COS 702 Bonus problem

Amoeba domain is defined as follows:

$$\partial\Omega = \left\{ (r\cos\theta, r\sin\theta) : r = e^{\sin\theta}\left(\sin^2 2\theta\right) + e^{\cos\theta}\left(\cos^2 2\theta\right) \right\},$$

Uniformly distributed 150 points on the curve of the boundary of amoeba and plot it (see Figure 1.

Hint: We first calculate the length of the curve from

$$S = \int_0^{2\pi} \sqrt{r(\vartheta)^2 + r'(\vartheta)^2} d\vartheta. \tag{1}$$

This integral may be evaluated using the MATLAB© routine routine `quadl` which evaluates an integral using adaptive Lobatto quadrature within a user-prescribed accuracy.

In order to place M points on the boundary, we take the length of each subsegment to be S/M. We choose $\theta_1 = 0$ and then solve, serially, the nonlinear equations

$$F(t) = \sqrt{\left(r(t)\cos t - r(\theta_k)\cos\theta_k\right)^2 + \left(r(t)\sin t - r(\theta_k)\sin\theta_k\right)^2} - \frac{S}{M} = 0, \quad k = 1, \ldots, M-1, \tag{2}$$

to yield $t = \theta_k, k = 2, \ldots, M$, respectively. The solution of the nonlinear equations may be carried out using the MATLAB© routine `fzero`. The angles $\theta_k, k = 1, \ldots, M$ define *approximately* equally spaced points on the curve defined by

$$\boldsymbol{x}_k = (x_k, y_k) = r(\theta_k)(\cos\theta_k, \sin\theta_k), \quad k = 1, \ldots, M. \tag{3}$$

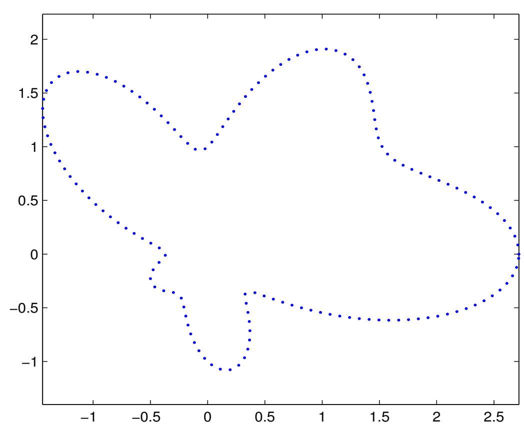

Figure 1: The profile of amoeba domain.