Assignment 5:

Objective: In this problem we need to construct an amoeba like domain bounded by

$$\partial \Omega = \{ (r\cos\theta, r\sin\theta) : r = e^{\sin\theta} (\sin^2 2\theta) + e^{\cos\theta} (\cos^2 2\theta) \} \quad \text{............(1)}$$

After construction of the curve, we need to find the area of the curve. This problem deals with few parameters. So our object is to find the optimal values of the parameters. To verify the calculated area, we need to compare it with the exact result.

Finding the normal vector : Equation (1) is given in polar coordinate form. It can be converted into cartesian coordinates and can be calculated the normal vector of the function. Other than that, the normal vector of the function can be calculated by using gradient(F) [1] function of matlab. In this implementation the first approach is used.

Construction of the Amoeba curve : To construct the amoeba first auxiliary points along normals "inside" and "outside" are produced. Then set distance along normal at which to place new points. Then new points are created. Here, "original" points have rhs=0, "inside" points have rhs=-1, "outside" points have rhs=1. Next to this distance is measured from the centers and RBF coefficient is calculated. Pf is calculated by multiplying coefficient and the result of RBF. Then new points are generated in mesh grid to plot the 3D graph. Then data sites are plotted with interpolant (zero contour of 2D-fit Pf).

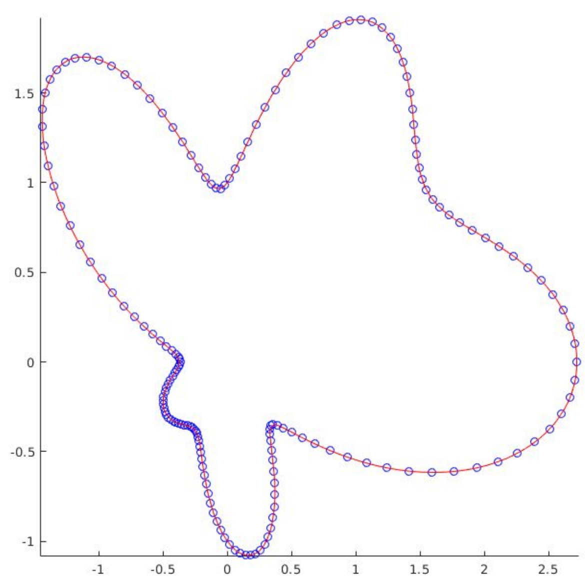

Fig 1. Plotting of amoeba in 2D

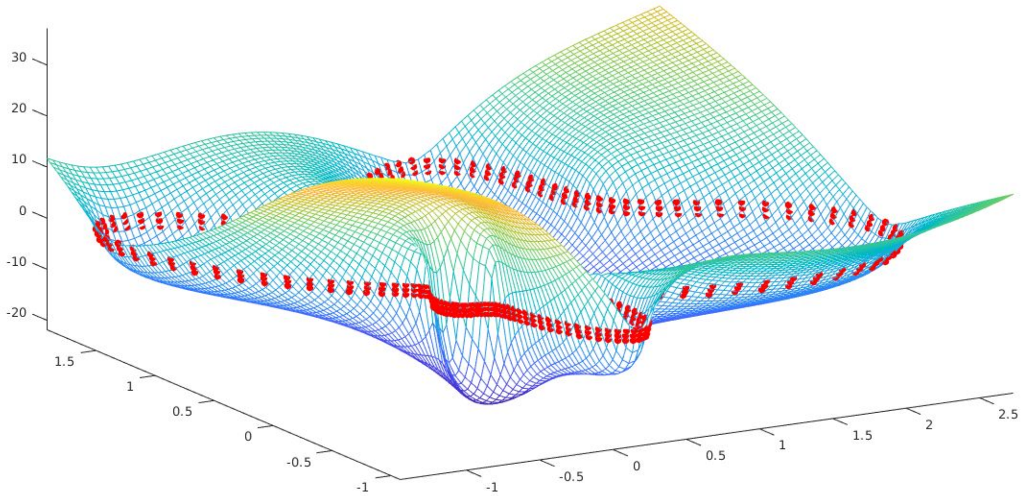

Fig 2. Plotting of amoeba in 3D

Boundary points:

In the present context total 4 different boundary points are used in the range of 50 to 200 with an interval of 40.

Calculating the area of Amoeba using Halton Sequence:

To find the area of an irregular shape using Halton sequence is very popular and effective method. In the present context the amoeba is constructed inside a rectangle of area 12.5523 square units. According to the process, *N* random points are generated in the rectangle. Some of them are inside the amoeba some of them are outside. The ratio of number of point inside the amoeba to *N* is the ratio of area of amoeba to rectangle. To determine the optimum value of *N* ten different values are tried.

For every boundary point, a set of Halton points are used to find the approximate area of the amoeba. The range of Halton points are used is 100 to 20,000 with an interval of 100. The best approximate area is calculated is 6.601236823901846, using 11400 halton points and 170 boundary points. The exact area of the amoeba is 6.601165460416832. The error is 7.136348501468603e-05.

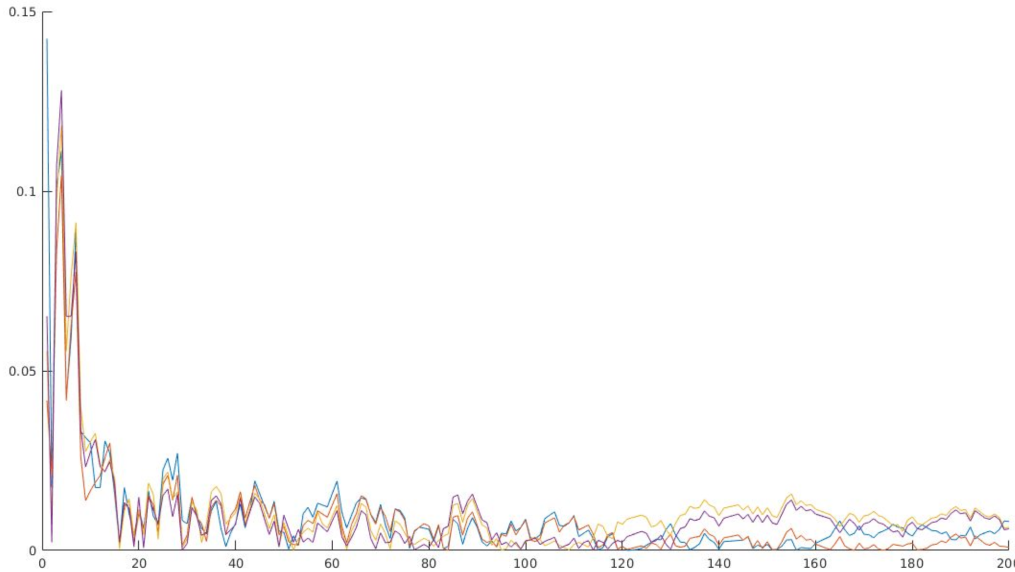

Fig 3. Error plot for different number of boundary points and Halton points

In Fig 3. we can visualize the error change of error with respect to the number of boundary points and Halton points. The X-axis denotes the 200 different number of Halton points, whereas Y-axis is representing the error levels. Four different curves are representing the changes of errors for four different values of boundary points with respect to different values Halton points.

Determining optimum value of C:

LOOCV algorithm is also used to find the best value of C. In this method the value of C = 60.000056286735890. The range of values for LOOCV method is used is same as the brute force method i.e 60 to 100.

Output in Matlab console:

Exact area of amoeba 6.601165.
Approximate area of amoeba 6.601237 using 11400 Halton points and 170 Boundary points.
Value of shape parameter calculated by LOOCV is 60.000056
Minimum error 0.000071.

Reference:

[1] https://www.mathworks.com/help/matlab/ref/gradient.html