

We apologize for not answering Reviewer 4's comments earlier, but we were not sent the review until now. We thank the reviewer for the careful read and helpful feedback. Responses to the reviewer's comments are addressed inline below.

----- REVIEW 4 -----

Q1) Definitions 1-6: Shouldn't the section curve restricted to be planar, because the resulting shape is a generalized cylinder. Currently, "joint planes" exist as many as the number of joints, but this should be unique. Also, this section plane should be parallel after the sweeping process. Can you simplify the definition from this point of view? The current definition has a freedom that is not used afterwards.

A1) No, the joint planes may be distinct for each joint, and the final folding need not be a generalized cylinder. It just happens to be the case that our construction has a common joint plane at all times (not a generalized cylinder though), but it breaks for relatively simple cases.

For instance, the strip narrowing gadget in Figure 4 is not a generalized cylinder. Also, if you fold the beginning of the strip narrowing gadget (Figure 4(a)-(c)) in half with a crease following the direction of the strip, the joint planes no longer coincide (in fact they become orthogonal). The photo linked below shows this.

<https://tinyurl.com/ydh5ll8u>

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Q2) What does it mean by "valid" in Proposition 1? Also, shouldn't it be "non-joint node" that the distance will not change? Wait, what happens when non-joint node collides with the joint? Does the node transfer to the adjacent segment or is merged to the joint? What is the purpose of "proposition"?

A2) This is now Definition 8. Good point, we changed it to non-joint nodes for clarification. The joint node is moves along the cross section, and the nodes that pass through it transfer to the adjacent segment.

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Q3) Definition 9: What is the definition of "folding"? Shouldn't it be the extruded surface instead of folding?

A3) This is now Definition 10. We are referring to the final folded state, which is formed by gluing the trapezoids together. This can be any straight crease origami folding (decomposed into trapezoids and degenerate trapezoids i.e. triangles).

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Q4) There are duplicate theorems and property numbers. (two Theorem 1, two property 1). I stopped following the theorems after this duplication. I would like to review them after the revision.

A4) Sorry about that. It has been fixed. We have also used unified numbering for Theorems, Lemmas, and Corollaries. Thank you for pointing this out.

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Title: Efficient Origami Construction of Orthogonal Terrains using  
Cross-Section Evolution  
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We thank the reviewers for the careful read and helpful feedback. Responses to the reviewer's comments are addressed inline below.

----- REVIEW 1 -----  
Overall evaluation: 2 (Minor revision)

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Q1) The phrase "cross section" should not have a hyphen. This is done correctly for most of the paper, but not in the title nor throughout page 1.

A1) We have removed the hyphen throughout.

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Q2) Quotation marks are not LaTeXed properly throughout the paper. For example, on Page 9 after Definition 14, it looks like the LaTeX code for the sentence is

We consider each "column" of...  
where the LaTeX code should be:  
We consider each ``column" of...

That will make the quotation marks be open and close in the correct way. This should be corrected throughout the paper.

A2) Fixed!

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Q3) Page 1, 2nd-to-last sentence of the second paragraph: "...though the construction is less inefficient than our construction applied to orthogonal terrains." Shouldn't this be, "... though the construction is less efficient than our construction applied to orthogonal terrains."

A3) Yup! Fixed.

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Q4) In Section 2, it is not stated what the ambient space is for the "1-dimensional cross section moving forward in time." Is this happening in 3D space? Or in a 2D plane? This should be stated explicitly. (It has implications for the later definition of the velocity vectors.)

A4) The 1-dimensional cross section sweeps accross the target surface in 3D. We have added this explicetely.

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Q5) In Section 2.1, Definition 2, should the last sentence say, "For a segment  $s_i$  in  $C$  we will denote its velocity as  $\hat{v}_i$ ." ?

A5) Yes! We have added the subscript to  $s_i$ .

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Q6) Also in Section 2.1: Two properties are listed. But these seem meant to be part of the definitions. Shouldn't they be stated as such? For example, Property 2 should be part of Definition 2, where the velocity vectors are defined --- that is, they should be defined as being orthogonal to the orientation vector of the segment. (Although see below for my comments on Property 3.)

A6) Yes, this was unclear. We have renamed these as Invariants, implying that

they are necessary properties for our construction.

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Q7) As for Property 1, I confess that I really do not understand what it is saying. The velocity vectors  $\hat{v}_s$  are assigned to segments according to Definition 2. But Property 1 describes non-joint points as "moving" when this has not been defined. Do the authors intend to associate each non-joint node in a sector  $s$  with the sector's velocity vector  $\hat{v}_s$ ? If so, then each non-joint node has a unit velocity vector assigned to it, and so of course they have the same velocity ... Do you see why this is confusing? Is Property 1 even necessary to state, since all non-joint nodes in a fixed segment share the same velocity vector?

A7) Yes, this is implied by the definition. We have changed the ambiguous wording 'move with' to 'have', which is the meaning you state. We name it so that it may be referenced in the statement of Theorem 1.

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Q8) In Definition 6, the authors state, "We define  $v_l^{\parallel}$  and  $v_l^{\perp}$  as the components of  $v_l$  coinciding with, and orthogonal to  $P$  respectively." Components of a vector usually refer to a specific coordinate system, and none is specified here. Do the authors really mean for  $v_l^{\parallel}$  to be the projection of  $v_l$  onto the plane  $P$ ? If so, that would be a way to define it that is independent of the coordinate system. On the other hand, do the authors really mean that we are defining a coordinate system at the joint  $J$  in 3D space where two axes lie in  $P$  and the third axis is orthogonal to  $P$ ? Then the  $P$  components and the orthogonal component makes more sense. If this is correct, then perhaps this should be stated more explicitly.

A8) We have restated these definitions according to the reviewer's suggestion.

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Q9) Again, Property 3 seems to be part of the definitions being given for joints and velocities. Stating these as Properties does make it easier for the authors to refer to them later. But if they want to do this, then I would recommend that the authors state somewhere that these Properties are part of the definitions and not things that should be proven (nor viewed as natural consequences of the definitions).

A9) We hope that the reclassification of Properties to Invariants adequately addresses this issue.

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Q10) In Definition 4, what is the difference, notationally, between  $J_i$  being not bolded and  $J_i$  being bold? Both seem to be points, which is equivalent to a vector in this context, yes?

A10) This was a typo. Fixed!

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Q11) In Definition 7, "For ever segment  $s$  in a cross section  $C...$ " should be "For every segment  $s...$ "

A11) Fixed!

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Q12) Is Proposition 1 really a proposition? In math, "propositions" are like theorems, in that they are statements that need to be proved. It looks to me like "Proposition 1" is actually defining what it means for a joint node to be "valid." If so, this should be a Definition, not a Proposition.

A12) You are right. We have changed to a defintion.

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Q13) On page 4, middle of the page, "Here,  $v$  is the magnitude of the plane velocity of  $J_i$ ." What does "plane velocity" mean here?

A13) We mean projection of  $J_i$  onto the joint plane  $\mathcal{P}$ . We have clarified in the text.

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Q14) In the middle of the proof of Lemma 2 (page 5), "Similarly, we consider a coordinate system with  $(\hat{v}_l, \hat{o}_l)$  as the basis." Don't we want  $(\hat{v}_r, \hat{o}_r)$  here?

A14) Yes! We have fixed the text.

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Q15) In Definition 9, the choice of the word "folding" to mean the unfolded piece of paper seems backwards and confusing. I recommend calling it the "unfolding" of the cross section interval.

A15) Here, we do mean the folding, not the unfolding, so the current terminology should suffice. Each  $Z_i$  is a trapezoid in  $\mathbb{R}^3$ , which we have clarified.

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Q16) On page 6, I am not sure why Proposition 2 is stated as a proposition. The statement, "The trajectory of a joint forms a crease in the folded state." is something the authors are asserting, not proving rigorously. My impression is that they view this statement as an immediate consequence of their cross section model, not as something to be proved. So why is this called a Proposition? A "Remark" or "Fact" would be a better descriptor.

A16) We have changed this descriptor to 'Remark'.

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Q17) On page 8, why are the Properties 1–6 being restated? Since they are previously defined in the paper, Theorem 1 on page 7 should merely say, "... such that Properties 1–6 hold for all segments and joints in each of the cross sections involved." This would save a lot of space.

A17) We have now simply referenced the Invariants.

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Q18) Page 9, start of Section 3.1: The  $E_{\{i,j\}}$  notation is not being used consistently. In the first sentence of this section, we see  $\{E_{i1}, E_{i2}, \dots, E_{in}\}$ . The first two terms there should have commas in the subscripts:  $E_{i,1}$  and  $E_{i,2}$ .

A18) Fixed!

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Q19) Page 10, bottom: It seems that Property 1 is being redefined. As far as I can discern, this has nothing to do with the Property 1 defined on page 2, and it is very bad form to refer to two different things using the same name. Perhaps the "Property 1" on page 10 should be called, "Property 7."

A19) Now that the previous properties are now Invariants, there is no longer a naming collision.

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Q20) Page 11: The second sentence of Section 3.2 seems to be missing a

close-parenthesis.

A20) Fixed!

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Q21) Page 13, last sentence reads, "We obtain the following primitive, as a consequence of Theorem 2." Should this be, "We obtain the following proposition, as a consequence of Theorem 2." ?

A21) Indeed, fixed!

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Q22) I also recommend, if there is room, the addition of a short Conclusion. (Perhaps there will be room for a conclusion if the statement of Theorem 1 is made shorter, as I suggested above.) If a conclusion can be included, I suggest stating something about any limitations this algorithm has or ways it could be improved (if any).

A22) We have added a conclusions indicating two directions for future work.

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