

## DIFFERENTIALLY PRIVATE LEARNING

An algorithm  $\mathcal{A}$  is said to be  $(\epsilon, \delta)$ -differentially private (DP) if

$$\mathbb{P}[\mathcal{A}(S_1) \in Q] \leq \exp(\epsilon) \mathbb{P}[\mathcal{A}(S_2) \in Q] + \delta$$

for all neighbouring datasets  $S_1, S_2$  and output sets  $Q$ .

**Existing Results:** Sample complexity of DP algorithms are dimension-dependent in the worst case.

In Semi-Private learning [1], the learner accesses

- ▶ **Private Labelled** dataset,
- ▶ **Public Unlabelled** dataset from nearby distribution

**This work:** Design Semi-Private learner for linear half-spaces that

1. Is **Computationally Efficient**
2. Admits **Dimension Independent** sample complexity
3. Performs well in **Challenging Practical Applications**

## THEORETICAL RESULTS

We exploit two properties of data distribution  $\mu$  (covariance  $\Sigma$ )

- ▶ **(A1) Large Margin:**  $\mu$  admits a classifier  $w^*$  with **margin  $\gamma$**
- ▶ **(A2) Low Rank:** Large Proj. of  $w^*$  on top- $k$  components of  $\Sigma$ .

**Algorithm 1** Unlabelled dataset  $(\mathbf{X}_U \in)$ , Labelled dataset  $(\mathbf{X}_L, Y_L)$ ,  $k$

- 1:  $\hat{\Sigma} \leftarrow \sum_{\mathbf{x} \in S_U} \mathbf{x} \mathbf{x}^\top$ ,  $\mathbf{A}_k \leftarrow$  top- $k$  principal components of  $\hat{\Sigma}$ .
- 2:  $\mathbf{X}_L^{\text{Proj}} \leftarrow$  Project  $\mathbf{X}_L$  on  $\mathbf{A}_k$ .
- 3:  $\hat{\mathbf{w}}_{\epsilon, \delta} \leftarrow$  Run Noisy-SGD on  $(\mathbf{X}_L^{\text{Proj}}, Y_L)$  with privacy parameters  $\epsilon, \delta$

**Guarantees on  $\hat{\mathbf{w}}_{\epsilon, \delta}$**

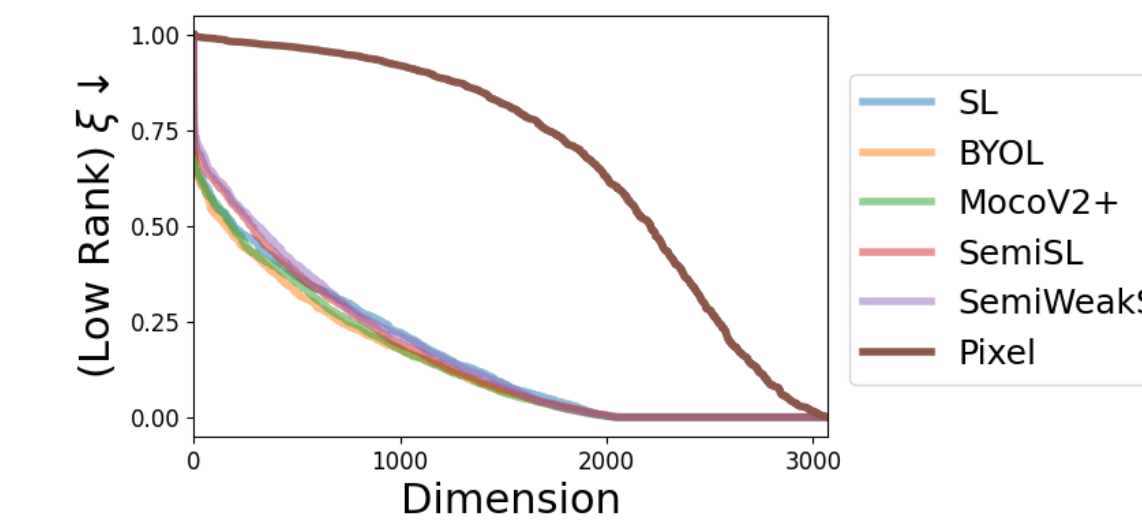
- ▶ **Privacy:**  $\hat{\mathbf{w}}_{\epsilon, \delta}$  is  $(\epsilon, \delta)$ -DP.
- ▶ **Accuracy:** For  $\alpha, \beta \geq 0$ ,  $|\mathbf{X}_U| = O\left(\frac{1}{\gamma^2}\right)$  and  $|\mathbf{X}_L| = \tilde{O}\left(\frac{\sqrt{k}}{\alpha \epsilon \gamma}\right)$ ,

$$\mathbb{P}[\text{Error}(\hat{\mathbf{w}}_{\epsilon, \delta}) \leq \alpha] \geq 1 - \beta$$

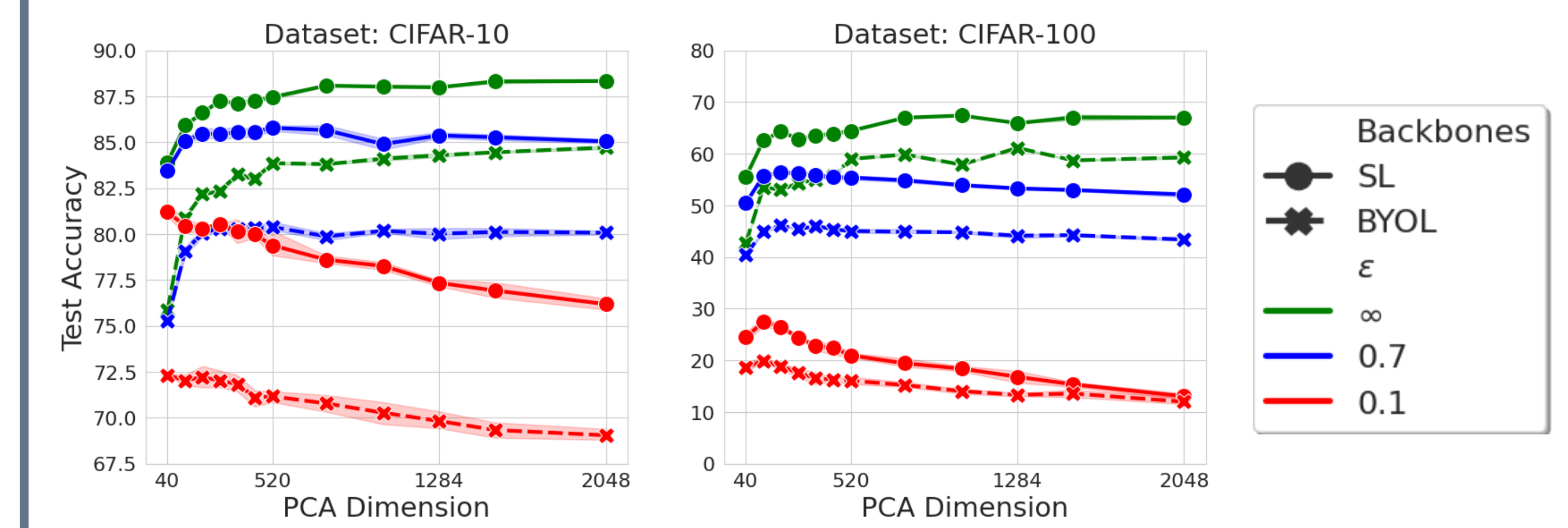
## EXPERIMENTAL SETTING

We consider

- ▶ **5 pre-training** methods on Imagenet (as raw images fail at A2)
  - ▶ Low rank  $\implies$  small  $\xi_k$  for small  $k$
  - ▶ Images in **pixel space**  $\not\approx$  low rank.
  - ▶ **Pre-trained** features  $\approx$  low rank.
- ▶ **Seven diverse datasets**
  - ▶ 3 medical datasets (DermNet, PCAM, Pneumonia)
  - ▶ 4 object datasets (CIFAR10/100, Flower-16, GTSRB).
- ▶ **Strict privacy guarantees** with  $\epsilon < 1$



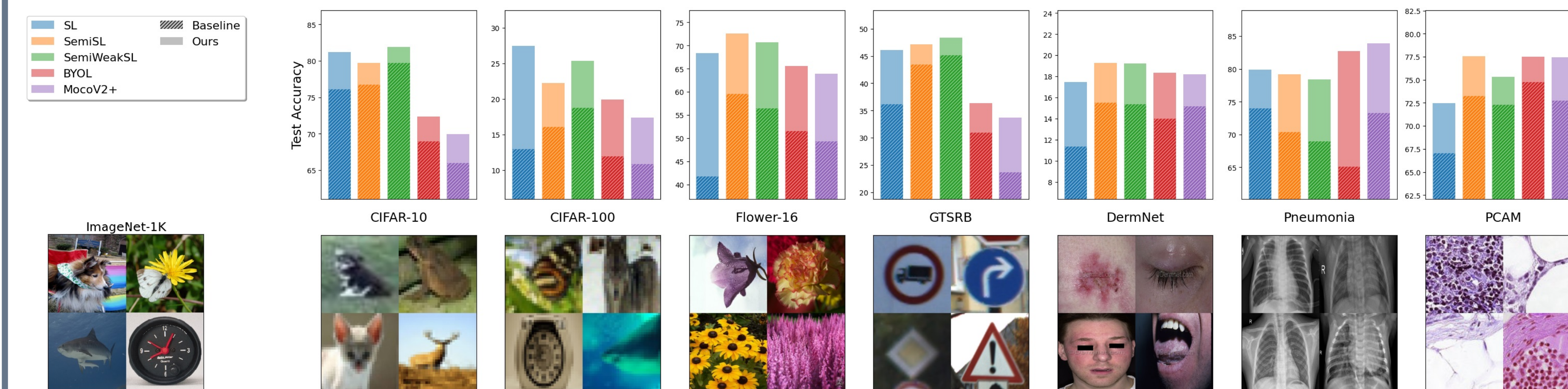
## EXPERIMENTS I: REDUCING DIMENSIONS



**Takeaway:**

- ▶ **Strict privacy** ( $\epsilon = 0.1$ ): Dimension  $\downarrow \implies$  Accuracy  $\uparrow$ .
- ▶ **Without privacy** ( $\epsilon = \infty$ ): Dimension  $\downarrow \implies$  Accuracy  $\downarrow$ .

## EXPERIMENTS II: COMPARING ALGORITHMS ACROSS DATASETS



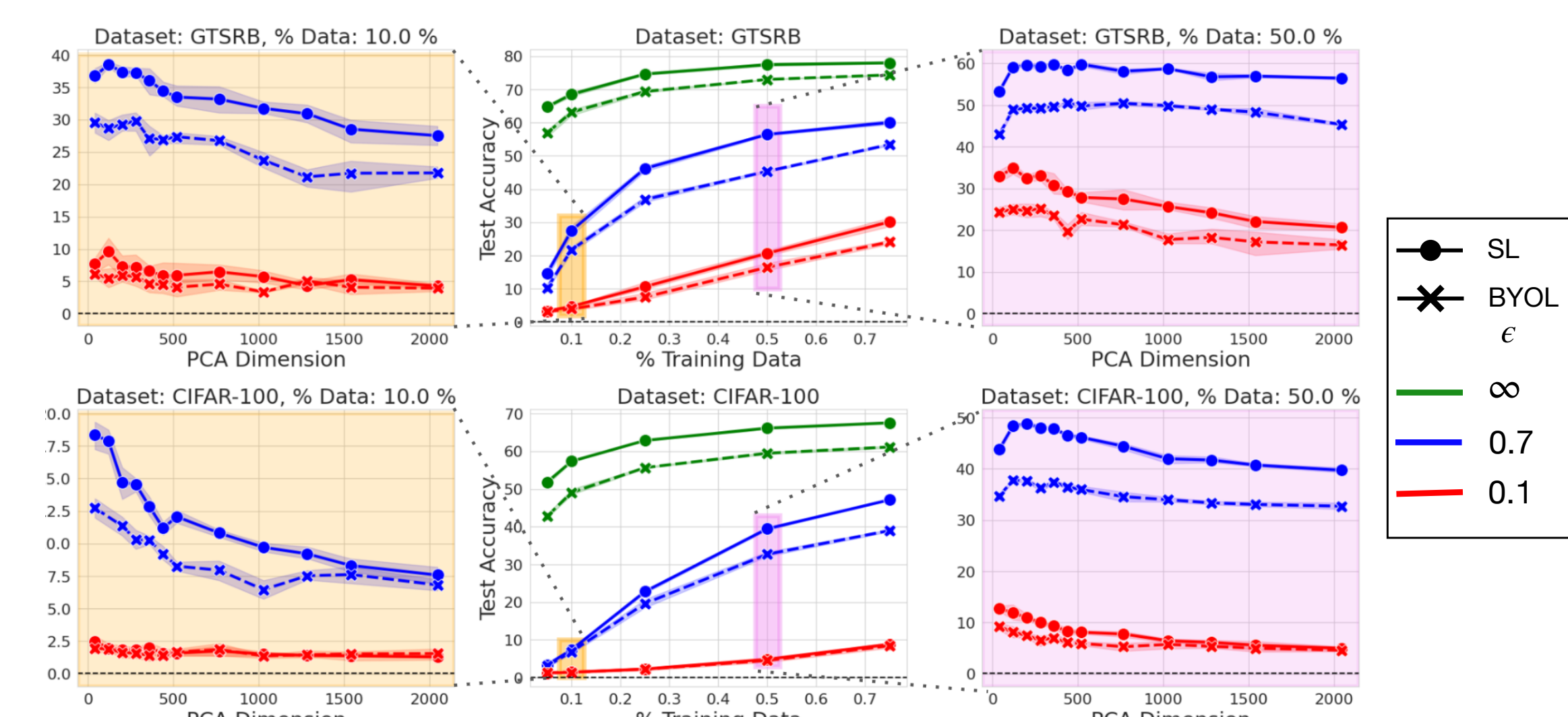
Comparison across datasets and pre-training for  $\epsilon = 0.1$ .

Methods	CIFAR10	CIFAR100
Baseline	84.89	50.65
JL [2]	84.4	50.56
AdaDPS [3]	83.49	33.9
GEP [4]	84.14	41.29
<b>OURS</b>	<b>85.89</b>	<b>55.86</b>

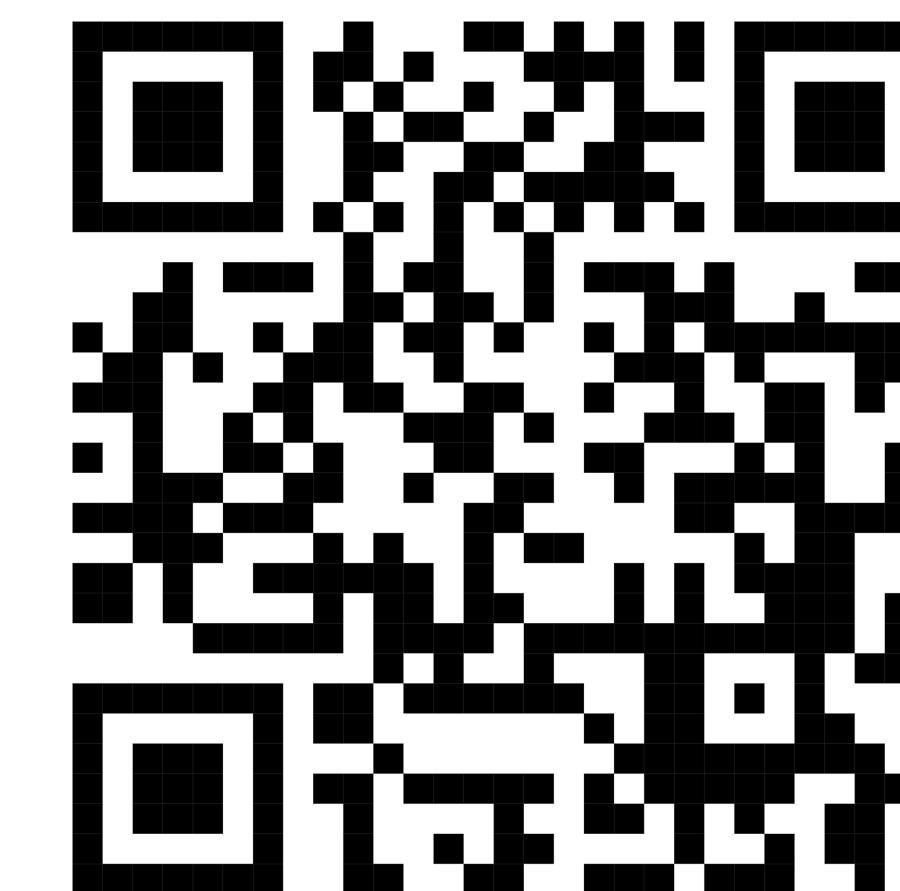
Different methods for  $\epsilon = 0.7$ .

## EXPERIMENTS III: LOW DATA SETTING

- ▶ With less private data, **DP-SGD** fails catastrophically.
- ▶ **Our algorithm** is robust to decreasing amount of private data.



## REFERENCES AND QR CODE FOR PAPER



[1] Alon, et al. "Limits of private learning with access to public data." Advances in neural information processing systems 32 (2019).

[2] Lê Nguyễn, et al. "Efficient private algorithms for learning large-margin halfspaces." Algorithmic Learning Theory. PMLR, 2020.

[3] Li, Tian, et al. "Private adaptive optimization with side information." International Conference on Machine Learning. PMLR, 2022.

[4] Yu, Da, et al. "Do not let privacy overbill utility: Gradient embedding perturbation for private learning." International Conference on Learning Representations (2021).