Paper Title

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Abstract: *The abstract must summarise the main content of the paper, and may be up to approximately 100 words in length. Unlike previous OSME publications, the abstract will be included in the proceedings. This abstract needs not be the same as that submitted for acceptance to the conference.*

1 Introduction

Outline

2 Extruded Surfaces (Preliminaries)

3 Cross Sections

We introduce a new method of origami construction, using cross section disgrams. Instead of beginning our construction from a 2-dimensional sheet of paper, we consider a 1-dimensional cross section moving forwards in time. A simple example is demonstrated in Figure 3.

Conservation of length

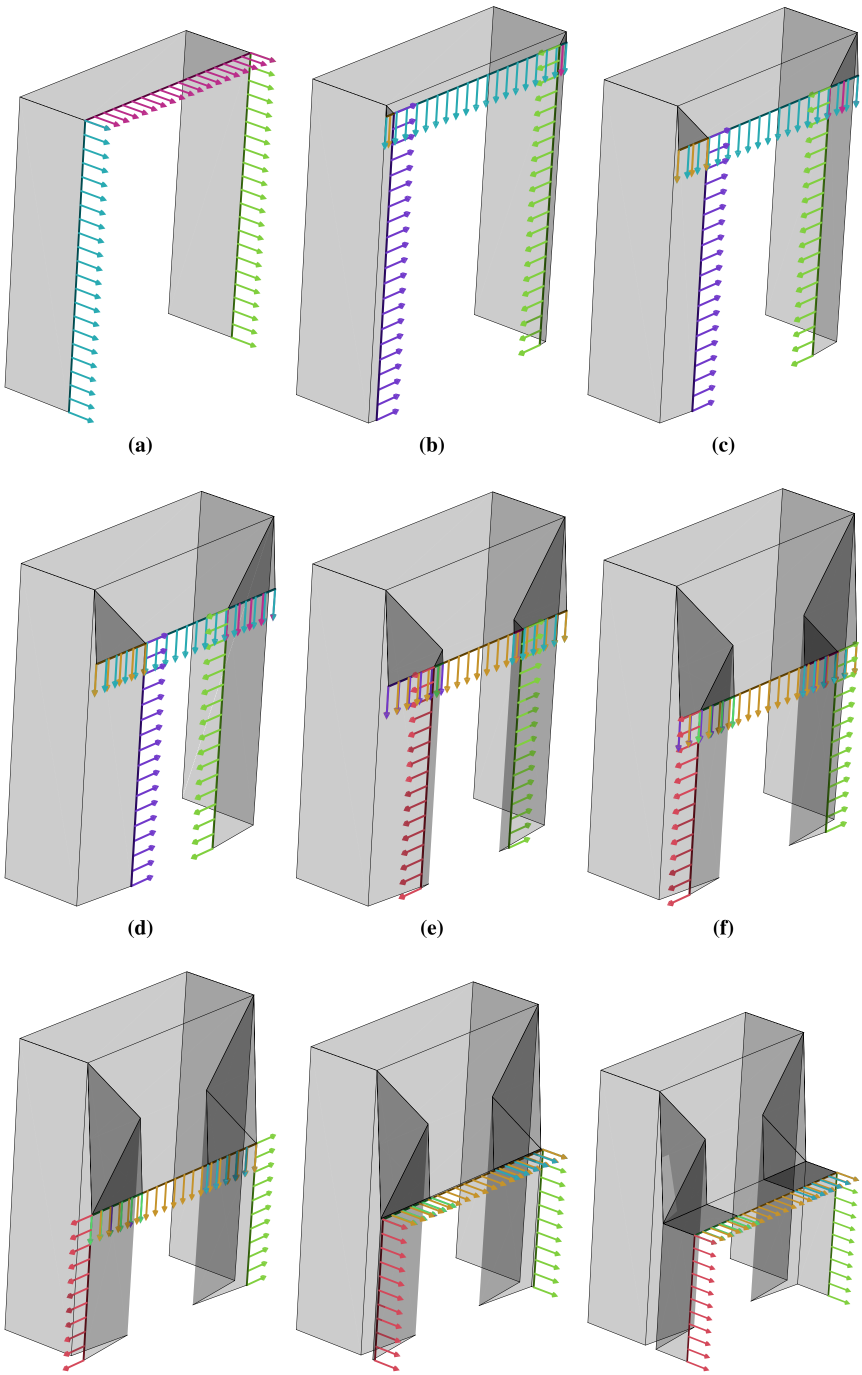
Definition 1. *A cross section is defined as an ordered list of line segments* $\langle s_1, s_2, \cdots, s_n \rangle$*, such that for every segment* s_i *(except the last), the right endpoint of* s_i *coincides with the left endpoint of* s_{i+1}*. Each line segment is associated with a velocity vector* $\mathbf{v}_i$ *of unit magnitude. Each segment is also associated with an unit orientation vector* $\mathbf{o}_i$*, which is the unit vector from the left endpint to the right endpoint.*

Lemma 2. *All points on a single segment move with the same velocity* v_i*.*

3.1 Joints

Definition 3. *A cross section with* n *segments is also associated with a list of joints* $\langle J_1, \cdots J_{n-1} \rangle$*, where* J_i *corresponds to the right endpoint of* s_i*. The velocity of a joint* J_i*, is* $\mathbf{J}_v = \mathbf{v}_i + \mathbf{v}_{i+1}$ *(i.e. the sum of the velocities of the corresponding segments).*

Definition 4. *Note that the trajectory of the joints form a crease ... angle is angle between direction vectors. Creases are also created when a segment changes direction... angle is change in direction vector.*



(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

Figure 1: *Level Shifting Gadget*

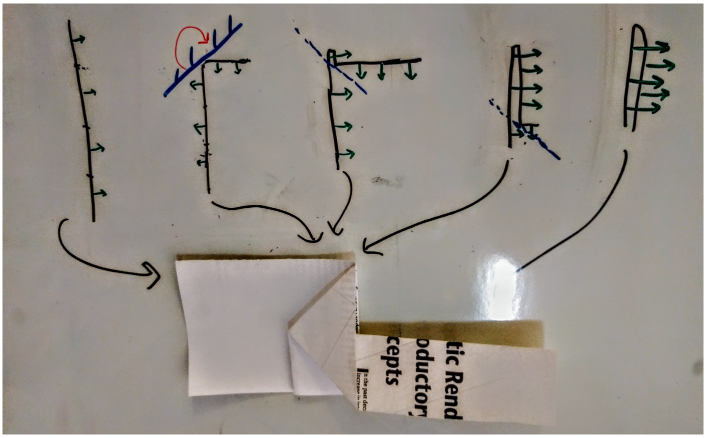


Figure 2: *A strip narrowing gadget constructed from cross sections.*

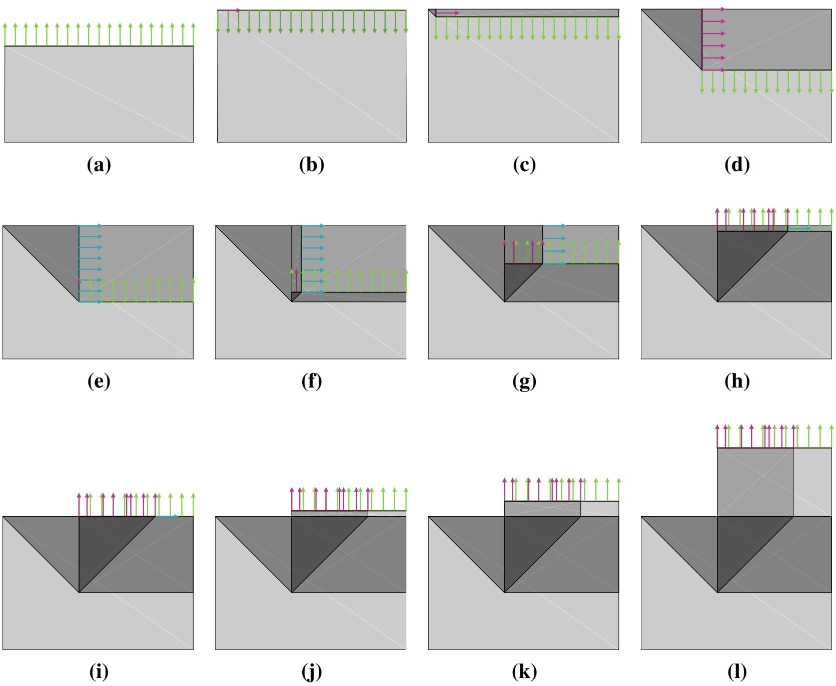


Figure 3: *Cross Section evolution of a strip narrowing gadget.*

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Definition 5. *A joint plane is the plane that coincides with both segments l and r associated with a particular joint J.*

Definition 6. *Consider a joint J associated with segments l and r, and joint plane $\mathscr{P}$, where $\mathbf{v}_l$ and $\mathbf{v}_r$ are the velocities of segments l and r. We define $\mathbf{v}_l^{\shortparallel}$ and $\mathbf{v}_l^{\perp}$ as the components of $\mathbf{v}_l$ coinciding with, and orthogonal to the joint plane respectively. Similarly, we define $\mathbf{v}_r^{\shortparallel}$ and $\mathbf{v}_r^{\perp}$, as the components of $\mathbf{v}_r$. Further, note that $\mathbf{v}_l^{\shortparallel}$ and $\mathbf{v}_r^{\shortparallel}$ have to be orthogonal to $\mathbf{o}_l$ and $\mathbf{o}_r$ respectively.*

Claim 7. *For a joint J associated with segments l and r, $\mathbf{v}_l^{\perp} = \mathbf{v}_r^{\perp}$.*

Corollary 8. *For a joint J associated with segments l and r, $\left\|\mathbf{v}_l^{\shortparallel}\right\| = \|\mathbf{v}_r^{\shortparallel}\|$.*

Claim 9. *A joint J corresponding to segments s and t is* valid *if the resulting evolution of the joint preserves distances.*

3.2 Time Travel

In the process of time travel, the lengths of segments may change. This can also be visualized as movement of the corresponding joint along one of the segments. The movement of a joint increases the length of one of its associated segments, and decreases the length of the other segment by the same amount (this ensures that the total length is preserved). Most velocity sequences are invalid. For example...

We will use some abuuse of notation, by using $\mathbf{v}_i, \mathbf{o}_i$ etc. to refer to the corresponsing parameters of s_i.

Definition 10. *For ever segment s, we associate a left velocity L_s, which indicates the rate at which s grows from its left endpoint. Similarly, we define a right velocity $R_s \ldots$*

Claim 11. *The right velocity of s_i is the negation of the left velocity of s_{i+1} i.e. $R_i = -L_{i+1}$. Furthermore, the left velocity of the first segment, and the right velocity of the last segment should be zero, i.e. $L_0 = R_n = 0$ This ensures that the total length of the cross section does not change.*

Consider a joint $\mathbf{J}_i$ corresponding to segments $l = s_i$ and $r = s_{i+1}$, at time $t = 0$. Henceforth, we will refer to L_l as L, and R_r as R. Without loss of genenerality, we assume that $L < 0$. At a later time t, let the new joint position be $\mathbf{J}_i'$. We define nodes a and b corresponding to $\mathbf{J}_i$ and $\mathbf{J}_{i+1}$ respectively. We also define the initial and final positions of a as $\mathbf{a}$, and $\mathbf{a}'$, and similarly for b, we define $\mathbf{b}$ and $\mathbf{b}'$. Let d be the *separation* between nodes a and b.

First, note that $\mathbf{a}, \mathbf{b}$ lie on the segment l, and $\mathbf{a}', \mathbf{b}'$ lie on the segment r, which implies that $\mathbf{b}-\mathbf{a}=d\cdot\hat{\mathbf{o}}_\mathbf{l}$, and $\mathbf{b}'-\mathbf{a}'=d\cdot\hat{\mathbf{o}}_\mathbf{r}$.

$$\mathbf{b}'-\mathbf{a}' = (\mathbf{b}+t\cdot\mathbf{v}_l^{\shortparallel}) - (\mathbf{a}+t\cdot\mathbf{v}_r^{\shortparallel}) \tag{1}$$

$$\implies \mathbf{b}'-\mathbf{a}' = (\mathbf{b}-\mathbf{a}) + t\cdot(\mathbf{v}_r^{\shortparallel}-\mathbf{v}_l^{\shortparallel}) \tag{2}$$

$$\implies d\cdot\hat{\mathbf{o}}_\mathbf{r} = d\cdot\hat{\mathbf{o}}_\mathbf{l} + t\cdot(\mathbf{v}_r^{\shortparallel}-\mathbf{v}_l^{\shortparallel}) \tag{3}$$

$$\implies \mathbf{v}_r^{\shortparallel}-\mathbf{v}_l^{\shortparallel} = \frac{d}{t}\cdot(\hat{\mathbf{o}}_\mathbf{r}-\hat{\mathbf{o}}_\mathbf{l}) \tag{4}$$

$$= R\cdot(\hat{\mathbf{o}}_\mathbf{r}-\hat{\mathbf{o}}_\mathbf{l}) = -L\cdot(\hat{\mathbf{o}}_\mathbf{r}-\hat{\mathbf{o}}_\mathbf{l}) \tag{5}$$

This implies that l is directed clockwise, and r is directed anti-clockwise. If the angle between the segments is θ, the magnitude of $\hat{\mathbf{o}}_\mathbf{r}-\hat{\mathbf{o}}_\mathbf{l}$ is $\sqrt{2+2\cos(\theta)}$, and $|\mathbf{v}_r^{\shortparallel}-\mathbf{v}_l^{\shortparallel}| = v\cdot\sqrt{2+2\cos(\pi-\theta)}$. Here, v is magnitude of the plane velocity of J_i.

What is plane velocity

$$R = \frac{d}{t} = \frac{|\mathbf{v}_r^{\shortparallel}-\mathbf{v}_l^{\shortparallel}|}{|\hat{\mathbf{o}}_\mathbf{r}-\hat{\mathbf{o}}_\mathbf{l}|} \tag{6}$$

$$= v\cdot\frac{\sqrt{\cos^2(\pi/2-\theta/2)}}{\sqrt{\cos^2\theta/2}} \tag{7}$$

$$= v\cdot\tan\left(\frac{\theta}{2}\right) \tag{8}$$

Segments and Velocities

Divide the cross section into disjoint line segments, which share endpoints.

Evolution of Adjacent Segments

Each *segment* has a direction associated with it (this should be perpendicular to the segment). With the passage of time, segments may get shorter or longer. This can result in a segment length becoming zero.

Note that time evolution is reversible.

Creation of New Segments

We may create a new segment at any point. Often the new segment has zero length. In the event that an existing segment reaches length zero, we may modify …

4 Orthogonal Graphs

In this section, we outline a construction of orthogonal graphs with arbitrary rational extrusion heights. In our construction, the cross section at will always be on the $x-z$ plane. This makes the analysis much simpler.

4.1 Grid Extrusion

To simplify the presentation, we will consider an uniform $X-Y$ grid, with arbitrary rational extrusion heights corresponding to every grid square.

Definition 12. *An $n \times m$ rational grid extrusion is a 3-dimensional structure, whose projection onto the $x-y$ plane forms an unit grid of size $n \times m$. Additionally, the unit face corresponding to the location $(i.j)$, is extruded E_{ij} units in the z direction, where E_{ij} is a rational number.*

4.1.1 Strip Extrusion

We also consider each "column" of a given grid extrusion separately as an individual *strip exrusion*. We will construct each of the n strips independently, and attachthem together with *strip connectors*.

4.2 Optimality

Under some suitable assumptions, our construction can be made $2+\varepsilon$ optimal, for arbitrarily small ε.

We define the maximum deltas along the y-axis as $D_j = \max_i \|E_{i,j} - E_{i,j+1}\|$, and let $Y = n + \sum_{i=1}^{n-1} D_j$ We also define the lowest and highest points along x-axis as $L_i = \max_j E_{ij}$ and $H_i = \max_j E_{ij}$, and let

$$X = n + \sum_{i=1}^{n} [(H_i - \min(L_i, L_{i+1})) + (H_i - \min(L_i, L_{i-1}))]$$

The terms in the summation account for the total length of all the necessary worst case vertical walls, and the n is for the top faces.

Claim 13. *The x-axis length of the strip of paper required to fold this shape can be made arbitrarily close to X.*

Claim 14. *The y-axis length of the strip of paper required to fold this shape will be exactly Y.*

First, we pick an ε, such that 2ε divides all extrusion heights. The construction will require paper of size $X' \times Y$, where $X' = X + 2\varepsilon(n-1)$.

There are two components to the construction

- n strips parallel to the y-axis. Each of these strips will fold to the corresponding strip in the extruded graph. The total area of these strips will be $X \times Y$.
- $n-1$ Intermediate strips, each of size $2\varepsilon \times Y$ to connect the main strips together.

The total area is therefore $X \times Y + (n-1)2\varepsilon \times Y = X' \times Y$. This can of course be made arbitrarily close to $X \times Y$.

4.3 Construction of Grid Extrusion

Given a strip of size $w \times Y$, we will consider the evolution of the cross sections along the length Y. Imagine that there are $T = \frac{Y}{\varepsilon}$ time steps over which the cross section evolves along the y-axis. Consider the decomposition of Y as

$$Y = (1 + D_1) + (1 + D_2) + (1 + D_) + \cdots + (1 + D_{n-1})$$

Here, the times corresponding to

- the i^{th} "1" is realized as the strip $i - 1 \leq y \leq i$
- D_i is the transition between $i - 1 \leq y \leq i$ and $i \leq y \leq i + 1$

4.4 Construction of Strip Extrusion

4.4.1 Level Shifting

4.5 Construction of Strip Connectors

4.5.1 Alignment with Level Shifts

5 Polygons

References

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