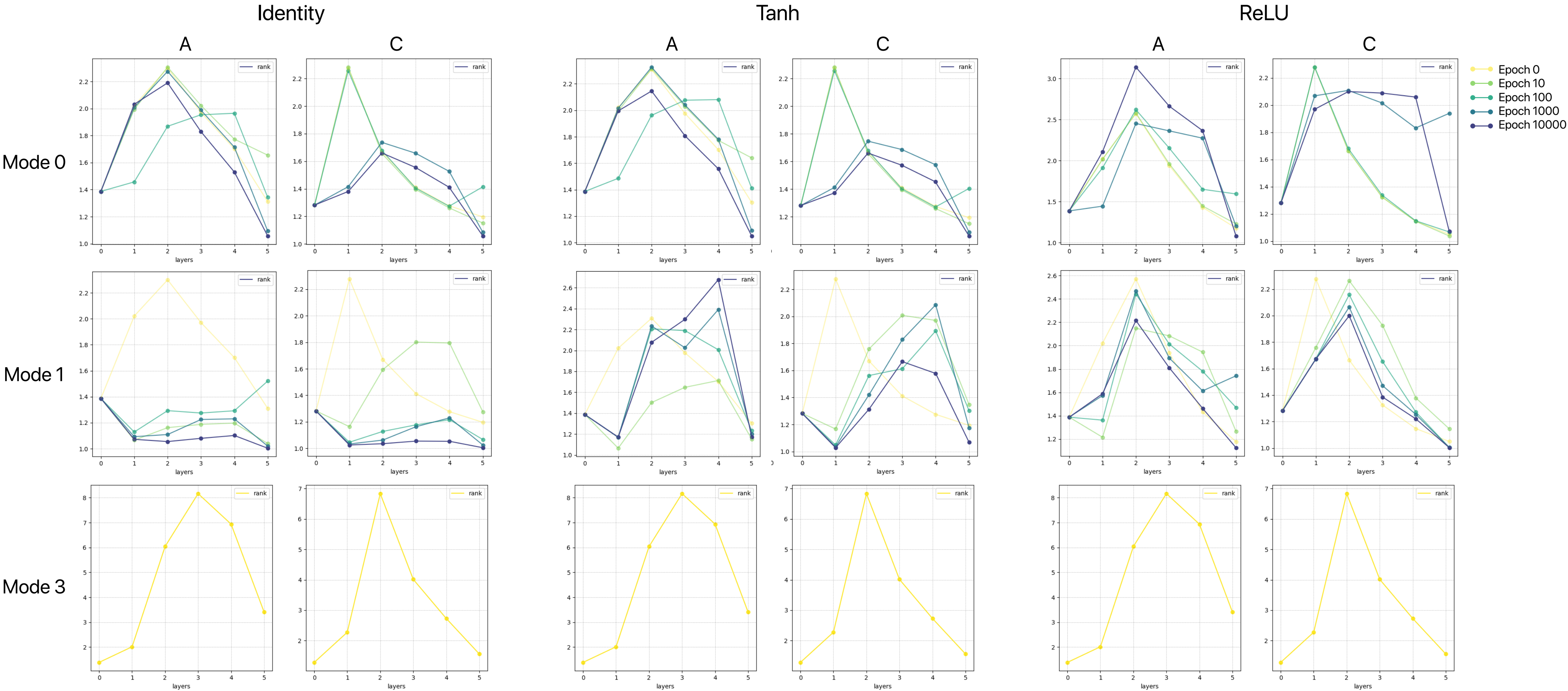
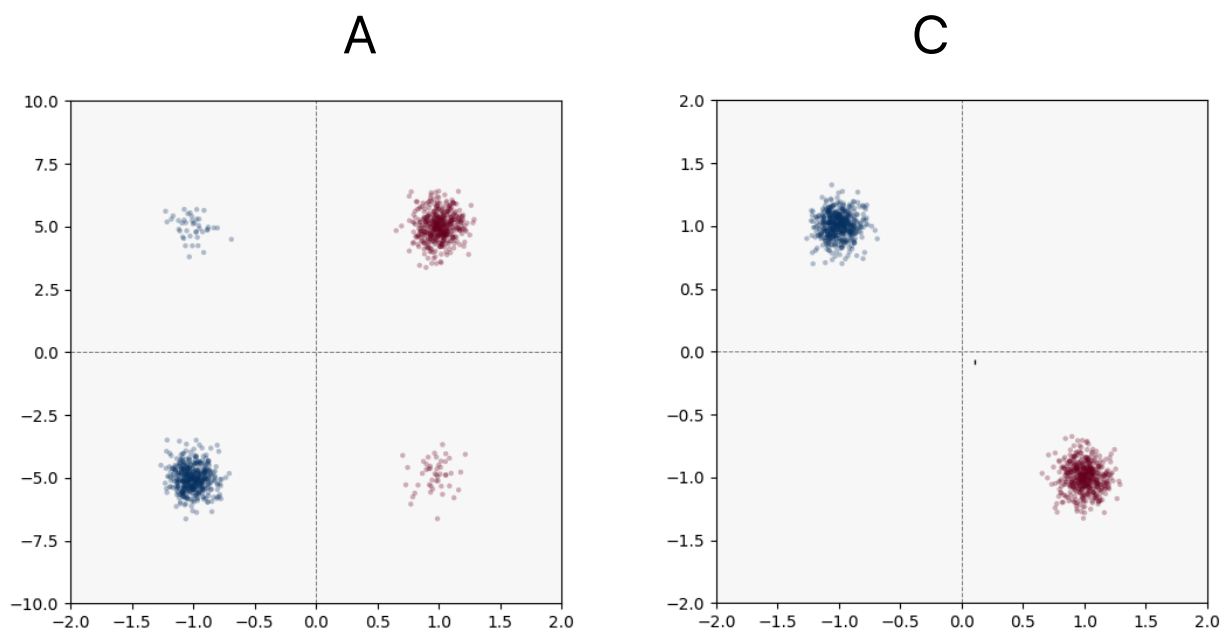


Setups:

	core corr	core gamma	spu corr	spu gamma	noise dim	noise gamma
Setting A	1	1	0.9	5	0	n/a
Setting B	1	1	0.9	5	40	1
Setting C	1	1	0	1	0	n/a
Setting D	1	1	0	1	40	1

Mode 0	Standard rich training; layer-wise representation rank computed
Mode 1	Lazy training; layer-wise representation rank computed
Mode 2	No training; Shred the NTRF matrix into chunks and computing the rank
Mode 3	No training; NTRF rank of separate l -layer networks are computed for $0 \leq l < 6$



Mode 1

A

C

A

C

A

C

Mode 3

A

C

A

C

A

C

Legend

Epoch 0

Epoch 10

Epoch 100

Epoch 1000

Epoch 10000

Hidden Manifold Model (HMM):

n : the number of data points

dl : the dimension of the latent space

da : the dimension of the ambient space

p : the probability of generating non-zero entries in the latent patterns

$W \in \mathbb{R}^{dl \times da}$: a random feature matrix drawn from a standard Gaussian

$Z \in \mathbb{R}^{n \times dl}$: a binary matrix sampled from a Bernoulli with parameter p

$b \in \mathbb{R}^{dl \times 1}$: the teacher weights drawn from a standard Gaussian

$X \in \mathbb{R}^{n \times da}$ denotes the inputs

$Y \in \mathbb{R}^{n \times 1}$ denotes the labels

$$X = \sigma \left(\frac{ZW}{\sqrt{dl}} \right) \quad Y = \begin{cases} 1 & \text{if } Zb^T > 0 \\ 0 & \text{o.w.} \end{cases}$$

Teacher:

A HMM with

$n = 2000$ (1000 for train and 1000 for valid)

$dl = 4$, $da = 512$, $p = 0.5$

$\sigma = \text{identity} / \tanh$

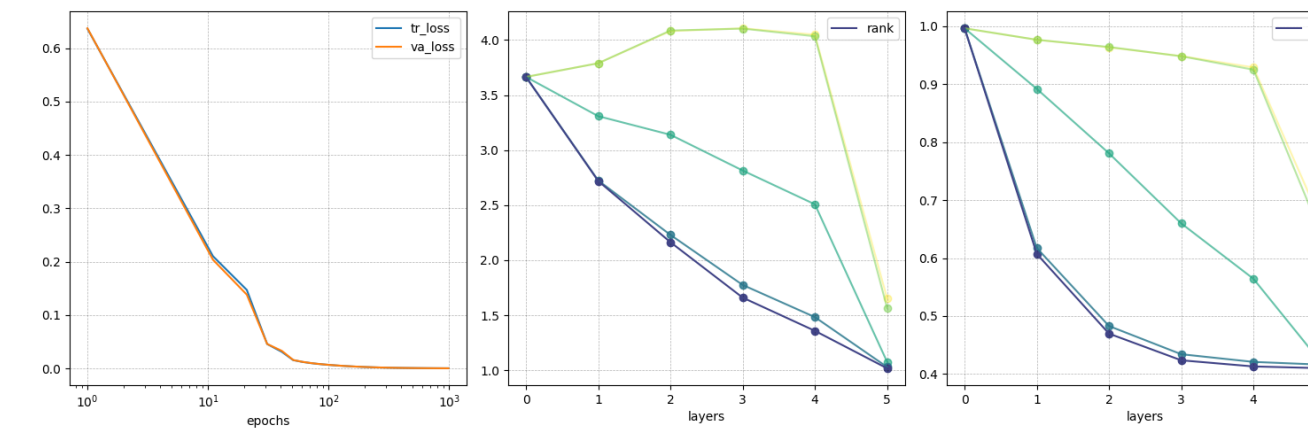
Student:

A 5-layer model: $\text{dims}=[512, 100, 100, 100, 100, 2]$

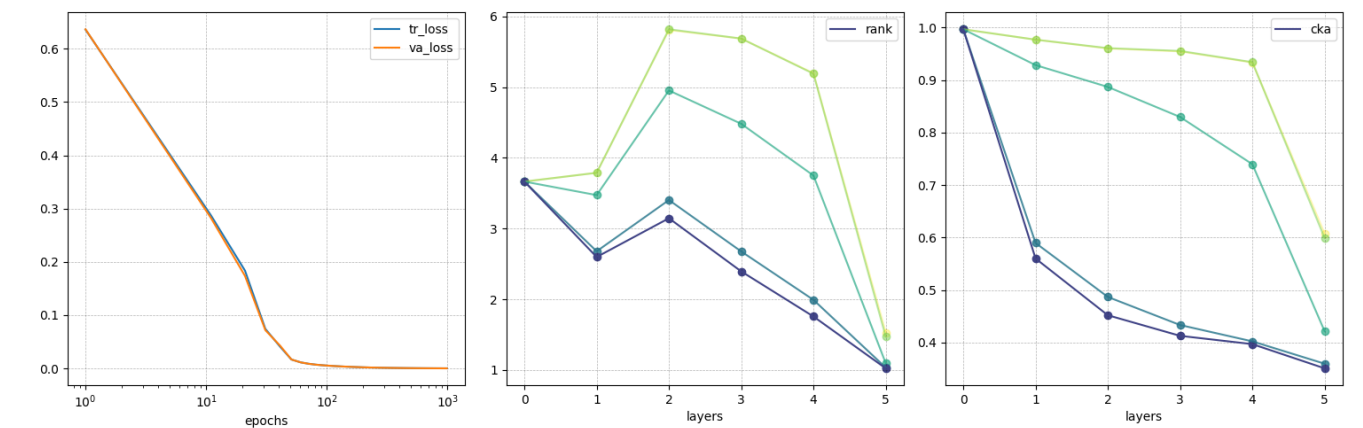
$\text{act_fn} = \text{identity} / \tanh$

Results:

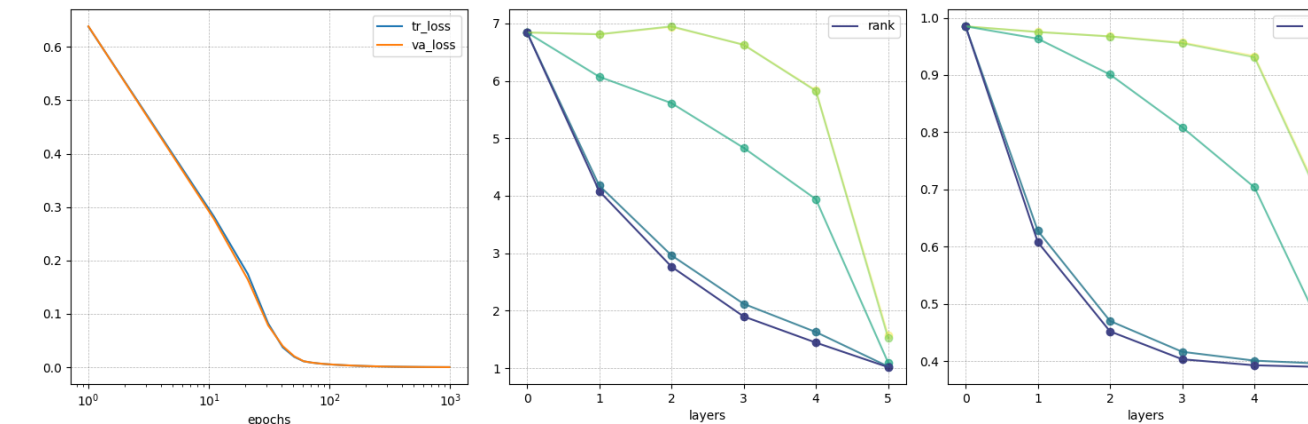
$\sigma = \text{identity}$, $\text{act_fn} = \text{identity}$



$\sigma = \text{identity}$, $\text{act_fn} = \tanh$



$\sigma = \tanh$, $\text{act_fn} = \text{identity}$



$\sigma = \tanh$, $\text{act_fn} = \tanh$

