Friction models for sliding dry, boundary and mixed lubricated contacts

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Abstract

Friction, lubrication, and wear have a strong influence on the performance and behavior of mechanical systems. This paper deals with different friction models for sliding contacts running under different conditions. The models presented are suited to different situations, depending on the type of contact, running conditions, and the behavior of interest. The models will be discussed from simulation and tribological points of view. The different types of friction models considered are:

- friction models for transient sliding under dry, boundary and mixed lubrication conditions,
- friction models for micro-displacements of engineering surfaces subjected to transient sliding,
- friction models often used in the simulation and control of technical systems,
- combined friction models that represent physical behaviors fairly well but are also suitable for use in simulating systems,
- friction models that take into account the stochastic nature of interacting surface asperities.

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1. Background

The general trend toward increased use of product models and simulations during both the development and use of products has created a need for powerful and relevant product models. Since friction has a very strong influence on the performance and behavior of mechanical systems, good representation and prediction of friction phenomena are important in all simulations and analyses [1].

The divisions of Machine Elements at the Royal Institute of Technology in Stockholm and at the Luleå Technical University in Luleå, along with a number of Swedish companies, are pursuing a research program named INTERFACE. The aim is to develop interface models that can be integrated with commercial simulation tools to improve their ability to predict behavior and performance during product realization.

This project draws on the work done by Sellgren [2], who developed general principles for modeling systems. His approach is modular, and lays down strict guidelines for behavioral models of machine elements, modules, and interfaces. Sellgren [2] defined an interface as an attachment relation between two mating faces. That definition is elaborated on by Andersson and Sellgren [3] in terms of an interaction relation between two functional surfaces. A functional surface is a carrier of a function, and in this paper a contact surface also considered.

The modeling principle proposed and elaborated on in Ref. [4] is illustrated by the link mechanism of the lifting system shown in Fig. 1. The different components of the mechanism are represented by CAD models, and the interfaces are represented by symbols from i1 to i6. An interface can be both a mechanical contact and an interface element, as when it is a bearing or a system component such as a coupling. This paper considers only mechanical contacts.

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Nomenclature

A	amplitude
F	friction force
F_{app}	external applied tangential force
F_c	Coulomb sliding friction force, $F_c = \mu N$
F_s	maximal static friction force in the Stribeck friction model
f	frequency
i	exponent in the Stribeck friction model and in the Dahl friction model
k	spring constant in the 1 DOF system
k_{sat}	constant in combined Coulomb and viscous friction model
k_{tanh}	constant in combined Coulomb and tanh friction model
k_v	viscous friction coefficient
m	mass of the sliding body in the 1 DOF system
N	normal load, for the 1 DOF system: $N = mg$
x	position of the sliding body in the 1 DOF system
x_0	position of the wall in the 1 DOF system
v	sliding speed
v_s	sliding speed coefficient in the Stribeck friction model
δ	maximal micro-slip before macro-sliding occurs
μ	coefficient of friction

Fig. 1. Structuring of the mechanical subsystem components and interfaces of the lifting unit [4].
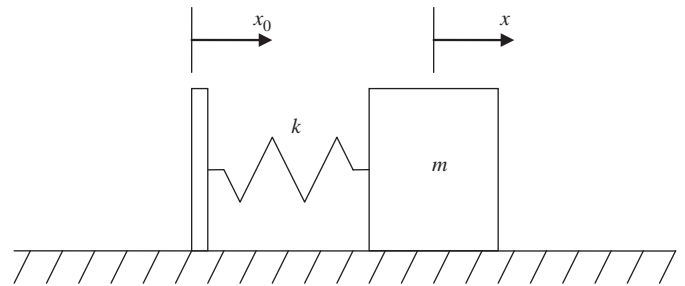

Fig. 2. A 1 DOF dynamic system with a moving wall, a spring, and a body sliding on another fixed body. The friction in the contact between the bodies can be represented by different models.

Friction in mechanical contacts is influenced by many parameters, including the geometry of the contact surfaces, their properties, the running conditions, and any lubricants used. This paper present different friction models for transient sliding contacts running under different conditions.

2. Common friction models for pure sliding and oscillating sliding contacts

The behavior of different friction models presented below was studied using the 1 DOF system shown in Fig. 2, where the motion of the left wall, x_0, is independent. It can thus also have an oscillating motion with different frequencies. The equation of motion for the system can be formulated as

$$m\ddot{x} = k(x_0 - x) - F, \tag{1}$$

where m is the mass of the moving body, x the position of the body, x_0 the independent position of the wall, k the spring constant, and F the friction force. The friction force on the body acts in the opposite direction to the sliding velocity of the upper body.

Numerical simulations of the system with the different friction models were carried out in Simulink/Matlab with

Table 1
Parameter values used in the simulations

Parameter	Value
m	1 (kg)
k	100 (N/m)
μ	0.1 (dimensionless)
k_{sat}, k_{tanh}	1000 (s/m)
δ	10^{-5} (m)

the solver ode23. The absolute and relative tolerance of the solver was set to 10^{-12} and 10^{-14}, respectively. The parameter values used in the simulation agree with Table 1.

2.1. Coulomb friction model

The most commonly used friction model is the Coulomb friction model, which can be formulated as

$$F = \begin{cases} F_c \sin(v) & \text{if } v > 0, \\ F_{app} & \text{if } v = 0 \text{ and } F_{app} < F_c, \end{cases} \tag{2}$$

where F is the friction force, $v = \dot{x}$ the sliding speed and F_{app} the applied force on the body. F_c is the Coulomb

sliding friction force defined as

$$F_c = \mu N, \qquad (3)$$

where μ is the coefficient of friction and N the normal load in the contact. The Coulomb friction model is often simplified as

$$F = F_c \, \text{sign}(v) \qquad (4)$$

although this can cause problems in simulations due to the properties of the sign function. The Coulomb friction model is illustrated in Fig. 3a.

Coulomb friction is often referred to as dry friction, but the model is used for dry contacts as well as boundary and mixed lubricated contacts. Although it is known that a Coulomb friction model does not always represent the friction behavior in a contact well, such models are often used to describe the friction in mechanical contacts. It must be remembered that the representation of friction data for different contacts by coefficients of friction does not necessarily mean that the contacts exhibit Coulomb friction behavior; rather, these coefficients represent a certain value for a particular running condition. In practice, however, these coefficients are often used as if the friction behavior is in accordance with Coulomb, which sometimes results in deviations and poor behavior prediction.

All in all, it is surprising that the use of the coefficient of friction is so common, since the use of a Coulomb friction model in analyses and simulations is often rather troublesome.

There are advantages of using a modularized model architecture of a system in a product realization process with natural question-driven simulations. Such an architecture makes it possible to assemble different component and interface models into a system model and to directly simulate the behavior of the system, without extra condition checks on the state of the system or interruptions in the model. However, in order to construct such a model, some extra considerations have to be taken into account in order to find useful friction models.

2.2. Viscous friction model

Since the equation of motion for dynamic systems is strongly non-linear with a Coulomb friction model, a viscous friction model is often used instead. Such a model is considerably easier to simulate, but the representation of the friction is often poor. A viscous friction model can be formulated as

$$F = k_v v, \qquad (5)$$

where F is the friction force, v the sliding speed, and k_v the viscous coefficient. In this case the equation of motion for the system in Fig. 2 is a linear differential equation, which can be solved both numerically and analytically. Although the simulation is easy to perform, the validity of the viscous model is doubtful. However, in some cases, such as full film contacts, the viscous model may offer the best representation of the behavior. In other cases, the viscous model is a poor representation but, by tuning the viscous coefficient, the model can represent such things as damping rather well under particular running conditions.

A step response with the 1 DOF system shown in Fig. 2 with viscous friction is shown in Fig. 4. The damping of the oscillations varies with the viscous friction coefficient.

2.3. Combined Coulomb and viscous friction model

Since the Coulomb friction model is problematic as regards both the analysis and simulation of a system's behavior, a combination of the viscous friction model and the Coulomb friction model could be advantageous. Such a model will have the following form:

$$F = \begin{cases} F_c \min(k_{sat} v, 1) & \text{if } v \geqslant 0, \\ F_c \max(k_{sat} v, -1) & \text{if } v < 0, \end{cases} \qquad (6)$$

where k_{sat} is a coefficient that determines the speed of the transition from $-$ to $+$ (see Fig. 3b).

The combined Coulomb and viscous friction model can easily be modeled in Matlab/Simulink by using the saturation block. The 'sat' function can be defined as
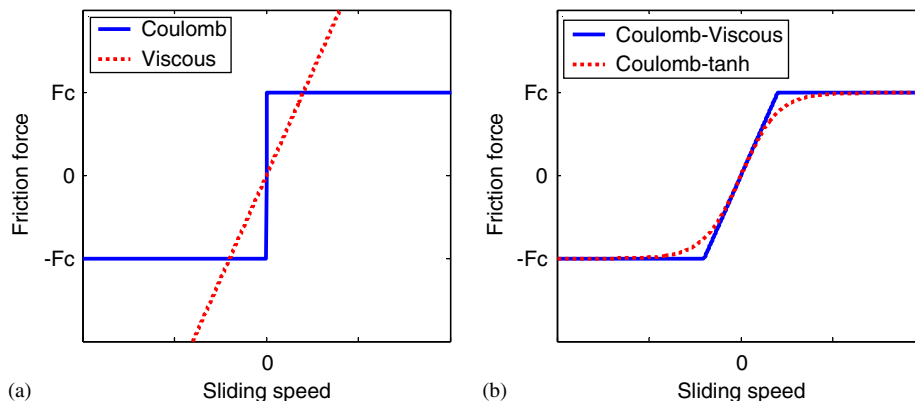

Fig. 3. (a) Coulomb and viscous friction forces as function of sliding speed, (b) combined Coulomb and viscous friction and combined Coulomb and tanh friction as function of sliding speed.
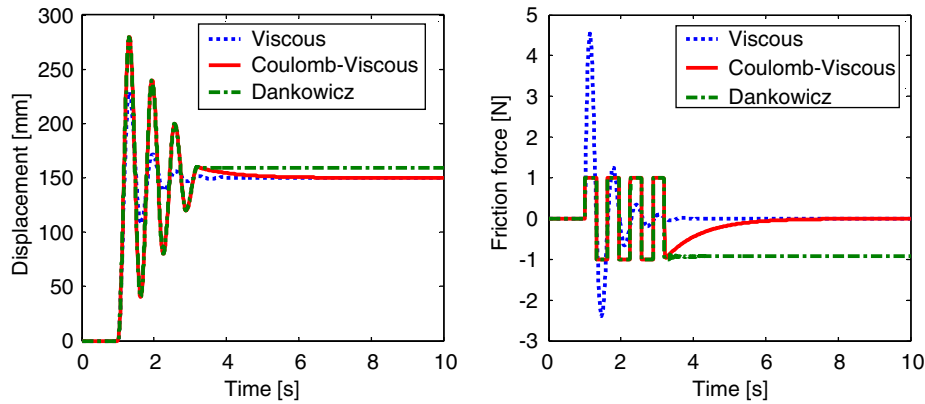
Fig. 4. Step response for the system in Fig. 2 with a viscous friction model (dot), a combined Coulomb–viscous friction model (solid) and a Dankowicz friction model (dash–dot).

follows:

$$\text{sat}(k_{\text{sat}}v) = \begin{cases} \min(k_{\text{sat}}v, 1) & \text{if } v \geqslant 0, \\ \max(k_{\text{sat}}v, -1) & \text{if } v < 0. \end{cases} \quad (7)$$

The combined Coulomb and viscous friction model can then be formulated as

$$F = F_{\text{c}}\, \text{sat}(k_{\text{sat}}v). \quad (8)$$

Using the combined Coulomb and viscous friction model eliminates the difficulty in determining the friction force at zero sliding speed both at start up and at direction change, although we still have a strongly non-linear system model to solve. However, simulation tests do not indicate any problem with the non-linearity and the step response shown in Fig. 4 behaved as expected. The combined Coulomb and viscous friction model is thus a plausible candidate to represent friction in sliding and oscillating contacts.

Another possibility, also used in some systems models, is to replace the sign function in the simplified Coulomb friction model shown in Eq. (4) with a tanh function, and to let the new model be valid for all v, as shown in

$$F = F_{\text{c}}\, \tanh(k_{\text{tanh}}v), \quad (9)$$

where k_{tanh} is a coefficient that determines how fast the tanh function changes from near -1 to near $+1$. This model behaves like other combined model, but is probably more numerically stable.

The main disadvantage with the proposed combined Coulomb–viscous friction model and combined Coulomb–tanh friction model is that they assume zero friction force at zero sliding speed. This means that friction force exists only when there is a motion. The consequences can be observed in the step response of the 1 DOF system in Fig. 2. The body tends to drift until the applied forces are zero. Furthermore step response simulations with small displacement results in a motion of the body even though the applied force is not large enough to overcome the Coulomb friction force. These observations are not

expected in a real case of Coulomb friction, where the friction force at zero sliding speed equals the applied forces.

Thus, although the combined friction models are convenient in simulation of oscillating motions they can give rise to inaccurate final position in simulations with small applied forces or if the friction force is supposed to hold a load over a longer time. In these cases we have to consider the small displacements (i.e. the micro-displacements) that are functions of the tangential deformations or displacements between the surfaces (see below).

2.4. Stribeck friction model

Most sliding contacts are lubricated. The friction force will then vary with the sliding speed depending on the extent to which the interacting contact surfaces are running under boundary, mixed, or full film lubrication. Even dry contacts show some behavior similar to that in lubricated contacts in that they have a higher static friction than dynamic or sliding friction. In lubricated sliding contacts, the friction decreases with increased sliding speed until a mixed or full film situation is obtained, after which the friction in the contact can either be constant, increase, or decrease somewhat with increased sliding speed due to viscous and thermal effects. This behavior was described some 100 years ago by Stribeck [5], whose name is often associated with sliding friction in lubricated contacts running under boundary, mixed, and full film conditions. The model can be formulated as

$$F = (F_{\text{c}} + (F_{\text{s}} - F_{\text{c}})e^{-(|v|/v_{\text{s}})^i})\text{sign}(v) + k_v v, \quad (10)$$

where F is the friction force, v the sliding speed, F_{c} the Coulomb sliding friction force, F_s the maximum static friction force, v_{s} the sliding speed coefficient, k_{v} the viscous friction coefficient, and i an exponent. The Stribeck friction model is illustrated in Fig. 5.

The Stribeck friction model can provide very good representation of the friction between sliding surfaces. It covers everything from Coulomb friction to viscous
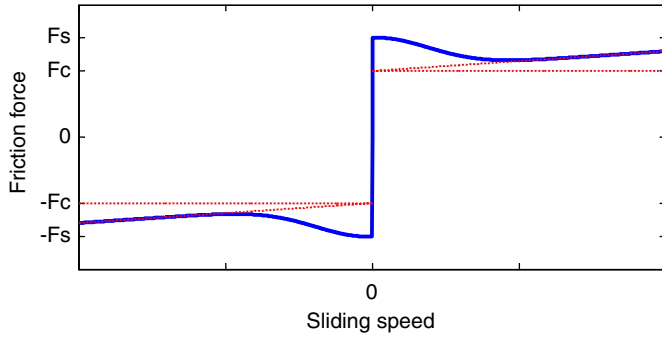
Fig. 5. Relation between friction force and sliding speed according to the Stribeck model.

friction, depending on the choice of parameter values. However, the Stribeck friction model presents the same problem as the Coulomb friction model when it comes to changing sliding direction when the friction value is dependent on the applied force and the contact surfaces are sticking to each other. We accordingly tried to formulate a combined Stribeck and viscous model and a combined Stribeck and tanh friction model, both accommodating the transition to pure sliding in either direction. We replaced the sign function with a sat function and a tanh function, respectively:

$$F = (F_c + (F_s - F_c)e^{-(|v|/v_s)^i} \mathrm{sat}(k_{\mathrm{sat}}v) + k_v v, \qquad (11)$$

$$F = (F_c + (F_s - F_c)e^{-(|v|/v_s)^i}) \tanh(k_{\tanh}v) + k_v v. \qquad (12)$$

These combined Stribeck models facilitate simulations in the same way as the combined Coulomb models, but also share their disadvantages (see above).

3. Friction at small displacements between sliding contacts

The main disadvantage of the Coulomb and Stribeck friction models is the undetermined friction force at zero-sliding speed. These friction models run into problems at the start of a motion and at the point where the motion reverses. Although the behavior at these points can be modeled by a sign function, which represents the behavior fairly well, the representation complicates simulations and necessitates extra condition checks of the system states or interruptions of the simulations. Various tricks can be used to overcome the problem, such as replacing the sign function with a viscous saturated function, or using a tanh function. These modifications improve the ability of friction models to simulate the behavior of systems, but do not represent small displacements very well. But small displacements are important in many high precision and control applications, and we therefore also have to look for a friction model that can handle them.

3.1. Micro-slip and friction at small displacements between interacting contact surfaces

The friction behavior of contact surfaces under pure or oscillating sliding conditions with very small displacements has been extensively studied at KTH Machine Elements for many years. All the results show that the transition from positive to negative motion or the start of a motion is always associated with a small relative tangential displacement between the interacting surfaces before full friction or gross slip is obtained. The displacement before gross slip occurs is often 1–10 μm. The explanation of this behavior is that before gross slip occurs, there is local sliding in the contact. This phenomenon used to be named micro-slip or micro-displacement.

Micro-slip models based on basic surface models represent the micro-slip phenomenon rather well, but are not always easy to use in dynamic simulations [6–8]. Drawing on existing models, Sellgren and Olofsson [9] developed a constitutive model for micro-slip in finite element analysis. The derived model is, however, rather complex to use in other simulation tools, for the displacements have to be set equal to zero at turning points. This problem can be overcome by formulating the micro-slip model as a differential equation according to the Dankowicz model (see below). Another way to accommodate micro-slip in contacts is by combining Coulomb friction with elastic deformation. The resulting elastic Coulomb is widely used in FE codes. While the elastic Coulomb friction model does not represent reality very well, it simplifies the numerical calculations.

3.2. Dankowicz friction model

Dankowicz [10] represented friction behavior by a first-order differential equation and a help variable z. The Dankowicz friction model has the following form:

$$\dot{z} = \dot{x}\left(1 - \frac{z}{\delta}\mathrm{sign}(\dot{x})\right),$$
$$F = \frac{F_{\max}}{\delta}z, \qquad (13)$$

where F is the friction force, z the help variable, δ represents the micro-slip displacement before gross slip, and $F_{\max}$ the maximum friction force, which here can be considered as the Coulomb sliding friction force.

In order to find out to what extent the Dankowicz model is numerically appropriate in different situations, an analysis was made using the 1 DOF system shown in Fig. 2. To reduce the number of simulations, the equation of motion and the friction model were made non-dimensional by introducing the non-dimensional variables:

$$X = \frac{x}{\delta}, \quad X_0 = \frac{x_0}{\delta}, \quad Z = \frac{z}{\delta}, \quad T = t\sqrt{\frac{k}{m}}$$

which yield the non-dimensional differential equations for the system in Fig. 2, with the Dankowicz friction

model,

$$\ddot{X} = (X_0 - X) - CZ,$$
$$\dot{Z} = \dot{X}(1 - Z\,\text{sign}(\dot{X})), \tag{14}$$

where

$$C = \frac{F_{\max}}{\delta k}. \tag{15}$$

Three different motions of the wall were analyzed: a step, an oscillating motion with continuously increasing frequency and an oscillating motion with a constant frequency. Each motion was first simulated with an amplitude to small to create macro-sliding of the body and then with an amplitude large enough to create macro-sliding. For each combination of motion type and amplitude, four cases were simulated: $C = 1$, $C = 10$, $C = 100$ and $C = 1000$.

The simulations were carried out without any numerical trouble, but showed that for very small displacements the friction model will behave as a linear spring and can lead to undamped high-frequent oscillations in the system response, see Fig. 6. This behavior suggests that there is no sliding whatsoever in the contact, just elastic deformation. This is not to be expected for a real sliding contact. More likely there would exist local sliding at any tangential load, no matter how small, leading to energy dissipation and damping in the contact.

3.3. Dahl friction model

The Dahl model [11] is frequently used in control engineering. Both Dahl and Dankowicz based their friction models on the fact that the friction force is a function of displacement only, and thus the time rate of friction can be expressed as

$$\frac{\mathrm{d}F(x)}{\mathrm{d}t} = \frac{\mathrm{d}F(x)}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}, \tag{16}$$

where $F(x)$ is a friction force. The general Dahl friction model has the form

$$\frac{\mathrm{d}F}{\mathrm{d}x} = \sigma_0 \left| 1 - \frac{F}{F_{\mathrm{c}}}\text{sign}(\dot{x}) \right|^{i} \text{sign}\left(1 - \frac{F}{F_{\mathrm{c}}}\text{sign}(\dot{x}) \right), \tag{17}$$

where F is the friction force and σ_0 is a coefficient. In the literature, the Dahl model is often simplified with the exponent $i = 1$:

$$\frac{\mathrm{d}F}{\mathrm{d}t} = \sigma_0 \left(1 - \frac{F}{F_{\mathrm{c}}}\text{sign}(\dot{x}) \right)\dot{x}. \tag{18}$$

In its simplified form the Dahl friction model shows to be equal to the Dankowicz model and will thus behave as a linear spring for small displacements.

3.4. Canudas de Wit et al. friction model

The Canudas de Wit et al. friction model [12,13] has many similarities with the Dahl and Dankowicz models. They, too, based their model on the fact that the friction force is dependent on deformation between the surfaces and formulated the model as follows, in its simplest form:

$$\dot{z} = \dot{x} - \frac{|\dot{x}|}{g(\dot{x})}z,$$
$$g(v) = \frac{1}{\sigma_0}(F_{\mathrm{c}} + (F_{\mathrm{s}} - F_{\mathrm{c}})e^{-(|v|/v_{\mathrm{s}})^2}),$$
$$F = \sigma_0 z + \sigma_1 \dot{z} + k_v \dot{x}, \tag{19}$$

where F is the friction force and σ_0 and σ_1 are coefficients. The friction model takes the Stribeck effect in consideration through the $g(v)$ function and the damping term ($\sigma_1 \mathrm{d}z/\mathrm{d}t$) in the friction force prevent the model from behaving as an linear spring at small displacements.

3.5. Combined Coulomb, Stribeck, viscous, and Dankowicz friction model for small transient motions

Based on the findings above, we propose that in many cases a combined friction model for sliding motions should be used, based on the following equations:

$$\dot{z} = \dot{x}\left(1 - \frac{z}{\delta}\text{sign}(\dot{x}) \right),$$
$$F = \left(1 + (\frac{F_{\mathrm{s}}}{F_{\mathrm{c}}} - 1)e^{-(|\dot{x}|/v_{\mathrm{s}})^i} \right)F_{\mathrm{c}}\frac{z}{\delta} + k_v \dot{x}. \tag{20}$$
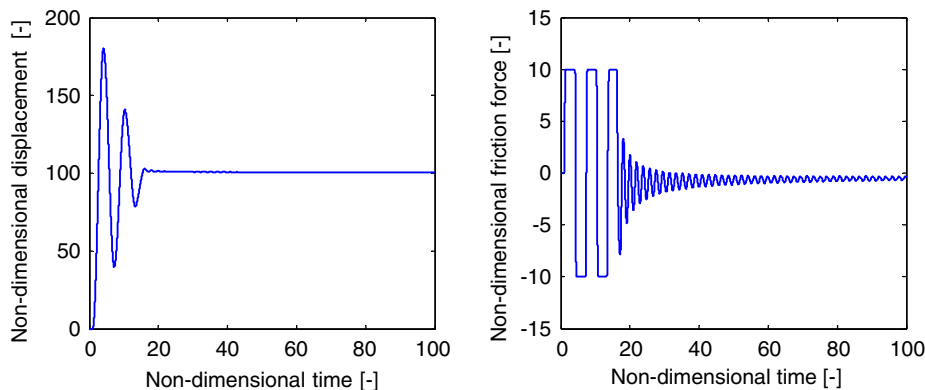

Fig. 6. Step response for the non-dimensional system with $C = 10$ and a step size large enough to create macro-sliding of the body.

This model incorporates micro-slip, Coulomb friction, Stribeck effects, and viscous friction. The δ-parameter can be determined from micro-displacement tests. The other parameters (i.e., F_c, F_s, v_s, i, and k_v) can be determined from dynamic friction tests. Normally we then make oscillating tests and identify the parameter from the test results.

4. Friction as a stochastic process

Friction between sliding contact surfaces is a result of stochastic interactions between rubbing asperities. The rubbing asperities cause friction by shearing surface materials, lubricants, or surface coatings. Yet while the friction force in nearly all friction tests is highly stochastic in nature, with significant variations in both amplitude and frequency, most friction models do not take these variations into account and instead represent the friction forces by a smooth mean value. Furthermore, test results seldom compensate for the dynamic behavior of the test equipment, although most friction measurements are subjected to mechanical filtering or amplification by the test machine dynamics.

The stochastic nature of friction can be represented in the friction model by adding noise to the smooth mean friction force. White noise is not recommended since it contains frequencies that are infinitely high. A stochastic signal with controllable frequencies and amplitude with a standard deviation can be created using the method described in Ref. [14]. The friction model can then be written as

$$F_{\text{Stoc}} = F_{\text{smooth}} + P(A, f), \tag{21}$$

where F_{Stoc} is the stochastic friction force, F_{smooth} is the friction force determined by any smooth friction model, $P(A, f)$ is a stochastic function representing the friction force noise, A is the amplitude standard deviation, and f is a typical frequency. Since the stochastic friction force variations are often produced by asperity interactions during sliding, the noise frequency may be a function of the sliding speed v.

5. Conclusions

There are advantages to generating behavioral models for mechanical systems using a modularized model structure [2,3]. The structure is built up of component, subsystem, and interface models. By connecting these models, a system model is constructed that can be used for simulations and analyses. In this paper, the term interface refers to mechanical contacts interacting under transient and oscillating sliding motions.

Friction has a strong influence on both the behavior and performance of a system. This paper presents different friction models and discusses their relevance and convenience as models in a modularized system model structure

and as part of a system model in simulations. The different friction models studied were as follows:

- commonly used friction models such as the Coulomb friction model, viscous friction model, and Stribeck friction model,
- combined friction models such as the Coulomb and viscous friction model, Coulomb and tanh friction model, Stribeck and viscous friction model, and Stribeck and tanh friction model,
- friction models for small displacements such as the Dankowicz model and models often used in control engineering such as the Dahl model and Canudas de Wit model,
- a friction model that takes into account considering the stochastic nature of friction.

Based on the study, following modeling recommendation can be made:

- In simulation where the exact final position is unimportant, friction could be modeled by a combined Stribeck and tanh model.
- When the exact final position is of importance in our simulation the use of a combined Stribeck and Dankowicz friction model is appropriate.
- If stochastic effects are important to the simulation a noise should be added to the smooth friction model.

Acknowledgments

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