Assignment 7: volume of Bumpy sphere

Using the method of fundamental solutions to reconstruct the surface of the bumpy sphere which is defined as follows:

$$\partial\Omega = \{(x, y, z) : x = \rho \sin\phi \cos\theta, y = \rho \sin\phi \sin\theta, z = \rho \cos\phi, 0 \le \theta \le 2\pi, 0 \le \phi \le \pi\}$$

where

$$\rho(\phi, \theta) = 1 + \frac{1}{6} \sin(6\theta) \sin(7\phi).$$

For the reconstruction of the surface, consider the following biharmonic equation

$$\Delta^2 u(x, y, z) = 0, \qquad (x, y, z) \in \Omega,$$
$$\mathrm{u(x, y, z)} = 1, \qquad \mathrm{(x, y, z)} \in \partial\Omega.$$

After the construction of the surface, then use quasi-Monte-Carlo Method to approximate the volume of the bumpy sphere.

Using MATLAB triple integral to find the volume of the bumpy sphere:

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integral3(fun,xmin,xmax,ymin,ymax,zmin,zmax)
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$$\int_0^{2\pi} \int_0^{\pi} \left(\int_0^{1+\sin(6\theta)\sin(7\phi)/6} \sin(\phi)\rho^2 d\rho \right) d\phi d\theta$$