

The background of the slide is a faded, light blue-tinted image of the University of Southern Mississippi's main building, featuring a large central dome and classical columns.

*Welcome
from the
Department of Mathematics
at the
University of Southern
Mississippi*

Adaptive Methods

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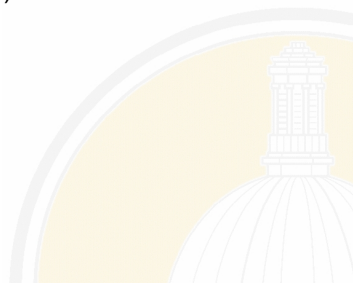
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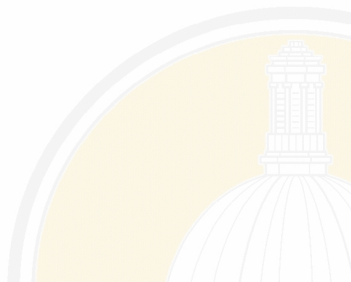
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- ▶ Introduction
- ▶ Knot Insertion (Chapter 21)
- ▶ Knot Removal (Chapter 21)
- ▶ Greedy One-Point Algorithm (Chapter 33)



An adaptive algorithm is an algorithm that changes its behavior based on information available at the time it is run. This might be information about computational resources available, or the history of data recently received.

—Wikipedia



Algorithm 21.2 (Page no. 181)

- ▶ Let data sites $\mathbf{x} = \{\mathbf{x}_1, \dots, \mathbf{x}_N\}$, data $f_i, i = 1, \dots, N$, and a tolerance tol be given.
- ▶ Choose M initial knots $\Xi = \{\xi_1, \dots, \xi_M\}$.
- ▶ Calculate the least squares fit

$$\wp_f(x) = \sum_{j=1}^n c_j \Phi(x, \xi_j)$$

with its associated error

$$e = \sum_{i=1}^N [f_i - \wp_f(x_i)]^2$$

.

While $e > tol$ do

- ▶ “Weight” each data point $x_i, i = 1, \dots, N$, according to its error component, i.e., let

$$w_i = |f_i - \rho_f(x_i)|, i = 1, \dots, N.$$

- ▶ Find the data point $x_v \notin \Xi$ with maximum weight w_ξ and insert it as a knot, i.e.,

$$\Xi = \Xi \cup \{x_v\} \text{ and } M = M + 1.$$

- ▶ Recalculate fit and associated error.

MATLAB implementaiton —> Program 21.1 (Book)

RBFKnotInsert2D.m

Algorithm 21.2 (Page no. 184)

- ▶ Let data sites $\mathbf{x} = \{\mathbf{x}_1, \dots, \mathbf{x}_N\}$, data $f_i, i = 1, \dots, N$, and a tolerance tol be given.
- ▶ Choose M initial knots $\Xi = \{\xi_1, \dots, \xi_M\}$.
- ▶ Calculate an initial fit

$$\wp_f(x) = \sum_{j=1}^n c_j \Phi(x, \xi_j)$$

with its associated least squares error

$$e = \sum_{i=1}^N [f_i - \wp_f(x_i)]^2$$

.

- “Weight” each knot $\xi_j, j = 1, \dots, M$, according to its least squares error, i.e., form

$$\Xi^* = \Xi \setminus \{\xi_j\},$$

and calculate the weights

$$w_j = \sum_{i=1}^N [f_i - \wp_f^*(x_i)]^2,$$

where

$$\wp_f^*(x) = \sum_{j=1}^{M-1} c_j \Phi(x, \xi_j^*),$$

is the approximation based on the reduced set of knots Ξ^* .

- Find the knot ξ_μ with lowest weight $w_\mu < tol$ and permanently remove it, i.e.,

$$\Xi = \Xi \setminus \{\xi_\mu\} \text{ and } M = M - 1.$$

- Recalculate fit and associated error.

MATLAB implementaiton —> Program 21.2 (Book)

Input data locations X , associated values of f , tolerance $tol > 0$

- ▶ Set initial residual $r_0 = P_f^X$, initialize $u_0 = 0, e = \infty, k = 0$
- ▶ Choose starting point $y_k \in X$



While $e > tol$ do

- ▶ Set $\beta = \frac{r_k(y_k)}{\Phi(y_k, y_k)}$
- ▶ For $1 \leq i \leq N$ do


$$r_{k+1}(x_i) = r_k(x_i) - \beta \Phi(x_i, y_k)$$

$$u_{k+1}(x_i) = u_k(x_i) + \beta \Phi(x_i, y_k)$$

end

- ▶ Find $e = \max_X |r_{k+1}|$ and the point y_{k+1} where it occurs
- ▶ Increment $k = k + 1$

end MATLAB implementaiton → Program 33.1 (Book)



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