Welcome

from the

Department of Mathematics

at the

University of Southern

Mississippi

Adaptive Methods

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- Introduction
- Knot Insertion (Chapter 21)
- Knot Removal (Chapter 21)
- Greedy One-Point Algorithm (Chapter 33)

An adaptive algorithm is an algorithm that changes its behavior based on information available at the time it is run. This might be information about computational resources available, or the history of data recently received.
—Wikipedia

Algorithm 21.2 (Page no. 181)

- Let data sites $\mathbf{x} = \{\mathbf{x_1}, ..., \mathbf{x_N}\}$, data $f_i, i = 1, ..., N$, and a tolerance tol be given.
- Choose M initial knots $\Xi = \{\xi_1, ..., \xi_M\}$.
- Calcuate the least squares fit

$$\wp_f(x) = \sum_{j=1}^{n} c_j \Phi(x, \xi_j)$$

with its associated error

$$e = \sum_{i=1}^{N} [f_i - \wp_f(x_i)]^2$$

.

While $e > tol$ do

- "Weight" each data point $x_i, i = 1, \ldots, N$, according to its error component, i.e., let

$$w_i = |f_i - \mathcal{P}_f(x_i)|, i = 1, \ldots, N.$$

- Find the data point $x_\nu \notin \Xi$ with maximum weight w_ξ and insert it as a knot, i.e.,

$$\Xi = \Xi \cup \{x_\nu\} \text{and} M = M + 1.$$

- Recaclulate fit and associated error.

MATLAB implementaiton —> Program 21.1 (Book)
RBFKnotInsert2D.m

Algorithm 21.2 (Page no. 184)

- Let data sites $\mathbf{x} = \{\mathbf{x_1}, \ldots, \mathbf{x_N}\}$, data $f_i, i = 1, \ldots, N$, and a tolerance tol be given.
- Choose M initial knots $\Xi = \{\xi_1, \ldots, \xi_M\}$.
- Calcuate an initial fit

$$\wp_f(x) = \sum_{j=1}^{n} c_j \Phi(x, \xi_j)$$

with its associated least squares error

$$e = \sum_{i=1}^{N} [f_i - \wp_f(x_i)]^2$$

.

- "Weight" each knot $\xi_j, j = 1, \ldots, M$, according to its least squares error, i.e., form

$$\Xi^* = \Xi \setminus \{\xi_j\},$$

and calculate the weights

$$w_j = \sum_{i=1}^{N} [f_i - \wp_f^*(x_i)]^2,$$

where

$$\wp_f^*(x) = \sum_{j=1}^{M-1} c_j \Phi(x, \xi_j^*),$$

is the approximation based on the reduced set of knots Ξ^*.
- Find the knot ξ_μ with lowest weight $w_\mu < tol$ and permanently remove it, i.e.,

$$\Xi = \Xi \setminus \{\xi_\mu\} \text{and} M = M - 1.$$

- Recalculate fit and associated error.

MATLAB implementaiton —> Program 21.2 (Book)

Input data locations X, associated values of f, tolerance $tol > 0$

- Set initial residual $r_0 = P_f^X$, initialize $u_0 = 0, e = \infty, k = 0$
- Choose starting point $y_k \in X$

While $e > tol$ do

- Set $\beta = \frac{r_k(y_k)}{\Phi(y_k, y_k)}$
- For $1 \leq i \leq N$ do

$$r_{k+1}(x_i) = r_k(x_i) - \beta\Phi(x_i, y_k)$$

$$u_{k+1}(x_i) = u_k(x_i) + \beta\Phi(x_i, y_k)$$

end

- Find $e = \max_X |r_{k+1}|$ and the point y_{k+1} where it occurs
- Inrement $k = k + 1$

end MATLAB implementaiton —> Program 33.1 (Book)

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