# Welcome from the Department of Mathematics at the University of Southern Mississippi

# RBF Hermite Interpolation

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## *Outline*

- *RBF Interpolation*
- *RBF Hermite Interpolation*
- *Comparison of results*
- *Conclusion*

## Surface Reconstruction Scheme

Assume that $f(\mathbf{x}) \approx s(\mathbf{x})$

To approximate $f$ by $s$ we usually require fitting the given

Data set $\{\mathbf{x}_i\}_1^N$ of pairwise distinct centres with the imposed

conditions $f(\mathbf{x}_i) = s(\mathbf{x}_i), \qquad 1 \le i \le N.$

The linear system $s(\mathbf{x}_i) = \sum_{j=1}^{N} a_j \varphi\left(\|\mathbf{x}_i - \mathbf{x}_j\|\right), \quad 1 \le i \le N,$

is well-posed if the interpolation matrix is non-singular

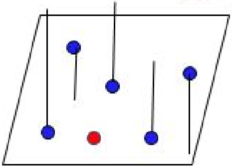
$$A_\varphi = \left[\varphi\|\mathbf{x}_i - \mathbf{x}_j\|\right]_{1 \le i, j \le N}$$

$$s(x_i)=\sum_{j=1}^{5} a_j\varphi\left(\left\|\mathbf{x}_i-\mathbf{x}_j\right\|\right),\quad 1\le i\le 5,$$

$$\begin{bmatrix}\varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15}\\ \varphi_{21} & \varphi_{22} & \varphi_{23} & \varphi_{24} & \varphi_{25}\\ \varphi_{31} & \varphi_{32} & \varphi_{33} & \varphi_{34} & \varphi_{35}\\ \varphi_{41} & \varphi_{42} & \varphi_{43} & \varphi_{44} & \varphi_{45}\\ \varphi_{51} & \varphi_{52} & \varphi_{53} & \varphi_{54} & \varphi_{55}\end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ a_3\\ a_4\\ a_5\end{bmatrix}=\begin{bmatrix}f(x_1)\\ f(x_2)\\ f(x_3)\\ f(x_4)\\ f(x_5)\end{bmatrix}$$

Interpolation Matrix

$$s(x_i)=\sum_{j=1}^{5} a_j\varphi\left(\left\|\mathbf{x}_i-\mathbf{x}_j\right\|\right)$$



Note: $\varphi_{ij}=\varphi\left(\left\|\mathbf{x}_i-\mathbf{x}_j\right\|\right)$

- This scheme use the data and the derivative informations of the data.
- Let us consider the data $(x_i, \lambda_i f)$ , $\quad i = 1, 2, ...N, \quad x_i \in \mathbb{R}^s$.
- Here, $\lambda_i$ are the continuous linear functionals and $f$ is some data function.

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- The generalized Hermite interpolant is of the form

$$P_f(x) = \sum_{j=1}^{N} c_j \lambda_j^{\xi} \phi(\| x - \xi \|), \quad x \in \mathbb{R}^s. \tag{1}$$

- The interpolant must satisfy the following relation:

$$\lambda_i P_f = \lambda_i f, \quad i = 1, 2, \ldots N. \tag{2}$$

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- The entries of the interpolation matrix has the form :

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- where $n_t$ is the number of test points in the domain and $\hat{u}_j$ and $u_j$ are the approximate and exact solution of the partial differential equation at the $j^{th}$ computational point respectively.

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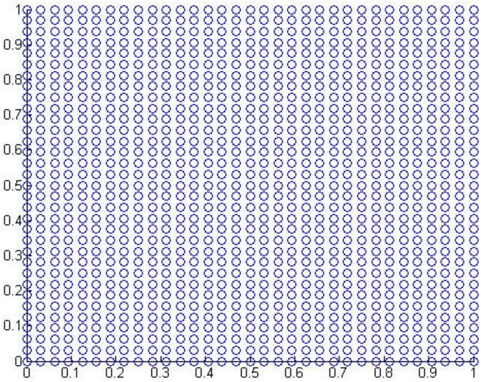
- For the regular domain, we chose the uniform $1089$ points on a unit square.



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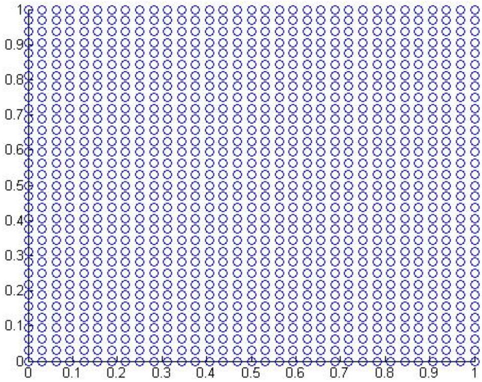
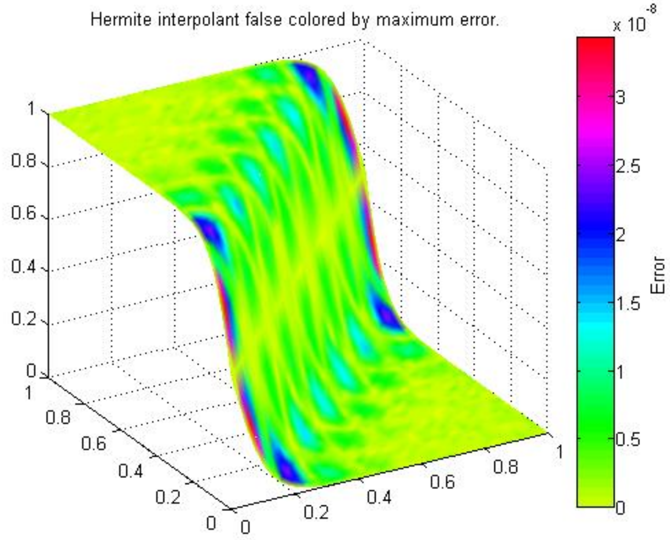
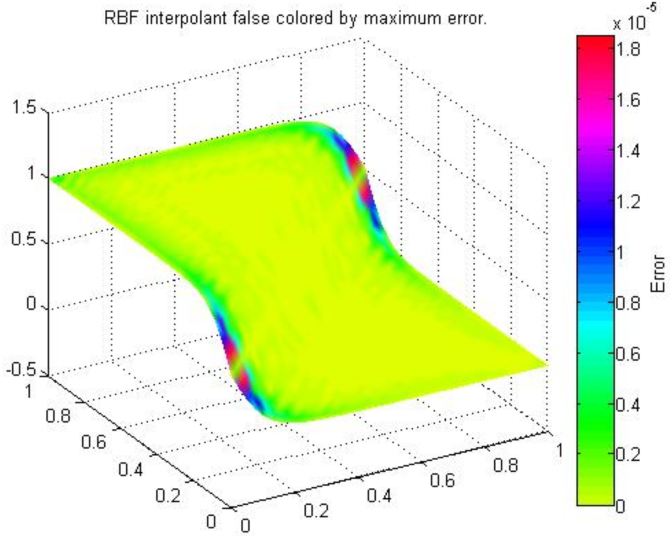
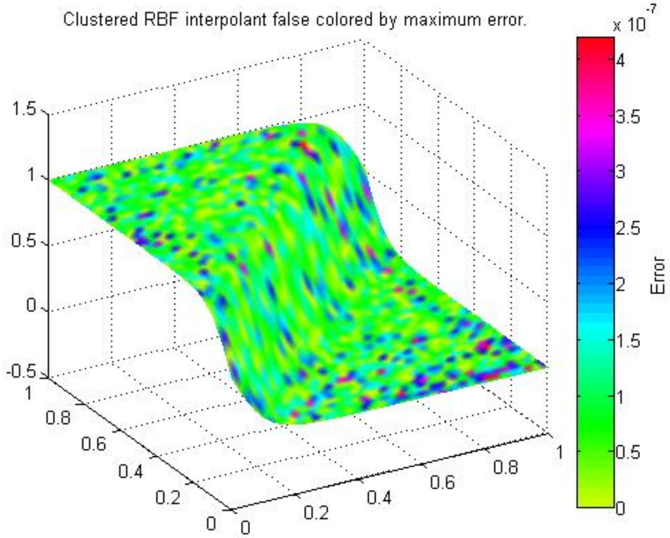
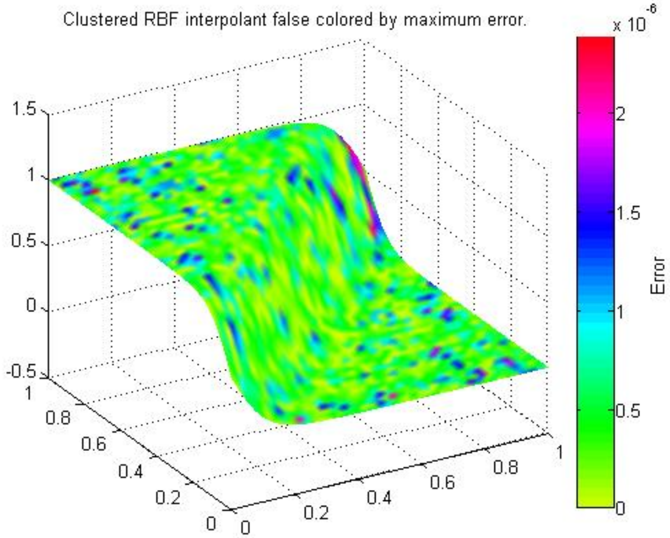



Hermite interpolant false colored by maximum error.
x 10^-8
Error
0
0.5
1
1.5
2
2.5
3
0
0.2
0.4
0.6
0.8
1
0
0.2
0.4
0.6
0.8
1
0
0.2
0.4
0.6
0.8
1


RBF interpolant false colored by maximum error.
x 10^-5
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Error
1.5
1
0.5
0
-0.5
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1


Clustered RBF interpolant false colored by maximum error.
1.5
1
0.5
0
-0.5
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
x 10^-7
4
3.5
3
2.5
2
1.5
1
0.5
0
Error


Clustered RBF interpolant false colored by maximum error.
x 10^-6
Error
1.5
1
0.5
0
-0.5
0.2
0.4
0.6
0.8
2

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