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 4 commits

 16 files changed

 0 commit comments


 3 contributors

	Commits on May 29, 2018		
	Amartya Shankha Biswas	Updates cross-sections	91a0b16
	Amartya Shankha Biswas	Fixed quotes	bd30ad4
	Commits on Jun 13, 2018		
	amartyashankha	Addressing comments.	9050091
	Commits on Jun 14, 2018		
	origamimagiro	revision doc and conclusions	7561d99

Showing 16 changed files with 505 additions and 933 deletions.

UnifiedSplit

2


 70SME-style.sty

47484950515253545556575859

474849505152535455565758596061

```
\newtheorem{prop}{Proposition}
\newtheorem{defn}{Definition}
\newtheorem{clm}{Claim}
+ \newtheorem{inv}{Invariant}
\newtheorem{theorem}{Theorem}
\newtheorem{lemma}{Lemma}
\newtheorem{property}{Property}
\newtheorem{corollary}{Corollary}
\newtheorem{proposition}{Proposition}
\newtheorem{definition}{Definition}
\newtheorem{claim}{Claim}
+ \newtheorem{remark}{Remark}
% link (url/links/citation) formatting
\RequirePackage[pdfusetitle]{hyperref}
```

14

 abstract/abstract.tex

81828384

818283

```
\title{Efficient Origami Construction of Orthogonal Terrains \\\-using Cross-Section Evolution}
```

```

84 | +using Cross_Section Evolution}
85 | 85 | \author{Amartya Shankha Biswas, Erik D. Demaine,
86 | 86 | and Jason S. Ku}
87 | 87 | \keywords{cross sections, orthogonal terrains, time evolution}
128 | 128 | \vspace{-0.2pc}
129 | 129 |
130 | 130 | In order to better communicate the algorithm and the final folded state
131 | 131 | -produced, we also introduce a new {\bf cross-section evolution} representation
132 | 132 | +produced, we also introduce a new {\bf cross-section evolution} representation
133 | 133 | of a folded isometry: a straight line is swept across the crease pattern of a
134 | 134 | -cross-section of the folded surface. The propagation of the cross-section
135 | 135 | +cross-section of the folded surface. The propagation of the cross-section
136 | 136 | between crease pattern vertices is uniquely determined by the initial
137 | 137 | -orientation of the cross-section, so the folded isometry can be constructed by
138 | 138 | -sweeping the line and locally modifying the cross-section when crossing crease
139 | 139 | +orientation of the cross-section, so the folded isometry can be constructed by
140 | 140 | +sweeping the line and locally modifying the cross-section when crossing crease
141 | 141 | pattern vertices during propagation. This representation not only simplifies the
142 | 142 | description of the 3D folded isometries constructed, but also provides a simpler
143 | 143 | framework to argue that the folded state does not self intersect, by propagating
144 | 144 | -planar cross-sections monotonically along a single direction. We then
145 | 145 | +planar cross-sections monotonically along a single direction. We then
146 | 146 | show that our construction's efficiency is within a small constant factor of
147 | 147 | any folding with optimal efficiency.
148 | 148 |
149 | 149 | \begin{figure}
150 | 150 | \centering
151 | 151 | \includegraphics[width=\linewidth]{Figures/fig2sm.pdf}
152 | 152 | - \caption{Snapshots of a cross-section evolution for two gadgets used to
153 | 153 | + \caption{Snapshots of a cross-section evolution for two gadgets used to
154 | 154 | construct orthogonal terrains. The sequence on top shows a level-shifting gadget
155 | 155 | that changes the height of a section via the use of auxiliary pleats to tuck
156 | 156 | away excess paper. The sequence on the bottom shows a paper-absorbing gadget

```

30 cross_sections/cross_sections.tex

```

3 | 3 |
4 | 4 | We introduce a new method of origami construction, using cross section diagrams.
5 | 5 | Instead of beginning our construction from a 2-dimensional sheet of paper, we
6 | 6 | -consider a 1-dimensional cross section moving forwards in time. A simple example
7 | 7 | -using strip narrowing\cite{strip_narrowing} is demonstrated in
8 | 8 | +consider a 1-dimensional cross section moving forwards in time through 3D space.
9 | 9 | +A simple example using strip narrowing\cite{strip_narrowing} is demonstrated in
10 | 10 | Figure~\ref{fig:strip_narrowing}.
11 | 11 |
12 | 12 | \subsection{Segments and Cross Sections}
13 | 13 | such that for every segment  $s_i$  (except the last),
14 | 14 | the right endpoint of  $s_i$  coincides with the left endpoint of  $s_{i+1}$ .
15 | 15 | Each segment  $s_i$  is also associated with a velocity vector  $\vec{v}_i$  of unit magnitude.
16 | 16 | -For a segment  $s_i$  we will also denote this velocity as  $\vec{v}_i$ .
17 | 17 | -\end{definition}
18 | 18 | -
19 | 19 | -\begin{definition}
20 | 20 | -\label{def:node}
21 | 21 | -Given a cross section  $C = \langle s_1, s_2, \dots, s_n \rangle$ , a node  $x$  denotes a point on one of the segments
22 | 22 |  $s_i$ .

```

```

32 | -A joint node is a node that resides on the endpoint of a segment.
33 | -The distance between two nodes on a cross section is defined as the overall length of cross section between the two
    | nodes.
    | 26 | +For a segment  $s_i$  in  $C$  we will also denote this velocity as  $\vec{\hat{v}}_i$ .
34 | 27 | \end{definition}
35 | 28 |
36 | 29 | \vspace{-1pc}
37 | -\begin{restatable}{pro}{UniformVelocity}
38 | -\label{pro:uniform_velocity}
39 | -All non-joint nodes on a segment  $s$  move with the same velocity  $\vec{\hat{v}}_s$ .
    | 30 | +\begin{restatable}{inv}{UniformVelocity}
    | 31 | +\label{inv:uniform_velocity}
    | 32 | +All non-joint nodes on a segment  $s$  have the same velocity  $\vec{\hat{v}}_s$ .
40 | 33 | \end{restatable}
41 | 34 | \vspace{-1pc}
42 | 35 |
43 | -\begin{restatable}{pro}{OrthogonalVelocity}
44 | -\label{pro:orthogonal_velocity}
    | 36 | +\begin{restatable}{inv}{OrthogonalVelocity}
    | 37 | +\label{inv:orthogonal_velocity}
    | 38 | The velocity  $\vec{\hat{v}}_s$  of segment  $s$  is orthogonal to its orientation  $\vec{\hat{o}}_s$ .
46 | 39 | \end{restatable}
47 | 40 |
    | 41 | +\begin{definition}
    | 42 | +\label{def:node}
    | 43 | +Given a cross section  $C = \langle s_1, s_2, \dots, s_n \rangle$ , a node  $x$  denotes a point on one of the segments
    | 44 |  $s_i$ .
    | 45 | +A joint node is a node that resides on the endpoint of a segment.
    | 46 | +The distance between two nodes on a cross section is defined as the overall length of cross section between the two
    | 47 | nodes.
    | 48 | +\end{definition}
    | 49 | +
48 | 48 | \input{cross_sections/joints}
49 | 49 | \input{cross_sections/evolution}
50 | 50 | \input{cross_sections/folding}

```

24 cross_sections/evolution.tex

```

13 | 13 |
14 | 14 | \begin{definition}
15 | 15 | \label{def:segment_length}
16 | 16 | -For ever segment  $s$  in a cross section  $C$ , we associate a left pace  $L_s$ ,
    | 16 | +For every segment  $s$  in a cross section  $C$ , we associate a left pace  $L_s$ ,
17 | 17 | which indicates the rate at which  $s$  shrinks from its left endpoint.
18 | 18 | Similarly, we define a right pace  $R_s$  grows from its right endpoint.
19 | 19 | Note that both these quantities can be negative.
25 | 25 |  $\vec{J}_v^L - \vec{\hat{v}}_{s_i} = L_{s_i} \cdot \vec{\hat{o}}_{s_i}$ , &&  $\vec{J}_v^R - \vec{\hat{v}}_{s_i} = R_{s_i} \cdot \vec{\hat{o}}_{s_i}$ .
26 | 26 | \end{align*}
27 | 27 |
28 | -\begin{proposition}
29 | -\label{prop:valid_joint}
    | 28 | +\begin{definition}
    | 29 | +\label{def:valid_joint}
30 | 30 | A joint  $J$  corresponding to segments  $s$  and  $t$  is \emph{valid} if and only if the evolution
31 | 31 | resulting from the velocities  $\vec{\hat{v}}_l$ ,  $\vec{\hat{v}}_r$ , and  $\vec{J}_v$  preserves distances between nodes.
32 | -\end{proposition}

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```

32 | +\end{definition}
33 | \vspace{-1pc}
34 | -\begin{restatable}{pro}{LeftRightPace}
35 | -\label{pro:left_right_pace}
36 | +\begin{restatable}{inv}{LeftRightPace}
37 | +\label{inv:left_right_pace}
38 | The right pace of  $s_{i+1}$  is equal to the left pace of  $s_i$ .
39 | This is to preserve the overall length of the cross section, and the distance between any two nodes.
40 | Furthermore, the left pace of the first segment, and the right pace of the last segment should be zero, i.e.,  $L_0 = R_n = 0$ 
41 | \end{align*}
42 | This is only possible if  $\vec{\hat{o}_l} \times \vec{\hat{o}_r}$  is oriented opposite to  $\vec{\hat{v}_l} \times \vec{\hat{v}_r}$ .
43 |
44 | -\begin{restatable}{pro}{SegmentOrientation}
45 | -\label{pro:SegmentOrientation}
46 | +\begin{restatable}{inv}{SegmentOrientation}
47 | +\label{inv:SegmentOrientation}
48 | Given two adjacent segments  $s_l$  and  $s_r$  in a cross section  $C$ , the vector
49 |  $\vec{\hat{o}_l} \times \vec{\hat{o}_r}$  must be oriented opposite to  $\vec{\hat{v}_l} \times \vec{\hat{v}_r}$ .
50 | \end{restatable}
51 |
52 | If the angle between the segments (between  $\vec{\hat{o}_l}$  and  $\vec{\hat{o}_r}$ ) is  $\theta$ ,
53 | the magnitude of  $\vec{\hat{o}_r} - \vec{\hat{o}_l}$  is  $\sqrt{2-2\cos(\theta)}$ ,
54 | and  $|\vec{\hat{v}_r} \times \vec{\hat{v}_l}| = v \sqrt{2-2\cos(\pi-\theta)}$ .
55 | -Here,  $v$  is magnitude of the plane velocity of  $J_i$ . Given  $\phi = \theta/2$ , we get --
56 | +Here,  $v$  is magnitude of the plane velocity (projection onto the joint plane
57 | +of  $\vec{J_i}$ .
58 | +Given  $\phi = \theta/2$ , we get --
59 | \begin{align*}
60 | -L = -R \Leftrightarrow \frac{dt}{dt} = \frac{\vec{\hat{v}_l} - \vec{\hat{o}_l}}{|\vec{\hat{v}_l} - \vec{\hat{o}_l}|} \cdot \frac{\vec{\hat{v}_r} \times \vec{\hat{v}_l}}{|\vec{\hat{v}_r} \times \vec{\hat{v}_l}|}
61 | - \frac{\vec{\hat{v}_l} \times \vec{\hat{v}_r}}{|\vec{\hat{v}_l} \times \vec{\hat{v}_r}|} \cdot \frac{\vec{\hat{o}_r} - \vec{\hat{o}_l}}{|\vec{\hat{o}_r} - \vec{\hat{o}_l}|}
62 | = v \frac{\sqrt{\sin^2(\pi/2 - \theta/2)}}{\sqrt{\sin^2(\theta/2)}}
63 | = v \cot(\phi).
64 | \end{align*}
65 |
66 | -\begin{restatable}{pro}{JointVelocity}
67 | -\label{pro:joint_velocity}
68 | +\begin{restatable}{inv}{JointVelocity}
69 | +\label{inv:joint_velocity}
70 | The velocity of a joint  $J_j$  associated with segments  $s_l$  and  $s_r$ , is a constant vector
71 |  $\vec{J_v}$ 
72 |  $= \frac{\vec{\hat{v}_l} - \vec{\hat{o}_l}}{|\vec{\hat{v}_l} - \vec{\hat{o}_l}|} \cdot \frac{\vec{\hat{v}_r} \times \vec{\hat{v}_l}}{|\vec{\hat{v}_r} \times \vec{\hat{v}_l}|}$ 

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22 cross_sections/folding.tex

```

28 | 28 |  $Z_s = LL'R'R$  is the corresponding trapezoid (Figure~\ref{fig:segment_trapezoid}).
29 | 29 | \end{lemma}
30 | 30 | \begin{proof}
31 | 31 | -By Property~\ref{pro:joint_velocity}, we know that the joint trajectories are straight lines, which form the non-parallel sides.
32 | 32 | +By Invariant~\ref{inv:joint_velocity}, we know that the joint trajectories are straight lines, which form the non-parallel sides.
33 | 33 | Since the non-joint nodes on a segment all have the same velocity, the initial and final segment positions form the parallel sides.
34 | 34 | \end{proof}

```

```

34 34
41 41 \begin{proof}
42 42 We consider the evolution of joint  $J$  for time  $T$  (Figure~\ref{fig:trapezoid_angles}).
43 43 First consider a coordinate system with  $(\vec{\hat{v}_l}, \vec{\hat{o}_l})$  as the basis.
44 44 -Since,  $\vec{\hat{v}_l} = \vec{v_l}^{\text{shortparallel}}$ , from Property~\ref{pro:joint_velocity},
44 44 +Since,  $\vec{\hat{v}_l} = \vec{v_l}^{\text{shortparallel}}$ , from Invariant~\ref{inv:joint_velocity},
45 45 we know that  $-R_l = \left| \vec{v_l}^{\text{shortparallel}} \right| \cdot \cot(\phi) = \vec{\hat{o}_v} \cdot \cot(\phi)$ .
46 46 So,  $\vec{J_v} = \vec{\hat{v}_l} - \vec{\hat{o}_v} \cdot \cot(\phi)$ . Therefore,
47 47 \begin{align*}
48 48 \quad \cot(\angle LJJ') &= \frac{\left| T \cdot R_l \right|}{\left| T \cdot (\vec{J_v} - R_l) \right|} = \cot(\phi) \\
49 49 \quad \implies \angle LJJ' &= \phi. \\
50 50 \end{align*}
51 51 -Similarly, we consider a coordinate system with  $(\vec{\hat{v}_l}, \vec{\hat{o}_l})$  as the basis.
51 51 +Similarly, we consider a coordinate system with  $(\vec{\hat{v}_r}, \vec{\hat{o}_r})$  as the basis.
52 52 Since  $\vec{J_v} = \vec{\hat{v}_r} - \vec{\hat{o}_r} \cdot \cot(\phi)$ , we get
53 53 $$
54 54 \cot(\angle RJJ') = \frac{\left| T \cdot L_r \right|}{\left| T \cdot (\vec{J_v} - L_r) \right|} = -\cot(\phi)
92 92 \label{def:interval_folding}
93 93 Consider a cross section interval  $\mathcal{C}$  formed from a cross section  $C$  evolving over time  $T$ .
94 94 By Lemma~\ref{lem:trapezoid}, the segments  $\angle s_1, s_2, \dots, s_n$  form trapezoids
95 95 - $\angle Z_1, Z_2, \dots, Z_n$  each of height  $T$ .
95 95 + $\angle Z_1, Z_2, \dots, Z_n$  each of height  $T$ , where  $Z_i$  represents the  $i^{\text{th}}$  trapezoid in folded
96 96 space.
96 96 The folding  $\mathcal{F_C^T}$  corresponding to  $\mathcal{C}$  is formed by successively gluing the trapezoids
97 97 - $Z_i$  to  $Z_{i+1}$  along the trajectory of joint  $J_i$  (for  $1 \leq i < n$ ) to form a connected shape.
97 97 + $Z_i$  to  $Z_{i+1}$  along the trajectory of joint  $\vec{J_i}$  (for  $1 \leq i < n$ ) to form a connected shape.
98 98 \end{definition}
99 99
100 100 \begin{definition}
105 105 Similarly, the final-boundary of  $\mathcal{F_C^T}$  is defined as the union of the final segments in  $\mathcal{C_F}$ .
106 106 \end{definition}
107 107
108 108 -\begin{theorem}
108 108 +\begin{restatable}{thm}{interval_strip}
109 109 \label{thm:interval_strip}
110 110 Consider a cross section interval  $\mathcal{C}$  formed from a cross section  $C$  evolving over time  $T$  to form a folding
111 111  $\mathcal{F_C^T}$ .
111 111 Further assume that the total length of cross section  $C$  is  $X$  units. Then,  $\mathcal{F_C^T}$  is isometric to a
112 112  $X \times T$  strip of paper.
112 112 -\end{theorem}
112 112 +\end{restatable}
113 113 \begin{proof}
114 114 By repeated use of Lemma~\ref{lem:trapezoid_gluing}, we know that  $\mathcal{F_C^T}$  is isometric to a trapezoid.
115 115 Let  $L, L'$  be the initial and final positions of the left (non-parallel) edge of the trapezoid, and
116 116 let  $R, R'$  be the initial and final positions of the right edge of the trapezoid.
117 117 Say that  $C$  comprises of segments  $\angle s_1, s_2, \dots, s_n$ .
118 118 -From Property~\ref{pro:left_right_pace}, we know that the left pace of  $s_0$  is zero.
118 118 +From Invariant~\ref{inv:left_right_pace}, we know that the left pace of  $s_0$  is zero.
119 119 So, the line  $LL'$  follows the trajectory of  $\vec{\hat{v}_0}$ , which is orthogonal to the segment  $s_0$ .
120 120 In other words, the left edge of the trapezoid has length  $T$ , and is orthogonal to the parallel edges.
121 121 Similarly, because the right pace of  $s_n$  is zero, the right edge of the trapezoid is also orthogonal.
122 122 Therefore,  $\mathcal{F_C^T}$  is isometric to a right angled trapezoid (i.e., a strip) of length  $X$  and width  $T$ .
123 123 \end{proof}
124 124
125 125 -\begin{proposition}
126 126 -\label{prop:joint_crease}

```

```

125 +\begin{remark}
126 +\label{rem:joint_crease}
127 127 The trajectory of a joint forms a crease in the folded state.
128 128 -\end{proposition}
128 128 +\end{remark}

```

20 cross_sections/joints.tex

```

3 3
4 4 \begin{definition}
5 5 \label{def:joints}
6 -A cross section with  $n$  segments is also associated with a list of joints  $\langle J_1, \dots, J_{n-1} \rangle$ ,
7 -where  $J_i$  corresponds to the right endpoint of  $s_i$  (same as left endpoint of  $s_{i+1}$ ).
8 -A particular joint  $J_i$  is associated with a left segment  $s_i$ , a right segment  $s_{i+1}$ , and a velocity  $\vec{J}_v$ .
6 +A cross section with  $n$  segments is also associated with a list of joints
7 + $\langle \vec{J}_1, \dots, \vec{J}_{n-1} \rangle$ ,
8 +where  $\vec{J}_i$  corresponds to the right endpoint of  $s_i$  (same as left endpoint of  $s_{i+1}$ ).
9 +A particular joint  $J_i$  is associated with a left segment  $s_i$ ,
10 +a right segment  $s_{i+1}$ , and a velocity  $\vec{J}_v$ .
9 11 \end{definition}
10 12
11 13 \begin{definition}
19 21 \label{def:joint_plane_velocity}
20 22 Consider a joint  $J$  associated with segments  $l$ ,  $r$ , and joint plane  $\mathcal{P}$ ,
21 23 where  $\vec{v}_l$  and  $\vec{v}_r$  are the velocities of segments  $l$  and  $r$ .
22 -We define  $\vec{v}_l^{\text{shortparallel}}$  and  $\vec{v}_l^{\text{perp}}$  as the components of  $\vec{v}_l$  coinciding
with,
23 -and orthogonal to  $\mathcal{P}$  respectively.
24 +We define  $\vec{v}_l^{\text{shortparallel}}$  and  $\vec{v}_l^{\text{perp}}$  as components of
25 + $\vec{v}_l$ , such that  $\vec{v}_l^{\text{shortparallel}}$  is the projection of  $\vec{v}_l$ 
26 +onto  $\mathcal{P}$ , and  $\vec{v}_l^{\text{perp}} = \vec{v}_l - \vec{v}_l^{\text{shortparallel}}$ 
27 +(orthogonal to  $\mathcal{P}$ ).
24 28 Similarly, we define  $\vec{v}_r^{\text{shortparallel}}$  and  $\vec{v}_r^{\text{perp}}$ , as the components of  $\vec{v}_r$ .
25 -By Property~\ref{pro:orthogonal\_velocity},  $\vec{v}_l^{\text{shortparallel}}$  and  $\vec{v}_r^{\text{shortparallel}}$ 
29 +By Invariant~\ref{inv:orthogonal\_velocity},  $\vec{v}_l^{\text{shortparallel}}$  and  $\vec{v}_r^{\text{shortparallel}}$ 
30 are orthogonal to  $\vec{o}_l$  and  $\vec{o}_r$  respectively.
31 This uniquely determines the direction of all velocity components.
32 \end{definition}
33
34 Henceforth, we will refer to the  $\vec{v}^{\text{shortparallel}}$  component as the \emph{joint plane velocity},
35 and the  $\vec{v}^{\text{perp}}$  component as the \emph{joint orthogonal velocity},
36
33 -\begin{restatable}{pro}{JointOrthogonalVelocity}
34 -\label{pro:joint_orthogonal_velocity}
37 +\begin{restatable}{inv}{JointOrthogonalVelocity}
38 +\label{inv:joint_orthogonal_velocity}
35 39 For a joint  $J$  associated with segments  $l$  and  $r$ ,  $\vec{v}_l^{\text{perp}} = \vec{v}_r^{\text{perp}}$ 
36 40 i.e. the joint orthogonal velocities have to be equal, such that the joint plane moves with a fixed velocity along it's
normal.
37 41 As a corollary,  $\left| \vec{v}_l^{\text{shortparallel}} \right| = \left| \vec{v}_r^{\text{shortparallel}} \right|$ .

```

20 cross_sections/multiple.tex

```

36 36
37 37 \begin{proposition}
38 38 \label{prop:creases}
39 -In addition to the creases formed along the trajectory of joints (Proposition~\ref{prop:joint\_crease}),

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```

39 +In addition to the creases formed along the trajectory of joints (Remark~\ref{rem:joint_crease}),
40 creases are also created when a segment changes velocity.
41 For instance, consider two adjacent cross sections  $\mathcal{C}$  and  $\mathcal{D}$ ,
42 with corresponding segments  $s_C \in \mathcal{C}_F$  and  $s_D \in \mathcal{D}_I$ ,
59 \label{thm:main}
60 Consider a cross section sequence  $\mathcal{C} = \langle \mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_m \rangle$  where
each
61 cross section interval  $\mathcal{C}_i$  evolves over time  $T_i$  to form a folding  $\mathcal{F}_i$ 
62 such that the following properties hold for all segments and joints in each of the cross sections involved.
63 \begin{itemize}
64 - \item[] \vspace{-1.6em} \texttt{UniformVelocity}
65 - \item[] \vspace{-1.6em} \texttt{OrthogonalVelocity}
66 - \item[] \vspace{-1.6em} \texttt{JointOrthogonalVelocity}
67 - \item[] \vspace{-1.6em} \texttt{LeftRightPace}
68 - \item[] \vspace{-1.6em} \texttt{SegmentOrientation}
69 - \item[] \vspace{-1.6em} \texttt{JointVelocity}
70 \end{itemize}
62 +such that Invariants~1-6 hold for all segments and joints in each of the cross sections involved.
63 +% \begin{itemize}
64 + % \item[] \vspace{-1.6em} \texttt{UniformVelocity}
65 + % \item[] \vspace{-1.6em} \texttt{OrthogonalVelocity}
66 + % \item[] \vspace{-1.6em} \texttt{JointOrthogonalVelocity}
67 + % \item[] \vspace{-1.6em} \texttt{LeftRightPace}
68 + % \item[] \vspace{-1.6em} \texttt{SegmentOrientation}
69 + % \item[] \vspace{-1.6em} \texttt{JointVelocity}
70 +% \end{itemize}
71 Then, the folding  $\mathcal{F}_{\mathcal{C}}$  obtained by successively gluing the final boundary of  $\mathcal{F}_i$  to the
initial boundary
72 of  $\mathcal{F}_{i+1}$  (for each  $1 \leq i \leq m$ ), is isometric to a  $X \times T$  strip of paper, where  $T = \sum T_i$ .
73 \end{restatable}

```

12 introduction.tex

```

19 exists above the terrain. This result improves an algorithm,
20 \cite{BoxPleating_Origami5} also presented at 50SME, applicable to a more
21 general class of inputs, providing a universal construction to fold general
22 -orthogonal polyhedra, though the construction is less inefficient than our
22 +orthogonal polyhedra, though the construction is less efficient than our
23 construction applied to orthogonal terrains. Our construction approach follows
24 three steps:
25
32 \vspace{-0.2pc}
33
34 In order to better communicate the algorithm and the final folded state
35 -produced, we also introduce a new {\bf cross-section evolution} representation
35 +produced, we also introduce a new {\bf cross_section evolution} representation
36 of a folded isometry: a straight line is swept across the crease pattern of a
37 folded surface, and we keep track of how the folding of the line evolves as a
38 -cross-section of the folded surface. The propagation of the cross-section
38 +cross_section of the folded surface. The propagation of the cross_section
39 between crease pattern vertices is uniquely determined by the initial
40 -orientation of the cross-section, so the folded isometry can be constructed by
41 -sweeping the line and locally modifying the cross-section when crossing crease
40 +orientation of the cross_section, so the folded isometry can be constructed by
41 +sweeping the line and locally modifying the cross_section when crossing crease
42 pattern vertices during propagation. This representation not only simplifies the
43 description of the 3D folded isometries constructed, but also provides a simpler

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44 44 framework to argue that the folded state does not self intersect, by propagating
45 45 -planar cross-sections monotonically along a single direction. We then
46 46 +planar cross-sections monotonically along a single direction. We then
47 47 show that our construction's efficiency is within a small constant factor of
any folding with optimal efficiency.

```

15 main.tex

```

57 57
58 58 \input{orthogonal/orthogonal}
59 59
60 60 +\section{Conclusion}
61 61 +
62 62 +This paper provides a universal construction to fold orthogonal terrains that is
63 63 +optimally efficient over its domain, improving upon previous constructions that
64 64 +were less efficient but applicable to a more general class of target shapes,
65 65 +i.e., orthogonal polyhedra. Some natural questions arise. Can one improve the
66 66 +efficiency of the construction for a more restricted set of terrains? For
67 67 +example, our lower bound is achieved by a maximum height-difference
68 68 +checkerboard; perhaps one can improve folding efficiency for terrains that are
69 69 +more slowly varying. Our construction only covers the terrain from above,
70 70 +allowing the folding to exist anywhere in the space below the terrain. This
71 71 +allocation is necessary when points of the terrain have more than  $2\pi$ 
72 72 +material at a point. What is the minimum area of paper that can exist away from
73 73 +the target terrain over all possible foldings?
74 74 +
60 75 \section*{Acknowledgments}
61 76
62 77 We thank Martin Demaine, Heng Yi Cheng, Aviv Ovadya, and Tomohiro Tachi

```

9 orthogonal/construction.tex

```

2 2 \label{sec:column_extrusion}
3 3
4 4 First, we consider a single column (Figure~\ref{fig:column_extrusion})
5 5 -of the orthogonal terrain  $\left\{ E_{i1}, E_{i2}, \dots, E_{in} \right\}$ .
6 6 +of the orthogonal terrain  $\left\{ E_{i,1}, E_{i,2}, \dots, E_{i,n} \right\}$ .
7 7 We denote the column extrusion heights as  $\left\{ H_1, H_2, \dots, H_n \right\}$ , where  $H_j = E_{i,j}$ .
8 8 Consider the decomposition of  $T$  into the following time intervals: (parallel to the  $x$ -axis):
9 9 \begin{align}
10 10 a sequence of  $2k$  horizontal segments that \emph{accordion} back and forth.
11 11 During each  $1$ -interval, all segments move along the positive  $y$  direction (Figure~\ref{fig:level_shift0}), to create
12 12 the  $i$ th level.
13 13 Subsequently, during the level shift, all segments move in the  $x$ - $z$  plane
14 14 (Figure~\ref{fig:level_shift1}, ~\ref{fig:level_shift2}).
15 15 -\begin{property}
16 16 +\begin{restatable}{pro}{AccordionEven}
17 17 \label{pro:accordion_even}
18 18 The number of accordion folds during horizontal evolution (along the  $y$  axis) must be even.
19 19 -\end{property}
20 20 +\end{restatable}
21 21
22 22 The top segment moves downwards in intervals of  $2\varepsilon$ .
23 23 During this process, the horizontal segments move downwards continuously
24 24 (Figure~\ref{fig:level_shift1}, ~\ref{fig:level_shift2}).
25 25 \end{align*}
26 26 %\geq 1 + 2\cdot\left( \max\left\{ H_i \right\} - \min\left\{ H_i \right\} \right) \&T \geq \left( m + \sum\limits_{i=1}^m \left| H_{i+1} - H_i \right| \right)

```



```

89 89 \end{theorem}
90 -
91 -\input{orthogonal/grid}
92 -\input{orthogonal/connector}

```

4 orthogonal/grid.tex

```

3 3
4 4 Now, we consider multiple column extrusions evolving in parallel.
5 5 Henceforth, we will refer to the evolution of column cross sections along the  $y$ -axis
6 6 -(the  $'1'$ 's in Equation~\ref{eq:column_decomposition} as \emph{horizontal evolution}.
7 7 +(the  $'1'$ 's in Equation~\ref{eq:column_decomposition}) as \emph{horizontal evolution}.
8 8 Meanwhile, a \emph{vertical transition} will refer to level shifting evolution in the  $x$ - $z$  plane.
9 9 Let  $\mathcal{C}^i$  be a valid cross section evolution corresponding
37 37 to the  $i$ th column in the grid extrusion (as defined in Section~\ref{sec:column_extrusion}).
38 38 up-down gadgets (each gadget \emph{stalls} for  $2\varepsilon$  time).
39 39 %Notice that this gadget requires at least one accordion segment. In fact, by Lemma~\ref{lem:accordion_even}, it
40 40 requires two segments.
41 41 %Given the worst case scenario, where  $E_{i,j}=E_{i,j+1}$  is the max height in column  $i$ ,
42 42 -We obtain the following primitive, as a consequence of Theorem~\ref{thm:column_extrusion}.
43 43 +We obtain the following proposition, as a consequence of Theorem~\ref{thm:column_extrusion}.
44 44 \begin{proposition}
45 45 \label{prop:accordion_layers}

```

4 orthogonal/orthogonal.tex

```

31 31 \label{fig:column_extrusion}
32 32 \vspace{-0.8em}
33 33 \end{wrapfigure}
34 34 -We consider each  $'column'$  of a given grid extrusion separately
35 35 +We consider each  $'column'$  of a given grid extrusion separately
36 36 as an individual \emph{column extrusion} (Figure~\ref{fig:column_extrusion}).
37 37 We will construct each of the  $n$  columns independently (Figure~\ref{fig:level_shift}),
42 42 and attach them together with \emph{column connectors} (Figure~\ref{fig:column_connector}).
43 43 %Imagine that there are  $T = \frac{\varepsilon}{\Delta t}$  time steps of length  $\Delta t$ , over which the cross section
44 44 evolves along the  $y$ -axis.
45 45 \input{orthogonal/construction}
46 46 +\input{orthogonal/grid}
47 47 +\input{orthogonal/connector}
48 48 \input{orthogonal/size}
49 49 \input{orthogonal/optimal}
50 50 %\input{orthogonal/assumption}

```

178 reviews_7osme.txt

227 reviews_7osme_response.txt

855 temp.vim

2 title.tex

```

... .. @@ -1 +1 @@
1 1 -\title{Efficient Origami Construction of Orthogonal Terrains using Cross-Section Evolution}

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1 | +\title{Efficient Origami Construction of Orthogonal Terrains using Cross-Section Evolution}

No commit comments for this range