Numerical Methods for Estimation Problems

***Abstract*— This electronic document is a live template. The various components of your paper [title, text, heads, etc.] are already defined on the style sheet, as illustrated by the portions given in this document.**

I. Introduction

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II. Linear Basis Function Regression

We will now examine regression models that use a linear combination of arbitrary basis functions. The closed form solution is actually available for such model classes (Bishop equation 3.15). For this section, our training data was generated using the function $y(x) = \cos(\pi x) + \cos(2\pi x)$ with added noise.

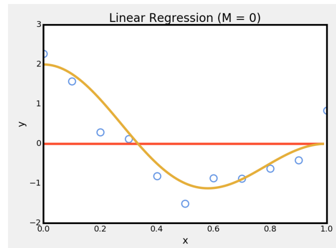
A. Polynomial basis functions

We first try linear regression with a simple polynomial basis $\phi_0(x) = 1$, $\phi_1(x) = x$, $\ldots$, $\phi_M(x) = x^M$. Note that for a given M there are $M + 1$ basis functions.

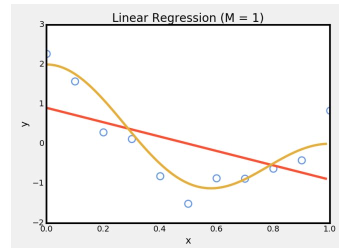
For $M = 0$, the result is a constant function. We can simplify the closed-form solution above to see that this is simply the mean of all the observations. As M increases, sum of squares error decreases and our estimate function comes closer to the points (Figure 1).

As the value of M increases, we see a greater tendency to overfit. At $M = 10$ the fitted curve goes through all the training points, resulting in zero error. This is not ideal, because the fitted curve does not generalize to test data, and would perform poorly if we tested it against more samples from the distribution.

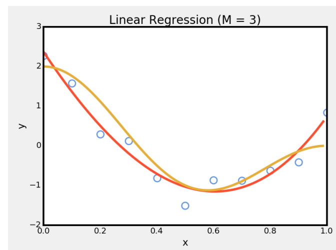
This demonstrates the need to restrict the complexity of our model class to the amount of training data at our disposal.



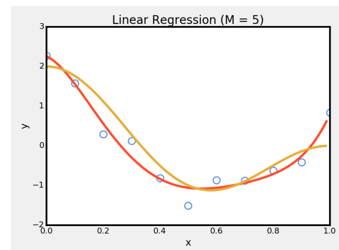
(a) $M = 0$



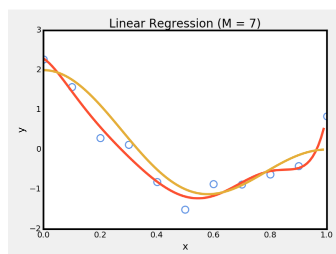
(b) $M = 1$



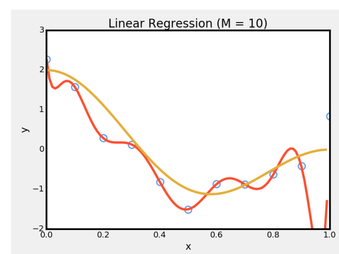
(c) $M = 3$



(d) $M = 5$



(e) $M = 7$



(f) $M = 10$

Fig. 1. Polynomials of orders M, shown in red, fitted to training data, shown as blue circles. The yellow curve shows the functin $y(x) = \cos(\pi x) + \cos(2\pi x)$ used to generate the data.

B. Closed-form and numerical derivative of SSE

Given a weight vector w, $M + 1$ basis functions $\phi_0(x)$, $\phi_1(x) \ldots \phi_M(x)$ and a data set (x_1, y_1), $(x_2, y_2) \ldots (x_n, y_n)$ we know that the sum of squares function is

$$\mathrm{SSE}(w) = \sum_{i=1}^{n} \left(y_i - \boldsymbol{w}^T \phi(x_i) \right)^2 \tag{1}$$

Here, the vector $\phi(x_i)$ is defined as

$$\phi(x_i) = \begin{bmatrix} \phi_0(x_i) & \phi_1(x_i) & \ldots & \phi_M(x_i) \end{bmatrix}^T$$

Taking the gradient of the SSE function w.r.t. w, we get

$$\nabla \mathrm{SSE}(w) = \sum_{i=1}^{n} 2 \left(y_i - \boldsymbol{w}^T \phi(x_i) \right) \phi(x_i)^T \tag{2}$$

We verified this closed form solution by using the numerical derivative from section I. At point w_{ml} (the most-likely weight vector we obtained in the previous section) we know that the gradient is supposed to be 0. We verified that our closed-form and numeric gradient at this point were also near 0. The results are tabulated in Table I. Note that our results become imprecise as the value of M increases. We also checked that the gradients were equal at other points.

M	$\lvert\nabla_{\text{closed-form}}\rvert$	$\lvert\nabla_{\text{numeric}}\rvert$
1	4.1×10^{-15}	2.5×10^{-7}
2	5.5×10^{-14}	1.9×10^{-7}
4	3.4×10^{-11}	1.2×10^{-7}
10	0.348	0.347

TABLE I

NORM OF GRADIENTS AT VARIOUS VALUES OF M

III. SOLVING LINEAR BASIS REGRESSION USING GRADIENT DESCENT

Before you begin to format your paper, first write and save the content as a separate text file. Keep your text and graphic files separate until after the text has been formatted and styled. Do not use hard tabs, and limit use of hard returns to only one return at the end of a paragraph. Do not add any kind of pagination anywhere in the paper. Do not number text heads-the template will do that for you.

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A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, sc, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as 3.5-inch disk drive.
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.
- Do not mix complete spellings and abbreviations of units: Wb/m2 or webers per square meter, not webers/m2. Spell out units when they appear in text: . . . a few henries, not . . . a few H.
- Use a zero before decimal points: 0.25, not .25. Use cm3, not cc. (bullet list)

C. Equations

The equations are an exception to the prescribed specifications of this template. You will need to determine whether or not your equation should be typed using either the Times New Roman or the Symbol font (please no other font). To create multileveled equations, it may be necessary to treat the equation as a graphic and insert it into the text after your paper is styled. Number equations consecutively. Equation numbers, within parentheses, are to position flush right, as in (1), using a right tab stop. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in

$$\alpha + \beta = \chi \tag{1}$$

Note that the equation is centered using a center tab stop. Be sure that the symbols in your equation have been defined before or immediately following the equation. Use (1), not Eq. (1) or equation (1), except at the beginning of a sentence: Equation (1) is . . .

D. Some Common Mistakes

- The word data is plural, not singular.
- The subscript for the permeability of vacuum ?0, and other common scientific constants, is zero with subscript formatting, not a lowercase letter o.
- In American English, commas, semi-/colons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an inset, not an insert. The word alternatively is preferred to the word alternately (unless you really mean something that alternates).
- Do not use the word essentially to mean approximately or effectively.
- In your paper title, if the words that uses can accurately replace the word using, capitalize the u; if not, keep using lower-cased.
- Be aware of the different meanings of the homophones affect and effect, complement and compliment, discreet and discrete, principal and principle.
- Do not confuse imply and infer.
- The prefix non is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the et in the Latin abbreviation et al..

- The abbreviation i.e. means that is, and the abbreviation e.g. means for example.

IV. USING THE TEMPLATE

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A. *Headings, etc*

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced. Styles named Heading 1, Heading 2, Heading 3, and Heading 4 are prescribed.

B. *Figures and Tables*

Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation Fig. 1, even at the beginning of a sentence.

TABLE II

AN EXAMPLE OF A TABLE

One	Two
Three	Four

We suggest that you use a text box to insert a graphic (which is ideally a 300 dpi TIFF or EPS file, with all fonts embedded) because, in an document, this method is somewhat more stable than directly inserting a picture.

Fig. 2. Inductance of oscillation winding on amorphous magnetic core versus DC bias magnetic field

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity Magnetization, or Magnetization, M, not just M. If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write Magnetization (A/m) or Magnetization A[m(1)], not just A/m. Do not label axes with a ratio of quantities and units. For example, write Temperature (K), not Temperature/K.

V. CONCLUSIONS

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

APPENDIX

Appendixes should appear before the acknowledgment.

ACKNOWLEDGMENT

The preferred spelling of the word acknowledgment in America is without an e after the g. Avoid the stilted expression, One of us (R. B. G.) thanks . . . Instead, try R. B. G. thanks. Put sponsor acknowledgments in the unnumbered footnote on the first page.

References are important to the reader; therefore, each citation must be complete and correct. If at all possible, references should be commonly available publications.

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