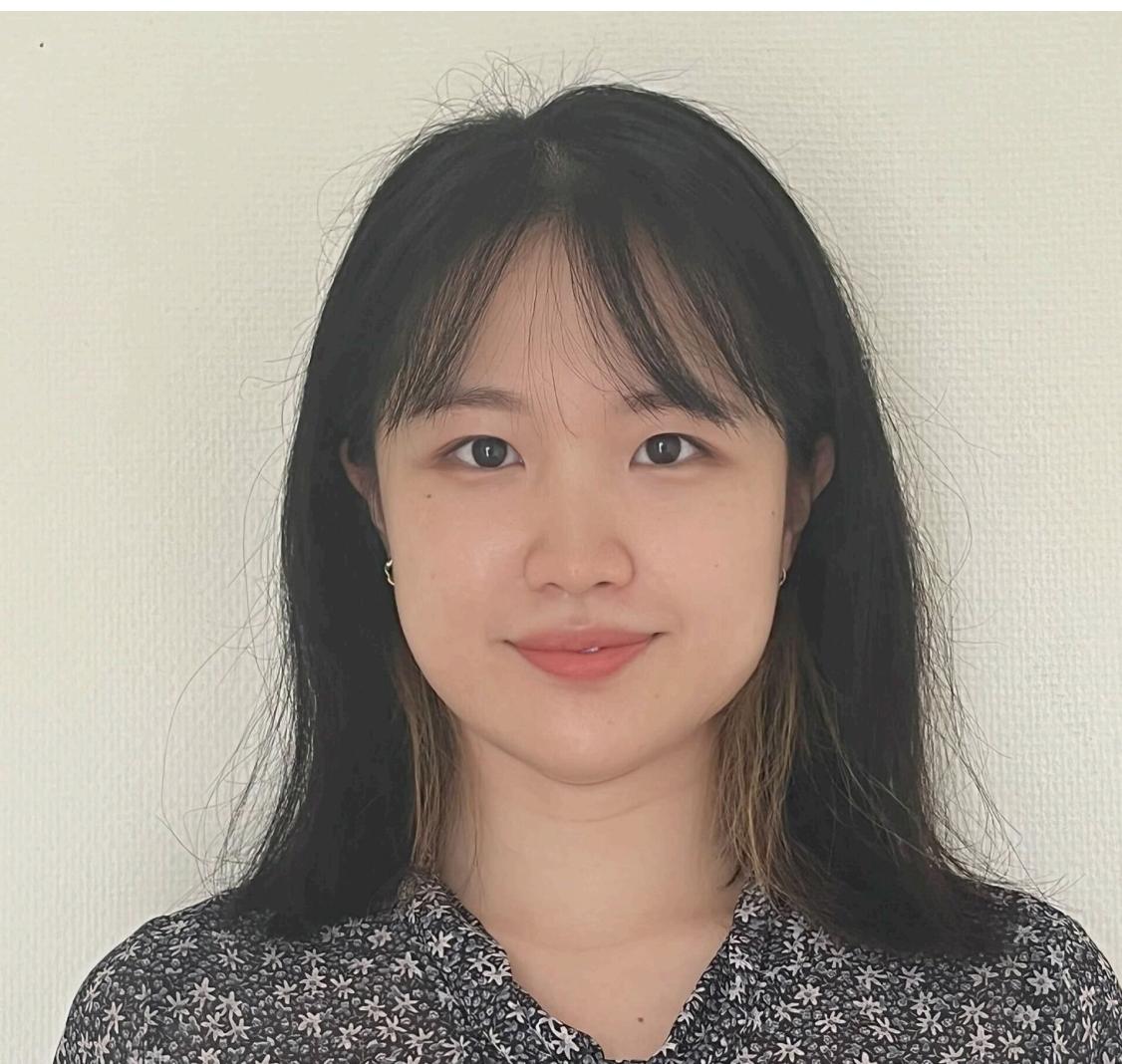


How unfair is private learning ?

Amartya Sanyal, Yaxi Hu, Fanny Yang



Amartya



Yaxi



Fanny

Privacy and Fairness

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Privacy and **Fairness** are both desirable properties in machine learning applications.



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THIS WORK: The interaction of Privacy and Fairness of nearly accurate algorithms.

Differential Privacy

Differential Privacy



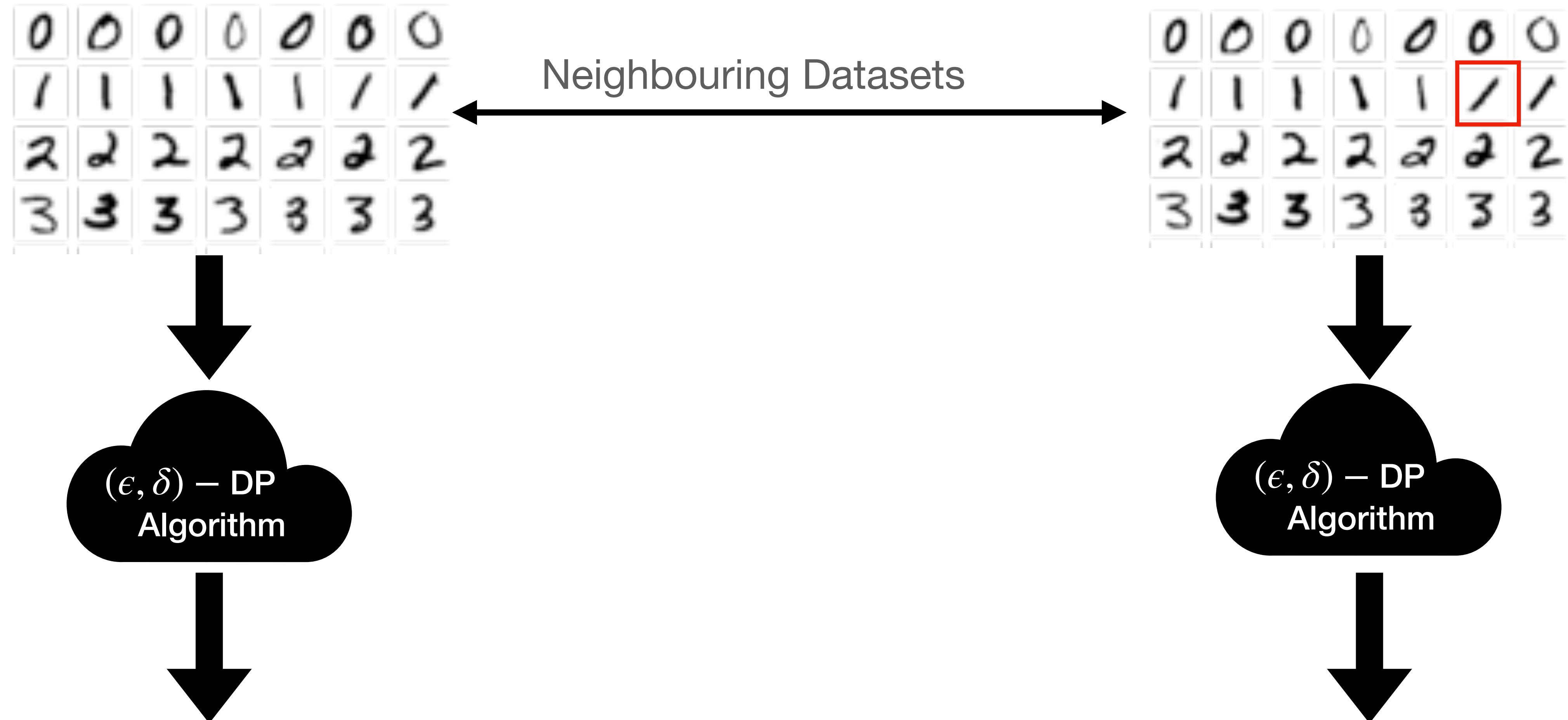
Differential Privacy



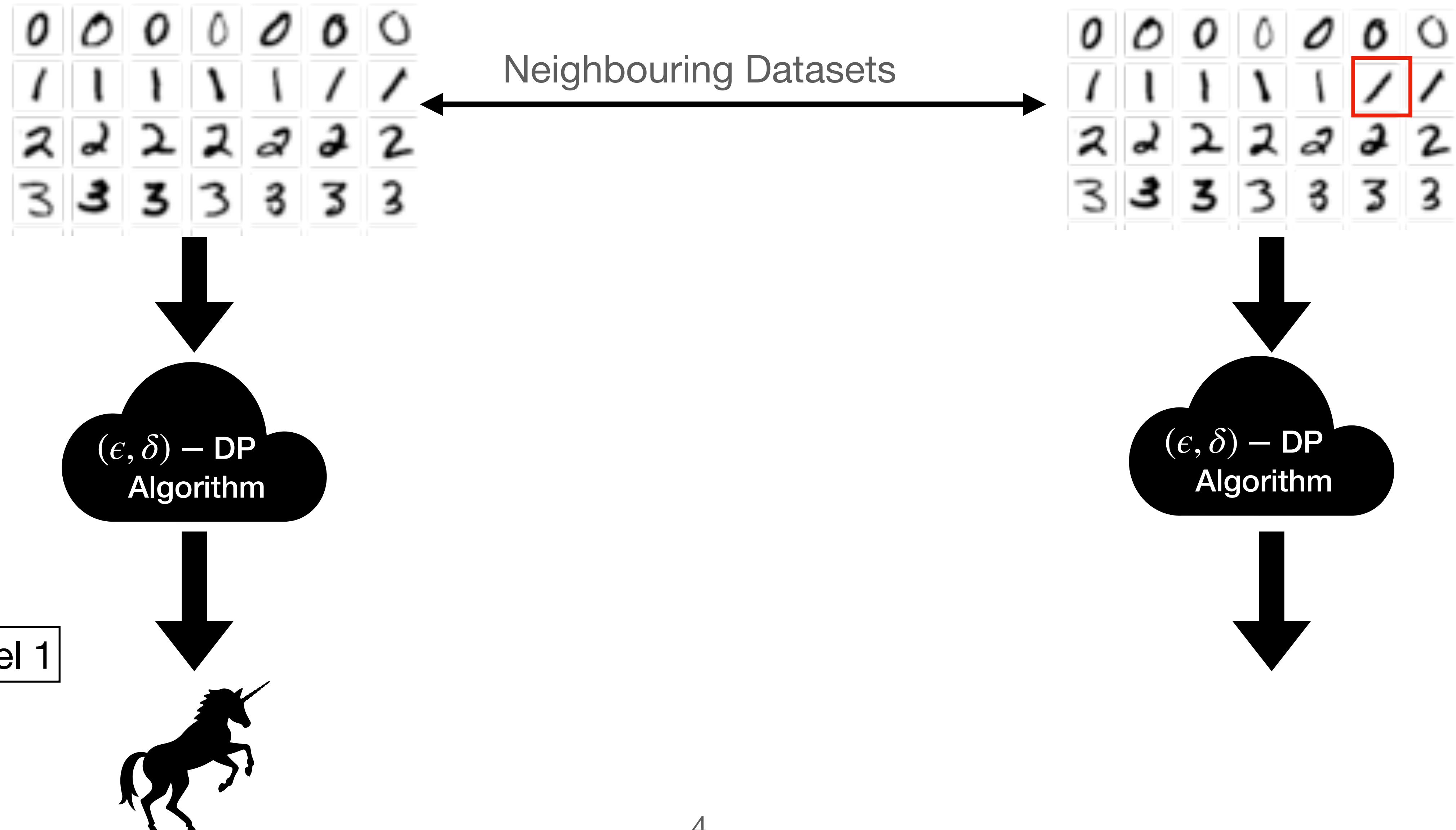
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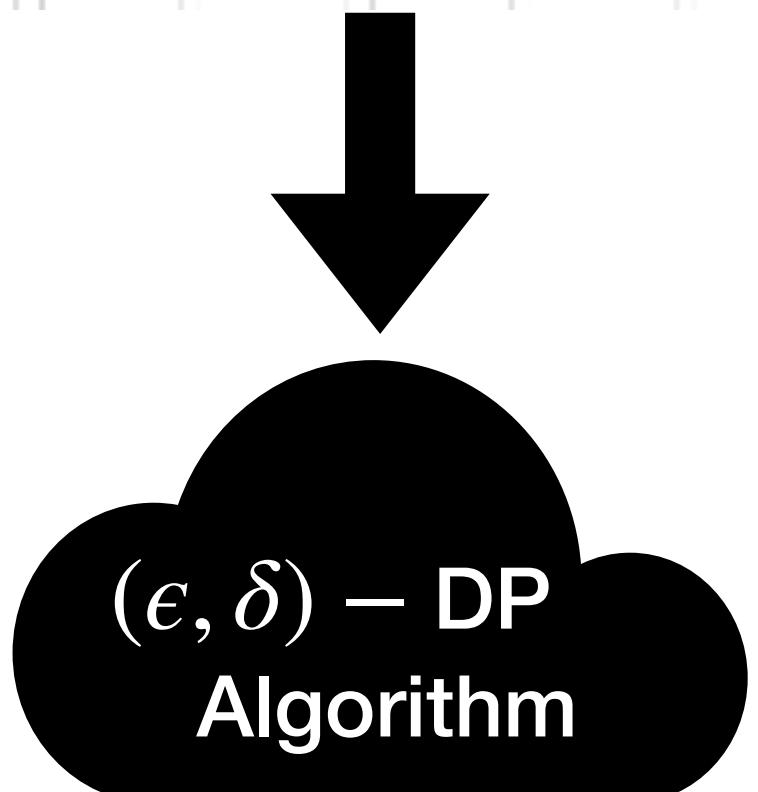
Differential Privacy



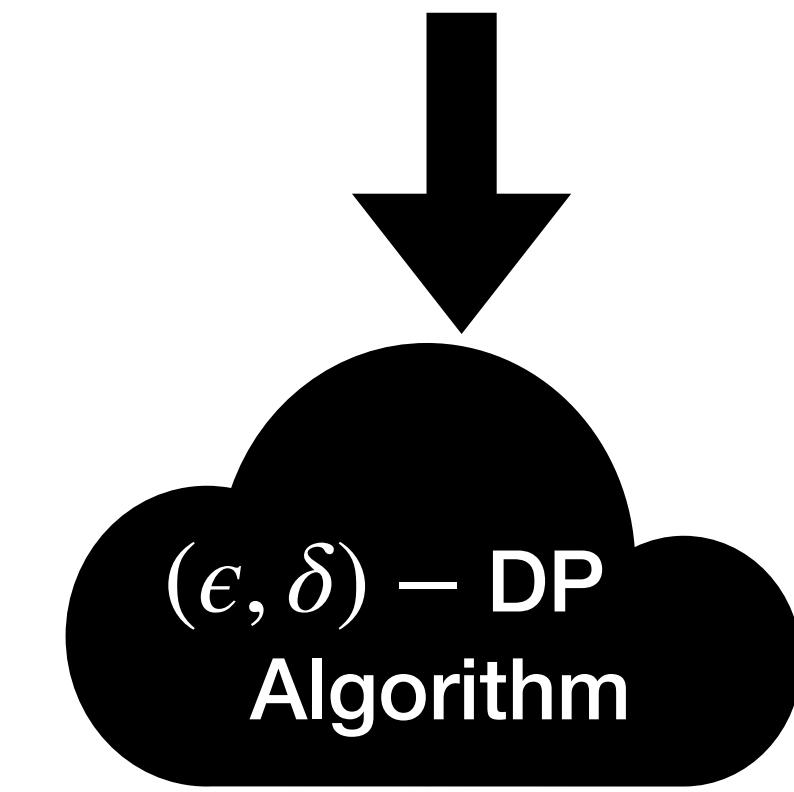
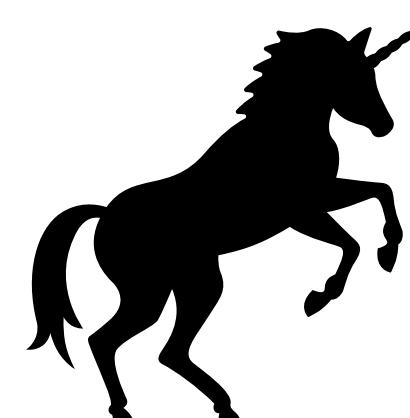
Differential Privacy



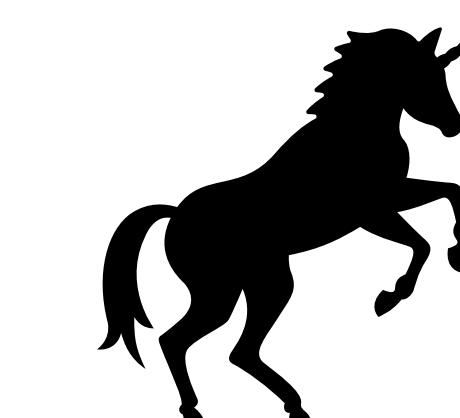
Differential Privacy



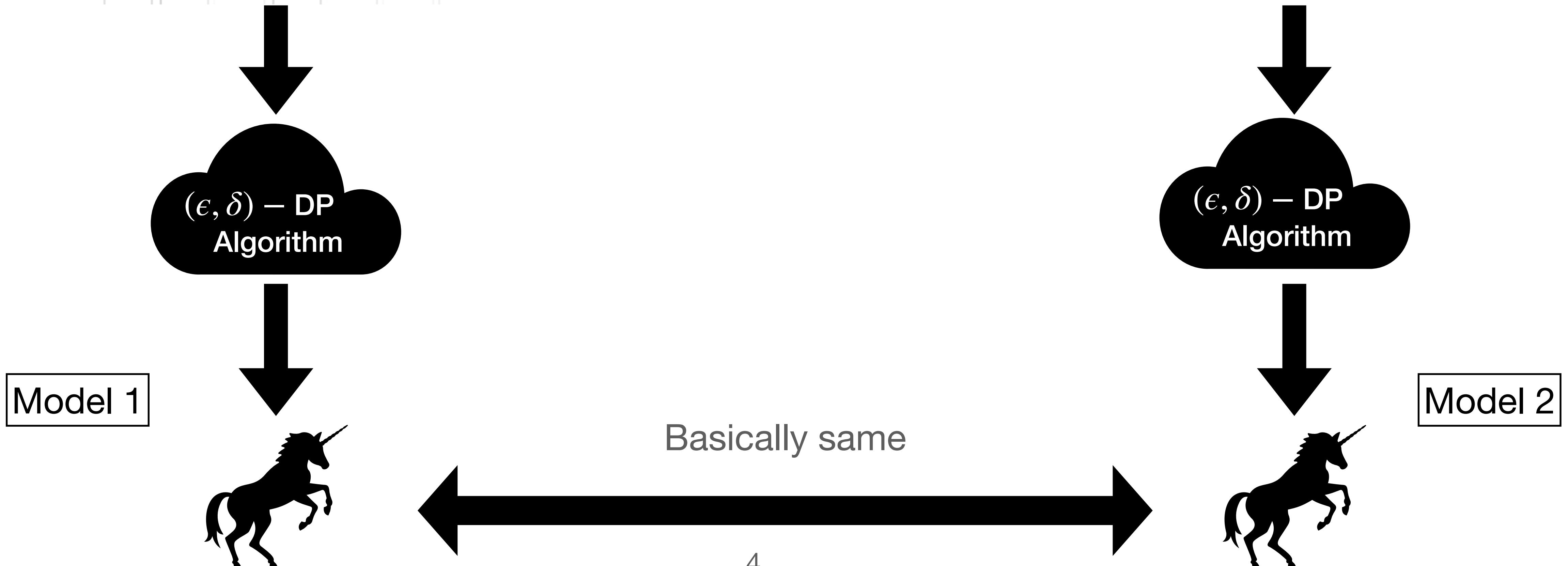
Model 1



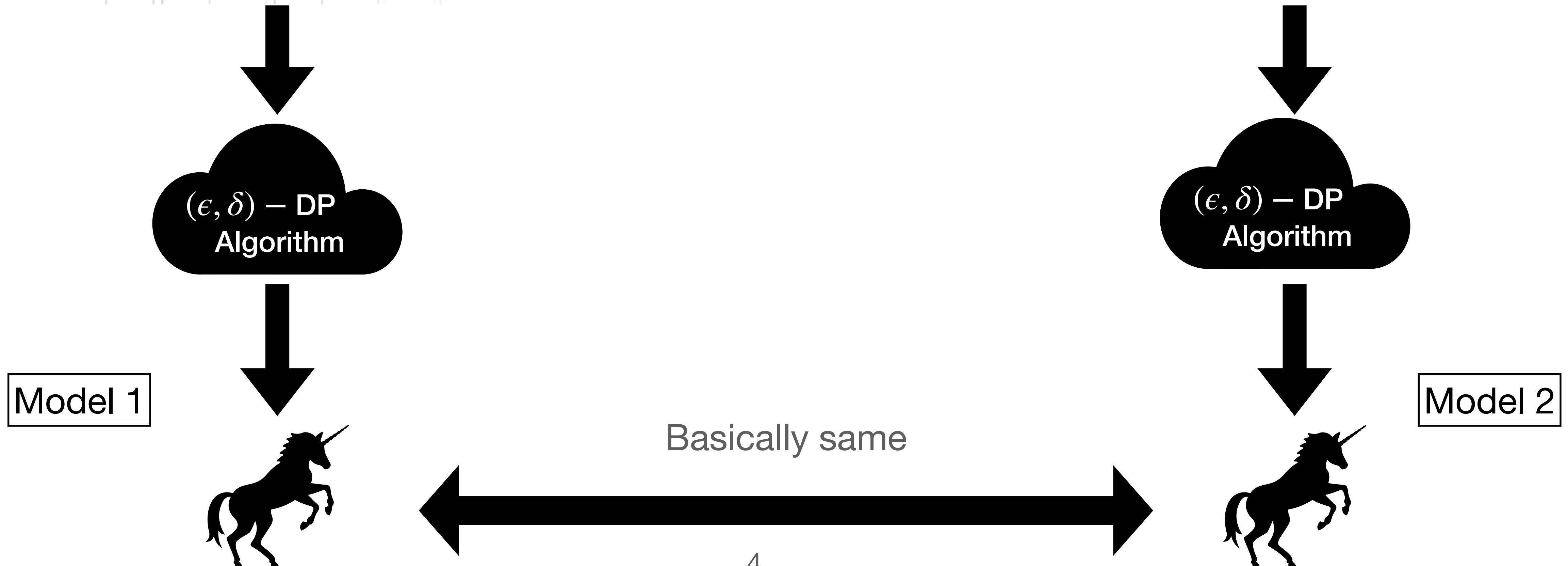
Model 2



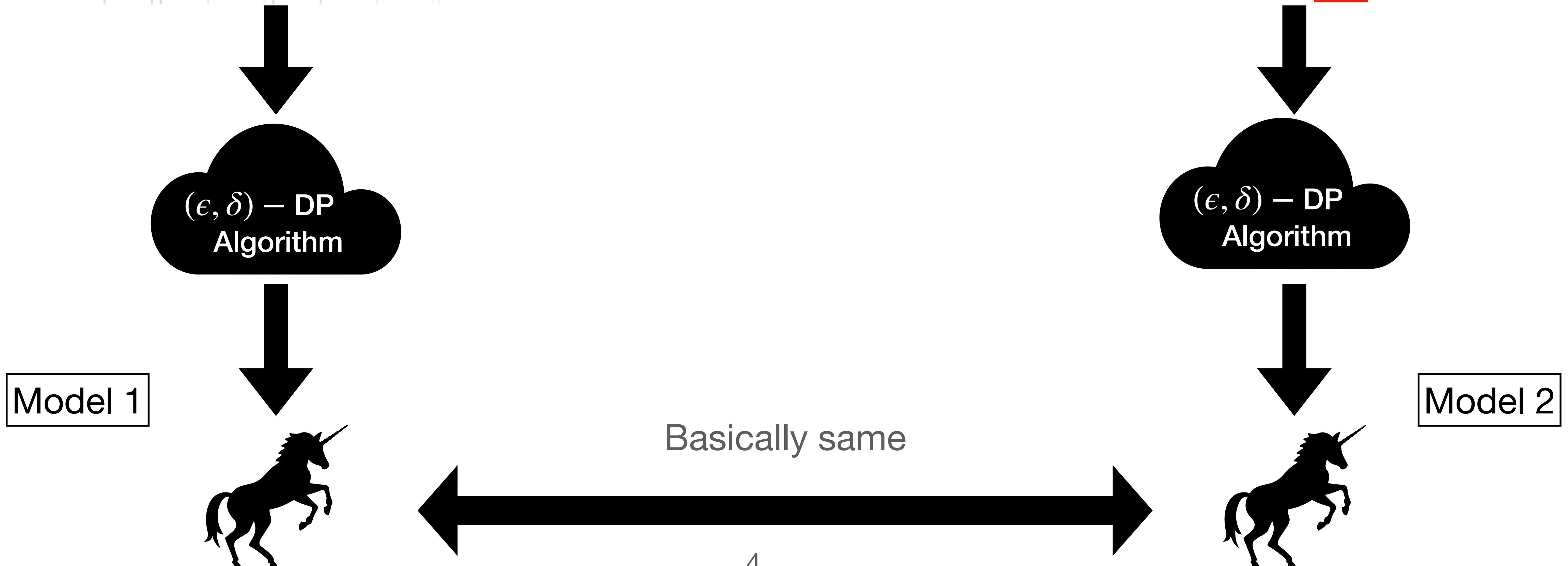
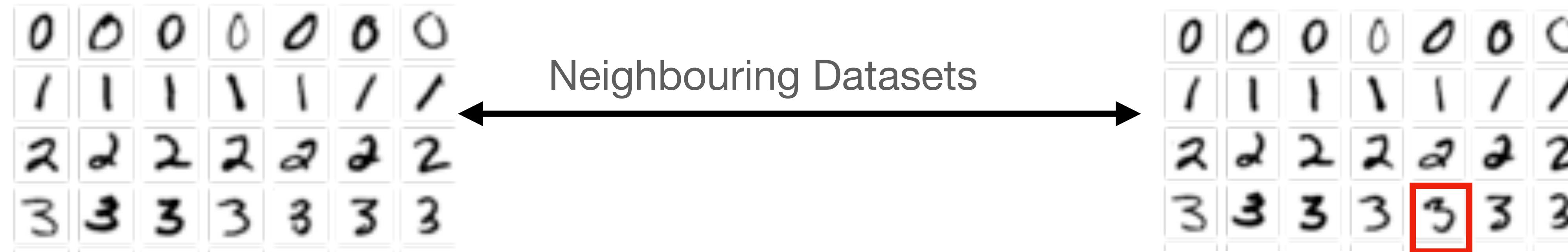
Differential Privacy



Differential Privacy



Differential Privacy



(Un) Fairness (Accuracy Discrepancy)

(Un) Fairness (Accuracy Discrepancy)

Genre

Thrillers

Superhero

B&W

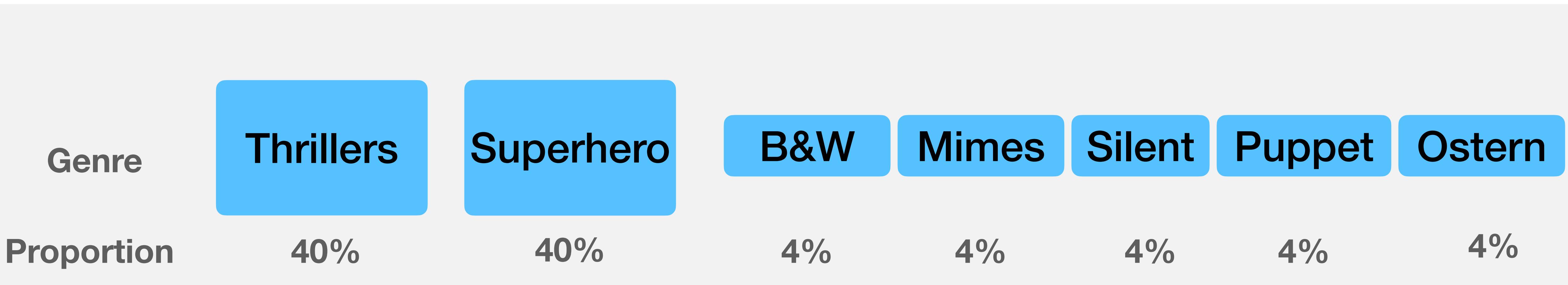
Mimes

Silent

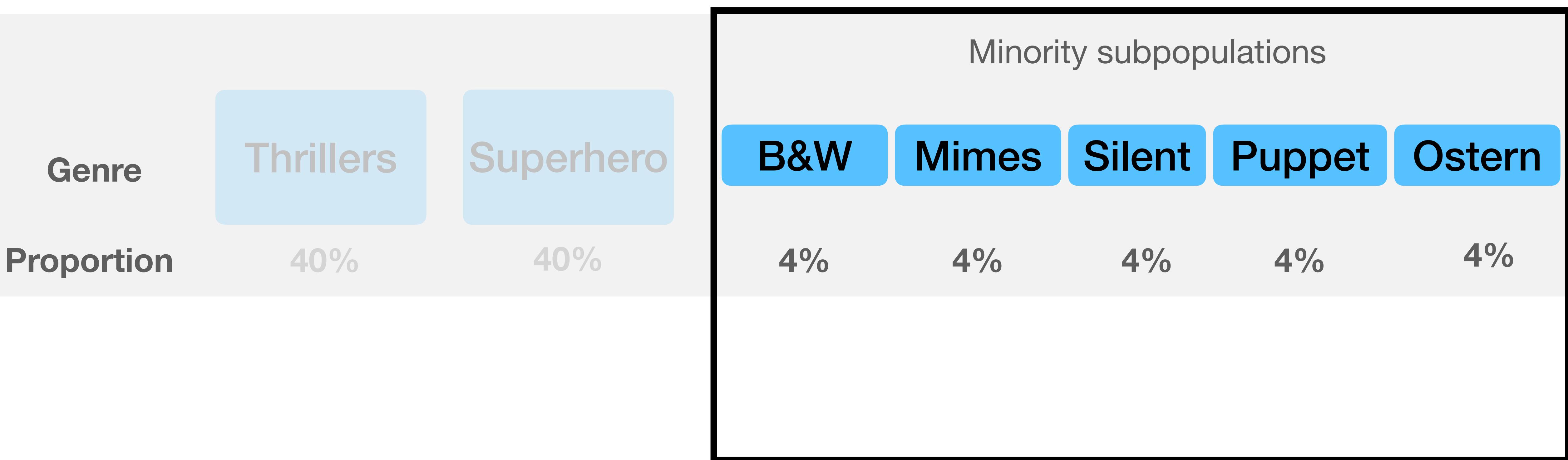
Puppet

Ostern

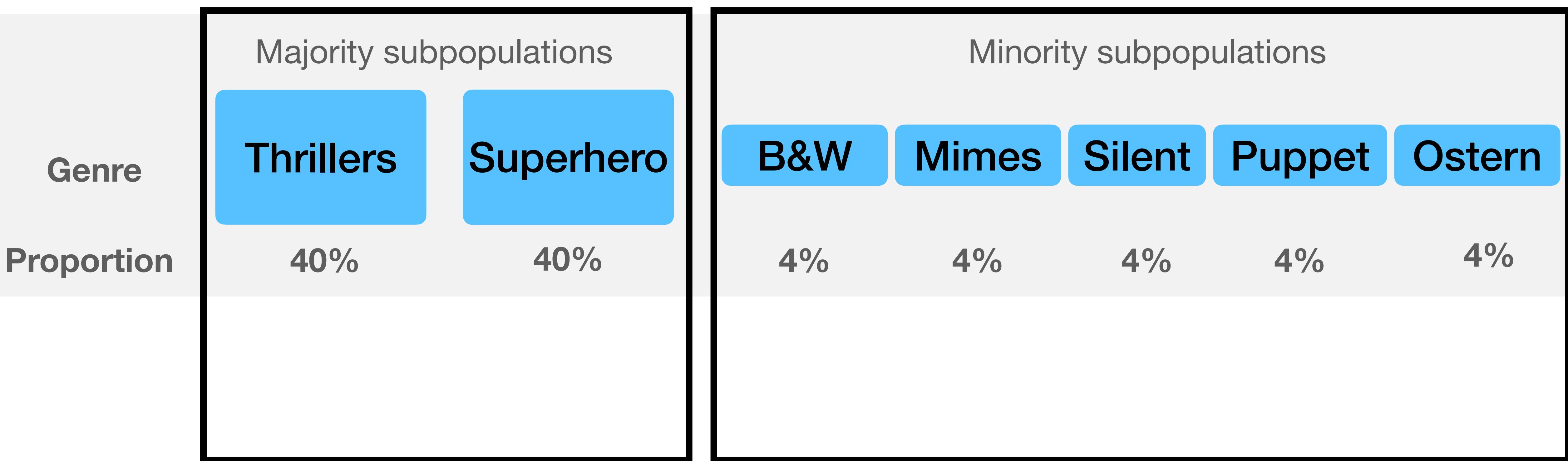
(Un) Fairness (Accuracy Discrepancy)



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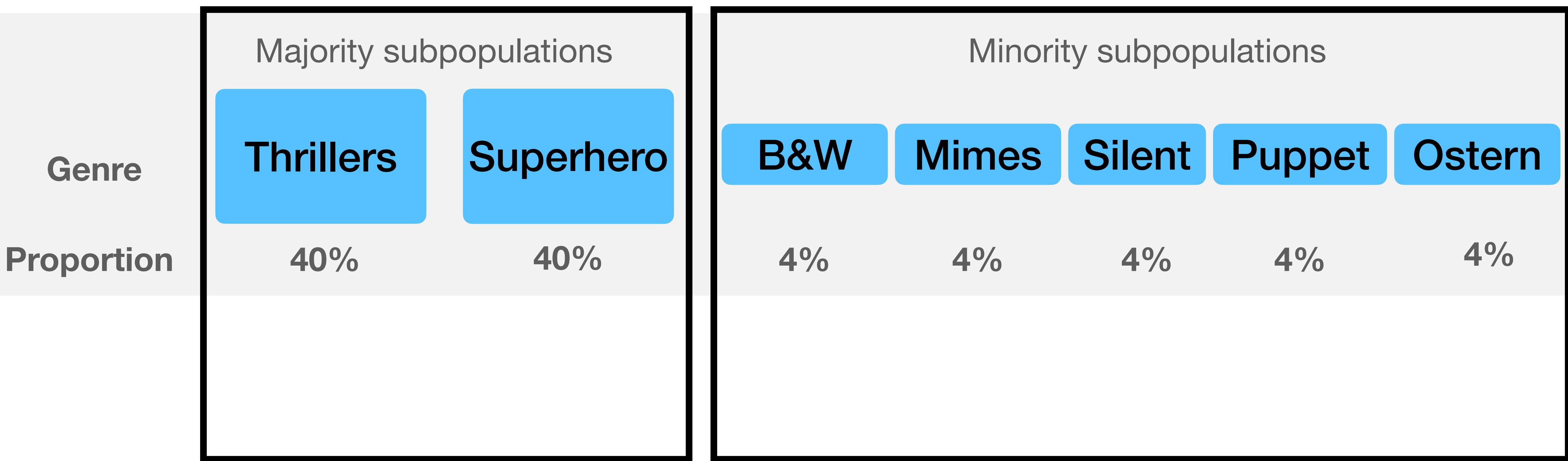


(Un) Fairness (Accuracy Discrepancy)



(Un) Fairness (Accuracy Discrepancy)

ML Problem: Is the movie safe to watch for kids ?



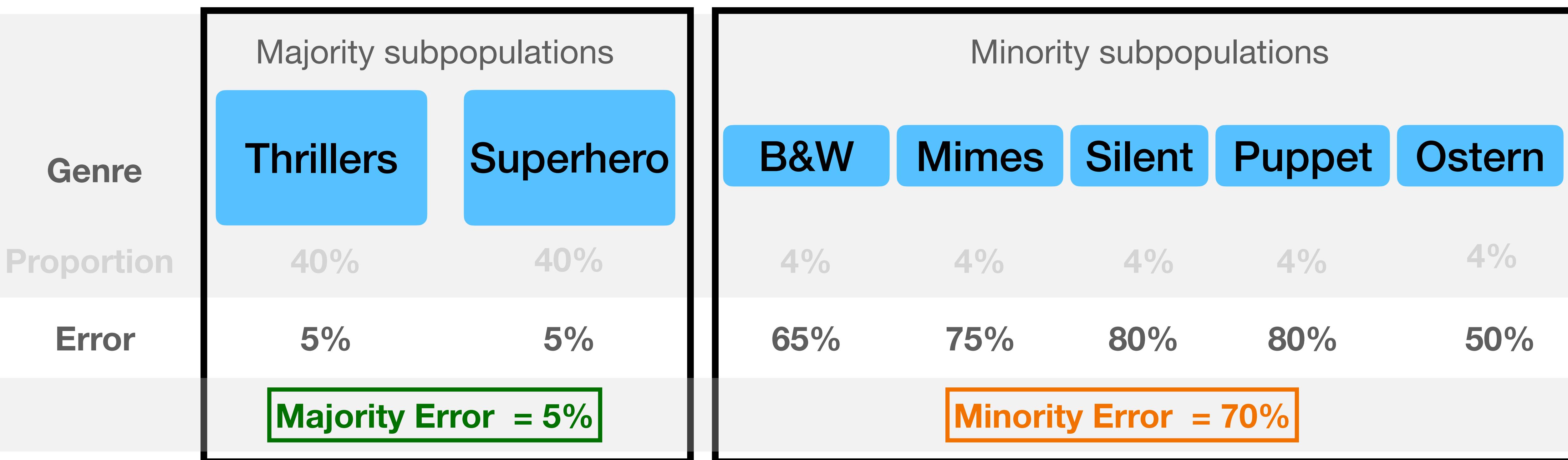
(Un) Fairness (Accuracy Discrepancy)

ML Problem: Is the movie safe to watch for kids ?

	Majority subpopulations		Minority subpopulations				
Genre	Thrillers	Superhero	B&W	Mimes	Silent	Puppet	Ostern
Proportion	40%	40%	4%	4%	4%	4%	4%
Error	5%	5%	65%	75%	80%	80%	50%

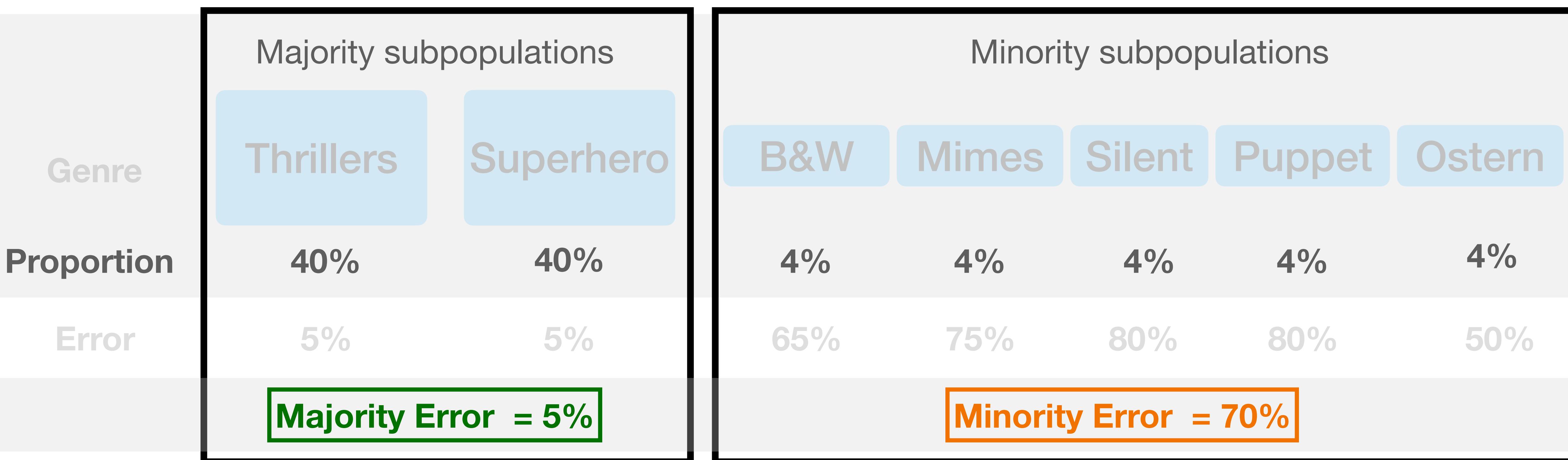
(Un) Fairness (Accuracy Discrepancy)

ML Problem: Is the movie safe to watch for kids ?



(Un) Fairness (Accuracy Discrepancy)

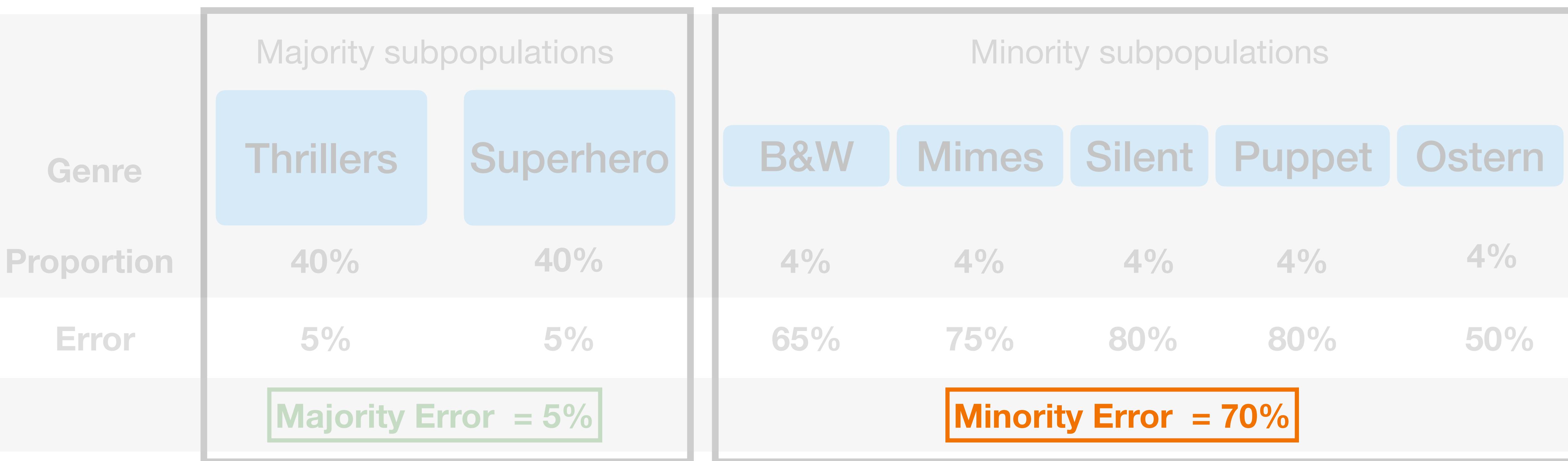
ML Problem: Is the movie safe to watch for kids ?



Total Error = 18%

(Un) Fairness (Accuracy Discrepancy)

ML Problem: Is the movie safe to watch for kids ?

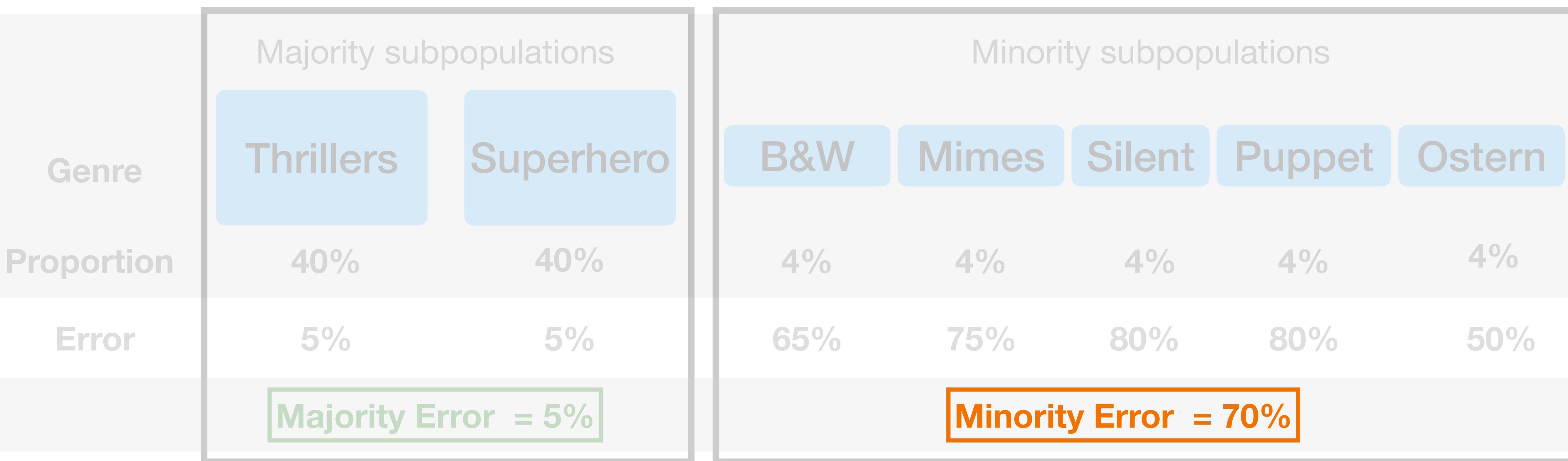


Total Error = 18%

Accuracy Discrepancy = Minority Error - Total Error

(Un) Fairness (Accuracy Discrepancy)

ML Problem: Is the movie safe to watch for kids ?



Total Error = 18%

Accuracy Discrepancy = $70 - 18 = 52\%$

Example dataset

CelebA

Example dataset

CelebA



Example dataset

CelebA

40 binary attributes with each image

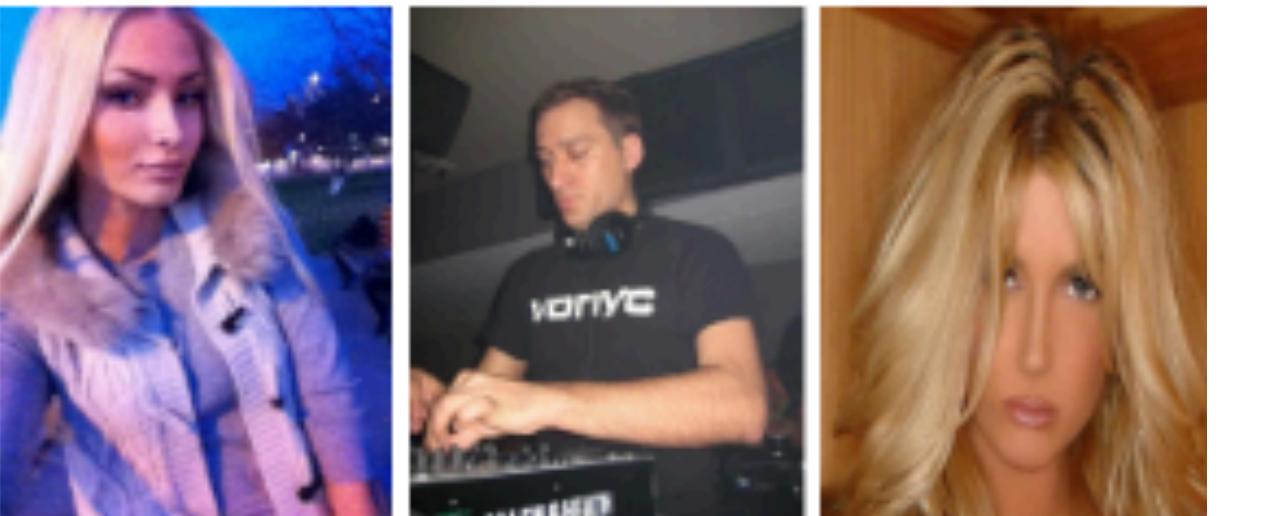
Eyeglasses



Bangs



Pointy Noise



Example dataset

CelebA

40 binary attributes -> 2^{40} subpopulations.

40 binary attributes with each image

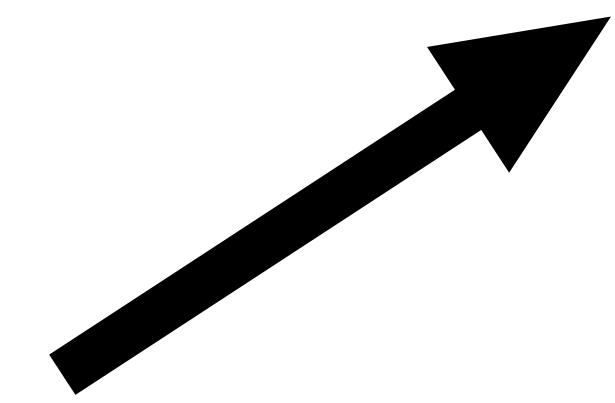
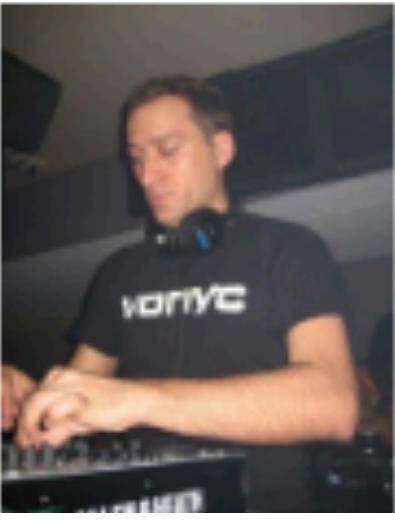
Eyeglasses



Bangs



Pointy Nose

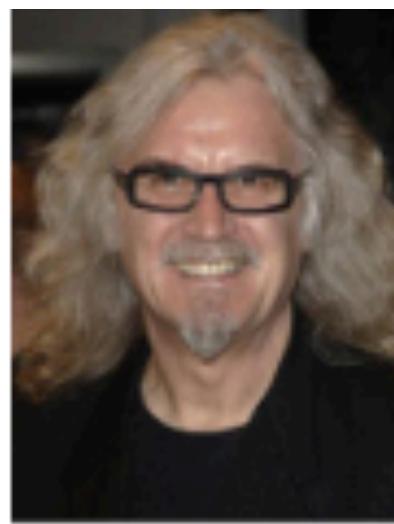


Example dataset

CelebA

40 binary attributes with each image

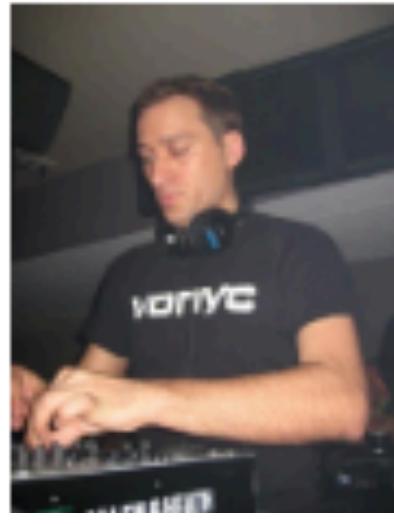
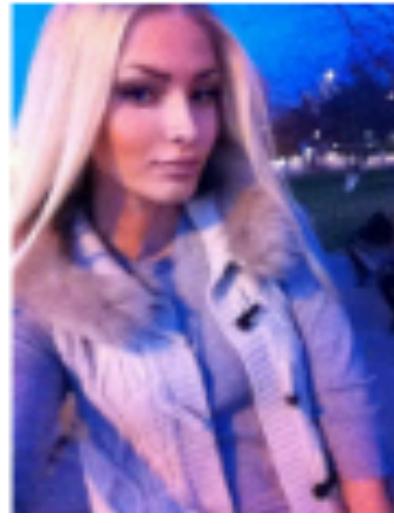
Eyeglasses



Bangs

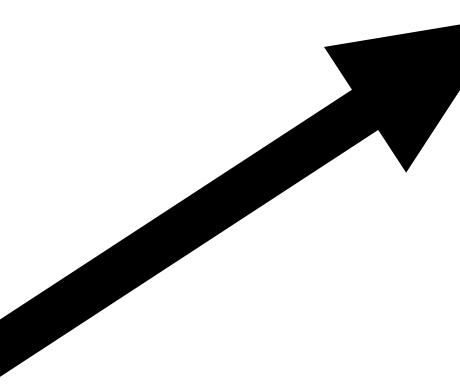


Pointy Nose



40 binary attributes -> 2^{40} subpopulations.

- Subpopulation 1: Eyeglasses, bangs, ..., pointy nose.



Example dataset

CelebA

40 binary attributes with each image

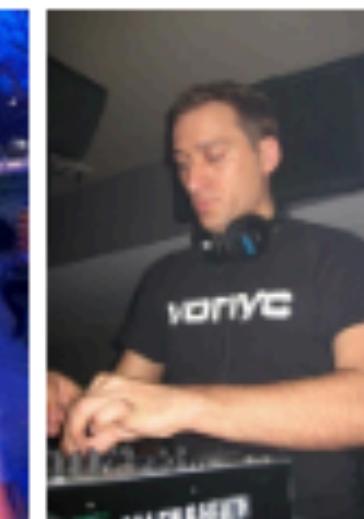
Eyeglasses



Bangs



Pointy Nose



40 binary attributes -> 2^{40} subpopulations.

- Subpopulation 1: Eyeglasses, bangs, ..., pointy nose.
- Subpopulation 2: No eyeglasses, bangs,.....,pointy noise.

Example dataset

CelebA

40 binary attributes with each image

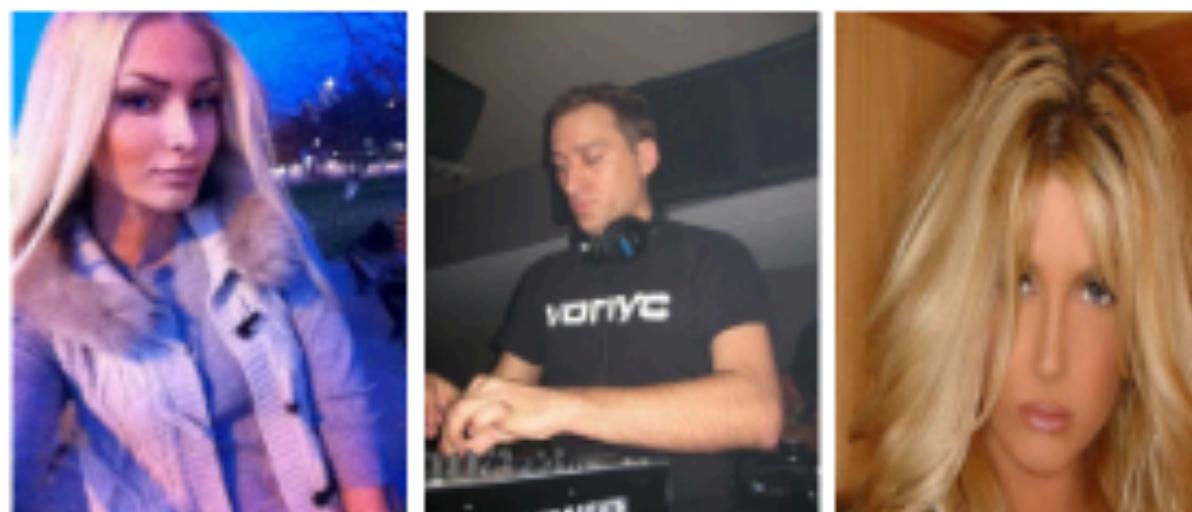
Eyeglasses



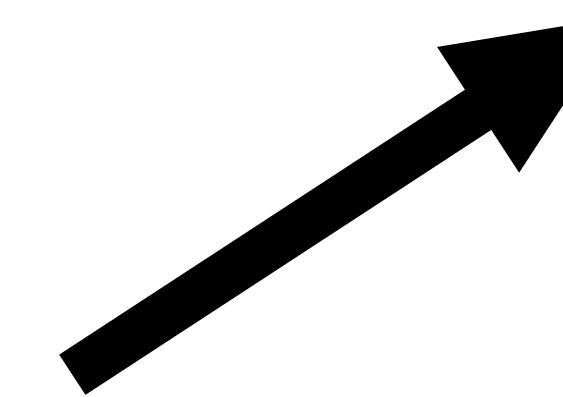
Bangs



Pointy Nose



40 binary attributes -> 2^{40} subpopulations.



- Subpopulation 1: Eyeglasses, bangs, ..., pointy nose.
- Subpopulation 2: No eyeglasses, bangs,.....,pointy noise.
- ...
- ...
- Subpopulation 2^{40} : No eyeglasses, no bangs,..., no pointy nose.

Example dataset

CelebA

40 binary attributes with each image

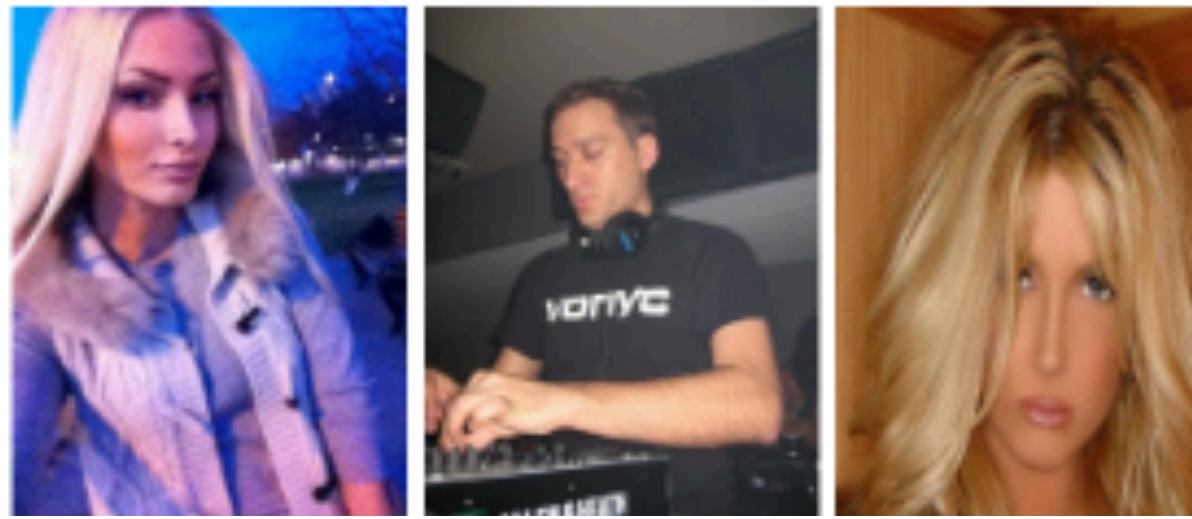
Eyeglasses



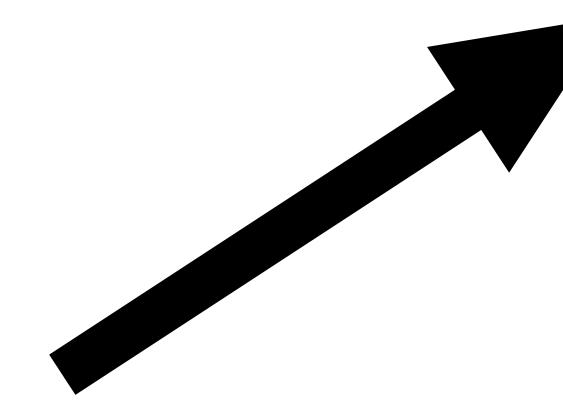
Bangs



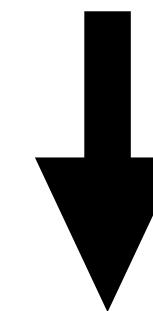
Pointy Nose



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Example dataset

CelebA

40 binary attributes with each image

Eyeglasses



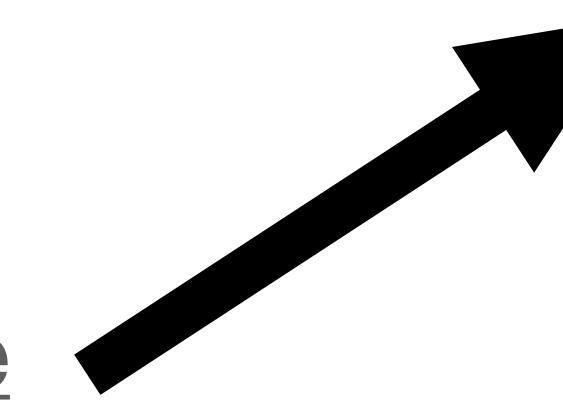
Bangs



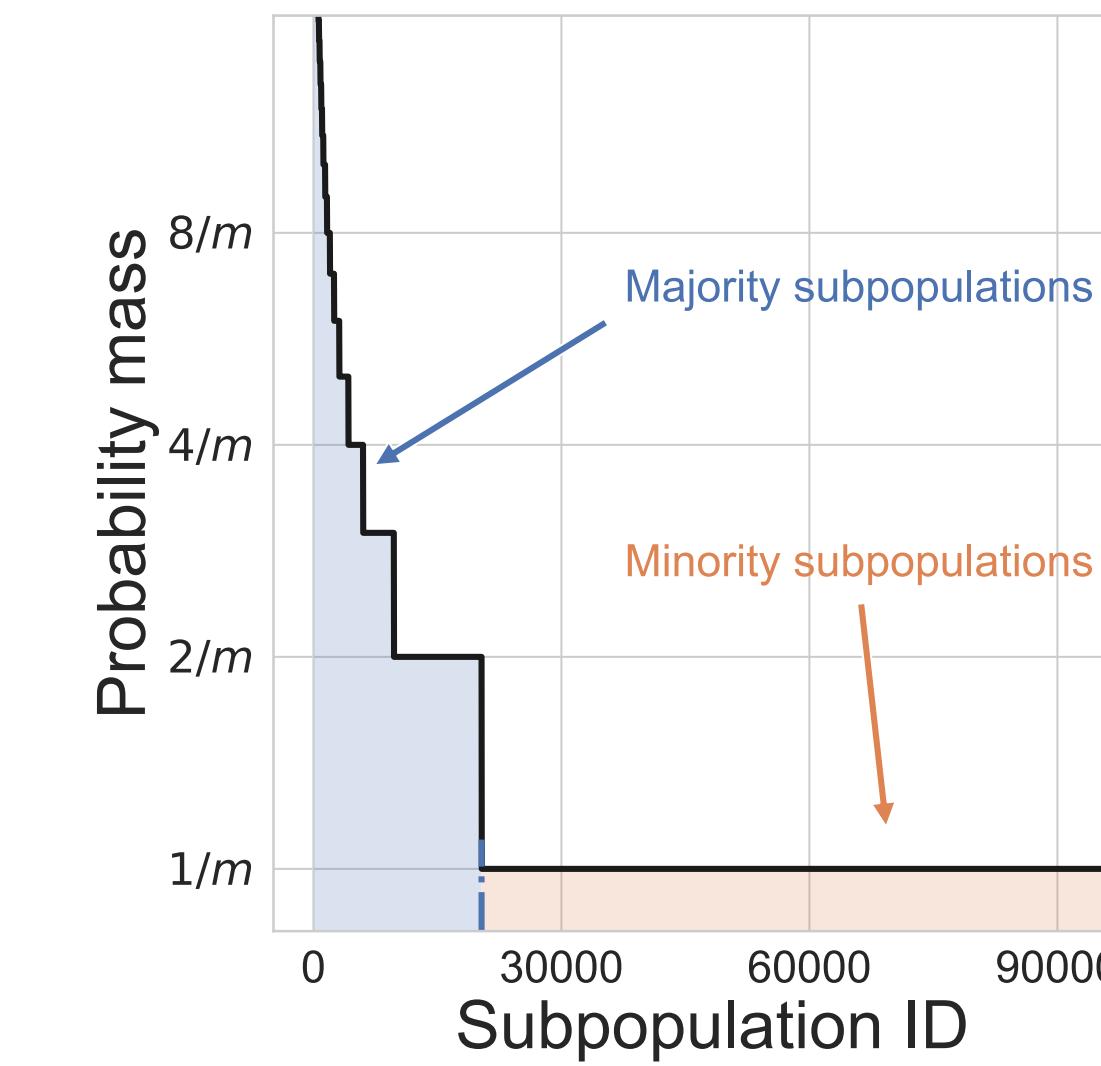
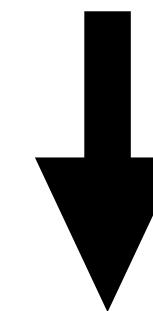
Pointy Noise



40 binary attributes -> 2^{40} subpopulations.



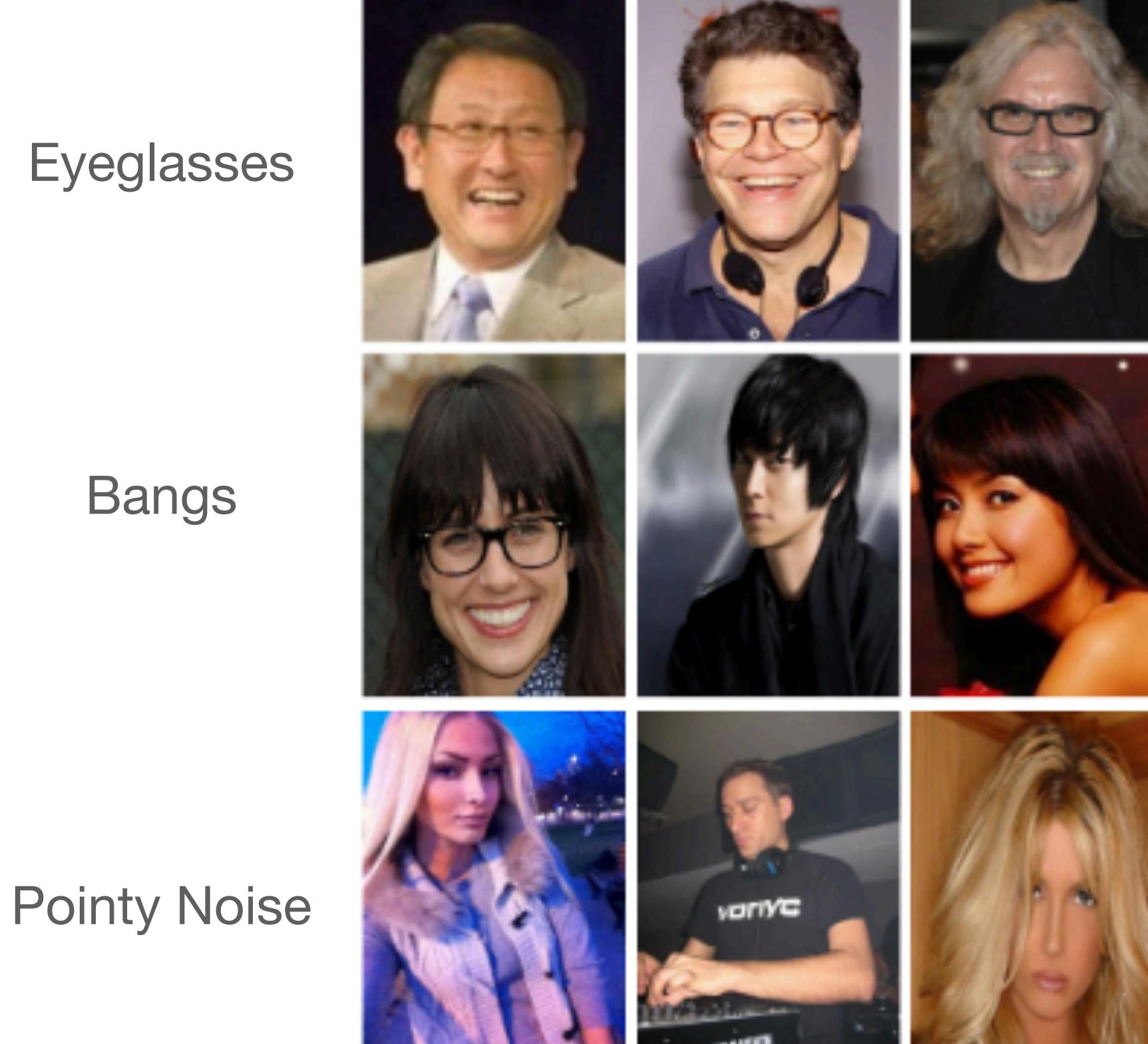
- Subpopulation 1: Eyeglasses, bangs, ..., pointy nose.
- Subpopulation 2: No eyeglasses, bangs,.....,pointy noise.
- ...
- ...
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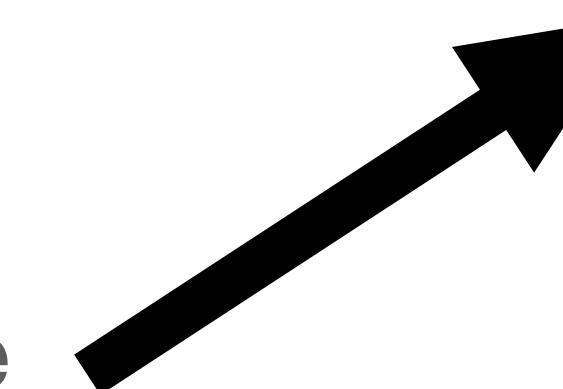
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CelebA

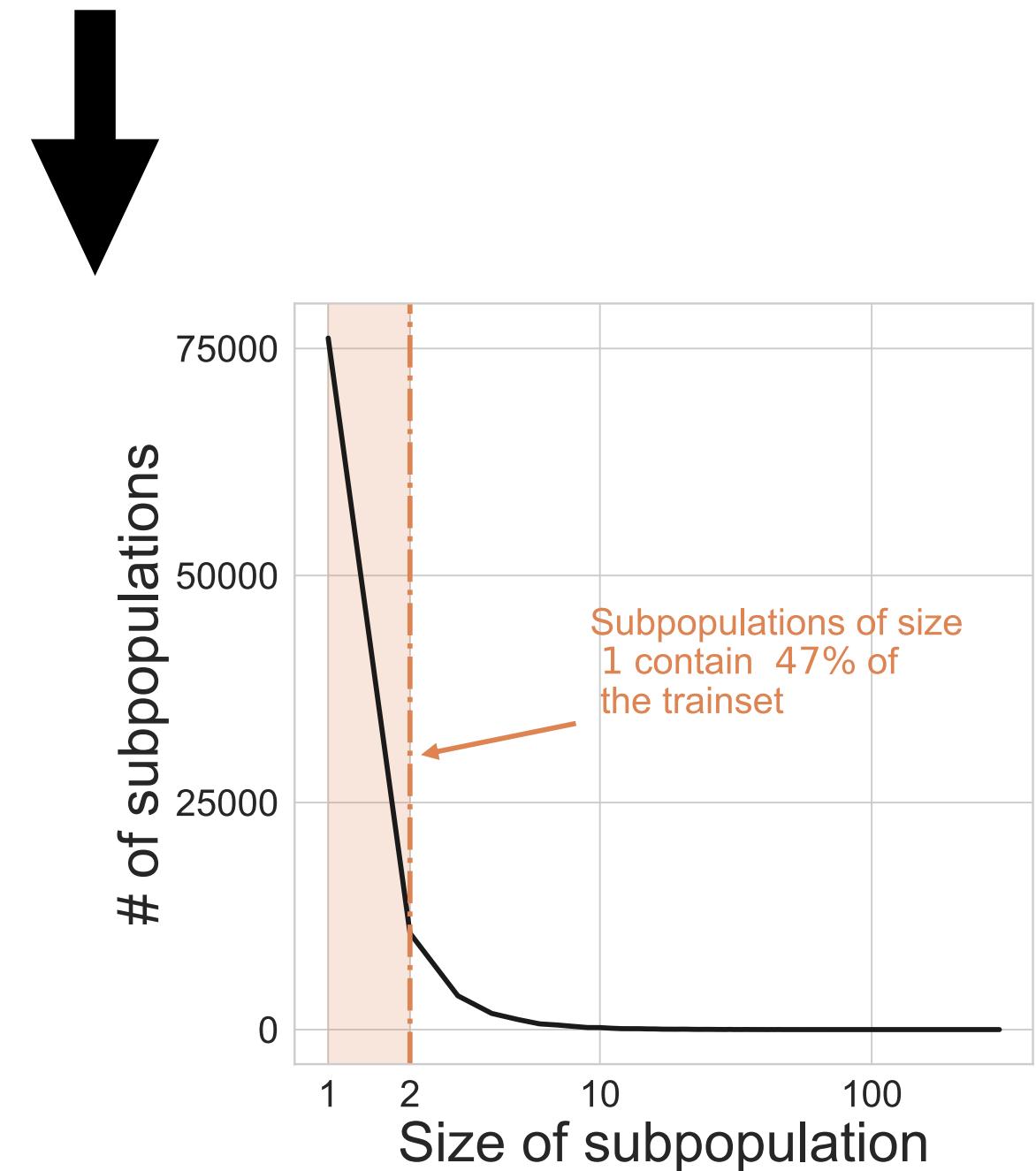
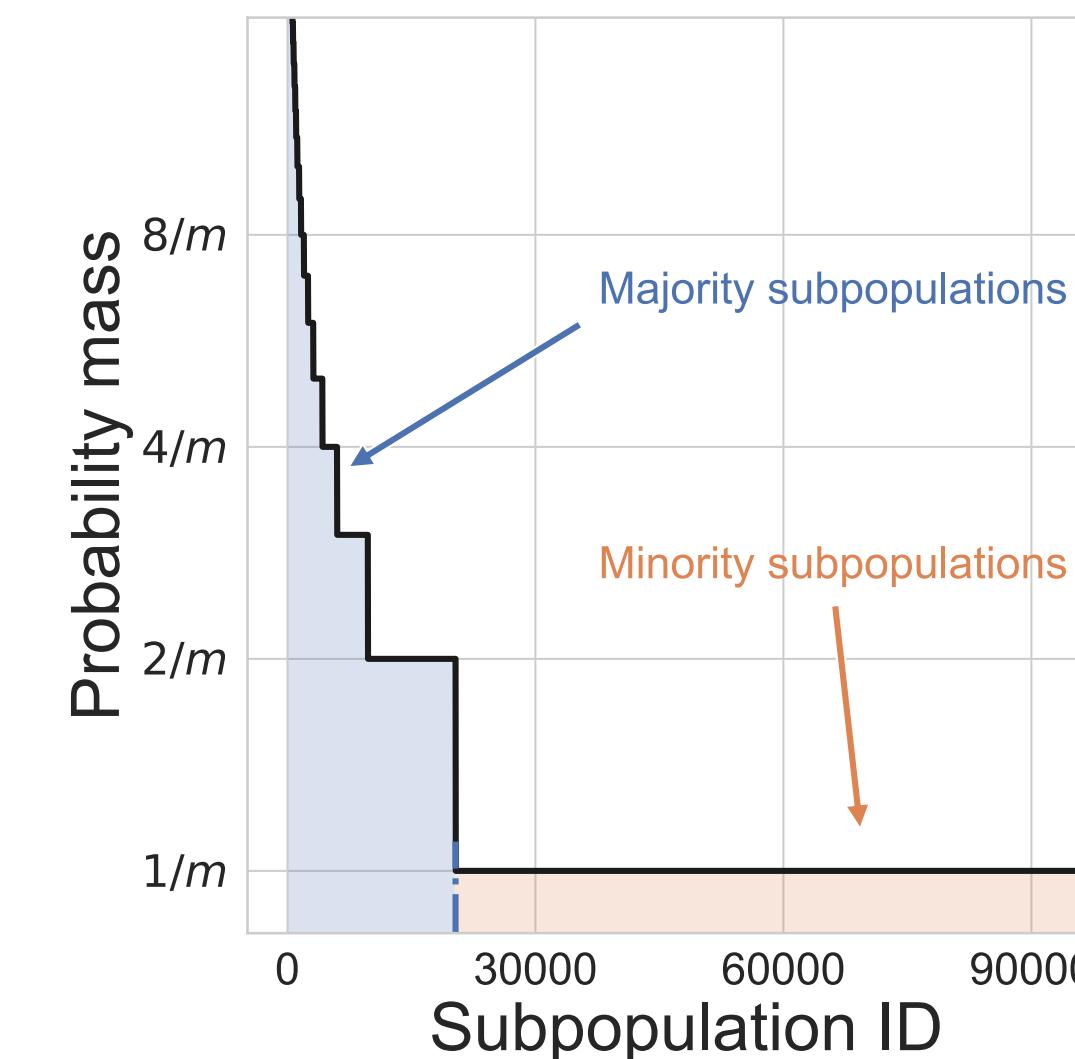
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- ...
- ...
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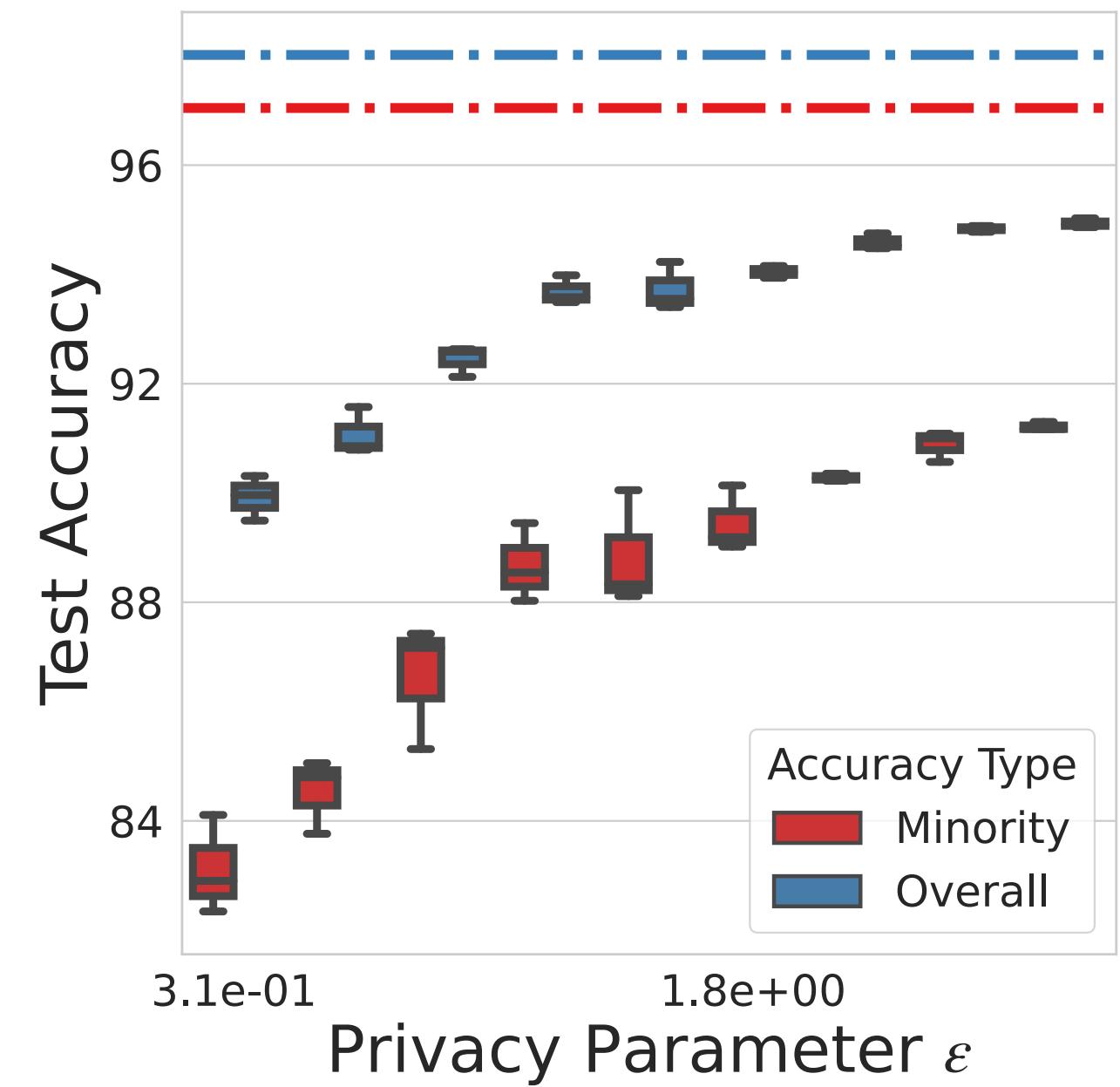


Privacy vs Fairness

CelebA

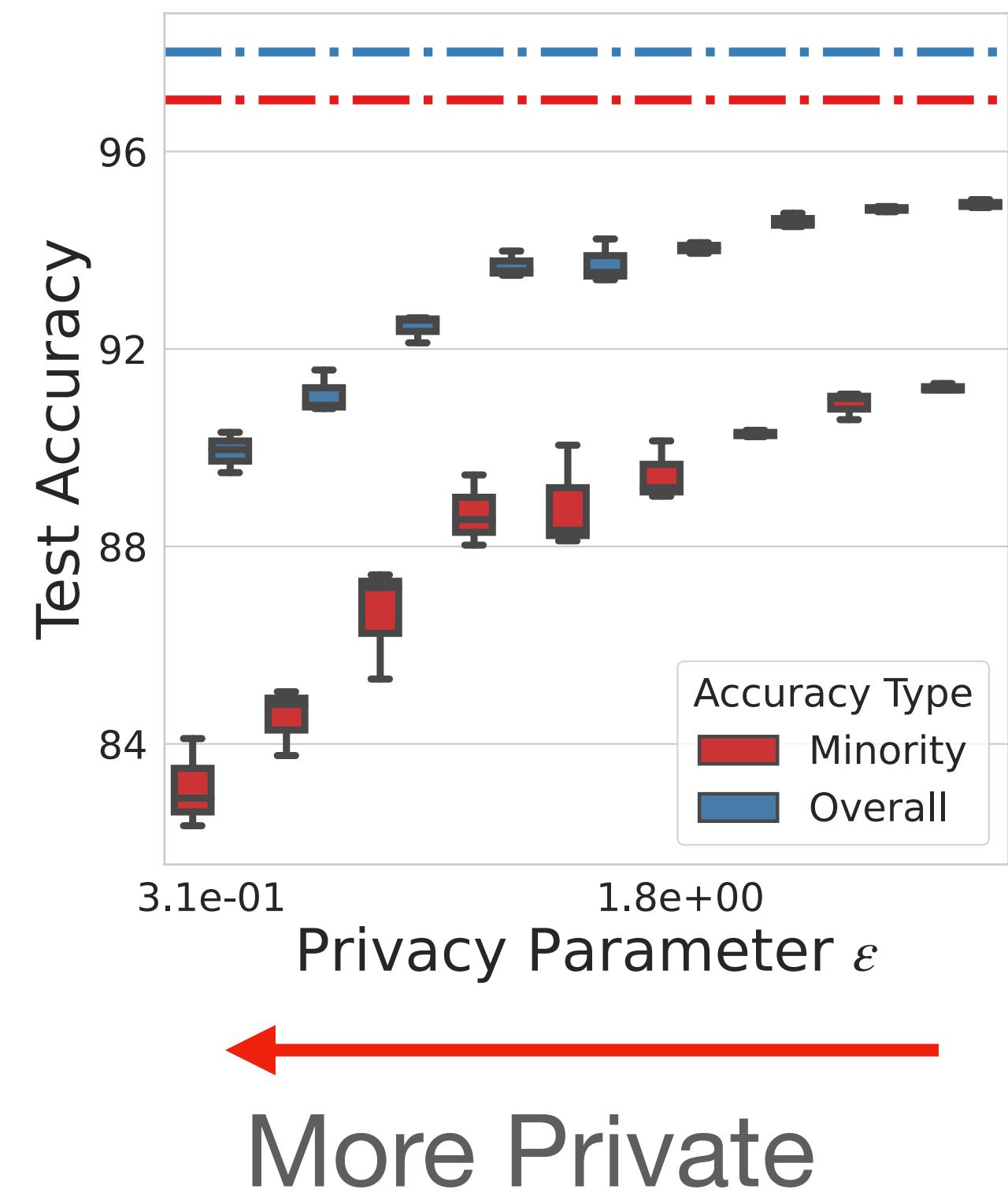
Privacy vs Fairness

CelebA



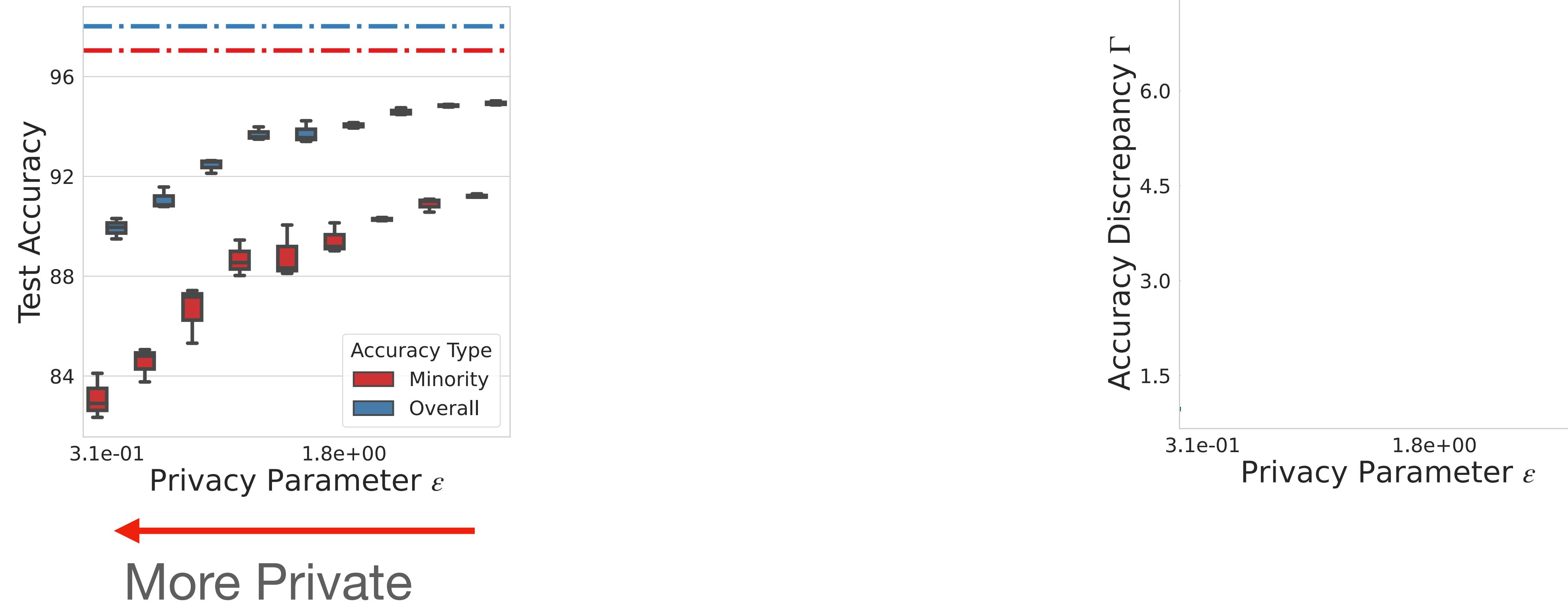
Privacy vs Fairness

CelebA



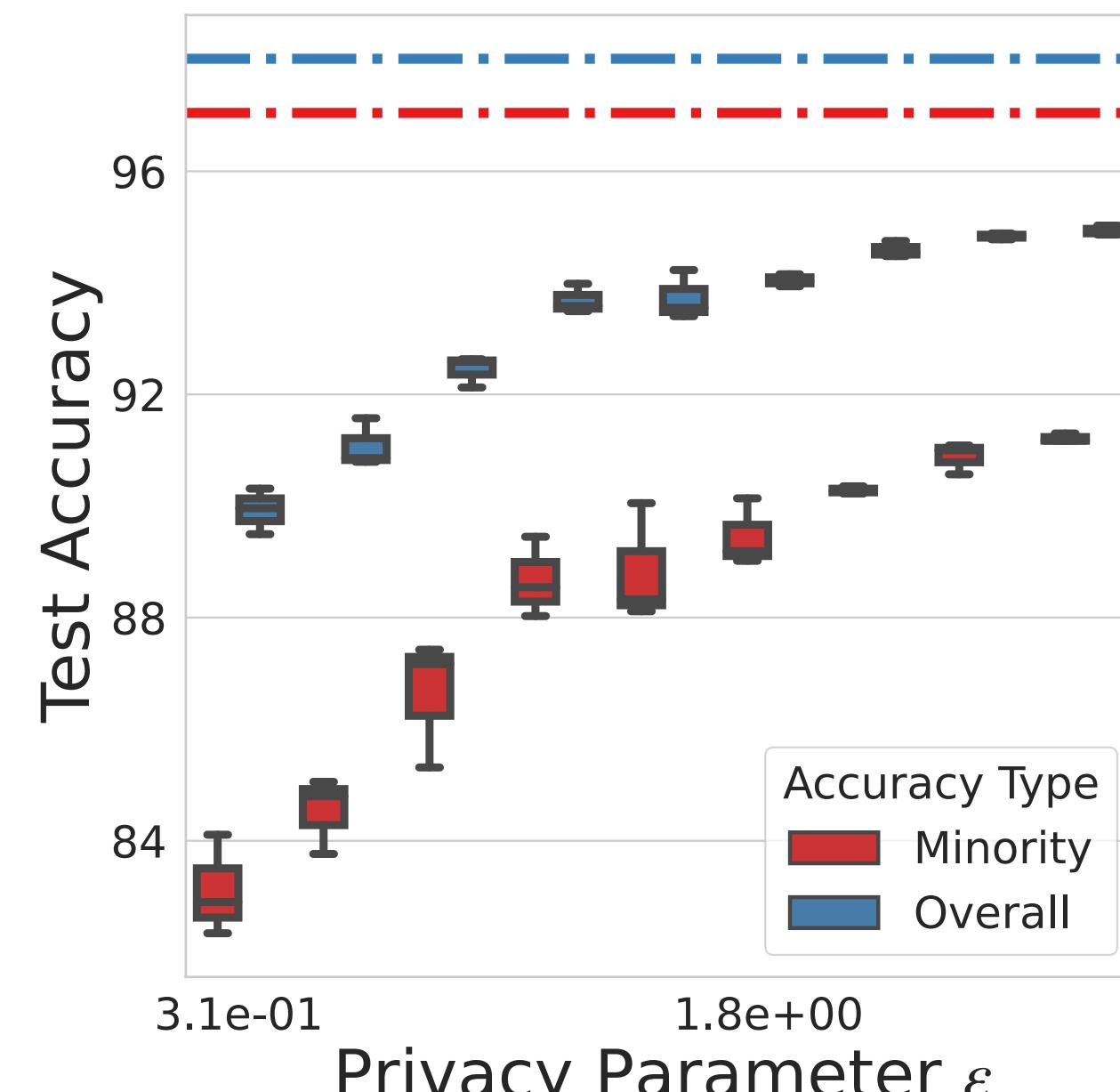
Privacy vs Fairness

CelebA

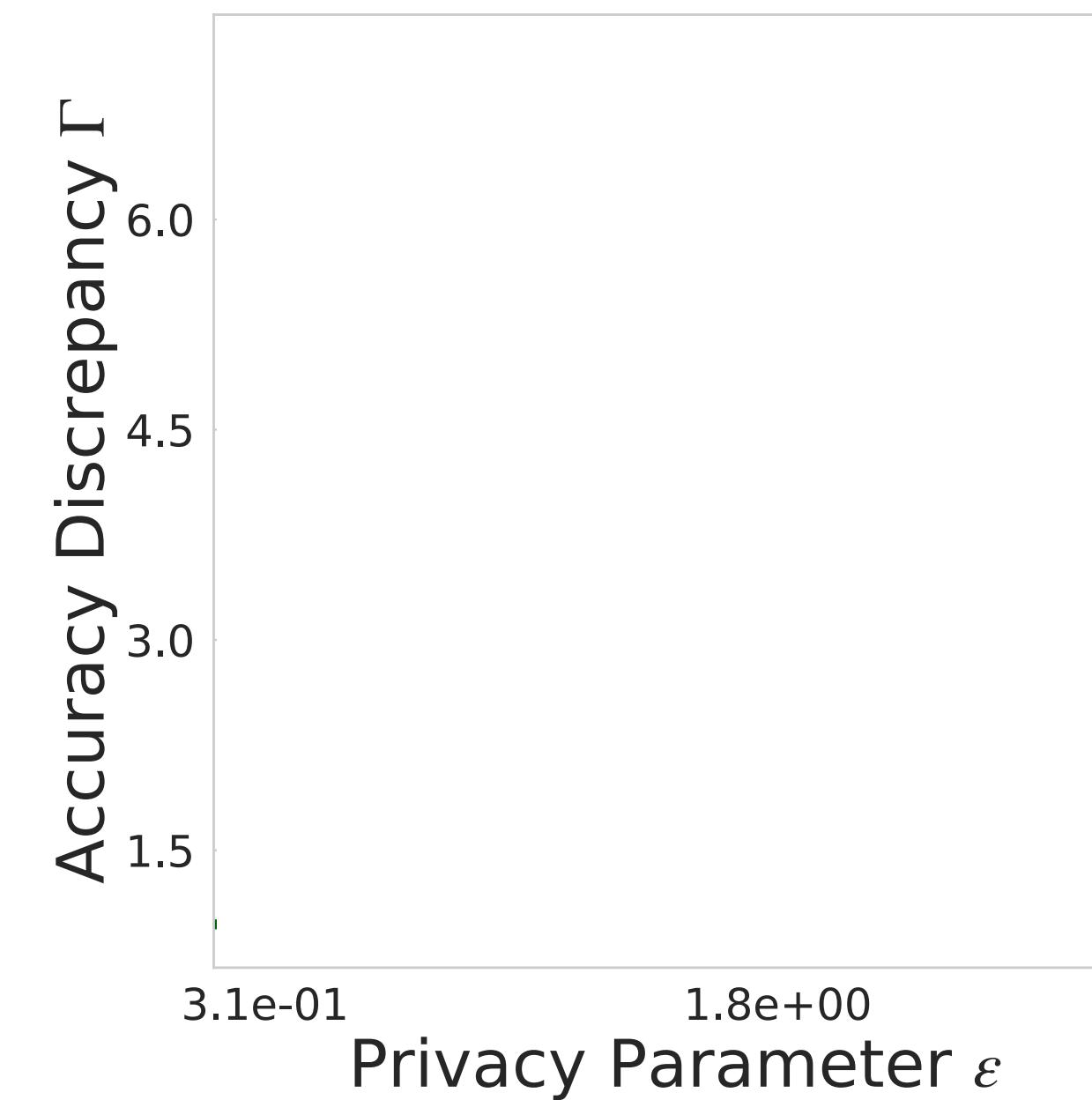


Privacy vs Fairness

CelebA



More Private



More Private

Privacy vs Fairness

CelebA

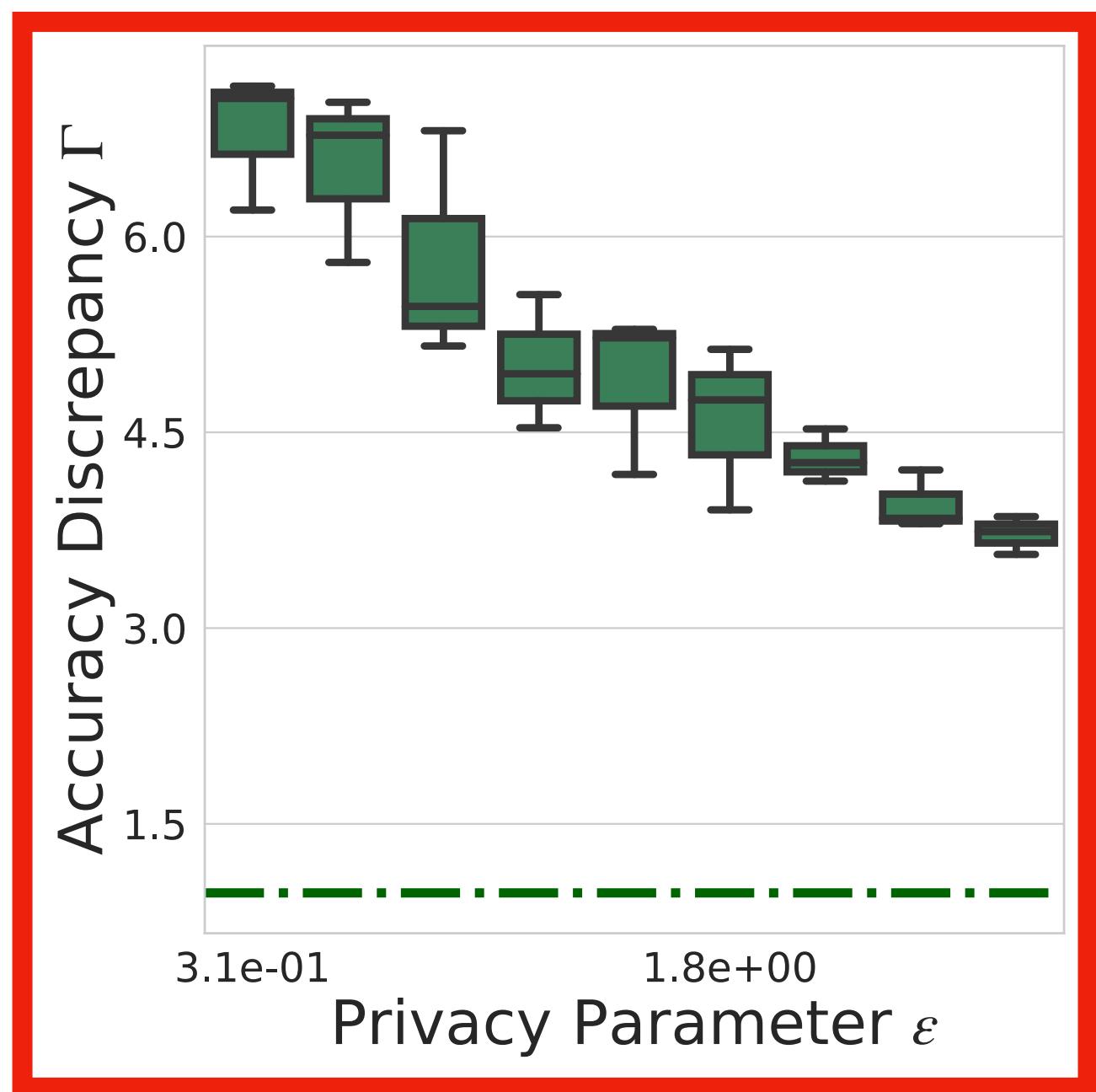


Privacy vs Fairness

CelebA

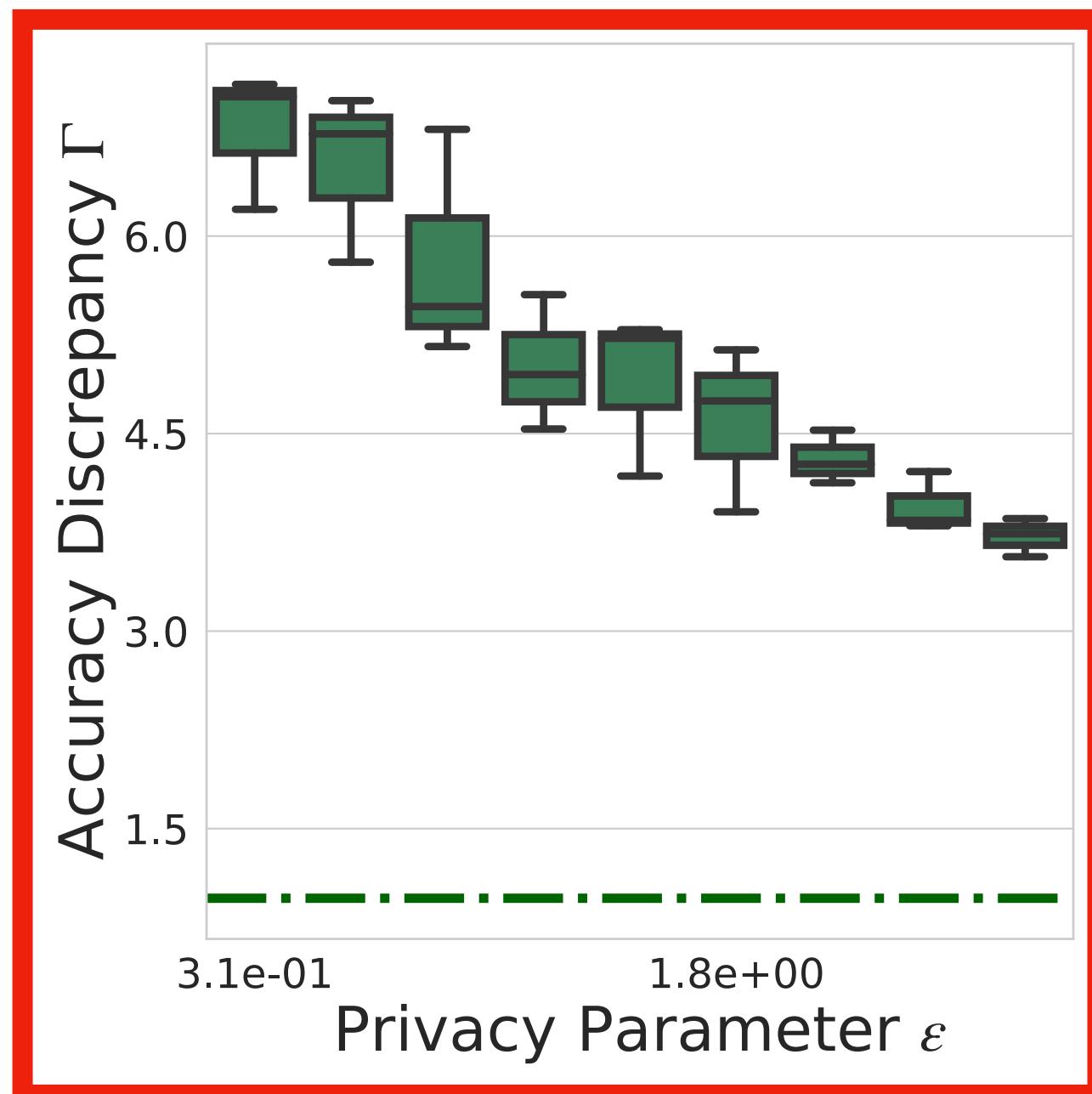


Is this trend systematic?



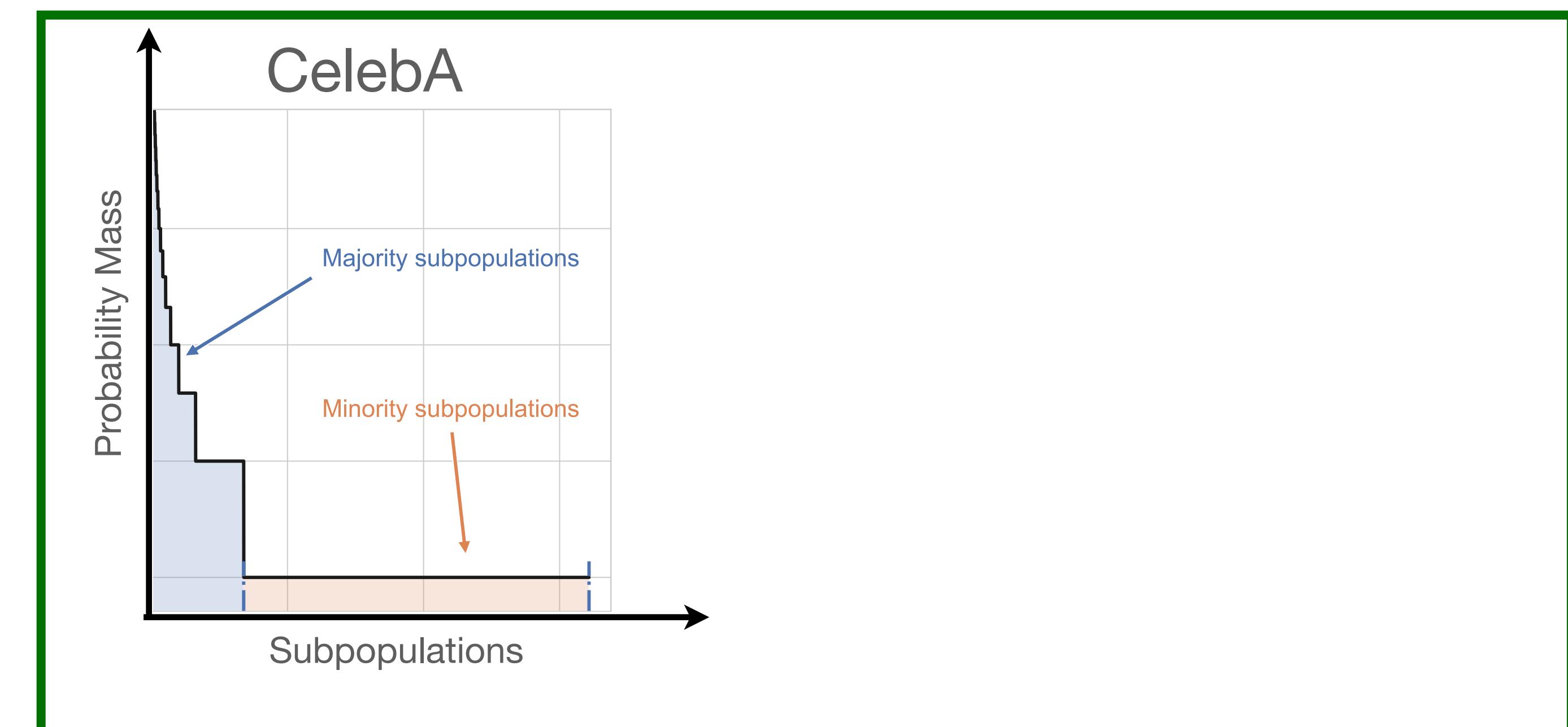
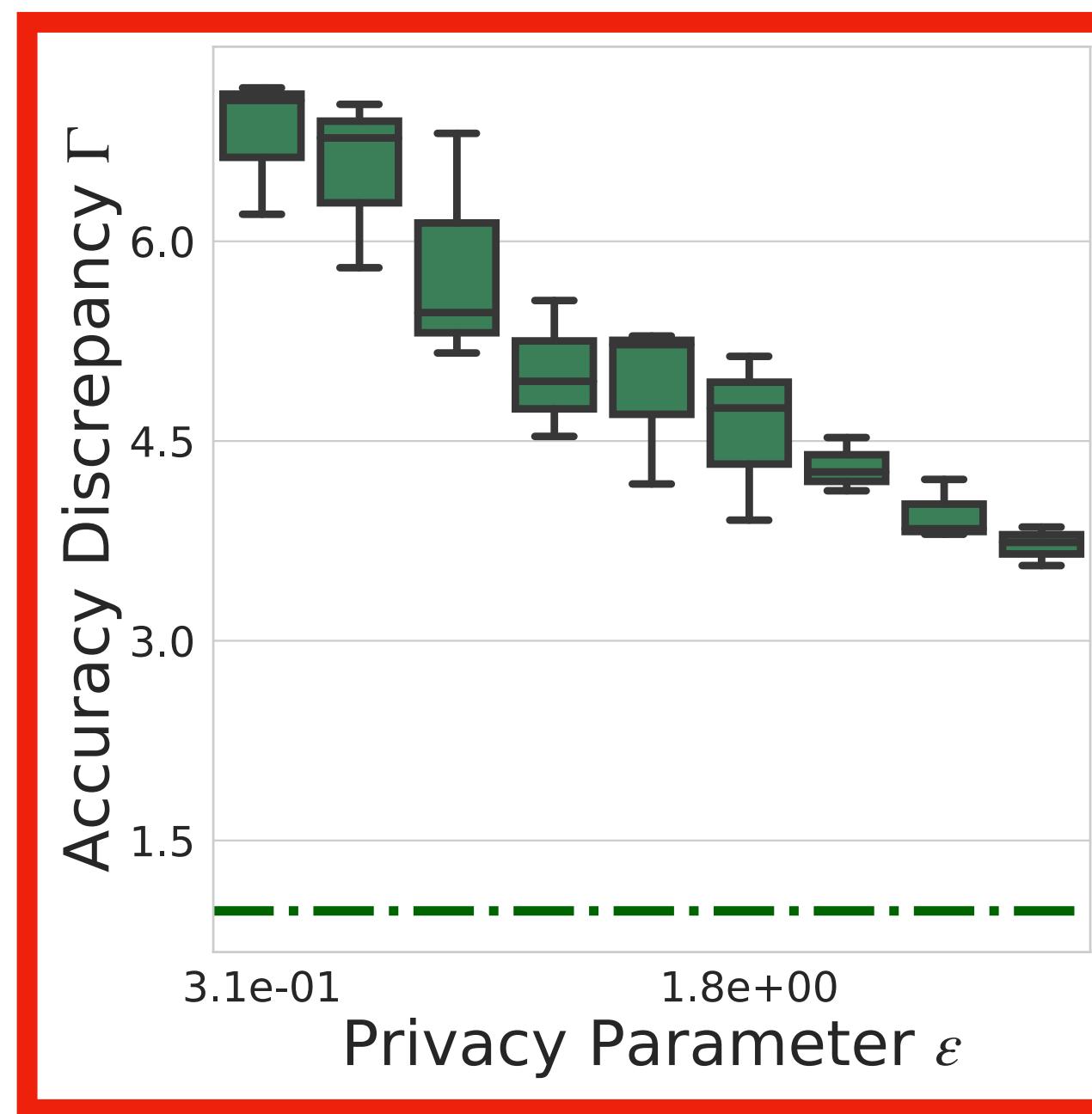
Is this trend systematic?

Main Contribution:
We prove **this trend** in a model-agnostic setting



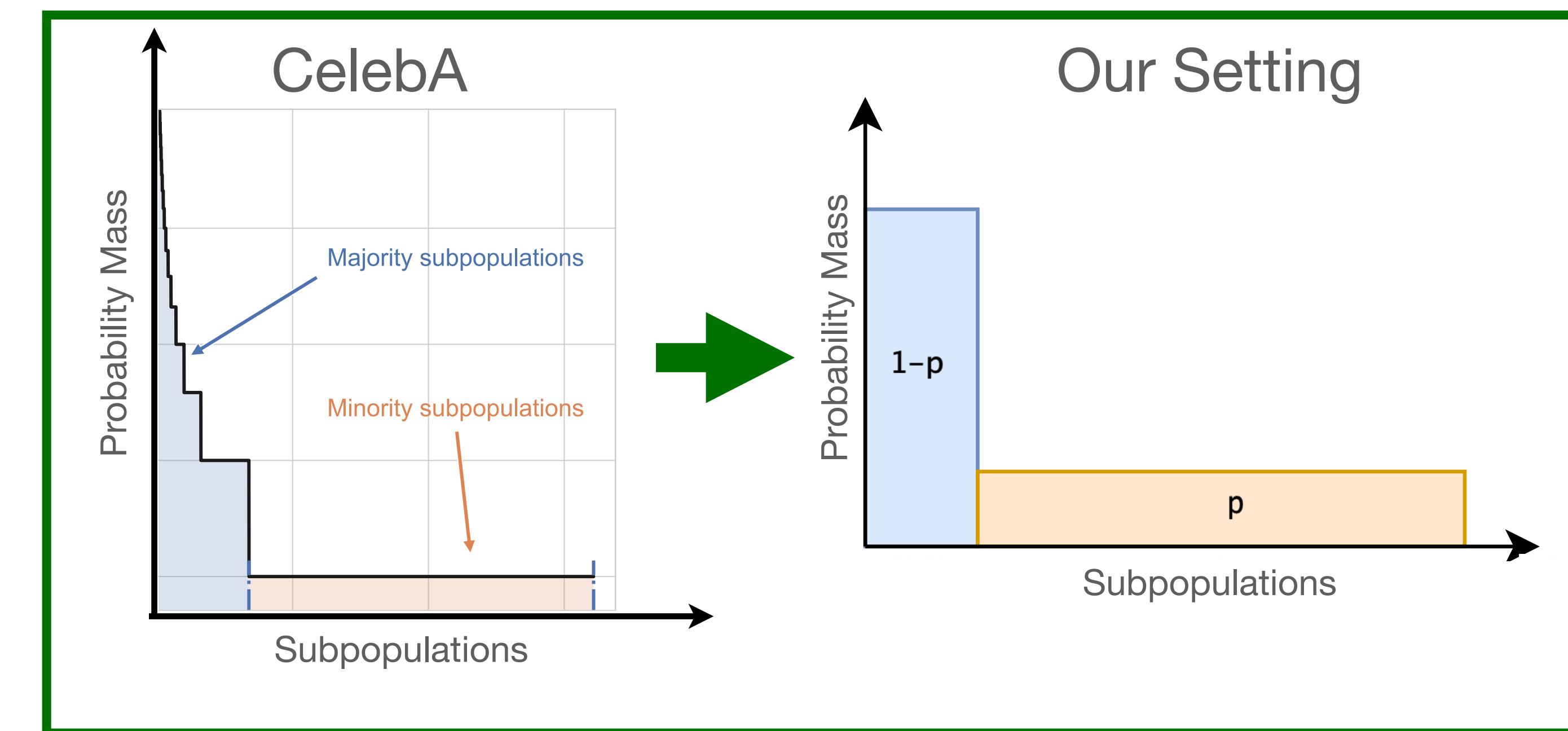
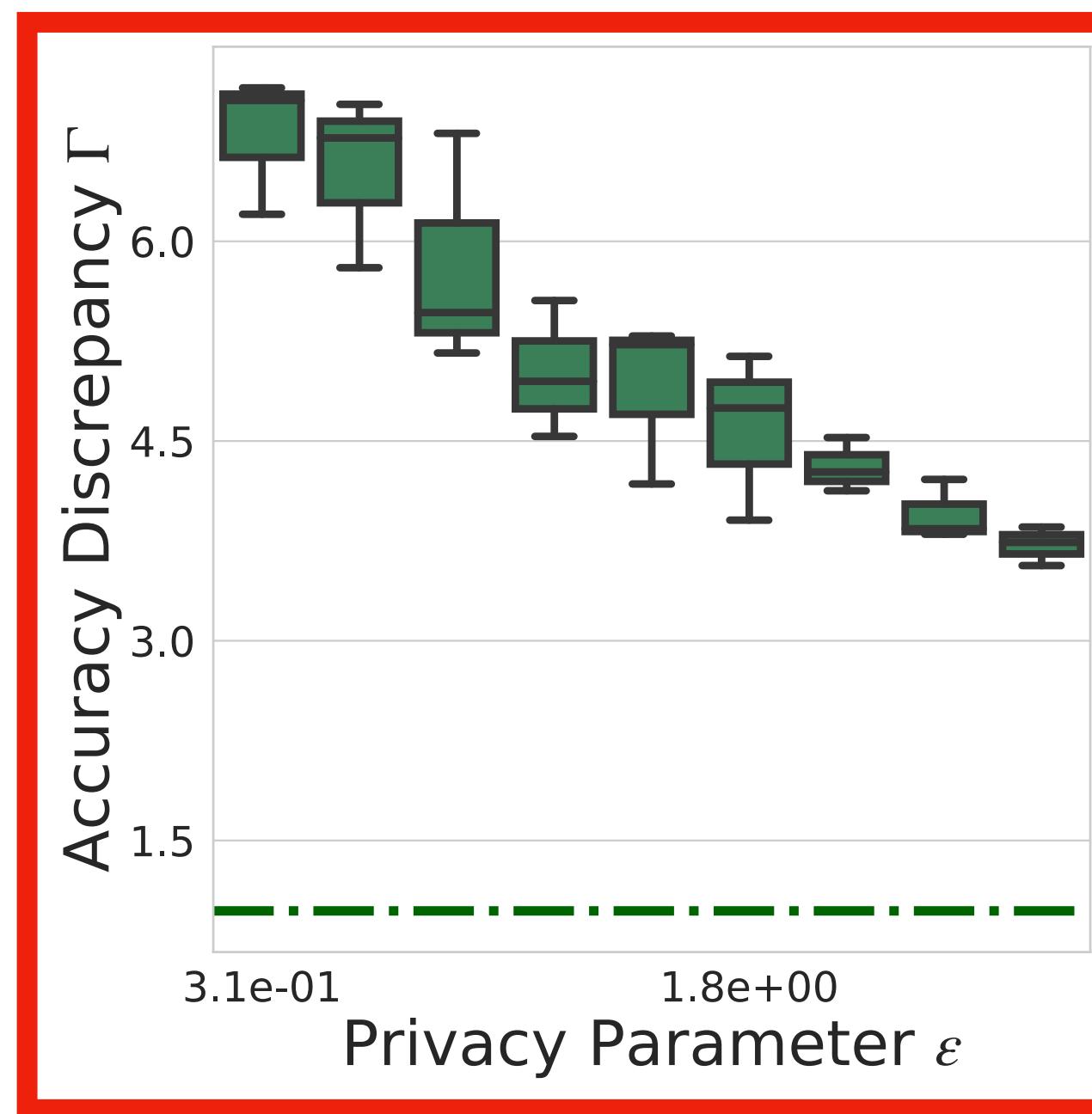
Is this trend systematic?

Main Contribution:
We prove **this trend** in a model-agnostic setting for **long-tailed distribution**.



Is this trend systematic?

Main Contribution:
We prove **this trend** in a model-agnostic setting for **long-tailed distribution**.



Definitions of error and fairness

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- Error

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- Error

$$\text{err}(A, \Pi, F) =$$

Definitions of error and fairness

- Error

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Learning Algorithm

Definitions of error and fairness

- Error

$$\text{err}(A, \Pi, F) =$$



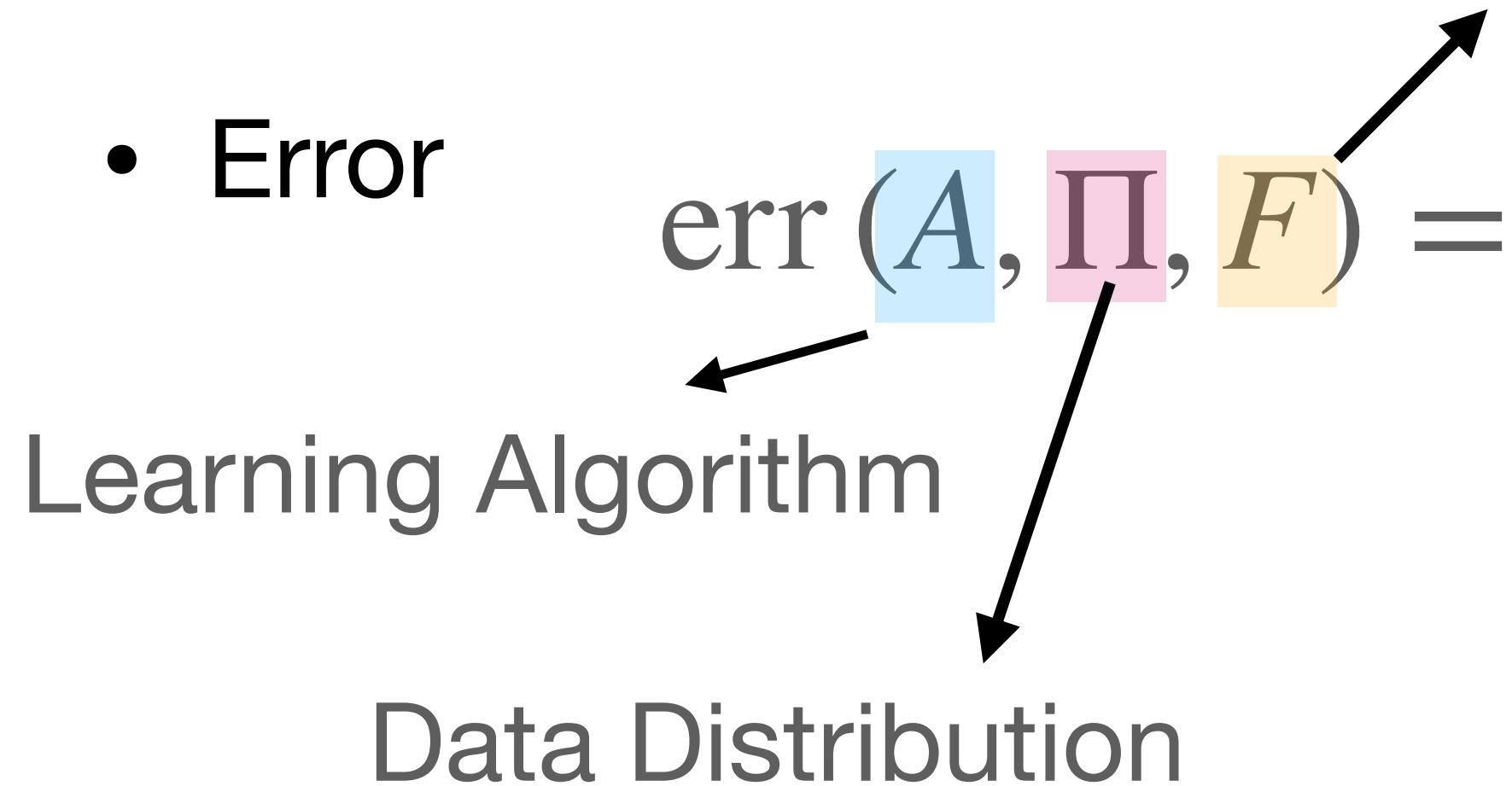
Learning Algorithm

Data Distribution

Definitions of error and fairness

Prior distribution over labelling functions $\subseteq Y^X$

- Error

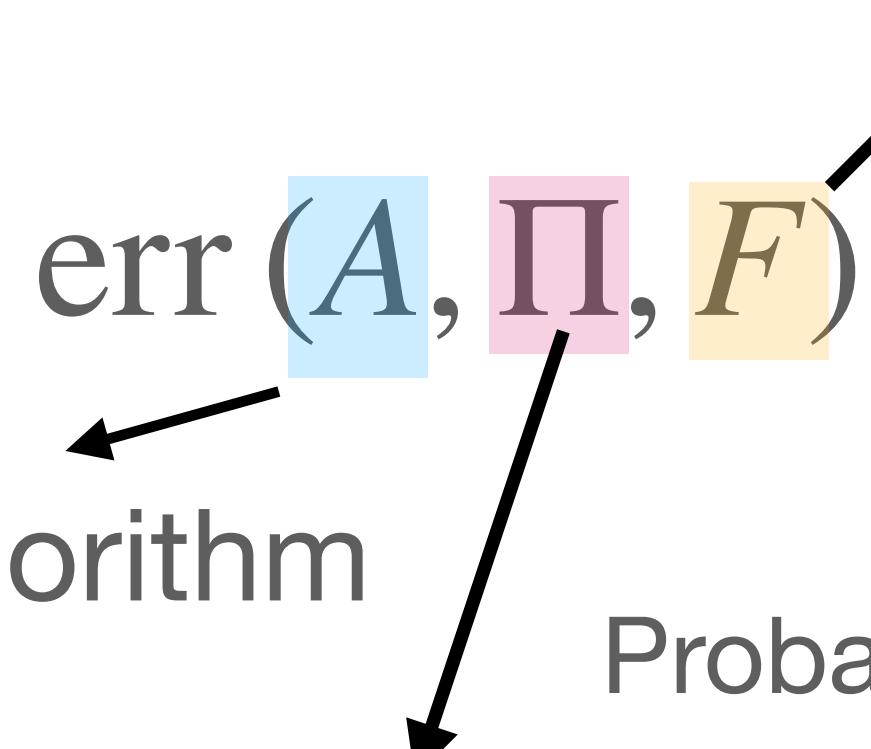


Definitions of error and fairness

Prior distribution over labelling functions $\subseteq Y^X$

- Error

$$\text{err}(A, \Pi, F) = \mathbb{P}[h(x) \neq f(x)]$$

Learning Algorithm 

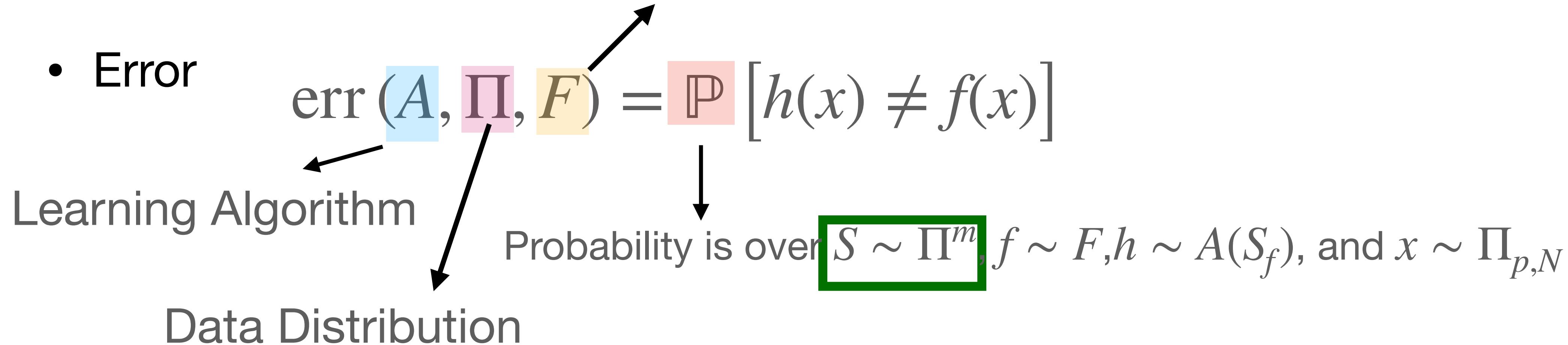
Data Distribution

Probability is over $S \sim \Pi^m, f \sim F, h \sim A(S_f)$, and $x \sim \Pi_{p,N}$

Definitions of error and fairness

Prior distribution over labelling functions $\subseteq Y^X$

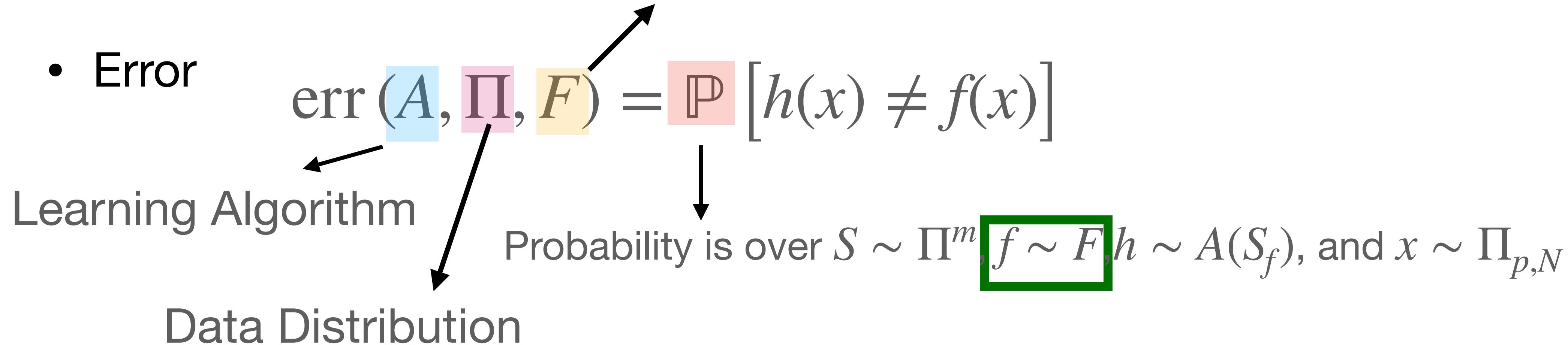
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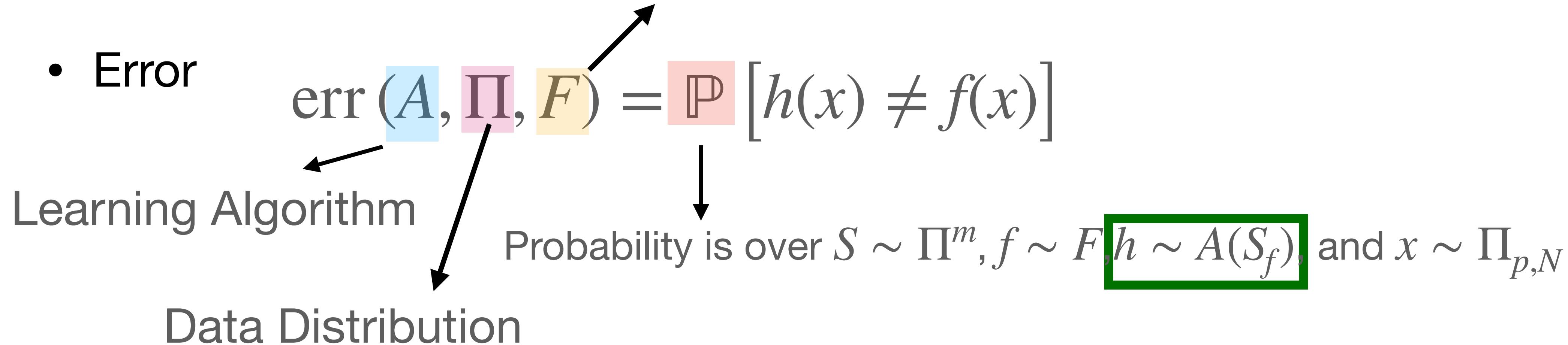
- Error



Definitions of error and fairness

Prior distribution over labelling functions $\subseteq Y^X$

- Error



Definitions of error and fairness

Prior distribution over labelling functions $\subseteq Y^X$

- Error

$$\text{err}(A, \Pi, F) = \mathbb{P}[h(x) \neq f(x)]$$

Learning Algorithm

The diagram illustrates the components of error. At the top, the formula $\text{err}(A, \Pi, F) = \mathbb{P}[h(x) \neq f(x)]$ is shown. Below it, three arrows point downwards to their respective definitions: 'Learning Algorithm' points to A , 'Data Distribution' points to x , and 'Probability is over' points to the entire expression $S \sim \Pi^m, f \sim F, h \sim A(S_f)$.

Probability is over $S \sim \Pi^m, f \sim F, h \sim A(S_f)$, and $x \sim \Pi_{p,N}$

Definitions of error and fairness

Prior distribution over labelling functions $\subseteq Y^X$

- Error

$$\text{err}(A, \Pi, F) = \mathbb{P}[h(x) \neq f(x)]$$

```
graph TD; A[A] --> err["err(A, Π, F)"]; Pi[Π] --> err; F[F] --> P["P[h(x) ≠ f(x)]"]; err --> P; subgraph "Learning Algorithm" [Learning Algorithm]; A; Pi; end; subgraph "Data Distribution" [Data Distribution]; F; end;
```

Learning Algorithm

Data Distribution

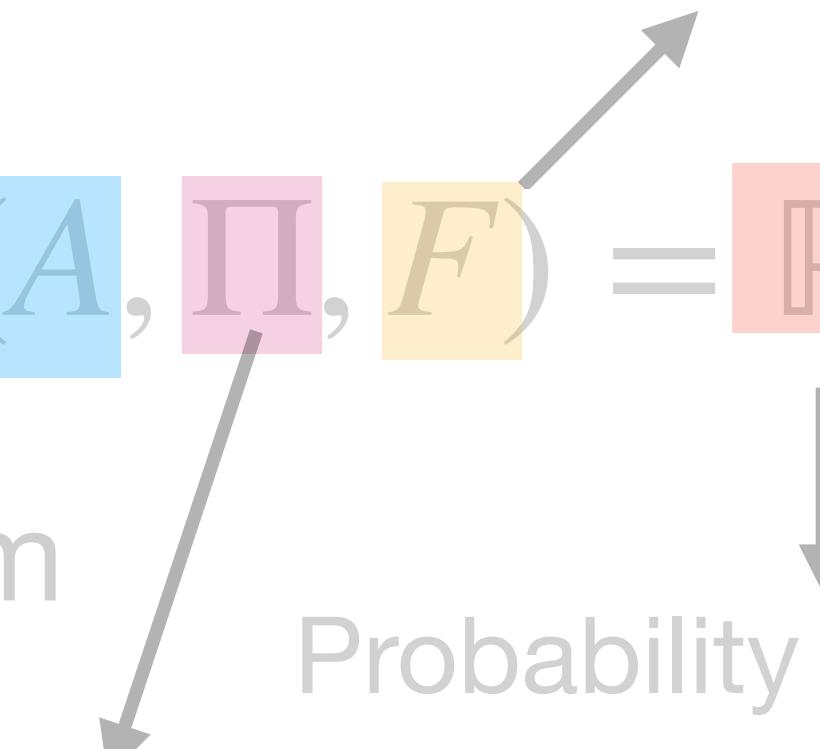
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Definitions of error and fairness

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Learning Algorithm 

Data Distribution

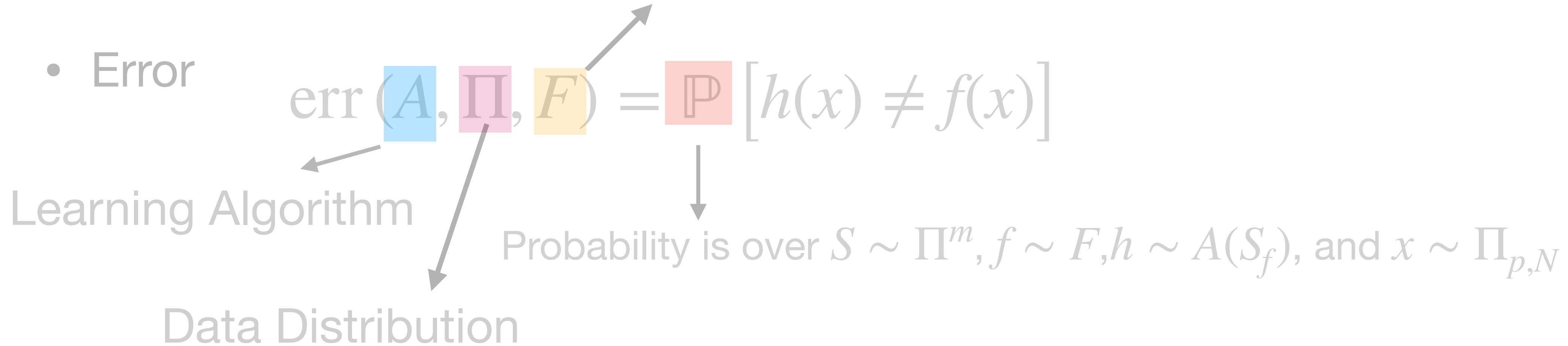
Probability is over $S \sim \Pi^m, f \sim F, h \sim A(S_f)$, and $x \sim \Pi_{p,N}$

- Accuracy Discrepancy

Definitions of error and fairness

Prior distribution over labelling functions $\subseteq Y^X$

- Error



- Accuracy Discrepancy

$$\Gamma(A, \Pi, F) = \text{err}_{\text{Minority}}(A, \Pi, F) - \text{err}(A, \Pi, F)$$

Definitions of error and fairness

Prior distribution over labelling functions $\subseteq Y^X$

- Error

$$\text{err}(A, \Pi, F) = \mathbb{P}[h(x) \neq f(x)]$$

Learning Algorithm

Data Distribution

The diagram illustrates the components of error. At the top, a grey arrow points from the text "Prior distribution over labelling functions" to a light grey box containing the expression $\mathbb{P}[h(x) \neq f(x)]$. Below this, two arrows point downwards to the right. One arrow originates from the text "Learning Algorithm" and points to a light blue box containing A . The other arrow originates from the text "Data Distribution" and points to a light yellow box containing F . A third arrow originates from the text "Prior distribution over labelling functions" and points to a light pink box containing Π .

- Accuracy Discrepancy

Marginalised over minority subpopulations

$$\Gamma(A, \Pi, F) = \boxed{\text{err}_{\text{Minority}}(A, \Pi, F)} - \text{err}(A, \Pi, F)$$

Privacy at the cost of fairness

Privacy at the cost of fairness

Consider any **(ϵ, δ) -DP algorithm** that obtains low error on a long-tailed distribution.

Privacy at the cost of fairness

Consider any **(ϵ, δ) -DP algorithm** that obtains low error on a long-tailed distribution.

(Informal Theorem A) We prove an asymptotic lower bound for accuracy discrepancy which

- (Privacy) Increases with privacy parameter ϵ .

Privacy at the cost of fairness

Consider any **(ϵ, δ) -DP algorithm** that obtains low error on a long-tailed distribution.

N : # Minority subpopulations
 m : # Training points

(Informal Theorem A) We prove an asymptotic lower bound for accuracy discrepancy which

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Consider any **(ϵ, δ) -DP algorithm** that obtains low error on a long-tailed distribution.

(Minority Subpopulations) Let $\frac{N}{m} \rightarrow c$ as $N, m \rightarrow \infty$.

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- (Privacy) Increases with privacy parameter ϵ .
- (Long-tailed) Increases with (relative) # of minority subpopulations c .

Privacy at the cost of fairness

Consider any **(ϵ, δ) -DP algorithm** that obtains low error on a long-tailed distribution.

(Minority Subpopulations) Let $\frac{N}{m} \rightarrow c$ as $N, m \rightarrow \infty$.

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- (Long-tailed) Increases with (relative) # of minority subpopulations c .
- (Label prior) Increases with entropy of the label prior.

Thank you