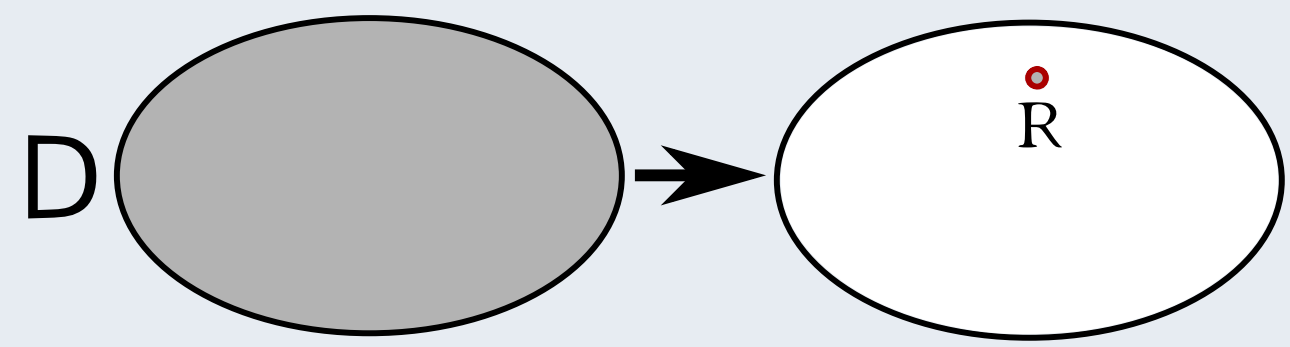


Local-Access Generators

Amartya Shankha Biswas, Ronitt Rubinfeld, Anak Yodpinyanee
CSAIL, MIT

Partial Sampling from a Distribution

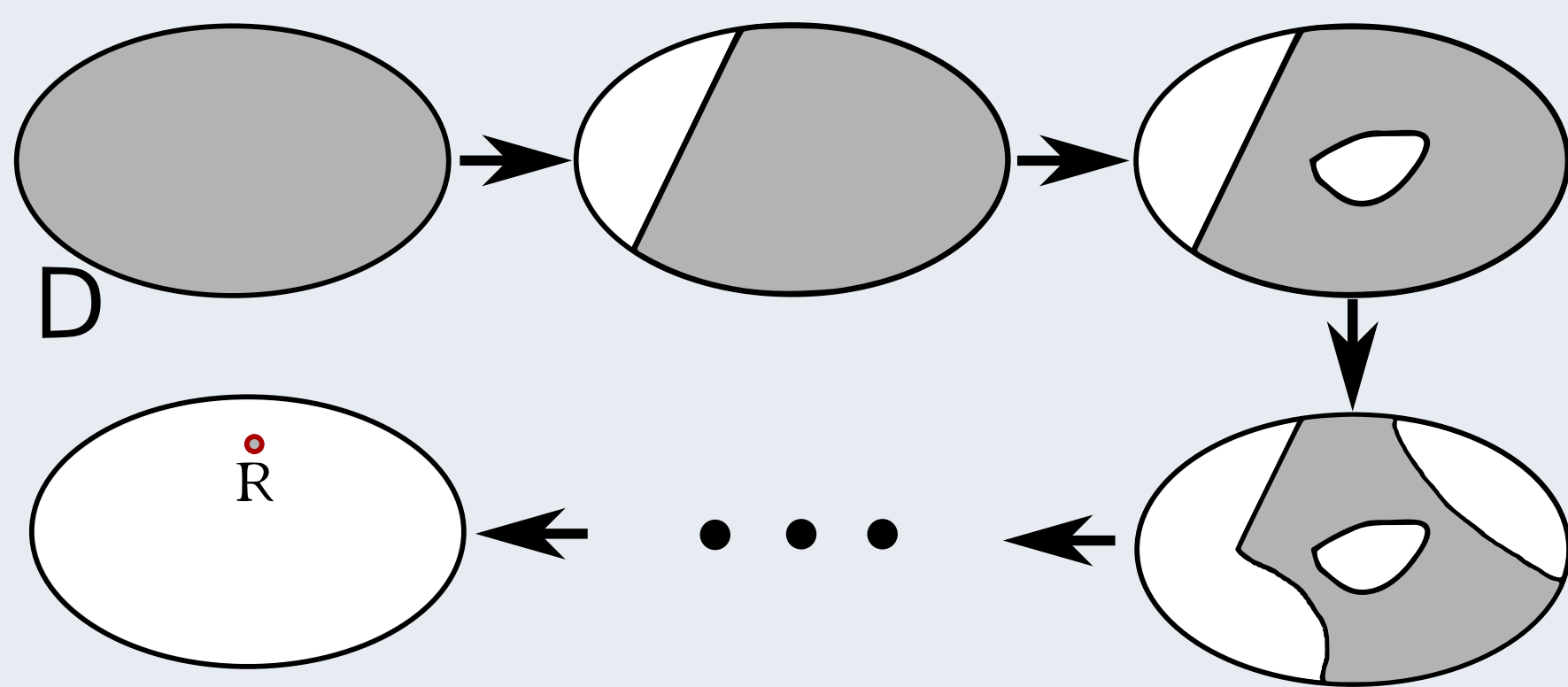
Full Sampling $R \sim D$ in $\mathcal{O}(T)$ time



Do we need to spend $\mathcal{O}(T)$ upfront?

N steps of *Partial Sampling*

Each partial step should take $\tilde{\mathcal{O}}(T/N)$ time.



Problem Statement

A local-access generator of a random object $R \sim D$, provides indirect access to R' with a *query oracle* s.t.

- All query responses (*partial samples*) are **consistent**
- The **distribution** of R' is ϵ -close to D in L_1 distance

Bucketing-Generator & Random-Nighbor Queries

Problem: next-neighbor cannot “jump” to a random potential neighbor of v

Bucketing Divide each row of the adjacency matrix into contiguous buckets
 \Rightarrow random neighbor of $v \approx$ random neighbor in a random bucket of v

Problem: Do NOT know $\deg(v)$: Must return each neighbor with prob. $1/\deg(v)$

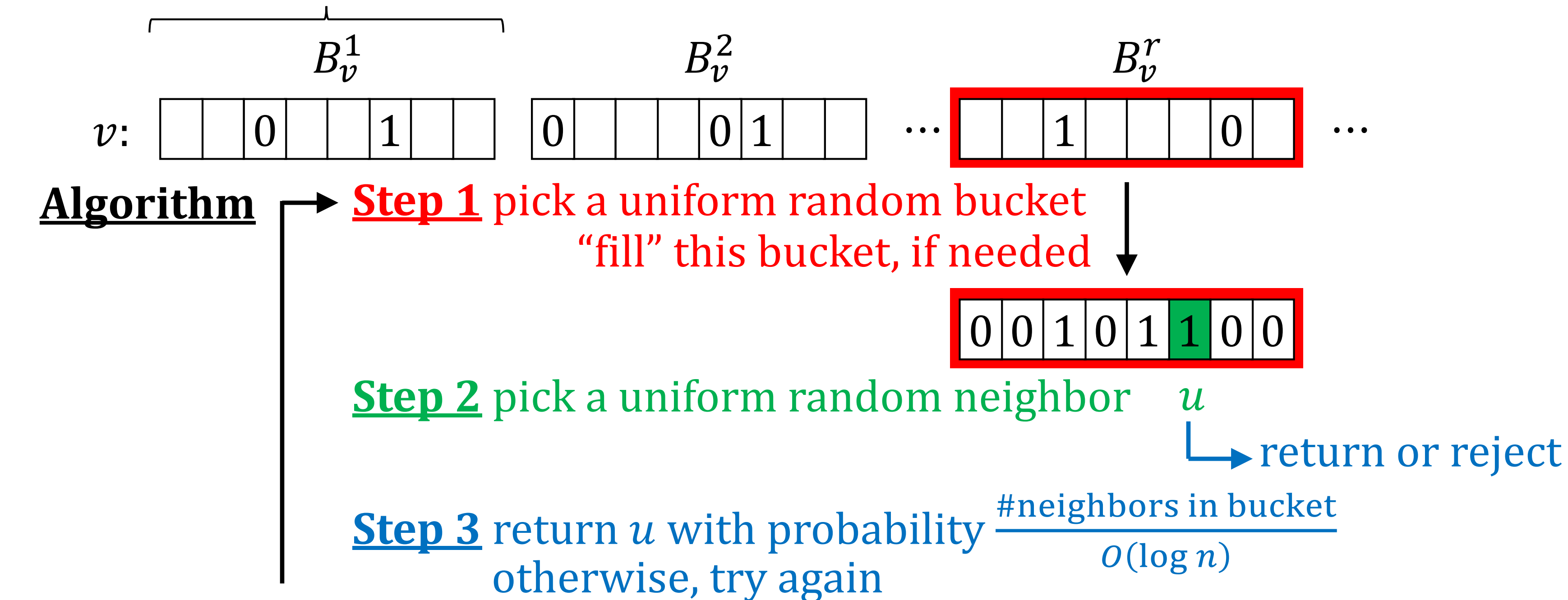
Rejection Sampling Normalize probability of returning any specific neighbor

Problem: next-neighbor cannot “jump” to a random potential neighbor of v

\Rightarrow suffice to show that **any neighbor** is returned with the **equal** probability

#neighbors in each bucket

$\sim \Theta(1)$ in expectation, $O(\log n)$ max w.h.p. \Rightarrow #buckets \sim #neighbors



$$\Pr[u \text{ returned}] = \frac{1}{\text{\#buckets}} \times \frac{1}{\text{\#neighbors in bucket}} \times \frac{\text{\#neighbors in bucket}}{O(\log n)} \sim \frac{\Omega(1/\log n)}{\text{\#neighbors}}$$

$\Pr[\text{some neighbor returned}] \sim \Omega(1/\log n) \Rightarrow O(\log n)$ tries suffices

Data Structure Buckets contains set of known neighbors, and “filled” marker
 \Rightarrow “fill” with expected $\Theta(1)$ next-neighbor queries $\left. \begin{array}{l} O(\log n) \text{ time per query} \\ \tilde{O}(m+n) \text{ space usage} \end{array} \right\}$
 \Rightarrow random-neighbor succeeds in $O(\log n)$ tries

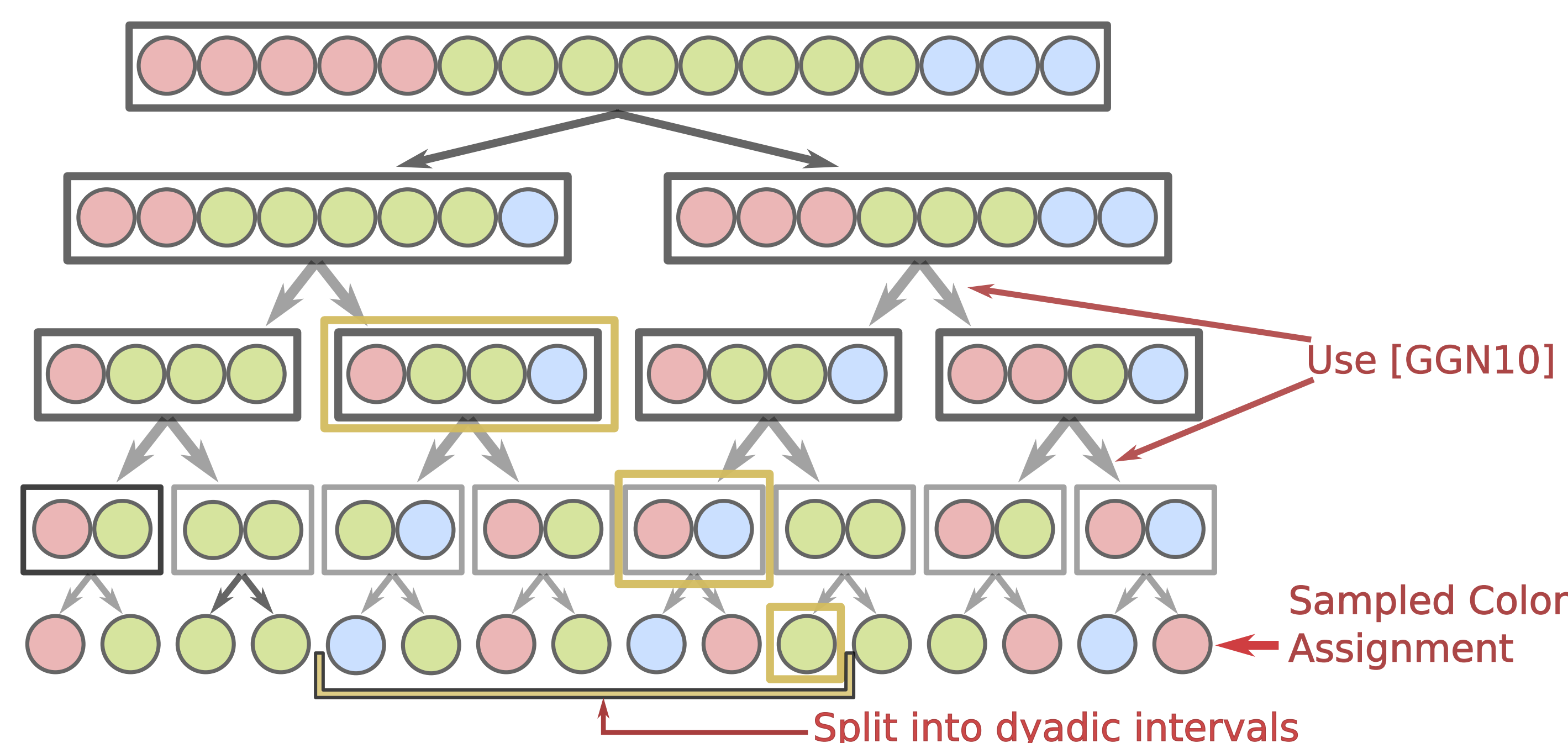
Stochastic Block Model

Communities $\{C_i\}_{i \in [r]}$ partition V : If $u \in C_i, v \in C_j$, then $\mathbb{P}_{(u,v) \in E} = p_{ij}$.

Given sizes of each community C_i and a range of length ℓ

- Count number of occurrences of each community in any contiguous range
- Sample from *Multivariate Hypergeometric Distribution*

$$\Pr[\mathbf{S}_\ell^C = \langle s_1, \dots, s_r \rangle] = \frac{\binom{C_1}{s_1} \cdot \binom{C_2}{s_2} \cdots \binom{C_r}{s_r}}{\binom{B}{\ell}} \quad \text{where } \ell = \sum_{i=1}^r s_i \text{ and } B = \sum_{i=1}^r C_i$$



Multivariate Hypergeometric Distribution

[GGN10] solves the special case of $r = 2$ and $B = 2\ell$.

COUNTING-GENERATOR

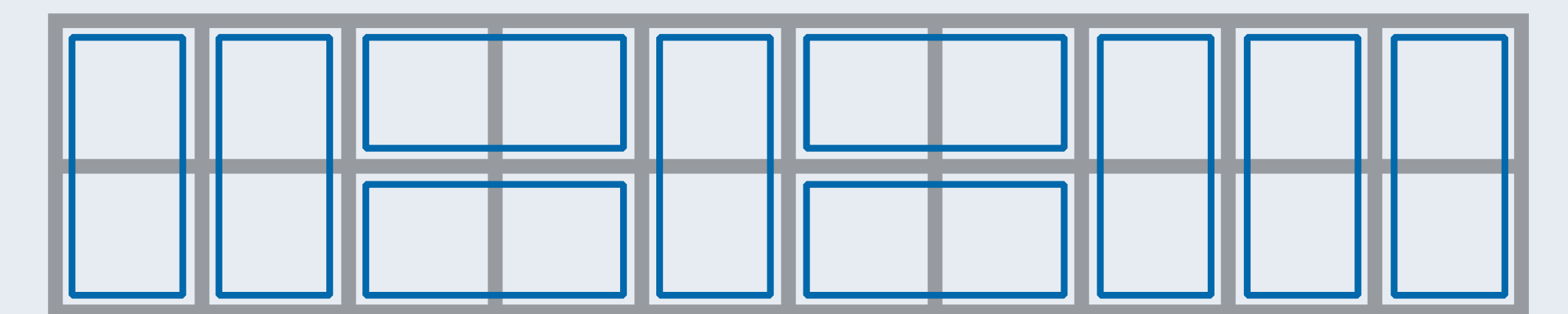
- **Extending to $B \neq 2\ell$:** Divide ℓ into dyadic segments.
- **Extending to $r > 2$:** Make a tree with a leaf for each C_i . Every branch in the tree is equivalent to a 2-splitting.

- Use COUNTING-GENERATOR to sample community counts
- Run the BUCKETING-GENERATOR as before.

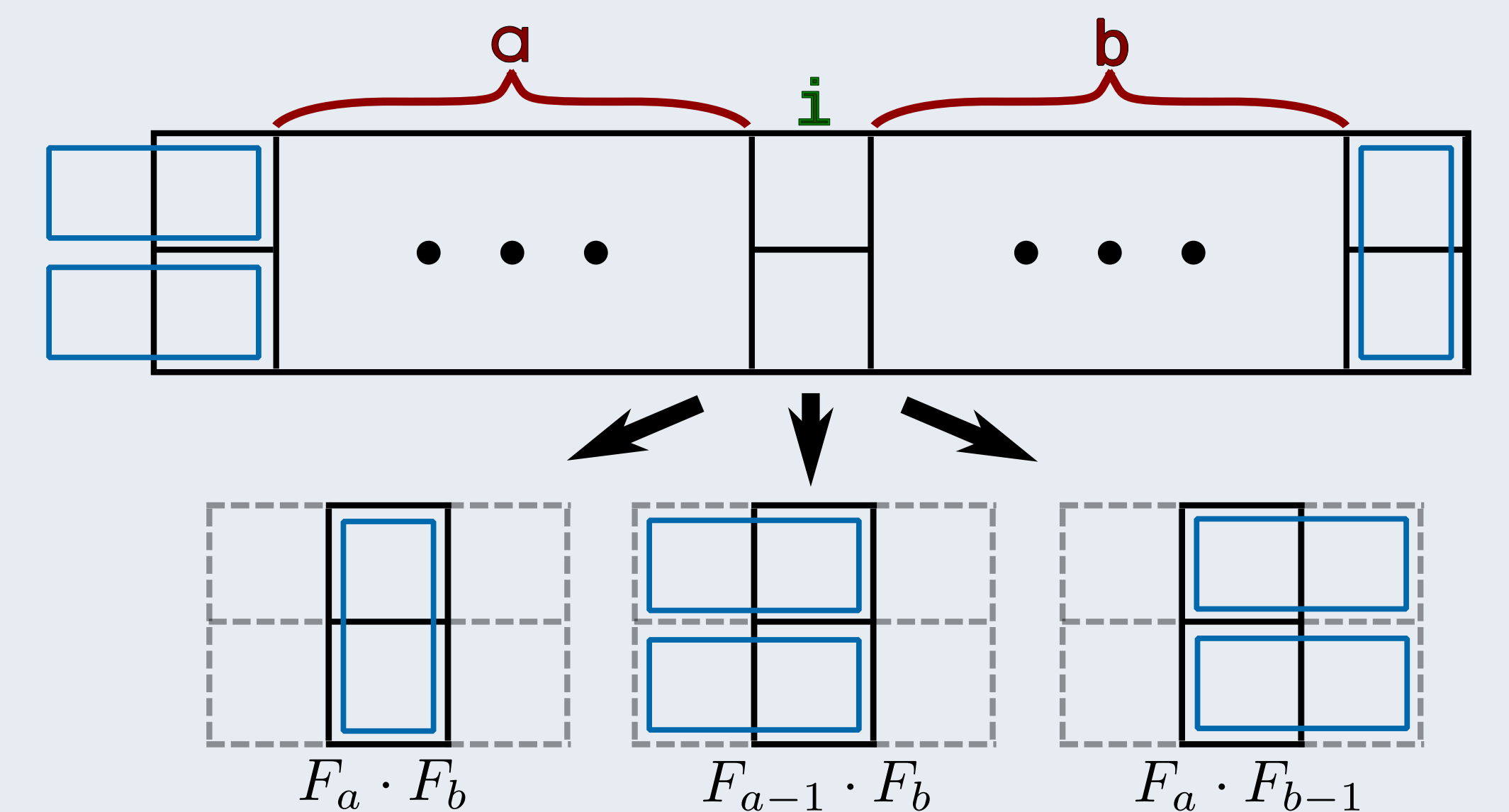
Work in Progress

Domino Tiling

A $2 \times n$ grid tiled with dominoes: F_n tilings possible.



Query: Domino at position i : *vertical* OR *horizontal*?



Sufficient to approximate F_c/F_{c-1} : Use ϕ if $c = \Omega(\log(n))$

Open: $k \times n$ grid for $k = \omega(1)$ and Dimer model.

Trivial Example - Sampling $G(n, p)$

Model: N vertex undirected graph: edge probability p

Query Model: Given vertices u, v , is $(u, v) \in E$?

- Just a collection of $\frac{N(N-1)}{2}$ Bernoulli RVs with bias p .

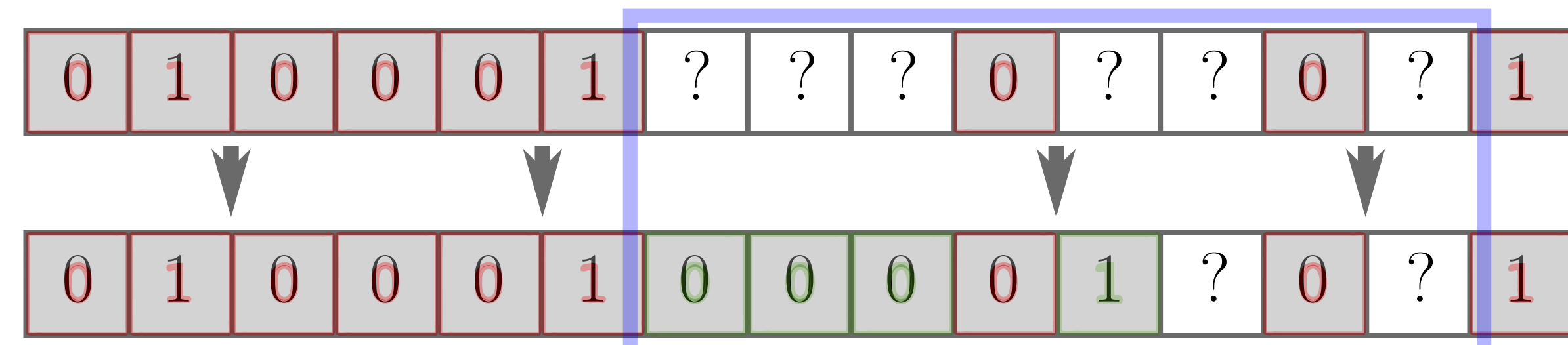
Find Next-Neighbor (*skip-sampling*)

Adjacency List query: Return neighbors of v in order.

$$\mathbb{P}[k \text{ non-neighbors before next-neighbor}] = p(1-p)^k$$

- Can sample from this distribution in $\tilde{\mathcal{O}}(1)$ time [ELMR17]
- Avoid sampling each 0 separately

Issue: Adjacency matrix is symmetric So, each zero must also appear in the corresponding column of v



If the sampled neighbor is a 0, discard and resample.

Cannot afford too many re-samplings.