Nonlinear Prices in Supply Chains

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Motivation I

- Huge literature studying the positive and normative effects of firm-level distortions on macroeconomic outcomes.
- Huge IO literature on price discrimination in imperfect competition settings.
- Big, recent literature on supply chains with firm-to-firm transactions.

We integrate the three to evaluate how price discrimination in supply chains affects resource allocations, firm profit distribution, firm entry, and welfare.

Motivation II: Price discrimination

- Under linear pricing, markup works as a uniform tax in production
    - Generates resource misallocation due to heterogeneous factor Marginal Revenue Products
- Under price discrimination, average price is not fully allocative
    - Markups are no longer a sufficient statistic for resource misallocation
- However, firm profits distribution is distorted, affecting firm entry
- Nontrivial welfare effects relative to linear prices: Increase in firm-level production (smaller tax on production) vs lower firm entry (entry wedge)

This paper: What we do

- Descriptive evidence: To inform the model
  - Firm-to-Firm transactions for Chile descriptive statistics
  - Provide evidence of price discrimination focused (second and third-degree)
- Theory: Based on descriptive evidence
  - Supply chain model with endogenous market structure
  - Endogenous firm participation in steady state
- Model quantification and counterfactuals
  - Calibrate the model using Chilean data
  - Three scenarios: Efficient, linear, and nonlinear prices.
  - Compare welfare and its drivers

This paper: What we find

Descriptive Evidence

- Price discrimination is prevalent in the data; second outweighs third-degree

Model: Nonlinear vs Linear Prices

- Nonlinear prices increase expected firm-level output but decrease firm entry
- Increase in firm-level outputs dominates; hence welfare is higher under nonlinear prices
- Under price discrimination, markups' negative effects on welfare are around 65% relative to linear price setups

Literature

Aggregate market power effects

Hsieh and Klenow (2014), Baqaee and Farhi (2021), Edmond, Midrigan, and Xu (2023), De Loecker, Eeckhout, and Mongey (2022), Boehm, Oberfield, South and Waseem (2024)

Heterogenous firm-level market power and aggregate implications

Peter and Bornestein (2024), Burstein, Cravino, and Rojas (2024)

Agenda

1. Descriptive evidence
2. Model
3. Model Quantification and Counterfactuals
4. Conclusion

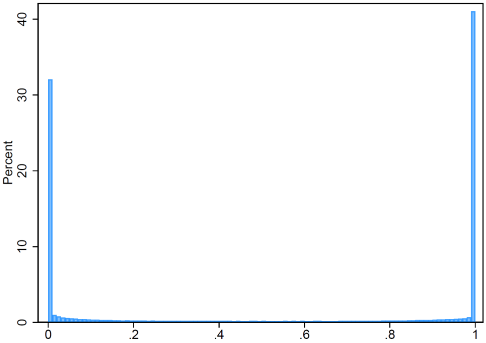
Data sources

Invoice example

- Invoice transactions for the universe of Chilean formal firms for 2018
  - Around 1.3 billion transactions
  - More than 10 million different products
  - Data on prices and quantities for every product transacted
- Merged with the firm's accounting balance sheet data
  - Sales, materials, investment, 6-digit industry
  - Employer-employee: Wages, headcount employees
  - Capital stock and investment

Sales partition: Number of firms

$$X_i = \begin{cases} 0 & \text{if all sales go to final consumers} \\ 1 & \text{if all sales go to other firms} \end{cases}$$



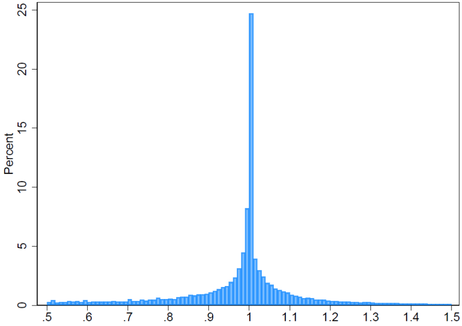
- More than 70% of firms sell only to final consumers or to other firms

Sales partition: Sales shares (excluding exports)

Sector (sales)	**All to final consumer**	**All to other firms**
Firm population	0.13	0.71
Agriculture (2%)	0.04	0.65
Mining (1%)	0.44	0.25
Manufacturing (15%)	0.05	0.77
Utilities (3%)	0.20	0.51
Construction (8%)	0.03	0.88
Retail and Wholesale (32%)	0.13	0.70
Transport (10%)	0.17	0.72
Financial Services (18%)	0.20	0.67
Real Estate Services (1%)	0.27	0.37
Business Services (7%)	0.09	0.81
Personal Services (2%)	0.70	0.10

Price dispersion

- Seller i, product g price and mean: p_{ig} , $\bar{p}_{ig}$
- $\mu_{ig} = \frac{p_{ig}}{\bar{p}_{ig}}$



- Variance of $\log \mu_{ig} = 0.65$ (excluding products with one transaction)
- No price dispersion in around 25% of transactions

Price observable determinants

$$\ln \; p_{ijgm} = \ln G + \ln \psi + \ln \xi_m + \ln \omega_{ijgm}$$

	(1)	(2)	(3)
FE Seller-product	Yes	No	No
FE Seller-product-buyer	No	Yes	No
FE Seller-product-quantity	No	No	Yes
FE month	Yes	Yes	Yes
Residual variance	0.46	0.45	0.38
Observations	1.2 billions	1.2 billions	1.2 billions
Adjusted R^2	0.8835	0.8883	0.9035
Partial adjusted R^2	–	0.041	0.172

- Where G is the price mean across *ijgm* and ψ are different fixed effects
- Quantity fixed effects, rather than buyer fixed effects, account for a larger portion of unexplained price variance

Price variance decomposition

$$\ln p_{ijgm} = \ln G + \ln \psi_{ig} + \ln \theta_j + \ln \gamma_q + \ln \xi_m + \ln \epsilon_{ijgm}$$

	$\frac{\text{var}(\theta_j)}{\text{var}(\theta_j+\gamma_q)}$	$\frac{\text{var}(\gamma_q)}{\text{var}(\theta_j+\gamma_q)}$	$\frac{2\text{cov}(\theta_j,\gamma_q)}{\text{var}(\theta_j+\gamma_q)}$
Main	0.39	0.77	-0.16
Lower bound	0.23	0.61	
Upper bound	0.55	0.93	

- ψ_{ig}, θ_j, and γ_q are seller-product, buyer, and quantity (1,000 quantiles) fixed effects
- Variation in quantities explains a higher share of the price variance on observables, but buyer dispersion is relevant too

Quantity discounts evidence

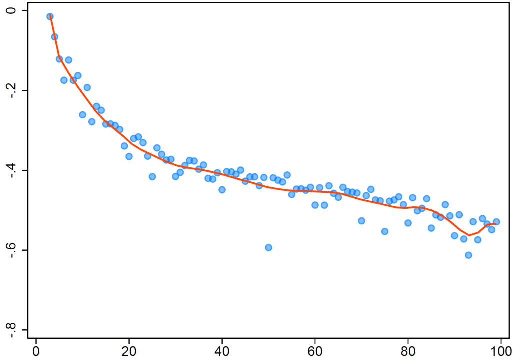
$$\ln p_{ijgm} = \beta_0 + \beta_1 \ln q_{ijg} + \ln \psi_{ijg} + \ln \xi_m + \ln \epsilon_{ijgm}$$

	(1)	(2)	(3)	(4)
ln q	-0.248	-0.188	-0.213	-0.224
Standard errors	(0.00002)	(0.00025)	(0.00005)	(0.00004)
FE Seller x product x buyer	Yes	Yes	Yes	Yes
FE month	Yes	Yes	Yes	Yes
High price products	No	Yes	No	No
Ex-manufacturing products	No	No	Yes	No
Ex-Retail and wholesale	No	No	No	Yes
Observations (millions)	1246	61	932	480
Adjusted R2	0.8981	0.7007	0.7561	0.7854

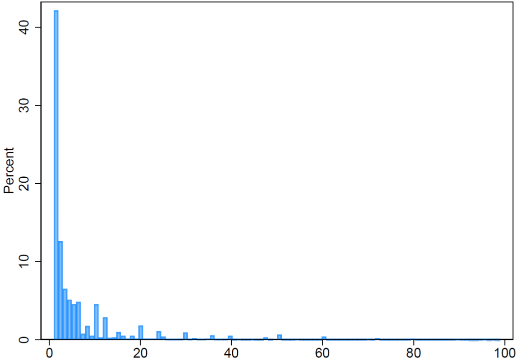
- Assuming we observe equilibrium objects in the data, unit price decreases with quantity By industry

Quantity discounts by quantity bins (1-100, 98% of transactions)

Panel A. Quantity discounts fixed effects



Panel B. Quantity traded histogram



- Quantity discounts are largest for initial units, and the marginal discount diminishes with quantities. Deciles regression

Quantity-price menus

- Under quantity-price menus: fix quantity, price variance should be zero
- Group X:{Seller-Product-Month + Quantity, Buyer}

$$C_X = \frac{\text{Standard deviation} p_X}{\text{median} p_X}; \quad X \in \{\text{Seller-Product-Month + Quantity, Buyer}\}$$

	C_X				
	p10	p25	Median	p75	p90
Seller - product - month	0.00	0.04	0.13	0.54	4.11
Seller - product - month - quantity	0.00	0.01	0.08	0.35	2.06
Seller - product - month - quantity - Buyer	0.00	0.00	0.00	0.04	0.45

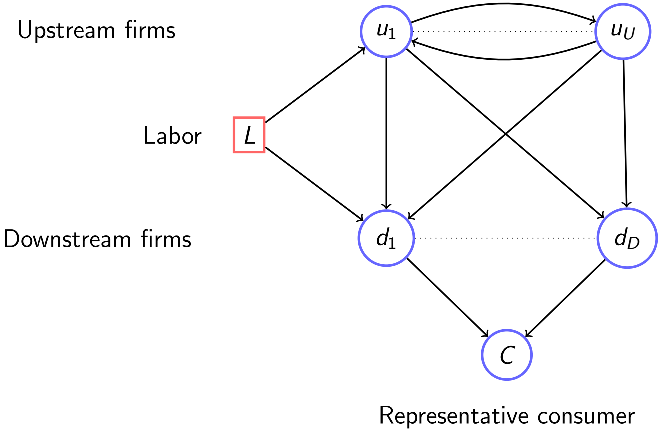
- C_x relatively small when controlling for quantity-buyer: Suggest quantity menus and quantity-seller menus

Recap of Empirical Evidence

1. Firms sell predominantly to other firms or final consumers
2. Evidence of quantity discounts (main factor) and buyer dispersion explaining price variance
3. Marginal discount decreases with quantity
4. Quantity and quantity seller menus seem prevalent (2on+3rd degree price discrimination)

Agenda

Supply Chain Structure



Setup: Representative Consumer

- CES aggregator on downstream varieties

$$Y = \left(\int_{z_0} q(z)^{\frac{\sigma-1}{\sigma}} M_z \mu(z) dz \right)^{\frac{\sigma}{\sigma-1}}$$

- M_z is the mass of downstream firms and $\mu(z)$ is the density of type z firms
- Offer labor and owns firms and receive their profits
- Budget constraint:

$$Y = wL + \Pi^U + \Pi^D$$

Setup: Firms

- Firm partition on Upstream (sells to other firms only) and downstream firms (sell to final consumers only)
- Call z downstream variety and ζ upstream variety productivity, $\gamma \in \{z, \zeta\}$
- Production function:

$$q_\gamma = \gamma \left(l(\gamma)^{\frac{\eta-1}{\eta}} + m(\gamma)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad \gamma \in \{z, \zeta\}$$

- m is a CES bundle of upstream varieties:

$$m(\gamma) = \left(\int_{\zeta_0} m(\gamma, \zeta)^{\frac{\sigma-1}{\sigma}} M_\zeta \mu(\zeta) d\zeta \right)^{\frac{\sigma}{\sigma-1}}$$

- Free entry into both firms groups, M_z, M_ζ are endogenous

Assumptions

1. Upstream firms charge nonlinear prices to either other upstream or downstream firms (2nd degree)
2. Prices schemes to both firm types can differ (3rd degree)
3. Downstream firms charge linear prices with constant markup to the representative consumer with normalized to 1 price
4. Firms productivity follows a Pareto distribution with tail parameter κ_z and κ_ζ
5. Upstream firms take other firms' nonlinear prices as given (no strategic interaction)
6. All firms are connected
7. Total labor is exogenous

Upstream profit maximization problem

- Mechanism Design Problem of Seller Firm:
  - Chooses a transfer T and and quantities to sell m, separately for upstream and downstream firms
  - Faces constant marginal cost

$$\max_{\substack{\{T(z,\zeta),m(z,\zeta)\},\\ \{T(\zeta',\zeta),m(\zeta',\zeta)\}}} \Pi^U = \mathbb{E}_z\left[T(z,\zeta) - c\, m(z,\zeta)\right] M_z + \mathbb{E}_\zeta\left[T(\zeta',\zeta) - c\, m(\zeta',\zeta)\right] M_\zeta$$

- Subject to:
  - Individual Rationality (IR): Firms receive non-negative surplus from buying
  - Incentive Compatibility (IC): Firms self-select into their tailored bundle

Upstream price scheme

Proposition 1: A flat fee and a linear component describe the solution to the mechanism design problem of the seller: solution strategy

$$T(\gamma,\zeta) = F_\gamma(\zeta) + p(\zeta)m(\gamma,\zeta), \quad \gamma \in \{z,\zeta\}$$

Flat fee: Extract surplus of lower (upstream or downstream) type

Linear component: $p(\zeta) = \frac{\rho}{\rho-1}c(\zeta), \quad \rho = \frac{\sigma\kappa}{\sigma-1}$

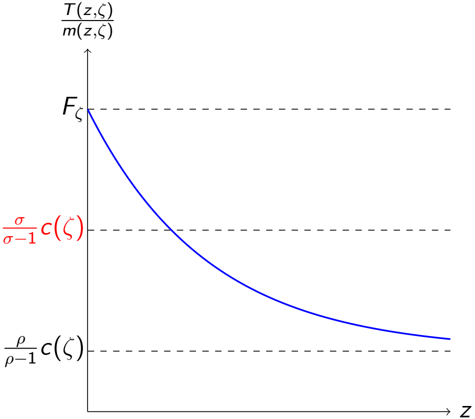
- Each upstream firm can charge two flat fees, one for upstream and one for downstream firms.
- The markup is lower than the linear price markup as $\rho > \sigma$ is a condition for finite output.

Upstream unit price scheme to downstream firms

The unit price paid by downstream firms can be computed as:

$$\frac{T(z,\zeta)}{m(z,\zeta)} = p(\zeta) + \frac{F(\zeta)_z}{m(z,\zeta)}$$

- The flat fee unit price share is decreasing in firm productivity
- Complete market coverage (all types have positive virtual utility details)



General Equilibrium Results: 3 wedges

1. Intensive margin: markup as an output tax
2. Input ratio: labor to materials ratio is distorted because of network structure and markups
3. Extensive margin: Firm creation ratio (upstream to downstream firm masses) is distorted because of firm profits distortions

General Equilibrium Results: Downstream firms NLP vs LP model

In this economy, GDP is given by:

$$Y = \left(\int_{z_0} q(z)^{\frac{\sigma-1}{\sigma}} M_z \mu(z) dz \right)^{\frac{\sigma}{\sigma-1}} \propto \underbrace{M_z}_{\text{Channel 1}} \underbrace{\mathbb{E}_z[q(z)]}_{\text{Channel 2}}$$

- NLP generates higher output per firm because of lower "output tax": $\mathbb{E}_z[q(z)]^{\text{NLP}} > \mathbb{E}_z[q(z)]^{\text{LP}}$ (Theorem)
- On average firms capture a larger share of total demand; lower mass of firms in NLP: $M_z^{\text{NLP}} < M_z^{\text{LP}}$ (Theorem)
- Effects on GDP will depend on what channels dominate

Agenda

Two counterfactuals

Efficient pricing (As in Baqaee and Farhi, 2021)

- Firms must charge markups to incentivize the optimal entry level
- But markup distorts input choices by acting as a uniform tax on production
- To mitigate this distortion, an output subsidy is required to restore undistorted marginal-cost , conditional on entry
- The subsidy is paid via a lump sum tax to the representative consumer

Linear prices from monopolistic competition

- Firms charge a linear markup over marginal cost

Assumptions and parametrization

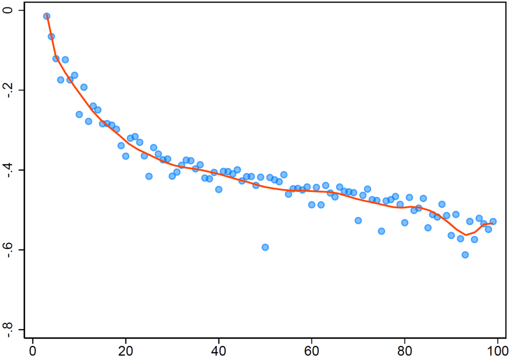
- $\eta = 1$ to ensure equilibrium existence and uniqueness
- $\implies$ Cobb-Douglas production functions

Model Parameters

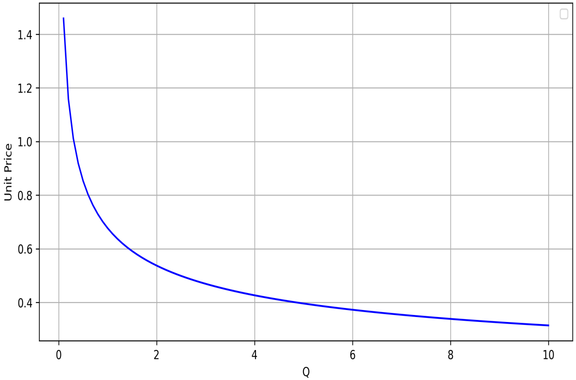
	Value	**Source**
Labor share in production (α)	0.38	Calibrated from data
Material bundle elasticity (σ)	3	Assumed
Firm exit rate (δ)	0.15	Calibrated from data
Entry cost (c_e)	0.16	Assumed
Pareto tail (κ)	2.4	Calibrated from data

Unit prices: Data vs calibrated model

Panel A. Data



Panel B. Model



Model Quantification: Ratio vs Efficient Pricing

	Linear Pricing	**Nonlinear Pricing**
Total Firms (M)	1.27	1.11
Downstream Firms expected output ($\mathbb{E}_z[q(z)]$)	0.48	0.86
Labor to materials per firm ratio (l/m)	2.25	0.96
Firm mass D to U ratio (M_D/M_U)	0.55	0.77
Welfare ($Y = GDP$)	0.62	0.76

- **Intensive margin wedge:** In LP, mean firm size is half relative to efficient pricing, while in NLP, it is only 15% smaller.
- **Input ratio wedge:** While the NLP ratio is close to efficient pricing, in the LP model, firms intensively replace materials with labor.
- **Extensive margin wedge:** NLP makes downstream firms larger; fewer enter, LP adds double marginalization, making downstream firms entry even smaller

Larger output per firm dominates lower entry; welfare is higher in NLP relative to LP

Agenda

Conclusion

- Price discrimination is prevalent in the data
- Different pricing models have different aggregate effects predictions
  - Linear price models overestimate aggregate markup costs
- Accounting for observed price discrimination, markups aggregate costs are around half relative to a linear price setup

Invoice Example

Go Back

KITCHEN CENTER

KITCHEN CENTER SPA

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Sucursales:	
Casa Costanera:	Av. Nueva Costanera 3900, Vitacura
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Mall Plaza Maule:	Circunvalación 1055, Local 226/227, Talca - Teléfono: (56-2) 24117748
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FECHA VENCIMIENTO : 03/08/2022

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FORMA DE PAGO : Contado

COD. VENDEDOR :

Dirección Origen: Camino lo Boza 8887

Comuna : Pudahuel Ciudad : Santiago

Dirección Destino: Los Misioneros 1923

Comuna : Providencia Ciudad : Santiago

Orden de Venta: 793325

Número de OC:

CÓDIGO	DETALLE	CANTIDAD	PRECIO UNITARIO	PRECIO ÍTEM
13452	Lavaplatos FDV Small Acqua bajo cubierta	1	92.428,57	92.429
14761	Encimera FDV Design 4T GLTX 65 BUT 2.0	1	142.848,74	142.849
14265	Campana Kubli Neu Slider	1	100.831,93	100.832
19110	Horno FDV Design	1	201.672,27	201.672
13377	Lavavajillas FS FDV Element 14C	1	243.689,07	243.689
14917	Griferia FDV CONICA FLEX	1	84.025,21	84.025
10232	Transporte - Providencia	1	15.529,41	15.529

Q Deciles regression

Go Back

$$\ln\ p_{ijgm} = \beta_0 + FE_{\text{Q deciles}} + \ln\psi_{ijg} + \ln\xi_m + \ln\epsilon_{ijgm}$$

FE	log p	SE	Min q	Max q	Mean q	Sd Q
D5	-0.17	0.00007	1	2	1.96	0.14
D6	-0.29	0.00008	2	4	3.37	0.53
D7	-0.41	0.00009	4	6	5.47	0.53
D8	-0.51	0.00011	6	12	9.29	1.59
D9	-0.65	0.00012	12	30	20.8	4.99
D10	-1.01	0.00007	30	+1 Billion	18,014	3,679,468

Quantity discounts evidence by sector

▸ Go Back

Sector	β_1	se	N obs (millions)	Adjusted R2
Agriculture	-0.480	(0.0002)	8.2	0.83
Mining	-0.375	(0.0008)	0.8	0.84
Manufacturing	-0.161	(0.0003)	313.6	0.92
Utilities	-0.174	(0.0005)	18.9	0.86
Construction	-0.722	(0.0006)	3.7	0.82
Retail and Wholesale	-0.181	(0.0002)	766.1	0.90
Transport and ICTs	-0.489	(0.0003)	39.3	0.85
Financial Services	-0.492	(0.0004)	15.5	0.79
Real Estate Services	-0.773	(0.0014)	1.7	0.81
Business Services	-0.562	(0.0003)	16.9	0.96
Personal Services	-0.498	(0.0010)	2.9	0.74

Solution Strategy & Mechanism

▸ Go Back

- Initial Conjectures:
  - Two-part tariff: $T(\gamma, \zeta) = p(\zeta)m(\gamma, \zeta) + F_\gamma(\zeta)$
  - Revenue functions: $R(z) = z^\theta y(l(z), m(z))^\theta A_z$
  - Buyer valuation: $\tau(\gamma) = \gamma^{(\sigma-1)/\sigma} p_m m(\gamma_0)^{1/\sigma}$
  - Distribution: τ follows Pareto with tail parameter $\rho = \frac{\kappa\sigma}{\sigma-1}$

- Verification Process:
  - Applied envelope theorem for IC constraints
  - Derived monotonicity conditions
  - Confirmed individual rationality via zero surpluses for the lowest type

Optimal Mechanism Characteristics

Go Back

- Pricing Structure:
  - Linear price: $p(\zeta) = \frac{\rho}{\rho-1} c(\zeta)$
  - Fixed fee: $F_\gamma(\zeta) = \text{Revenue}(\gamma_0) \frac{1}{\left(\frac{\alpha}{1-\alpha}\right)^\eta \left(\frac{p m}{w}\right)^{\eta-1} + 1} \left(\frac{1}{\sigma}\right)$
- Properties:
  - Complete market coverage (all types served)
  - Markup lower than standard monopolistic competition pricing
  - Information rents decrease with type concentration (κ)
- Quantity Allocation:
  - $m(\tau, \zeta)$ optimally scales with buyer type
  - Allocation satisfies both IC and IR constraints

Virtual Utility

Go Back Virtual Utility: The profits a seller gets when including a firm type γ

- Positive term: Profits from serving firm type γ
- Negative term: Information rents given to higher types to prevent mimicking lower types
- If the virtual utility is positive, serving every firm type γ is optimal.
- We can show that serving upstream and downstream lowest type is optimal under a Pareto distribution.