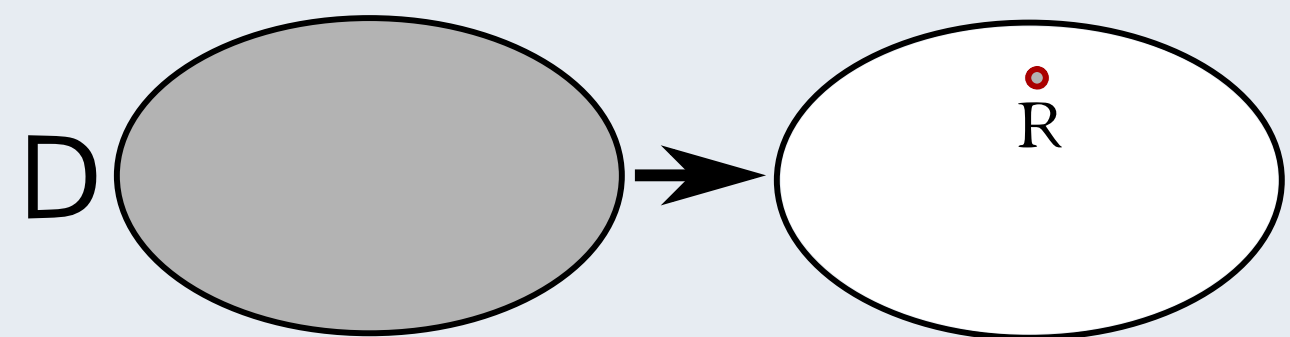


Local-Access Generators

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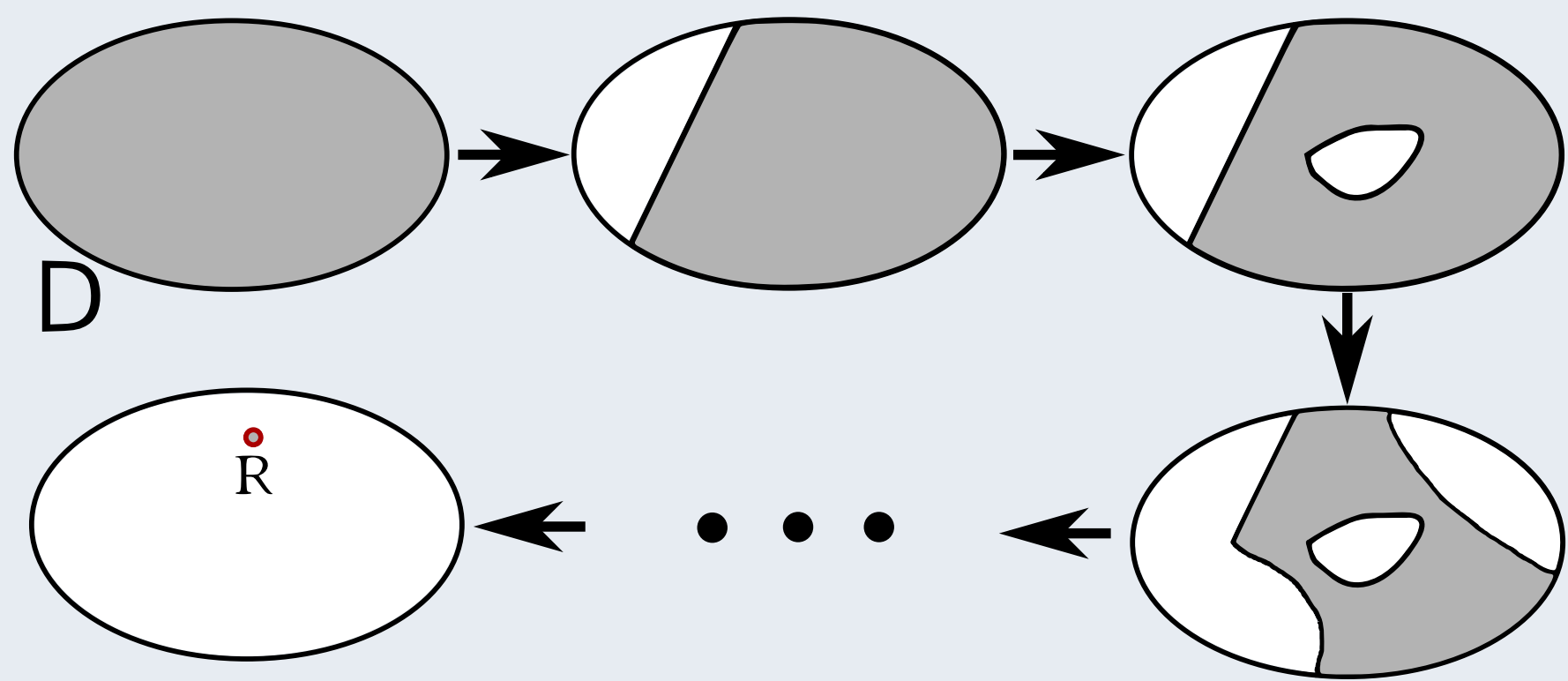
Partial Sampling from a Distribution

Full Sampling $R \sim D$ in $\mathcal{O}(T)$ time



Do we need to spend $\mathcal{O}(T)$ upfront?

Partial sampling: each step should take $\tilde{\mathcal{O}}(T/N)$ time.



Problem Statement

A local-access generator of a random object $R \sim D$, provides indirect access to R' with a *query oracle* s.t.

- All query responses (*partial samples*) are **consistent**
- The **distribution** of R' is ϵ -close to D in L_1 distance

Sampling $G(n, p)$: Vertex-Pair queries

Vertex-Pair: Given vertices u, v , decide whether $(u, v) \in E$.
Trivial: just a collection of $\binom{n}{2}$ Bernoulli RVs with bias p .

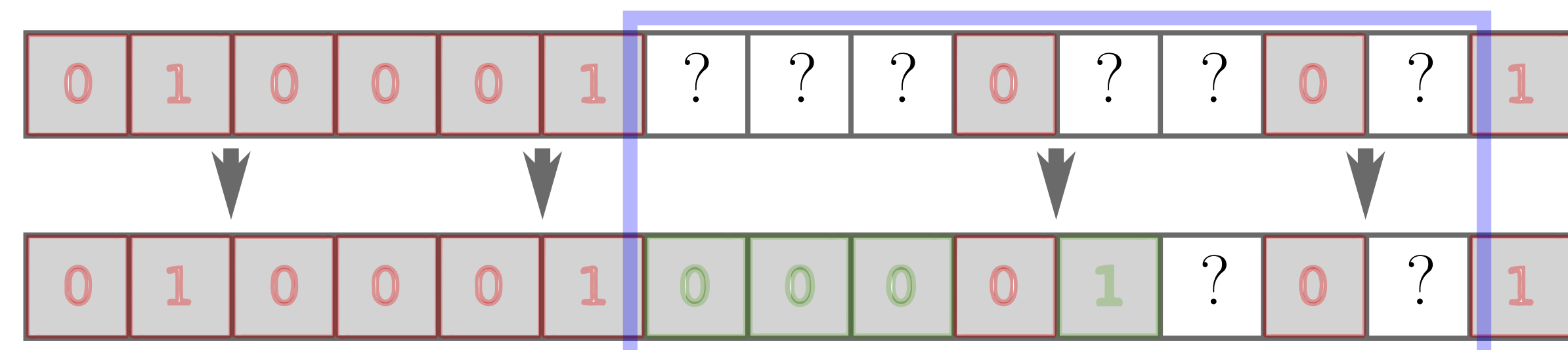
Next-Neighbor queries (skip-sampling)

Next-Neighbor: Return neighbors of v in order.

Issue: Naïve sampling spends $1/p$ time sampling the 0's
Idea: can compute **Next-Neighbor**'s distribution's CDF from
 $\mathbb{P}[k \text{ non-neighbors before next-neighbor}] = p(1-p)^k$

- **Skip-sampling** Draw from **Next-Neighbor** distribution
- Can sample from this distribution in $\tilde{\mathcal{O}}(1)$ time [ELMR17]
- Further analysis required for finite-precision arithmetic

Issue: Adjacency matrix is symmetric; we need to **record all generated 0's** in the corresponding column of v



Issue: if the sampled neighbor is already 0, must re-sample
 \Rightarrow may hit 0's many times – **too many re-samplings**

Random-Neighbor queries (Bucketing-Generator)

Random-Neighbor: Return a neighbor of v uniformly at random.

Issue: next-neighbor can't jump to a random potential neighbor of v

Bucketing Divide each row of the adjacency matrix into contiguous buckets
 \Rightarrow random neighbor of $v \approx$ random neighbor in a random bucket of v

Issue: do **not** know $\deg(v)$; must return each neighbor with probability $1/\deg(v)$

Rejection Sampling Return **any** neighbor with the **same** probability

#neighbors in each bucket
 $\sim \Theta(1)$ in expectation, $\mathcal{O}(\log n)$ max w.h.p. \Rightarrow #buckets \sim #neighbors

v :

0	1
---	---

0	0	1	0
---	---	---	---

 ...

1	0
---	---

 ...

Algorithm

- Step 1** pick a uniform random bucket
"fill" this bucket, if needed

0	0	1	0	1	1	0	0
---	---	---	---	---	---	---	---
- Step 2** pick a uniform random neighbor u
 \rightarrow return or reject
- Step 3** return u with probability $\frac{\text{\#neighbors in bucket}}{\mathcal{O}(\log n)}$
otherwise, try again

$$\mathbb{P}[\text{return } u] = \frac{1}{\text{\#buckets}} \times \frac{1}{\text{\#neighbors in bucket}} \times \frac{\text{\#neighbors in bucket}}{\mathcal{O}(\log n)} \approx \frac{\Omega(1/\log n)}{\text{\#neighbors of } v}$$

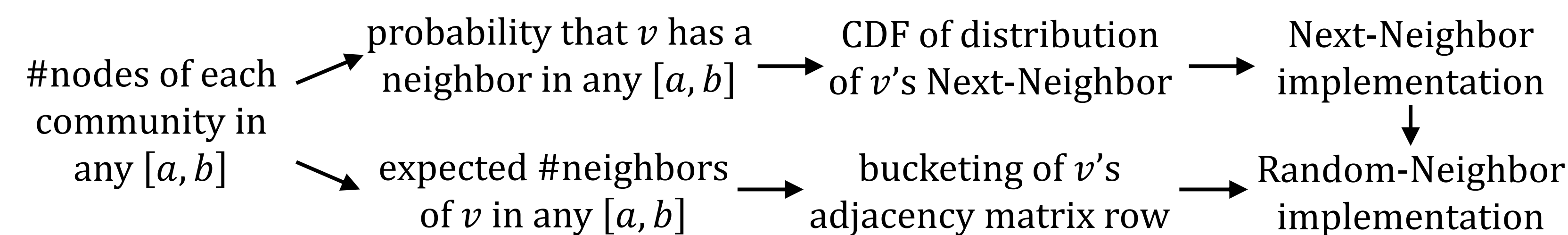
$$\mathbb{P}[\text{return any neighbor}] \approx \Omega(1/\log n) \Rightarrow \mathcal{O}(\log n) \text{ iterations suffice}$$

Data Structure bucket maintains its known neighbors and a **filled** marker
"fill" with expected $\mathcal{O}(1)$ **Next-Neighbor** queries $\left. \vphantom{\begin{array}{l} \text{"fill" with expected } \mathcal{O}(1) \text{ Next-Neighbor queries} \\ \text{Random-Neighbor succeeds in } \mathcal{O}(\log n) \text{ tries} \end{array}} \right\} \begin{array}{l} \mathcal{O}(\log n) \text{ expected time} \\ \tilde{\mathcal{O}}(n+m) \text{ total space usage} \end{array}$

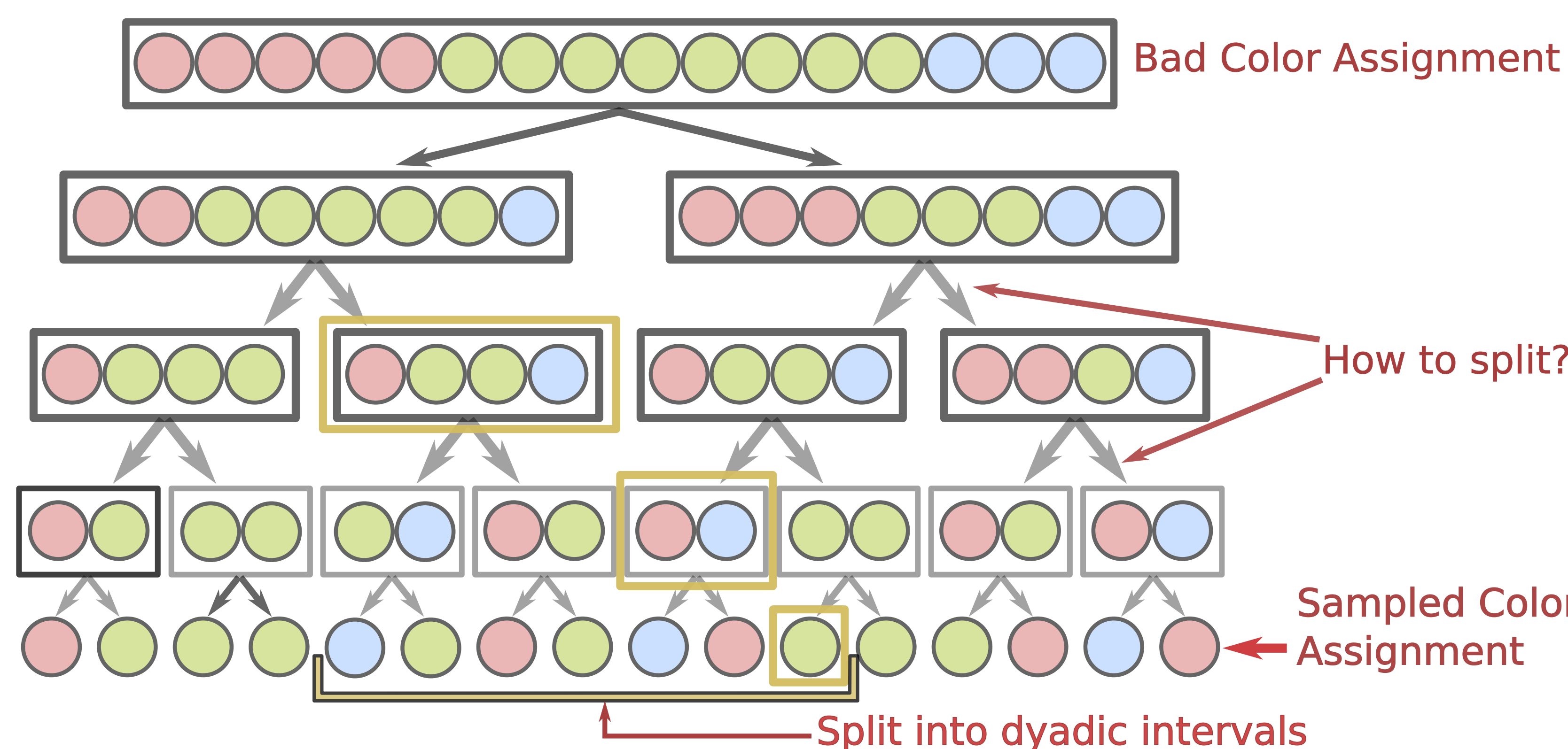
Stochastic Block Model (Counting-Generator)

Model: Each v is assigned to a *random* community (color) from C_1, \dots, C_r .
 \Rightarrow If $u \in C_i, v \in C_j$, then $\mathbb{P}[(u, v) \in E] = p_{ij}$. ($|C_i|$'s are given as input)

Idea #1 use #nodes of each color in *any* contiguous range to can generate SBM



Idea #2 implement a **Counting-Generator** to answer counting queries
 \Rightarrow BBST, split on-the-fly with *Multivariate Hypergeometric Distribution*



Multivariate Hypergeometric Distribution

[GGN10] solves the special case of $r = 2$ and $B = 2\ell$.

Counting-Generator

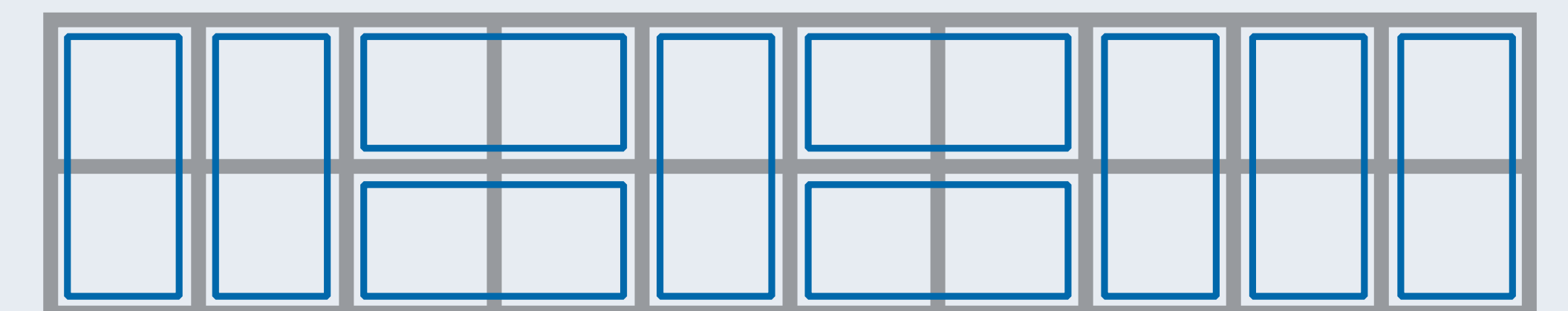
- **Extending to $B \neq 2\ell$:** Divide ℓ into dyadic segments.
- **Extending to $r > 2$:** Make a tree with a leaf for each C_i . Every branch in the tree is equivalent to a 2-splitting

- Use COUNTING-GENERATOR to sample community counts
- Run the BUCKETING-GENERATOR as before

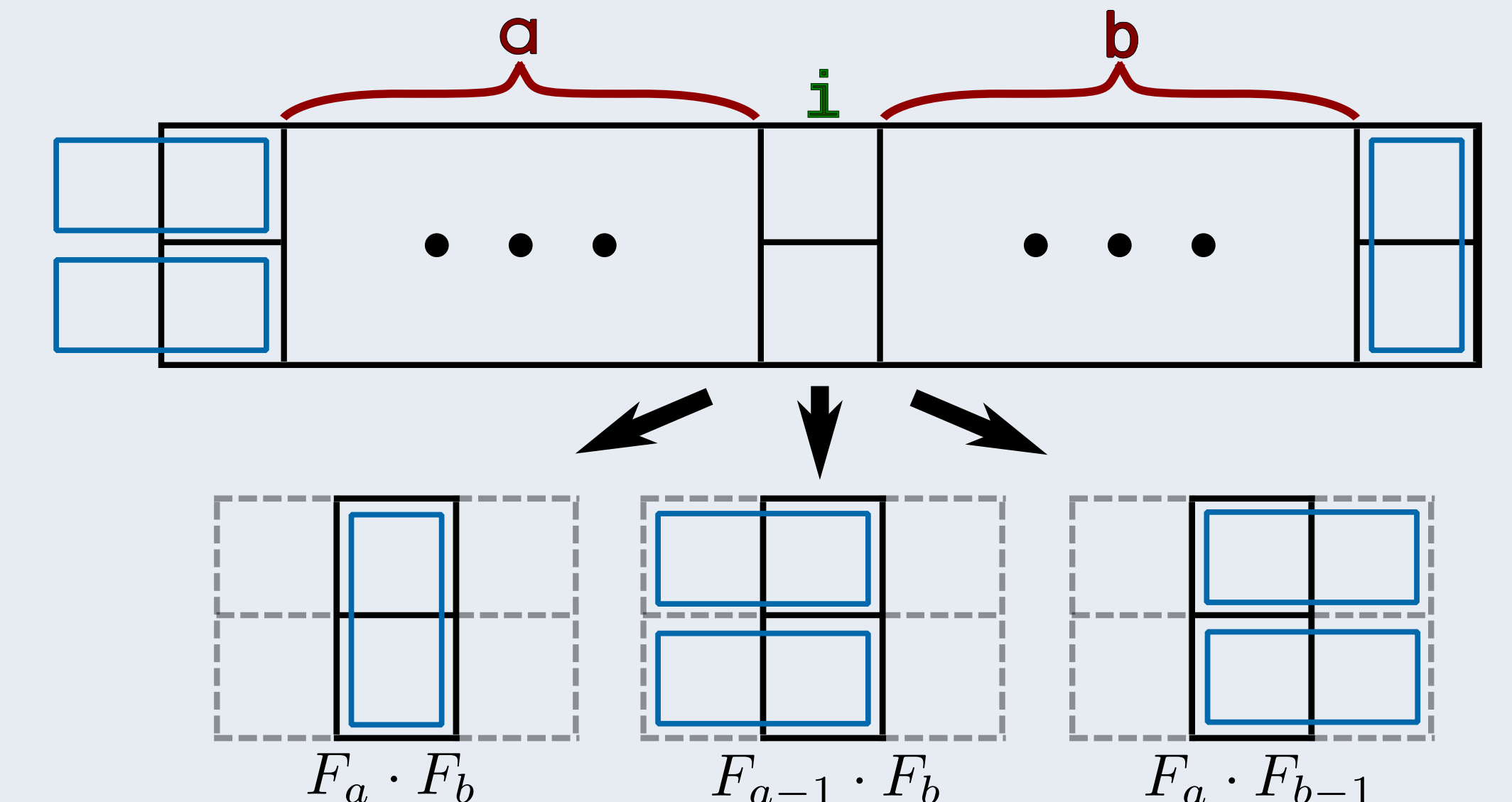
Work in Progress

Random Domino Tiling

A $2 \times n$ grid tiled with dominoes: F_n tilings possible.



Query: i^{th} column configuration: *vertical, left, right?*



Sufficient to approximate F_c/F_{c-1} : Use ϕ if $c = \Omega(\log(n))$

Open: $k \times n$ grid for $k = \omega(1)$ and Dimer model

Random Coloring: Glauber Dynamics

Consider a *uniform* random coloring of the input graph.

Query: What is v 's color in the random coloring?

Global Algorithm (Glauber Dynamics) for $k > 2\Delta$

- Sample $r = \mathcal{O}(n \log n)$ (vertex, color) pairs $\langle (v_i, c_i) \rangle_{i \in [r]}$
- For steps $i = 1, \dots, r$
 - If no neighbor of v_i has color c_i , set v_i 's color to c_i
 - Else, do nothing

Local Algorithm for $k = \Omega(\Delta \log n)$

- Locally sample *all* occurrences of (v, \star) : implemented efficiently with the proposed **Counting-Generator**
- Sample (w, \star) if necessary, where w is neighbor of v
- Query tree is of size $\mathcal{O}(1)$ for $k = \Omega(\Delta \log n)$

Open: $k = o(\Delta \log n)$