discounts

$$\log p_{ijg} = \alpha_2 + \beta_2 \log q_{ijg} + \pi + \epsilon_2 \quad (3)$$

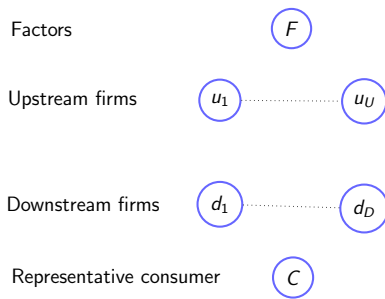
	(1)	(2)	(3)	(4)	(5)	(6)
$\log q$ (SE)	-0.199 (0.00002)	-0.237 (0.00005)	-0.160 (0.00007)	-0.117 (0.0002)	-0.243 (0.00007)	-0.214 (0.00008)
FE Seller x produt	Yes	Yes	No	Yes	Yes	Yes
FE Buyer	No	Yes	No	No	No	No
FE Seller x produt x buyer	No	No	Yes	No	No	No
High price products	No	No	No	Yes	No	No
Ex-Manufacturing products	No	No	No	No	Yes	No
Ex-retail products	No	No	No	No	No	Yes
Observations (around)	1.2 billions	1.2 billions	1.2 billions	62 millions	900 millions	800 millions
R2	0.89	0.90	0.90	0.76	0.72	0.82

- Unit price decreases with quantity

# Theory: Setup

- A representative consumer draws utility from goods produced by a continuum of downstream firms
- Downstream combine factors and material inputs from a continuum of upstream firms in production

Final demand details



## Theory: Downstream firms

- Downstream firms (measure  $D$ ) produce using different recipes
- Each decreasing returns to scale recipe uses labor  $l_{du}$  and capital  $k_{du}$  independently for each upstream firms material  $m_{du}$
- $z_u$  is upstream firm  $u$  product quality

$$y_d = z_d \int_{u_1}^{u_U} \left[ (l_{du}^\eta k_{du}^{1-\eta})^\gamma (z_u m_{du})^{1-\gamma} \right]^\xi dU$$

- ★ Separable demands for factors and materials from upstream firms
- ★ Engage in perfect competition charging normalized  $P = 1$  to the representative consumer

## Theory: Upstream firms

- Chose a menu of transfers and quantities  $(t, q)$  that maximizes its expected profits
- Based on revelation mechanisms, offers a tailored  $t, q$  to each downstream firm type

$$\begin{aligned} \underset{\{t, q\}}{\text{Max}} \pi_u &= \int_{\underline{z}_d}^{\bar{z}_d} m_d(z_d) [p_d(m_d) - c_u] dD \\ \text{s.t. } y_{du} &\in \max_{m_d > 0} [y_d - c_d \geq 0] \text{ , } \forall z_d \end{aligned}$$

- The set of constraints implies that each downstream firm  $z_d$  must prefer buying their allocation relative to:
  - ★ Not buying the product: Individual Rationality constraint
  - ★ Other positive quantity: Incentive Compatibility constraint



# Theory: Price schedule

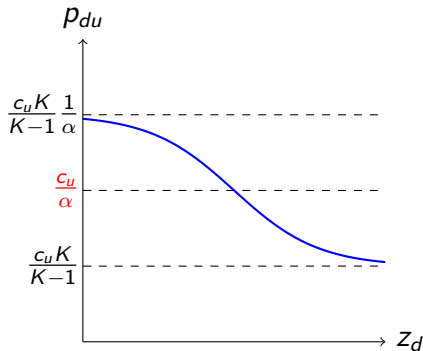
- The optimal mechanism depends on  $z_d$  distribution
- And interest case is when  $z_d$  are Pareto; material allocations will depend on the shape parameter  $K$

solution details

$$p_{du} = \left( \frac{\alpha}{c_u} \frac{K-1}{K} \right)^{-1} \left[ \alpha + (1-\alpha) \left( \frac{z_d}{z_d} \right)^{\frac{\alpha}{1-\alpha}} \right]$$

- $\alpha = \frac{\xi(1-\gamma)}{1-\gamma\xi} < 1$ ; output-material  $u$  elasticity

Analytical expressions



★ Low  $z_d$ s pay higher prices

# Theory: Dynamics

- Upstream firms measure  $U_t$  is exogenous, downstream firms measure  $D_t$  is endogenous; downstream firms have exogenous probability  $\delta$  of surviving
- Every period entrants choose to enter downstream and pay  $c_d^e$
- After paying costs, productivity is revealed, and firms decide to produce or exit
  - ★ Productivity evolves exogenously according to a Markov process (Hopenhayn, 1992), with a conditional distribution  $F(z'_d|z_d)$
  - ★ Every period, conditional on past productivity, firms receive a productivity shock (we assume it remains Pareto)
  - ★ Expected discounted profits are increasing in the current firm productivity  $z_d$

# Theory: Dynamics, value functions

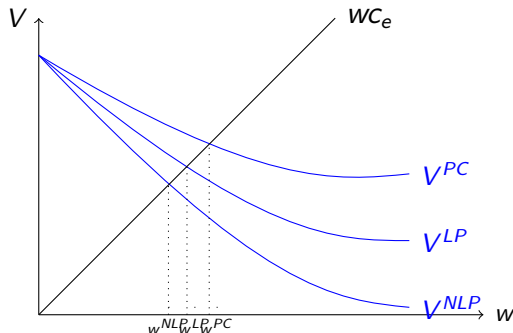
- With no discounting, value functions for entrants in a steady state are:  
 $V_{dt} = E[\pi_{dt}]$  Analytical expressions

- In steady state, the zero profit condition for entry implies:  $\frac{V}{w} = c_e$

- The results is based on assuming :  
 $K > \frac{1}{1-\alpha}$

★  $w^{NLP} < w^{LP} < w^{PC}$

- ★ Firm value is smaller under nonlinear prices

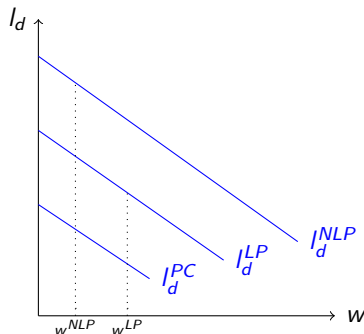


# Theory: Dynamics, GE labor allocations

- Downstream firms use more labor in a non-linear price setup relative to linear prices and perfect competition setups Analytical expressions

- Homologous result for upstream:

$$I_u^{PC} < I_u^{LP} < I_u^{NLP}$$



- ★ At any wage,  $I_d^{PC} < I_d^{LP} < I_d^{NLP}$ , but recall  $w^{NLP} < w^{LP} < w^{PC}$
- ★ Firms use more labor under nonlinear prices

# Theory: Dynamics, mass of firms

- Labor market clearing condition (recall  $M_u$  exogenous):

$$\begin{aligned} 1 &= M_d(1 - \delta)c_e + M_d \underbrace{\iint l_{du} dD dU}_{L_d} + M_u \underbrace{\int l_u dU}_{L_u} \\ &= M_d[(1 - \delta)c_e + L_d] + M_u L_u \end{aligned}$$

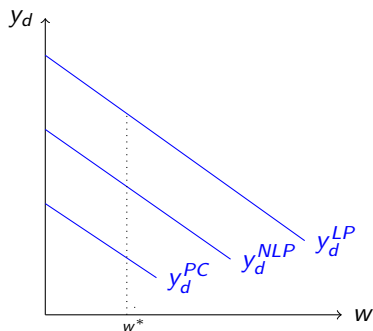
- We have that  $I_d^{PC} < I_d^{LP} < I_d^{NLP} \implies L_d^{PC} < L_d^{LP} < L_d^{NLP}$
- Hence,  $M_d^{LP} > M_d^{NLP}$ 
  - ★ The mass of firms downstream in steady state is smaller in a non-linear price setup relative to linear prices and perfect competition setups

# Theory: Dynamics, downstream firms output

- At any wage,  $y_d^{PC} < y_d^{NLP} < y_d^{LP}$
- Each downstream output produce more under non-linear-prices relative to linear prices

Analytical expressions

- But as  $M_d^{LP} > M_d^{NLP}$ ; we can show that  $M_d^{LP} Y^{LP} > M_d^{NLP} Y^{NLP}$



- ★ Total output and consumption are smaller in a nonlinear setup relative to a linear price setting.
- ⇒ Welfare is smaller in nonlinear pricing setups relative to linear price setups

# Next steps

- Develop on a fully quantitative model
  - ★ Calibrate all parameters using Chilean data
- Main challenges to align the theory with data
  1. Supply chain with arbitrary number of ledgers
  2. Sellers strategic interaction competing in second degree price discrimination (eg. supply function equilibrium)

# Appendix



## Theory: Final demand [Go Back](#)

- The representative consumer preferences over a final goods aggregator ( real GDP)

$$\sum_{t=0}^{\infty} \beta^t \log(C_t), \quad C_t = \int y(z_d) dD_t ,$$

$C_t$  is a final good aggregator,  $0 < \beta < 1$  is the discount factor,  $y(z_d)$  is downstream firm of productivity  $z_d$  and  $D_t$  is downstream firms measure

- The representative consumer owns the firms and receives profits and capital rents, which are reinvested to support capital accumulation facing a budget constraint:

$$P_t C_t + K_{t+1} - (1 - \psi) K_t = \Pi_t + R_t K_t + W_t,$$

$P_t = 1$  is the price of the final good aggregator; the representative consumer offers labor at wages  $W_t$ .  $K_t$  is aggregate capital, with price  $R_t$ ,  $\psi$  is the depreciation rate. Aggregate profits  $\Pi_t$  are the sum of all firms' profits.

# Non-linear price solution based on Mussa and Rosen (1978)

► Go Back

- Each downstream firm price menu can be pinned down from the following first-order condition (marginal product equals marginal cost):

$$\underbrace{z_u Y'_{z_u}(q_j(z_d))}_{MP_{qz_d}} = c_u \underbrace{\frac{z_d}{z_d - [h(z_d)]^{-1}}}_{\text{Separation cost}}, \quad \forall z_d$$

- $h(z_d)$  is the hazard rate; the measure of firms above productivity  $z_d$
- Marginal product equals marginal cost, but the marginal cost includes a shadow cost of separating downstream firm types
- Downward quantity distortion except the highest-productivity one
- To ensure that firms with higher productivity are still willing to purchase their targeted bundle, unit price decreases with  $z_d$ .
- The higher  $h(z_d)$ , the larger the measure of firms above  $z_i$ ; the lower the separation cost

## Theory: Allocations [▶ Go Back](#)

- If downstream firms productivities have a Pareto distribution with shape parameter  $K$ , allocations to downstream firms  $m_{du}$ , transfers  $T(z_u, z_d)$  and downstream profits  $\pi_d$  are: [solution details](#)

$$m_{du} = \left[ \frac{\tilde{A} \tilde{z}_{du}^\alpha z_d}{c} \frac{K-1}{K} \right]^{\frac{1}{1-\alpha}}$$

$$T(z_u, z_d) = \tilde{A} \tilde{z}_{du} \left[ \tilde{z} m_{du}^\alpha - \left( \frac{\tilde{A} \tilde{z}_{du}^\alpha}{c_d} \frac{K-1}{K} \right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) (z_d^{\frac{\alpha}{1-\alpha}} - \underline{z}_d^{\frac{\alpha}{1-\alpha}}) \right]$$

$$\pi_d = \left( \int_{u_1}^{u_U} \tilde{z}_{du} dU \right)^{\frac{1}{1-\alpha}} (\tilde{z} \tilde{A})^{\frac{1}{1-\alpha}} \left( \frac{c_u}{\alpha^2} \right)^{\frac{\alpha}{\alpha-1}} (1-\alpha)$$

# Theory: Linear prices and perfect competition [Go Back](#)

## Linear prices

- Based on downstream firm demands, upstream firms will offer the following quantity, prices, and markup:

$$m_{du}^{LP} = \left( \frac{c_u}{\tilde{z}\tilde{A}\tilde{z}_{du}\alpha} \frac{1}{\alpha} \right) , \quad p_{du}^{LP} = \frac{c_u}{\alpha} , \quad \mu_{du}^{LP} = \frac{1}{\alpha}$$

## Perfect competition

$$m_{du}^{PC} = \left( \frac{c_u}{\tilde{z}\tilde{A}\tilde{z}_{du}\alpha} \right) , \quad p_{du}^{PC} = c_u , \quad \mu_{du}^{PC} = 1$$

## Theory: Dynamics, value functions [Go Back](#)

$$V_d^{PC} = \tilde{A} \left( \int_{u_1}^{u_U} \tilde{z}_{du} dU \right)^{\frac{1}{1-\alpha}} \left( \frac{c_u}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} (1-\alpha) \frac{K}{K - \frac{1}{1-\alpha}} z^{\frac{1}{1-\alpha}}$$

$$V_d^{LP} = V_d^{PC} \alpha^{\frac{\alpha}{1-\alpha}}$$

$$V_d^{NLP} = V_d^{PC} \left( \frac{K}{K - \frac{1}{1-\alpha}} - 1 \right) \left( \frac{K-1}{K} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{K}{K - \frac{1}{1-\alpha}} \right)^{-1}$$

## Theory: Dynamics, labor allocations [▶ Go Back](#)

$$I_d^{PC} = \xi \gamma \alpha \tilde{A}^{\frac{1}{\alpha}} \left( \int_{u_1}^{u_U} \tilde{z}_{du} dU \right)^{\frac{1}{1-\alpha}} \left( \frac{c_u}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} \frac{K}{K - \frac{1}{1-\alpha}} \underline{z}^{\frac{1}{1-\alpha}}$$

$$I_d^{LP} = I_u^{PC} \alpha^{\frac{\alpha}{1-\alpha}}$$

$$I_d^{NLP} = I_u^{PC} \left( \frac{K-1}{K} \right)^{\frac{\alpha}{1-\alpha}}$$

## Theory: Dynamics, output [Go Back](#)

$$y_d^{PC} = \tilde{A}^{\frac{1}{\alpha}} \left( \int_{u_1}^{u_U} \tilde{z}_{du} dU \right)^{\frac{1}{1-\alpha}} \left( \frac{\alpha}{c_u} \right)^{\frac{\alpha}{\alpha-1}} \frac{K}{K - \frac{1}{1-\alpha}} z^{\frac{1}{1-\alpha}}$$

$$y_d^{LP} = y_u^{PC} \alpha^{\frac{\alpha}{1-\alpha}}$$

$$y_d^{NLP} = y_u^{PC} \left( \frac{K-1}{K} \right)^{\frac{\alpha}{1-\alpha}}$$