Distance matrix

Let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$, $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}$.

$$r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

Find $DM = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} \\ r_{21} & r_{22} & r_{23} & r_{24} & r_{25} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} \\ r_{51} & r_{52} & r_{53} & r_{54} & r_{55} \end{bmatrix}$

$$xa = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} x_1 & x_1 & x_1 & x_1 & x_1 \\ x_2 & x_2 & x_2 & x_2 & x_2 \\ x_3 & x_3 & x_3 & x_3 & x_3 \\ x_4 & x_4 & x_4 & x_4 & x_4 \\ x_5 & x_5 & x_5 & x_5 & x_5 \end{bmatrix}$$

$= x * ones(1,5)$

$$xb = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} * \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix} = \begin{bmatrix} x_1 & x_1 & x_1 & x_1 & x_1 \\ x_2 & x_2 & x_2 & x_2 & x_2 \\ x_3 & x_3 & x_3 & x_3 & x_3 \\ x_4 & x_4 & x_4 & x_4 & x_4 \\ x_5 & x_5 & x_5 & x_5 & x_5 \end{bmatrix}$$

$= ones(5,1) * x'$

Similarly, $ya = y * ones(1,5)$

$yb = ones(5,1) * y'$

```
DM = sqrt((xa - xb).^2 + (ya - yb).^2);
```

or

```
DM = sqrt( (x * ones(1,5) - ones(5,1) * x').^2
     + (y * ones(1,5) - ones(5,1) * y').^2 );
```

Using 'for' loop

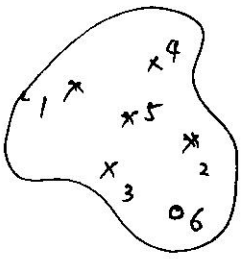
```
DM = zeros(5,5)
for i = 1:5
    for j = 1:5
        DM(i,j) = sqrt((x(i)-x(j))^2 + (y(i)-y(j))^2);
    end
end
```

```
DM = zeros(5,5)
for i = 1:5
    DM(i,:) = Sqrt((x(i)-x').^2 + (y(i)-y').^2);
end
```

Radial basis functions interpolation.

$$f(x,y) = \sum_{i=1}^{n} a_i \phi\left(\|(x_i,y_i)-(x,y)\|\right)$$

$$= \sum_{i=1}^{n} a_i \phi(r_i)$$



Let $\{(x_i,y_i)\}_{i=1}^{5}$ be the given five points.

For the first point (x_1,y_1), we have

$$f(x_1,y_1) = a_1\phi(r_{11}) + a_2\phi(r_{12}) + a_3\phi(r_{13}) + a_4\phi(r_{14}) + a_5(\phi_{15}).$$

Similarly, we have

$$f(x_2,y_2) = a_1\phi(r_{21}) + a_2\phi(r_{22}) + a_3\phi(r_{23}) + a_4\phi(r_{24}) + a_5\phi(r_{25})$$

$$\vdots$$

$$f(x_5,y_5) = a_1\phi(r_{51}) + a_2\phi(r_{52}) + a_3\phi(r_{53}) + a_4\phi(r_{54}) + a_5\phi(r_{55})$$

In matrix form,

$$\begin{bmatrix} f(x_1,y_1) \\ f(x_2,y_2) \\ f(x_3,y_3) \\ f(x_4,y_4) \\ f(x_5,y_5) \end{bmatrix} = \begin{bmatrix} \phi(r_{11}) & \phi(r_{12}) & \phi(r_{13}) & \phi(r_{14}) & \phi(r_{15}) \\ \phi(r_{21}) & \phi(r_{22}) & \cdots & \vdots & \vdots \\ \phi(r_{31}) & \vdots & \phi(r_{33}) & \vdots & \vdots \\ \phi(r_{41}) & \vdots & \cdots & \phi(r_{44}) & \\ \phi(r_{51}) & \cdot & \cdots & \cdot & \phi(r_{55}) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$$

$$\vec{F} = [\phi_{ij}]_{1\le i,j\le 5} \cdot \vec{a} \quad \Rightarrow \text{Find } \vec{a}.$$

Find $f(x_6,y_6)$

$$= a_1\phi(r_{16}) + a_2\phi(r_{26}) + a_3\phi(r_{36}) + a_4\phi(r_{46}) + a_5\phi(r_{56})$$