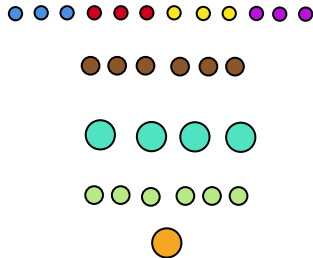
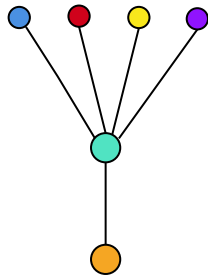


# Production networks inefficiencies

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IO pro-seminar Spring 2023. UCLA

June 7, 2023

# Motivation



## Motivation: Line network example IO Literature

### Upstream firm

Constant marginal cost:  $c_u$

Sells at:  $t_m$

### Midstream firm

Constant marginal cost:  $t_m$

Sells at:  $t_d$

### Downstream firm

Constant marginal cost:  $t_d$

Faces demand:

$$q = a - p$$



**Final consumers**

## Motivation: Line network example (2)

Firms engage in Nash Bargaining to split the surplus, where the buyer's bargaining power is  $b$ , and the seller is  $(1 - b)$ . For simplicity, I assume that the negotiation is only over prices so that equilibrium objects are:

If  $b = \frac{1}{2}$ :

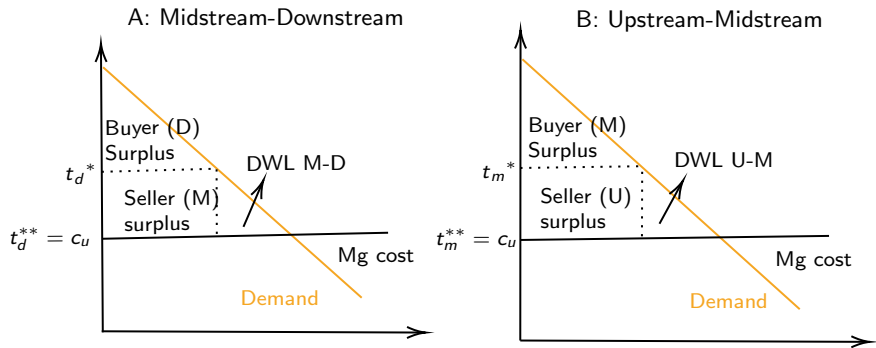
$$t_m^* = \frac{p + 2c_u}{3} \quad ; \quad t_d^* = \frac{2p + c_u}{3}$$
$$q^* = \frac{a - c_u}{4} \quad ; \quad p^* = \frac{3a + c_u}{4}$$

While if  $b = 1$  (buyer has all bargaining power, or integrated firms case):

$$t_m^{**} = t_d^{**} = c_u$$
$$q^{**} = \frac{a - c_u}{2} \quad ; \quad p^{**} = \frac{a + c_u}{2}$$

Then  $q^{**} > q^*$  and  $p^{**} < q^*$

# Motivation: Line network example DWL



Total inefficiency:  $DWL\ M-D + DWL\ U-M$

# This work

**What are the drivers of network formation inefficiencies? To provide an answer I will try to:**

- (a) Characterize the firm's bargaining process at the product level.
- (b) Estimate multi-product production functions.
- (c) Describe production network formation.

## **Goals**

- (a) For a given network structure, assess the economy's overall performance affected by heterogeneous bargaining weights.
- (b) Describe how bargaining-generated inefficiencies affect production network formation, and thus aggregate efficiency.

# Related Literature

## **Bargaining**

Collard-Wexler et al (2019), Ho and Lee (2019), Grossman et al (2023)

This work: Firm-Product level bargaining weights.

## **Multi-product Production function estimation**

De Loecker, Goldberg, Khandelwal, and Pavcnik (2016), Dhyne, Petrin, Smeets, and Warzynski (2023)

This work: Multi-product firms with transaction-specific markups.

## **Production networks formation**

Oberfield (2018), Acemoglu and Azar (2020), Dhyne et al. (2023), Arkolakis (2023)

This work: Bargaining

# Electronic invoice Example

KITCHEN  
CENTER

**KITCHEN CENTER SPA**  
IMPORTACIÓN Y DISTRIBUCIÓN DE ELECTRODOMÉSTICOS

FDV SIMPLE COOK Cuisinart GUBBLI Delonghi Janssen SUPERFOOD Arista LOPRA

Casa Matriz:  
Sucursales:  
Casa Costanera:  
Mall Parque Arauco:  
Mall Plaza Los Dominicos:  
Mall Borneaventura:  
Mall Plaza La Serena:  
Mall Harna Arauco:  
Outlet Park Vial:  
Mall Plaza Maule:  
Concepción:  
Temuco:  
Mall Fashion Temuco:  
Servicio Técnico:  
Centro de Distribución:  
Vila del Mar:  
Alto Las Condes:  
Outlet El Salto:

Av. El Salto 3485, Recoleta, Santiago  
Av. Nueva Costanera 3900, Viña del Mar  
Av. Kennedy 5413 Local 572, Las Condes - Teléfono: (56-2) 24117777 - Fax: (56-2) 24117711  
Padre Hurtado Sur 879, Local A2080/2076, Las Condes - Teléfono: (+56 2) 24117738  
San Ignacio 900 Local 12, Quilicura - Teléfono: (+56 2) 24117769  
Av. Huasthuat 105 BA 190, La Serena - Teléfono: (56-2) 24117732  
Av. Libertad 1348, Local 10-11/102, Villa del Mar - Teléfono: (56-2) 24117767/68  
Camino Internacional 2440 Local 72, Villa del Mar  
Circunvalación 1055, Local 226/227, Talca - Teléfono: (56-2) 24117748  
Peicavi 2567, Local 2, Talca - Teléfono: (56-2) 2411 7716 / 17  
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Bulnesen Ortega 01700, Local 1168-170, Temuco - Teléfono: (+56 2) 24117714  
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Av. El Salto 3480, Recoleta, Santiago.

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**FECHA EMISIÓN : 01/08/2022**

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Comuna : Providencia

Ciudad : Santiago

FECHA EMISIÓN :

FECHA VENCIMIENTO : 03/08/2022

TIPO DESPACHO :

FORMA DE PAGO : Contado

COD. VENDEDOR :

Orden de Venta: 793325

Número de OC:

CÓDIGO	DETALLE	CANTIDAD	PRECIO UNITARIO	PRECIO ÍTEM
13452	Lavaplatos FDV Small Acqua bajo cubierta	1	92.428,57	92.429
14761	Encimera FDV Design 4T GLTX 65 BUT 2.0	1	142.848,74	142.849
14265	Campana Kubli Neu Slider	1	100.831,93	100.832
19110	Horno FDV Design	1	201.672,27	201.672
13377	Lavavajillas PS FDV Element 14C	1	243.689,07	243.689
14917	Grifería FDV CONICA FLEX	1	84.025,21	84.025
10232	Transporte - Providencia	1	15.529,41	15.529

info@kitchencenter.cl



## Firm level Networks stats

In June 2018, there were 322,971 sellers and 1,582,618 buyers.

	In-degree (N suppliers)	Out-degree (N clients)
Mean	6.30	30.09
Median	2	2
Sd	18	731
p1	1	1
p25	1	1
p75	6	7
p90	16	28
p95	24	67
p99	58	344
Max	3,188	133,019

The distribution of outdegrees is much more unequal than indegrees, consistent with the properties of the U.S. input-output tables, as documented in Acemoglu et al. (2012).

## Firm level Product stats

In June 2018, there were 11,196,897 different products sold.

	N products sold	N products bought
Mean	41.22	43.83
Median	4	6
Sd	536	188
p1	1	1
p25	1	2
p75	12	26
p90	45	111
p95	116	216
p99	626	535
Max	92,142	101,360

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# Bargaining process

## Challenges

- Model firms payoffs.
- Describe contracts.
- Identify bargaining actions.

**Today presentation:** I will try to inform how to face these challenges using standard IO techniques and Chilean data.

## Link formation bargaining Game (1)

Firm  $i$  generates its total output  $Q_i$  by choosing capital  $K_i$ , labor  $L_i$ , its set of suppliers  $S_i$ , and intermediate goods  $X_i$ . Its Hicksian Productivity is  $A(S_i)$ , thus  $Q_i = F(K_i, L_i, S_i, X_i, A(S_i))$ .

Firms choose which providers to buy from ( $S_i$ ). A contract,  $\mathbb{C}$  is a combination of actions,  $a_i(S_i)$ , and transfers (positive or negative) from firm  $i$  to firm  $j$   $t_{ij}(S_i)$ .

Downstream profits (buyers profit) is  $\pi_i^D(\mathbb{C})$ , while upstream profits (seller profits) are  $\pi_j^U(\mathbb{C})$ .

The set of contracts with non-negative gains to trade for both the seller and the buyer is (the contract value of disagreeing is  $\mathbb{C}_0$ ):

$$\begin{aligned}\mathbb{C}_{ij}^+ = & \{ \pi_i^D(\mathbb{C}_{ij}, \mathbb{C}_{-ij}) - \pi_i^D(\mathbb{C}_0, \mathbb{C}_{-ij}) \geq 0 \} \\ & \cap \{ \pi_i^U(\mathbb{C}_{ij}, \mathbb{C}_{-ij}) - \pi_j^U(\mathbb{C}_0, \mathbb{C}_{-ij}) \geq 0 \}\end{aligned}$$

## Link formation bargaining Game (2)

Bilateral actions  $a_{ij}$ , include at least prices,  $p_{ijg}$  and quantities,  $q_{ijg}$ , agreements for different goods  $g$  that firms could trade.  $\phi$  denotes the outside option of nontrading.  $b_{ij} \in [0, 1]$  represents the buyer's bargaining weight.

Profits from any bilateral relationship will then be ( $k$ : buyers):

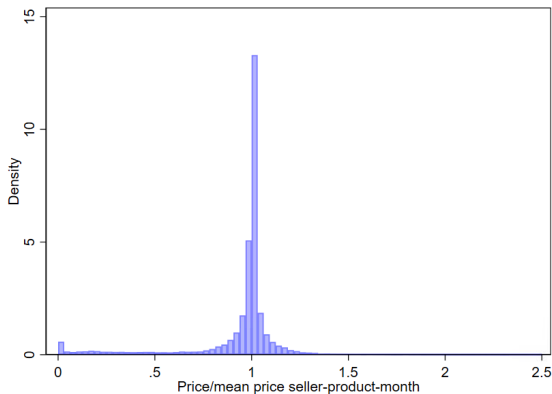
- Buyer  $\pi_i^D = \sum_g \sum_k x_{ikg}(p_{ikg} - c_{ig})$
- Seller:  $\pi_{ij}^U = \sum_g \sum_k x_{jig}(p_{jig} - c_{jg})$

Assume for simplicity that firms engage in Nash equilibrium in Nash bargains (Nash-in-Nash), meaning that each pair of firms negotiates its actions by taking all other actions as given. Hence, the total surplus of the bilateral relationship can be expressed as:

$$\text{Max}_{\{a_{ij}\}} \underbrace{\left[ \pi_i^D - \phi_i \right]^{b_{ij}}}_{\text{buyer surplus}} \underbrace{\left[ \pi_{ij}^U - \phi_j \right]^{1-b_{ij}}}_{\text{seller surplus}}$$

# Over which actions do firms bargain? (1)

**Price variation by seller-product-month**



## Over which actions do firms bargain? (2)

Non-linear pricing? Reg Log P on Log Q

	(1)	(2)	(3)	(4)
Log q	-0.1048	-0.0579	-0.1621	-0.1037
(SE)	0.00008	0.00006	0.00009	0.00007
Seller FE	Yes	Yes	Yes	Yes
Product FE	Yes	Yes	Yes	Yes
Buyer FE	No	No	Yes	Yes
Q > 1	No	Yes	No	Yes
R <sup>2</sup>	0.9112	0.9489	0.9246	0.9655
N obs	72,178,033	44,656,779	71,809,764	44,496,689



## Over which actions do firms bargain? (3)

Data suggests that:

- A given seller charges different prices of the same product to different buyers.
- Non-linear prices explain between 6 and 16% of the price changes.
- I assume the remaining share comes from buyer and seller market power.

## Link formation bargaining Game: Surplus share (1)

To inform the bargaining weights, the first step is to make the simplest approach possible by setting the following simplified assumptions:

- 1 Buyers choose the quantity of each intermediate input.
- 2 There is only one action in bilateral trade, price agreement, one for each good traded:  $a_{ij} = P_{ij} = (p_{ijg=1}, p_{ijg=2}, \dots, p_{ijg=G})$ .
- 3 Prices include all transfers; recharges, discounts or any side payment:  $t_{ij} = P_{ij}$
- 4 Outside option of nontrading is set to be zero:  $\phi_i = \phi_j = 0$ .

The total surplus maximization problem is:

$$\text{Max}_{\{P_{ij}\}} \underbrace{\left[ \pi_i^D \right]^{b_{ij}}}_{\text{buyer surplus}} \underbrace{\left[ \pi_{ij}^U \right]^{1-b_{ij}}}_{\text{seller surplus}}$$

## Link formation bargaining Game: Surplus share (2)

Note that the difference between prices and marginal costs, product by product, will include the buyer market power downstream (markups) and also the buyer market power upstream (markdown).

If prices and marginal costs are observed, it is possible to recover product-specific bargaining weights as:

$$b_{gij} = \frac{\pi_{ig}^D}{\pi_{ig}^D + \pi_{gij}^U}$$

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## Multi-product firms Production function

Following Dhyne, Petrin, Smeets & Warzynski (WP 2022).

A given firm produce  $M$  outputs ( $q = q_1, \dots, q_M$ ) using  $N$  inputs ( $x = x_1, \dots, x_N$ ). For good  $g$  produced by the firm, let the output production of other goods be denoted by  $q_{-g}$ . For any  $(q_{-g}, x)$  define the the following transformation function:

$$q_g^* = F_g(q_{-g}, x) = \max q_g \mid (q_g, q_{-g}, x)$$

Assuming a Cobb-Douglas PF with three factors, a multi-product firm will produce output  $g$  as (lower case variables denote logs):

$$q_{gt} = \beta_0^g + \beta_K^g k_t + \beta_L^g l_t + \beta_M^g m_t^j + \gamma_{-g}^g q_{-gt} + \omega_{gt}$$

# Multi-product firms Marginal Costs (1)

Firms minimize their variable cost function:

$$\min_{\{x\}} [P_1, \dots, P_N] \cdot x \quad \text{s.t.} \quad f_j(q_{-g^*, x, K, \omega}) - q_g^* \geq 0$$

The FOCs yield:

$$P_n = \lambda_g \frac{\partial f_j(q_{-g^*, x, K, \omega})}{\partial x_n} \quad \forall n = 1, \dots, N$$

Is it possible to solve for output  $g$  marginal cost as the expenditure on input  $n$  divided by the output elasticity of input  $n$  times the quantity of output  $g$  that is produced

$$\lambda_g = \frac{P_n}{\frac{\partial f_g(q_{-g^*, x, K, \omega})}{\partial x_n}} = \frac{P_n x_n^*}{\beta_n^g q_g^*}$$

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## Model Based on Acemouglu and Azar, 2020

A representative **household** utility is  $u(C_1, \dots, C_n)$  and provides  $L$  labor units at a price  $w$ .

**Firm**  $i$  generates its total output  $Q_i$  by choosing capital  $K_i$ , labor  $L_i$ , its set of suppliers  $S_i$  and intermediate goods  $X_i$ . Its Hicksian Productivity is  $A(S_i)$ , thus  $Q_i = F(K_i, L_i, S_i, X_i, A(S_i))$ .

Firms are assumed to be cost minimizers and solve its problem in two steps. Firms first choose  $K_i, L_i, X_i$  taking  $S_i$  as given, and second they choose their set of suppliers  $S_i$ .



## Cost minimization (1)

In the **first step**, firms determine the unit cost function ( $U_i$ ) where  $P$  is the vector grouping input prices  $P_j$ .

$$U_i(S_i, A_i(S_i), P) = \min_{\{X_i, L_i, K_i\}} r_i K_i + w_i L_i + \sum_{j \in S_i} P_j X_{ij}$$
$$\text{s.t. } Q_i = F(K_i, L_i, S_i, X_i, A(S_i))$$

$U_i$  is conditioned on the set of inputs,  $S_i$  because this determines which prices matter for costs and captures the dependence of the technology.

## Cost minimization (2)

The **second step** of cost minimization is to choose the combination of inputs suppliers ( $\approx$  technology) to minimize the unit cost function,  $U_i$ :

$$S_i^* = \arg \min_{\{S_i\}} U_i(S_i, A_i(S_i), P)$$

Given this cost function and distortion  $\mu_i$ , the equilibrium price of input  $i$  is:

$$P_i^* = (1 + \mu_i) U_i(S_i^*, A_i(S_i^*), P)$$

# Equilibrium

An equilibrium consist of  $(P^*, S^*, C^*, L^*, K^*, X^*, Q^*)$ , such that:

- (a) Price is equal to marginal cost, inclusive of distortions.
- (b) Consumer maximization.
- (c) Cost minimization
- (d) Market clearing.

## Example: CES production function

CES production function with input-specific productivities:

$$Q_i = A_i(S_i) \left[ \alpha_{K_i} K_i^{\frac{\sigma-1}{\sigma}} + \alpha_{L_i} L_i^{\frac{\sigma-1}{\sigma}} + \sum_{j \in S_i} \alpha_{ij} (A_{ij} X_{ij})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Unit cost function:

$$U_i = \frac{1}{A_i} \left[ \alpha_{K_i}^{\rho} r^{1-\sigma} + \alpha_{L_i}^{\rho} w^{1-\sigma} + \sum_{j \in S_i} \alpha_{ij}^{\rho} \left( \frac{p_{ij}}{A_{ij}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

Cost effectiveness,  $\frac{p_{ij}}{A_{ij}}$ , will shape the process of links formation. Firms will choose a set of suppliers by balancing the high productivity and low prices tradeoff.

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## Transaction level distortions to firm-level distortions

Naive approach: (Seller firm  $i$ , buyer firm  $k$ , product  $g$ )

**Step 1:** Transaction level markup:  $\mu_{ikg} = \frac{p_{ijg}}{mc_{ig}}$

**Step 2:** Product level markup:  $\mu_{ig} = \sum_k \alpha_{ikg} \mu_{ikg}$

where  $\alpha_{ikg} = \frac{\text{sales}_{ikg}}{\text{sales}_{ig}}$

**Step 3:** Firm level markup:  $\mu_i = \sum_g \alpha_{ig} \mu_{ig}$

where  $\alpha_{ig} = \frac{\text{sales}_{ig}}{\text{sales}_i}$

Markups are the result between buyer and seller power bargaining.

## Firm level distortions to economy-wise distortions (1)

Based on Baqaee & Farhi, QJE 2020

**Revenue-based input-output matrix.**  $ij^{th}$  element is the expenditure of firm  $i$  on inputs from firm  $j$  as a share of firm  $i$  total revenue:

$$\Omega_{ij} \equiv \frac{p_j x_{ij}}{p_i y_i}$$

**Cost-based input-output matrix.** Captures the change in the marginal cost function of firm  $i$  ( $C_i$ ) when the price of firm  $j$  changes. Using Sheppard's Lemma is possible to express it as the expenditure of firm  $i$  on inputs from firm  $j$  as a share of firm  $i$  total costs:

$$\tilde{\Omega}_{ij} \equiv \frac{\partial \log C_i}{\partial \log p_j} = \frac{p_j x_{ij}}{\sum_{k=1}^M p_k x_{ik}}$$

Both are related by **markup harmonic mean matrix**.

$$\tilde{\Omega} = \text{diag}(\mu) \Omega$$

$\text{diag}(\mu)$  is a diagonal matrix with  $ii^{th}$  element:  $\frac{\# \text{firms}}{\sum_t (\mu_{it})^{-1}}$

## Firm level distortions to economic wise distortions (2)

**Leontief inverse matrix** capture both the direct and indirect firm exposures through the production networks:

$$\tilde{\Psi} \equiv (I - \tilde{\Omega})^{-1} = I + \tilde{\Omega} + \tilde{\Omega}^2 + \dots$$

**Sales shares**

$$b_{it} = \frac{p_{it} c_{it}}{\sum_{j=1}^N p_{jt} c_{jt}}$$

**Cost-based Domar Weight**; suppliers' relevance in final goods demand, both directly and through production networks.

$$\tilde{\Lambda}' \equiv b' \tilde{\Psi}$$



# Firm level distortions to economic wise distortions (1)

$$\underbrace{\Delta \log Y_t - \tilde{\Lambda}'_{t-1}(\Delta \log L_t + \Delta \log K_t)}_{\Delta \text{ Distorted Solow Residual}} \\ \approx \underbrace{\tilde{\lambda}'_{t-1} \Delta \log A_t}_{\Delta \text{ Technology}} - \underbrace{\tilde{\lambda}'_{t-1} \Delta \log \mu_t - \tilde{\Lambda}'_{t-1}(\Delta \log Sh_t^K + \Delta \log Sh_t^L)}_{\Delta \text{ Allocative Efficiency}}$$

- Distorted Solow Residual: Weighs factor changes by the cost-based Domar weight ( $\tilde{\Lambda}$ ) rather than by its share in aggregate income (Hulten theorem)
- Cost-based Domar weight corrects for the fact that consumers do not value distortions but rather real use of resources

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# Data Sources

- ① Aggregate production variables (monthly, 2005 to date-1):  
Sales, materials, investment, wagebil, employment headcount.
- ② Capital stock (yearly, 2005 to date-1):  
Capital stock using perpetual inventory methods combining yearly capital stock with monthly investment.
- ③ F2F electronic receipts (transaction level, 2014-to date)  
Price, quantity, and product.

# Data Cleaning

- Final sample does not include firms with a missing variable of sales, capital, wage bill, or materials.
- WinzORIZED labor, capital, and materials shares over sales at 1% of both distribution tails.
- Firms with negative value added (sales minus materials), less than two workers, or capital less than 10.000 CLP (USD 15) are excluded.

Around 120,000 firms a year in the final sample.

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## Motivation: Line network example (2)

Firms engage in Nash Bargaining to split the surplus, where the buyer's bargaining power is  $b$ , and the seller is  $(1-b)$ . For simplicity, I assume that the negotiation is only over prices so that the Nash products are:

$$\text{U-M bargaining: } \max_{\{t_m\}} \underbrace{(t_d - t_m)^b}_{\text{M surplus}} \underbrace{(t_m - c_u)^{1-b}}_{\text{U surplus}}$$

$$\text{FOC result: } (1-b)(t_d - t_m) = b(t_m - c_u) \quad (1)$$

$$\text{M-D bargaining: } \max_{\{t_d\}} \underbrace{(p - t_d)^b}_{\text{D surplus}} \underbrace{(t_d - t_m)^{1-b}}_{\text{M surplus}}$$

$$\text{FOC result: } (1-b)(p - t_d) = b(t_d - t_m) \quad (2)$$

## Motivation: Line network example (3)

The downstream firm sells to final consumers and chooses prices to maximize their profits:

$$\text{Max}_{\{p\}} \pi = (p - mc)q = (p - t_d)(a - p)$$

From FOC:  $p^* = \frac{a+t_d}{2}$  and  $q^* = \frac{a-t_d}{2}$