Functional data analysis: FPCA and MDS

Write a report that contains the results of the computations that you are asked to carry out below, as well as the explanation of what you are doing. The main text (2 or 3 pages) should include pieces of source code and graphical and numerical output.
Upload your answers in a .pdf document (use LaTeX or R Markdown, for instance) to ATENEA, as well as the source code (*.R or *.Rmd, for instance). Your work must be reproducible.

Bike sharing data

In Bike sharing fdata (ATENEA) you can find information on the bike-sharing rental service in Washington D.C., USA, corresponding to years 2011 and 2012. There are the following objects:

- `day`: Data frame. Bike sharing counts aggregated on daily basis (731 days, 16 variables per day).
- `list.fdata`: A list with the following `fdata` objects, all with `0:23` as `arvals` (the functions depend on the hour of the day):

`temp`: Normalized temperature in Celsius. The values are divided to 41 (max).

`atemp`: Normalized feeling temperature in Celsius. The values are divided to 50 (max).

`hum`: Normalized humidity. The values are divided to 100 (max).

`windspeed`: Normalized wind speed. The values are divided to 67 (max).

`casual`: count of casual users.

`registered`: count of registered users.

`cnt`: count of total rental bikes including both casual and registered.

For the following questions, you are free to use either the **fda** or **fda.usc** libraries, or both.

1. Perform FPCA on the `fdata` object `cnt`

1. For the functional data set `cnt`, do FPCA extracting the first 3 principal functions. Which percentage of variability is explained by each principal function?
2. Do graphical representations of the principal functions and give an interpretation to the three main modes of variation around the mean function.
3. Do scatterplots of the columns of the data frame `day` against the scores in the first 3 principal functions and try to get new insights about the interpretations of the principal functions from them.

2. Perform MDS on the `fdata` object `temp`

1. Compute the first derivatives of the data in the data set `temp`. Plot the new functional data set.

2. Compute the L_1 distance between the derivatives of the `temp` functions.

3. Perform MDS (classical metric scaling, to be specific) on the previous distance matrix, and extract the first 3 principal coordinates.

4. Do graphical representations of the 3 dimensions identified by MDS and give an interpretation to the three main modes of variation around the mean function. To be specific, for each dimension identified by MDS plot in the same graphic the first derivative of the temperature function for the days having minimum, median and maximum scores at the corresponding principal coordinates. First, you can plot the functional data set of derivatives in gray color, and then add the lines corresponding to minimum, median and maximum scores.

5. Do scatterplots of the columns of the data frame `day` against the scores in the first 3 principal coordinates and try to get new insights about the interpretations of the principal functions from them.