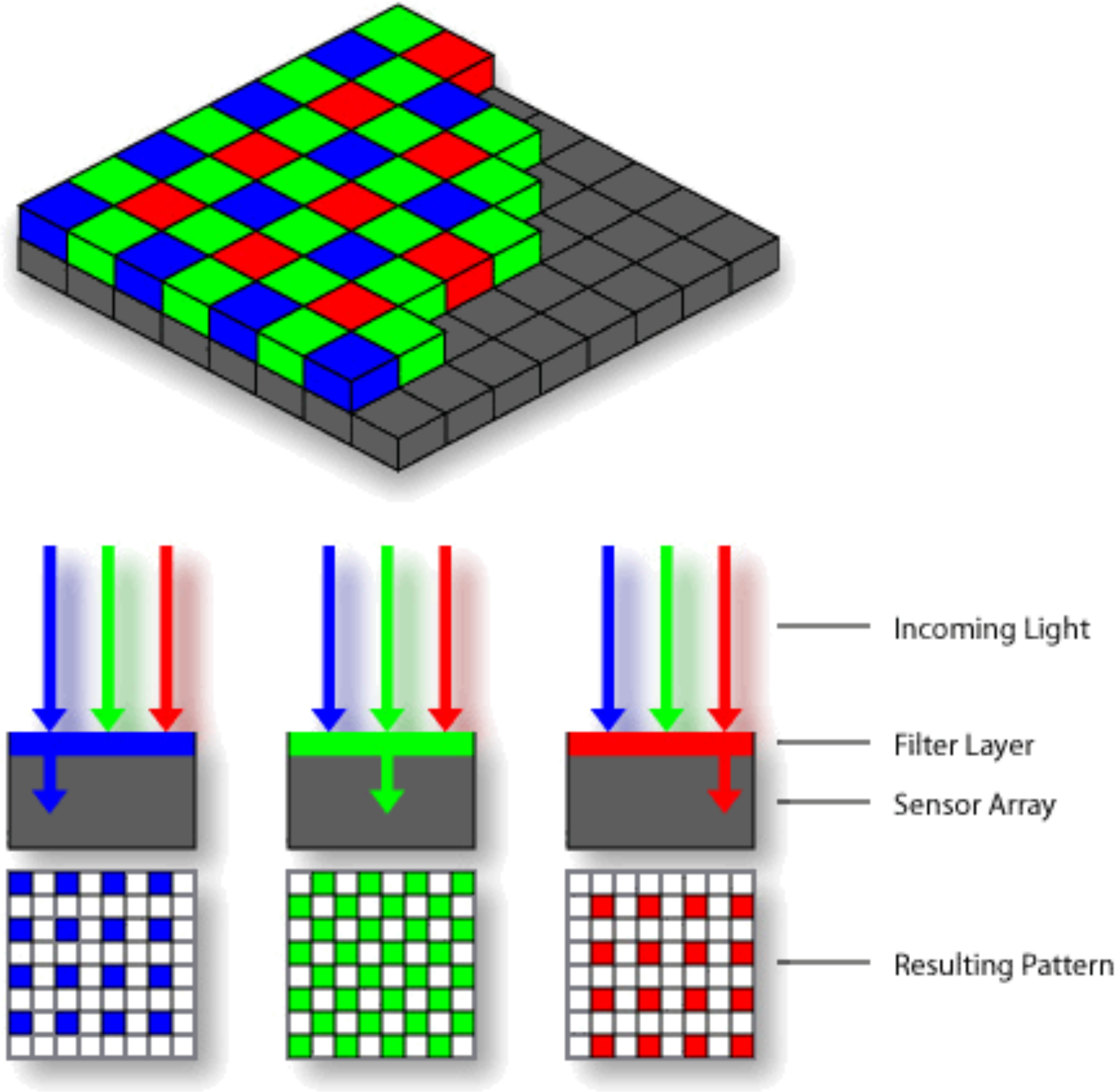
An Aside…


Incoming Light
Filter Layer
Sensor Array
Resulting Pattern

Astrometry

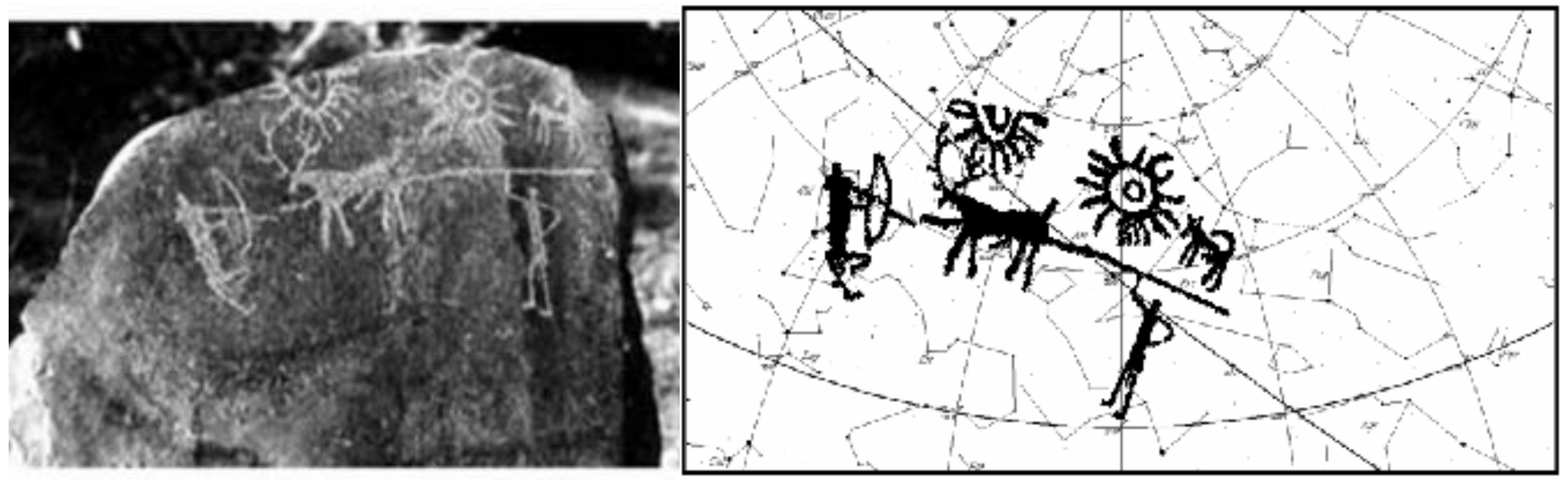
Alex Drlica-Wagner
DSFP Sep 16 2025

Thanks to Gary Bernstein

Ancient Astrometry

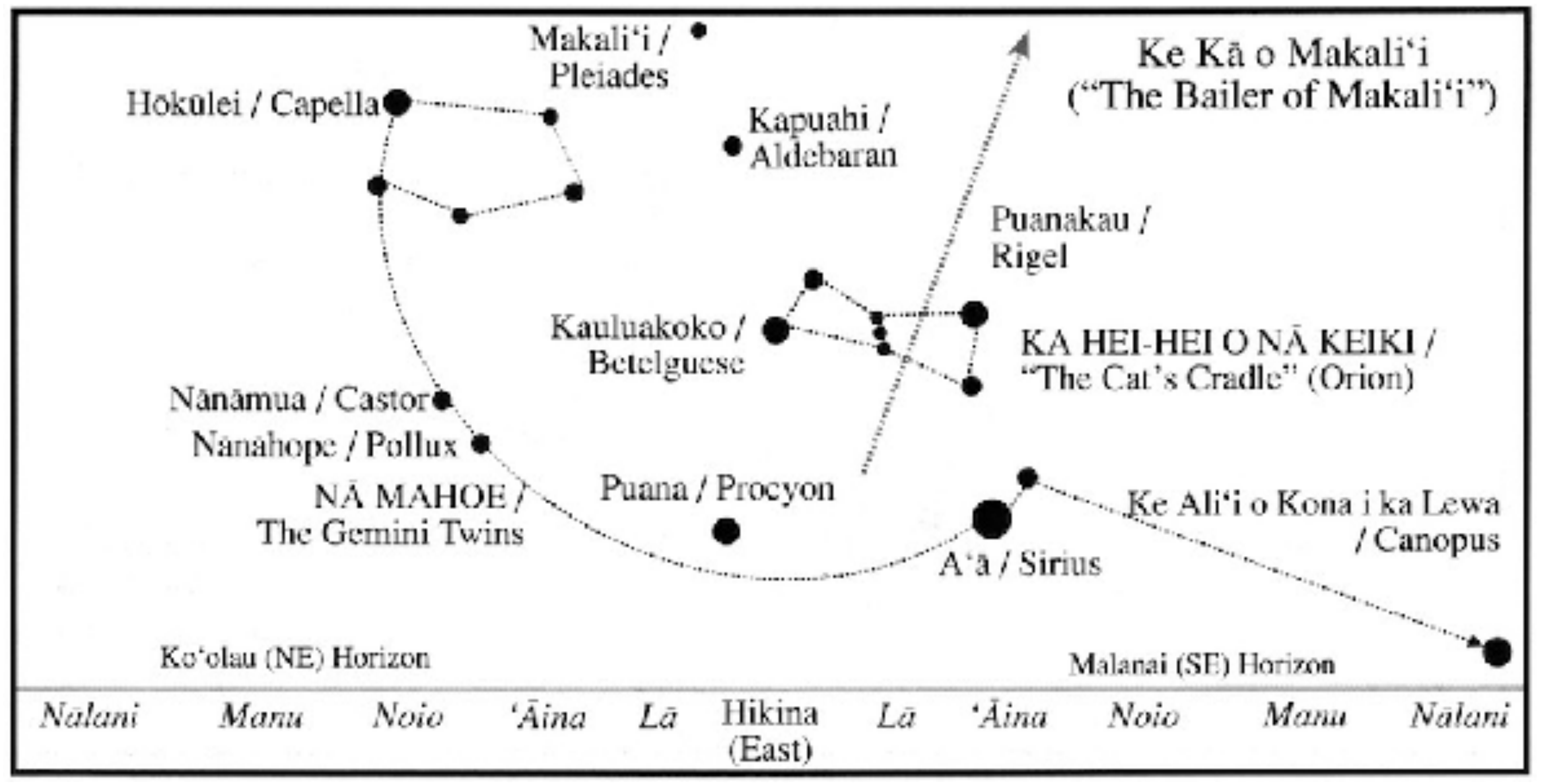
Aztec Sun Stone
(c. 1500 CE)

Petroglyph in Kasmir
(c. 4000-2000 BCE)



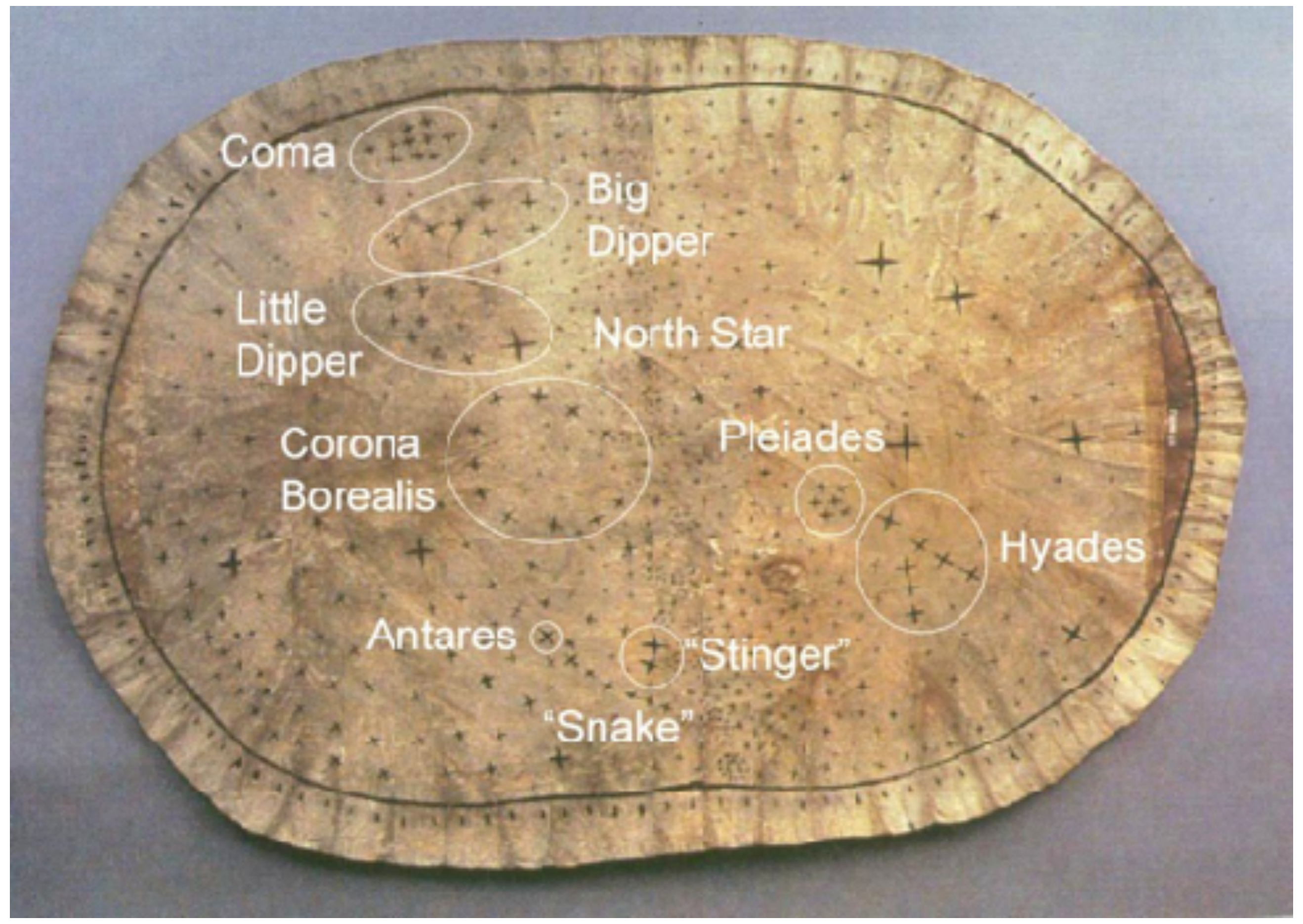
Dunhuang star chart
(c.700 CE)

Hawaiian Navigation
(c. 400 CE?)



Celestial Sphere of Ancient Greece
(c. 125 BCE)

Skidi Pawnee star chart
(c.1700 CE)



Early (Western) Sky Surveys

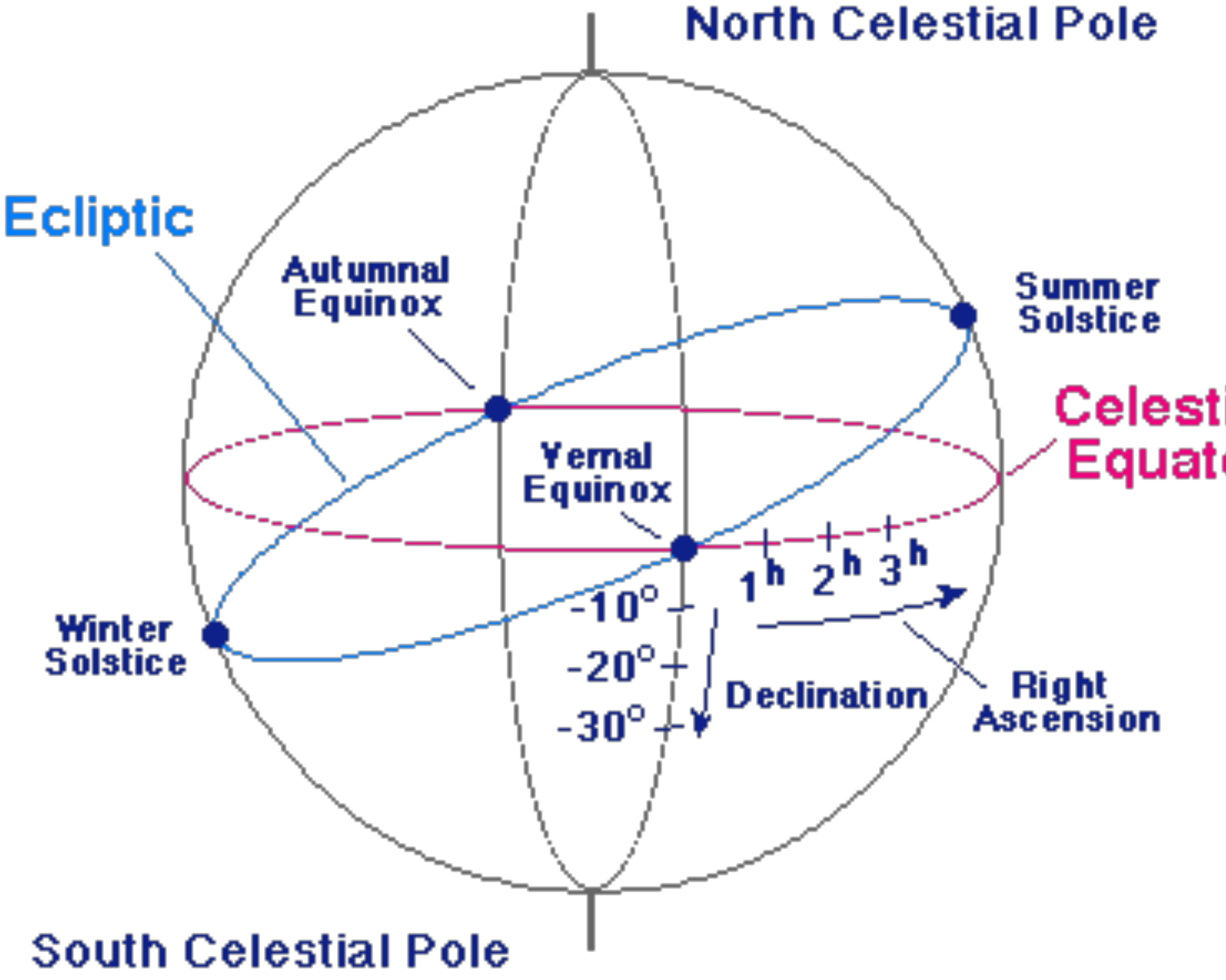
- Hipparchus (c.190 — c.120 BCE)
  - One of the first precise, quantitative catalogs of stars; ~1000 stars measured with ~30 arcmin accuracy (diameter of full Moon)
  - Measure precession of the equinoxes and length of the year to within 6 minutes
- Tycho Brahe (1546 — 1601)
  - Observations accurate to ~30 arc seconds (< $1/100^{\text{th}}$ of a degree or $1/60^{\text{th}}$ of the full Moon)
  - Precise observations of the planets throughout their orbits.
  - Funded by rich benefactors… a continuing trend

Celestial Equatorial Coordinate System

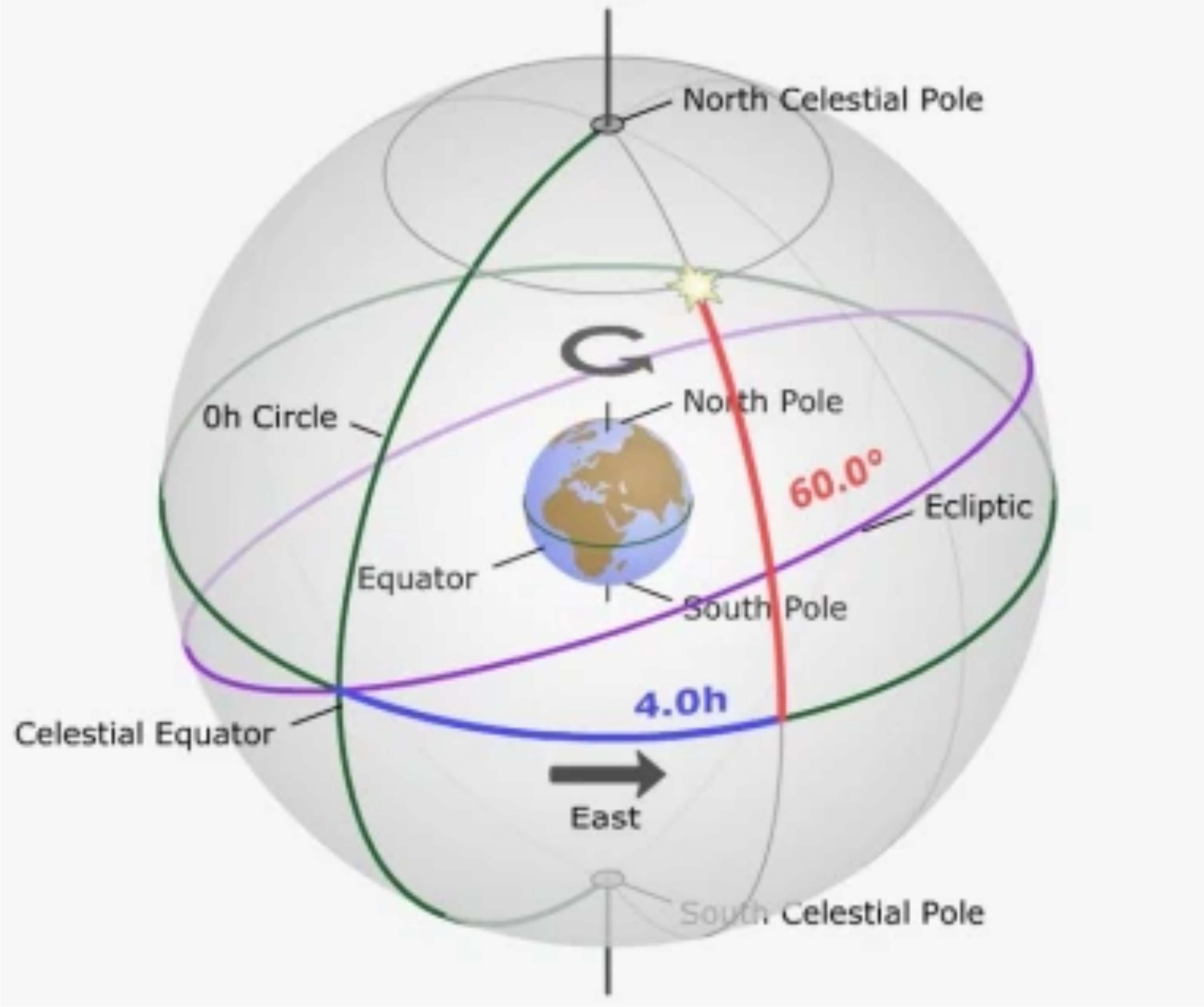
- Defined by an origin at the center of Earth, a fundamental plane consisting of the projection of Earth's equator onto the celestial sphere (forming the celestial equator), a primary direction towards the vernal (spring) equinox, and a right-handed convention.

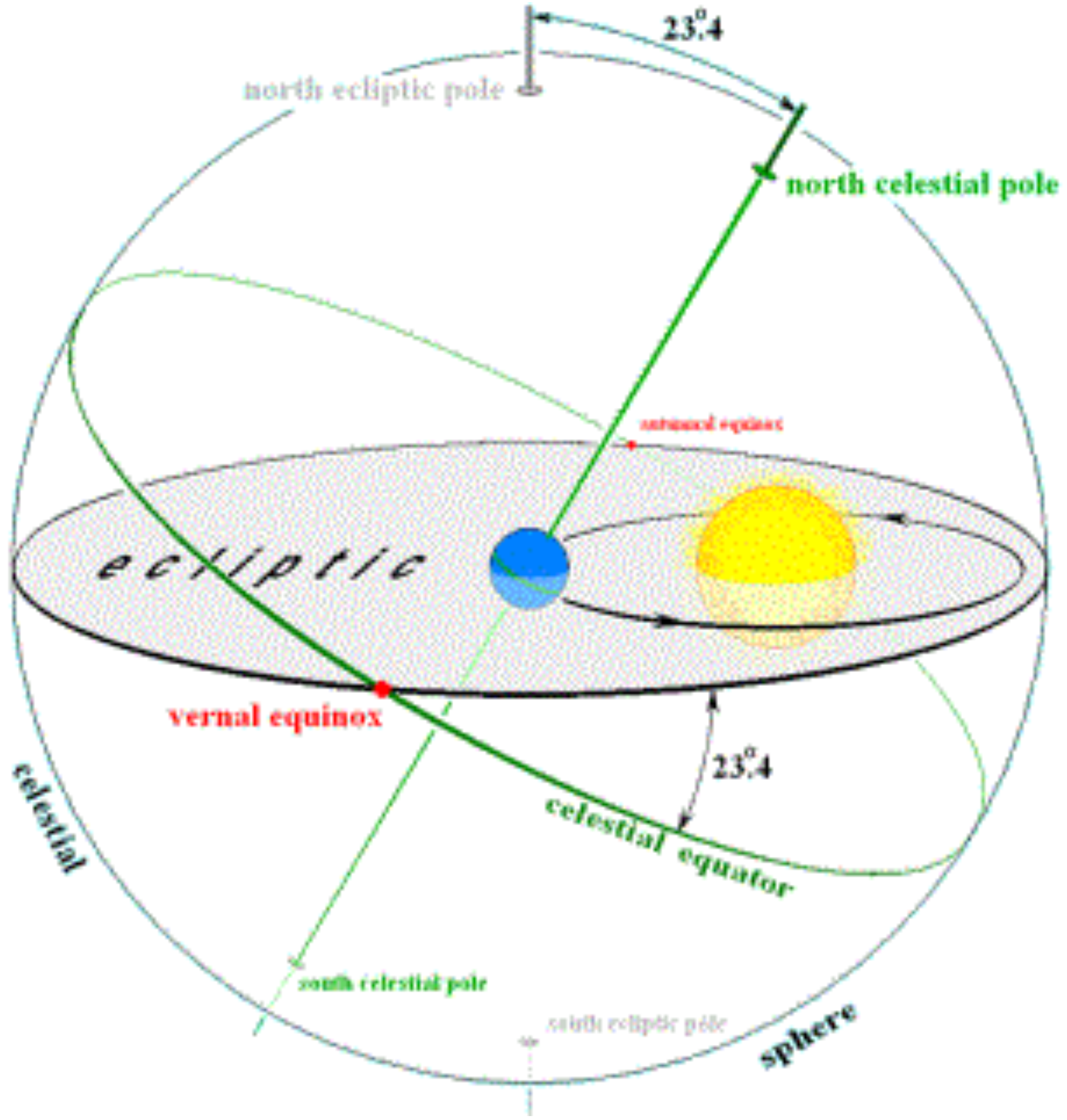
- Defines our conventional **Right Ascension** and **Declination** coordinates.

- Because of precession and nutation of the Earth's axis, the equatorial coordinate system must be defined at a specific **epoch**. The J2000.0 (January 1.5, 2000) epoch is commonly used.



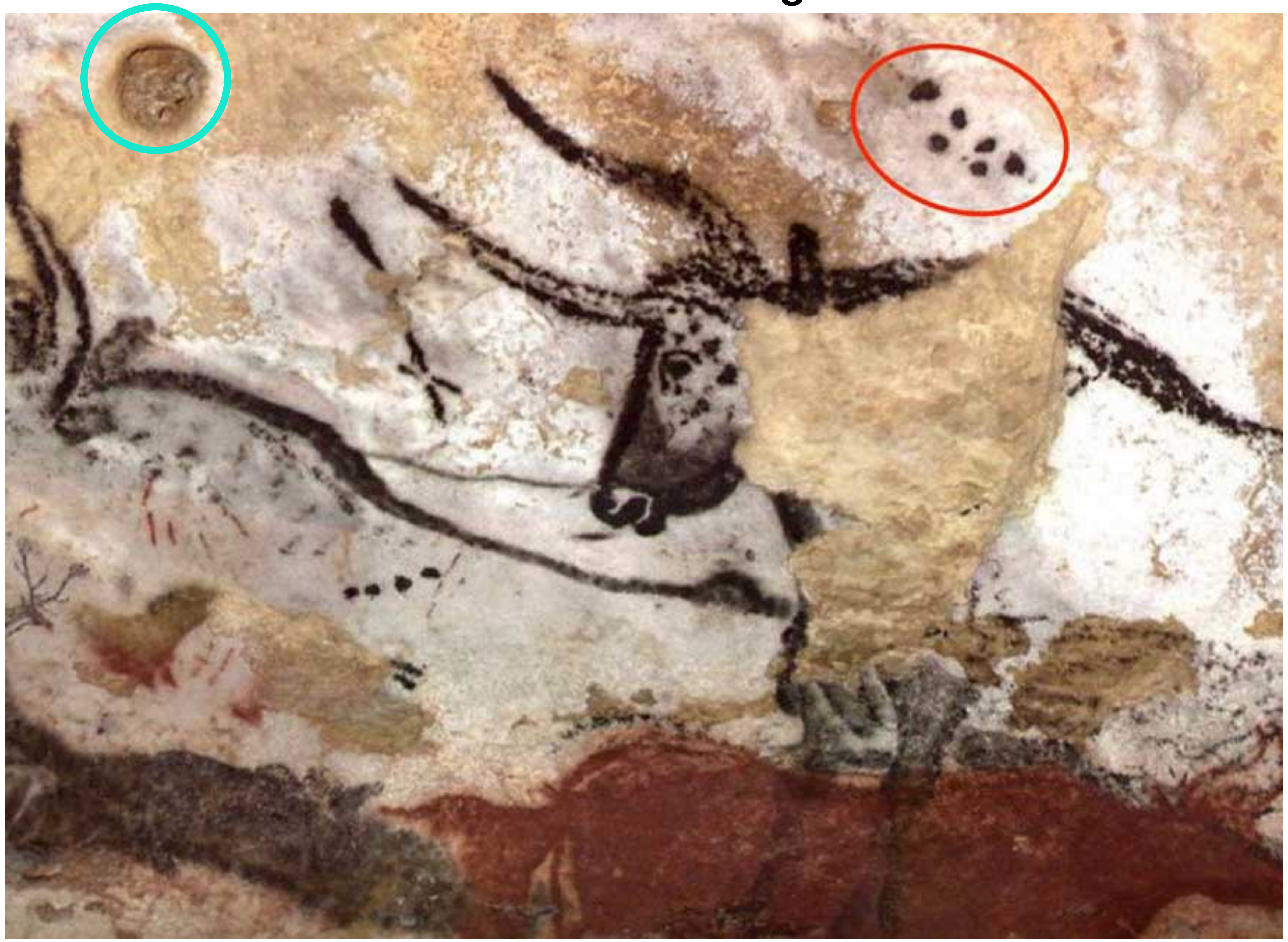
Celestial Equatorial Coordinate System



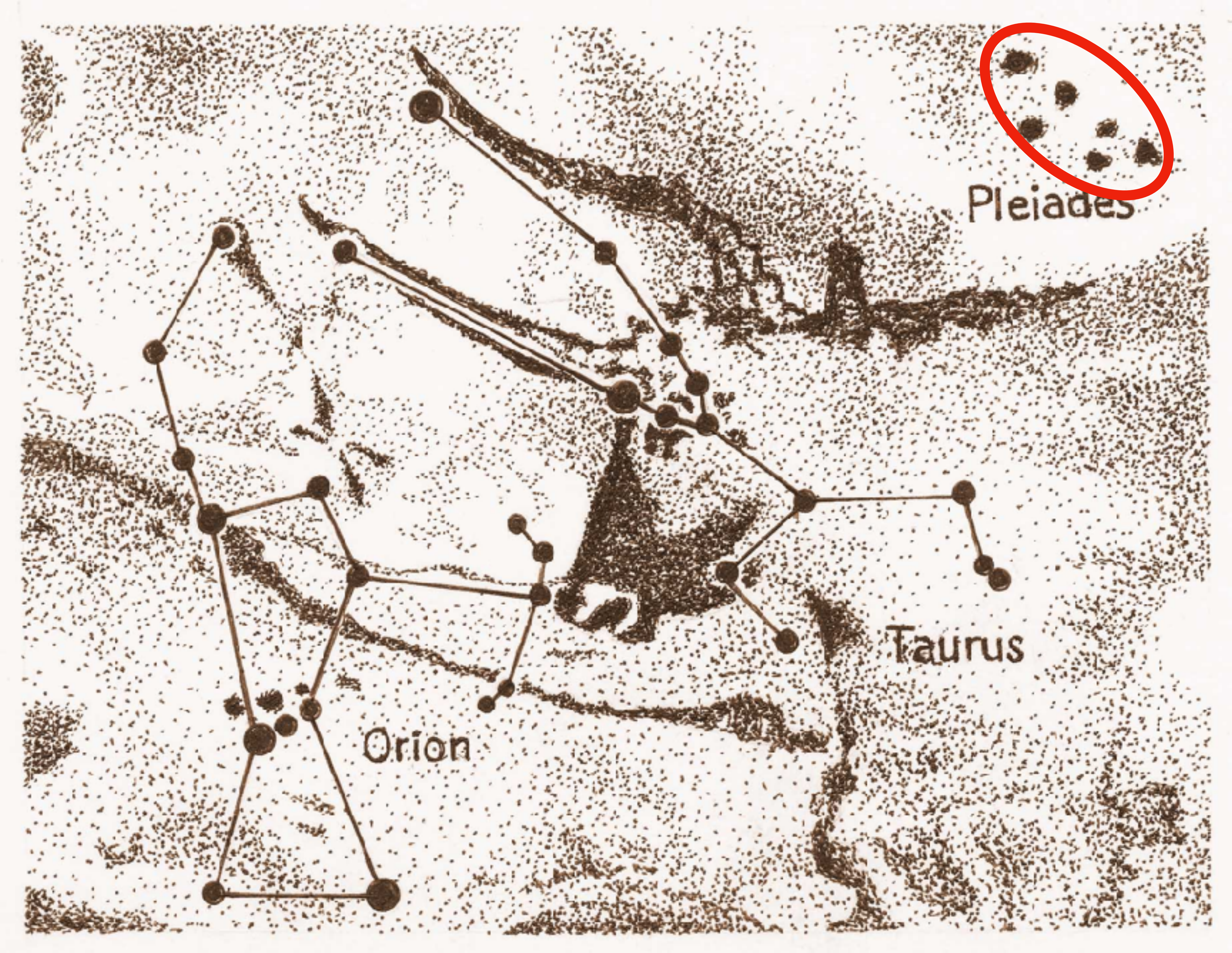


Ancient Astrometry

Lascaux Cave Painting



Modern Constellations



HIP 87671 - HR 6668 - HD 162817 - SAO 209446
Type: star
Magnitude: 6.10
Absolute Magnitude: -1.26
Color Index (B-V): 0.07
RA/Dec (J2000.0): 17h54m27.13s/-34°27'56.4"
RA/Dec (on date): 17h56m07.84s/-34°28'16.3"
HA/Dec: 9h27m41.42s/-34°28'16.3"
Az./Alt.: +276°18'23.4"/-59°14'02.9"
Gal. long./lat.: +355°12'26.4"/-4°26'35.9"
Supergal. long./lat.: +190°38'06.5"/+37°47'57.8"
Ecl. long./lat. (J2000.0): +268°50'06.1"/-11°01'55.2"
Ecl. long./lat. (on date): +269°11'15.0"/-11°02'07.0"
Ecliptic obliquity (on date): +23°26'19.2"
Mean Sidereal Time: 3h23m49.2s
Apparent Sidereal Time: 3h23m49.3s
Rise: 3h59m
Transit: 7h06m
Set: 10h13m
IAU Constellation: Sco
Distance: 967.82±18.96 ly
Proper motion: 4.86 mas/yr towards 159.8°
Proper motions by axes: 1.68 -4.56 (mas/yr)
Parallax: 3.370±0.050 mas
Spectral Type: B9.5/A0III
Solar Az./Alt.: +240°05'30"/+26°46'01"
Lunar Az./Alt.: +318°47'32"/-68°11'48"
Date and Time
Julian Day
2025 - 3 - 20
16 : 32 : 15
Pollux
Mars
Gemini
Jupiter
Uranus
Aries
Taurus
Aldebaran
Mercury
Sun
Ecliptic of Date
90°
60°
30°
Cetus
Orion
Betelgeuse
Bellatrix
Rigel
Canis Minor
Procyon
Monoceros
Lepus
Eridanus
Fornax
Sirius
Canis Major
Adhara
Columba
SE
S
SW
Earth, Bordeaux Observatory, 0 m
FOV 82.5°
13.28 FPS
2025-03-20 16:32:15 UTC+01:00

HIP 87671 - HR 6668 - HD 162817 - SAO 209446
Type: star
Magnitude: 6.10
Absolute Magnitude: -1.26
Color Index (B-V): 0.07
RA/Dec (J2000.0): 17h54m27.87s/-34°27'28.9"
RA/Dec (on date): 11h35m38.65s/-8°24'12.6"
HA/Dec: 10h48m45.24s/-8°24'12.6"
Az./Alt.: +331°37'59.6"/-50°26'19.1"
Gal. long./lat.: +356°12'54.8"/-4°26'30.0"
Supergal. long./lat.: +190°38'06.4"/+37°48'26.8"
Ecl. long./lat. (J2000.0): +268°50'15.1"/-11°01'27.6"
Ecl. long./lat. (on date): +177°54'18.8"/-10°09'11.3"
Ecliptic obliquity (on date): +24°08'34.6"
Mean Sidereal Time: 22h24m23.9s
Apparent Sidereal Time: 22h24m23.9s
Rise: 18h15m
Transit: 23h40m
Set: 5h10m
IAU Constellation: Sco
Distance: 967.82±16.96 ly
Proper motion: 4.86 mas/yr towards 159.8°
Proper motions by axes: 1.68 -4.56 (mas/yr)
Parallax: 3.370±0.060 mas
Spectral Type: B9.5/A0III
Solar Az./Alt.: +148°04'56"/+39°31'49"
Lunar Az./Alt.: +190°36'50"/+28°10'20"
Ecliptic of Date
Date and Time
Julian Day
-4600 – 4 – 26
10 : 31 : 19
Capella
Perseus
Auriga
Pollux
Triangulum
Cancer
Gemini
Uranus
Sun
Daytime Arietids
Aries
Mars
Mercury
Taurus
Aldebaran
Moon
Venus
Procyon
Canis Minor
Pisces
Saturn
Betelgeuse
Orion
Bellatrix
Monoceros
Rigel
Sirius
Earth, Bordeaux Observatory, 0 m
FOV 82.5°
-4600-04-26 10:31:19 UTC 00:02 (LMST)

129 Tau - HIP 27265 - HR 1985 - HD 38478 - SAO 94848
Type: star
Magnitude: 6.00
Absolute Magnitude: -1.30
Color Index (B-V): 0.06
RA/Dec (J2000.0): 5h46m32.91s/−15°50'52.7"
RA/Dec (on date): 23h55m48.66s/-9°18'23.3"
HA/Dec: 1h24m32.60s/-9°18'23.3"
Az./Alt.: +204°59'01.1"/+32°35'45.1"
Gal. long./lat.: +191°20'19.4"/-6°35'02.3"
Supergal. long./lat.: +357°29'55.1"/-54°12'13.4"
Ecl. long./lat. (J2000.0): +85°44'12.3"/-7°33'11.6"
Ecl. long./lat. (on date): +355°27'56.1"/-8°12'08.2"
Ecliptic obliquity (on date): +22°22'45.2"
Mean Sidereal Time: 1h20m21.3s
Apparent Sidereal Time: 1h20m21.3s
Rise: 9h48m
Transit: 15h10m
Set: 20h32m
IAU Constellation: Tau
Distance: 939.93±32.03 ly
Proper motion: 5.73 mas/yr towards 117.5°
Proper motions by axes: 5.08 -2.65 (mas/yr)
Parallax: 3.470±0.122 mas
Spectral Type: B8IIIpHg(Mn)
Solar Az./Alt.: +206°20'58"/−41°12'29"
Lunar Az./Alt.: +28°50'37"/-34°36'17"
Ecliptic of Date
Cancer
Pollux
Hydra
Procyon
Mercury
Gemini
Capella
Auriga
Perseus
Canis Minor
Daytime Arietids
Triangulum
Monoceros
Betelgeuse
Orion
Bellatrix
Aldebaran
Taurus
Venus
Aries
Pyxis
Canis Major
Rigel
Date and Time
Julian Day
-30200 – 10 – 11
16 : 34 : 17
Earth, Bordeaux Observatory, 0 m
FOV 82.5°
13.16 FPS
-30200-10-11 16:34:17 UTC-00:02 (LMST)

Celestial Reference Frame

- **The System:** International Celestial Reference System (ICRS)
  - Specifications for defining the coordinate system, including origin, fundamental planes/axes, along with constants, models, and algorithms for transforming observables.
  - The ICRS has its origin at the barycenter of the Solar System, with axes that are intended to "show no global rotation with respect to a set of distant extragalactic objects”
- **The Realization:** International Celestial Reference Frame (ICRF)
  - The set of identifiable fiducial sources/points on the sky along with coordinates that serves as the practical realization of the reference system

Observations enable realizations of the system.
Optical Gaia-CRF and radio ICRF3 are realizations of ICRS

J2000 vs ICRS

- The ICRS was intentionally defined to match closely with J2000.

- The axes are extremely closely aligned, differing by less than a tenth of an arcsecond. For many "ordinary" applications, they can be considered practically the same, but for the highest precision work done by Gaia, the ICRS is the superior choice.

Motion of Objects

- In addition to the motion of the Earth (which required us to choose an epoch for the equatorial reference system), objects on the sky move.

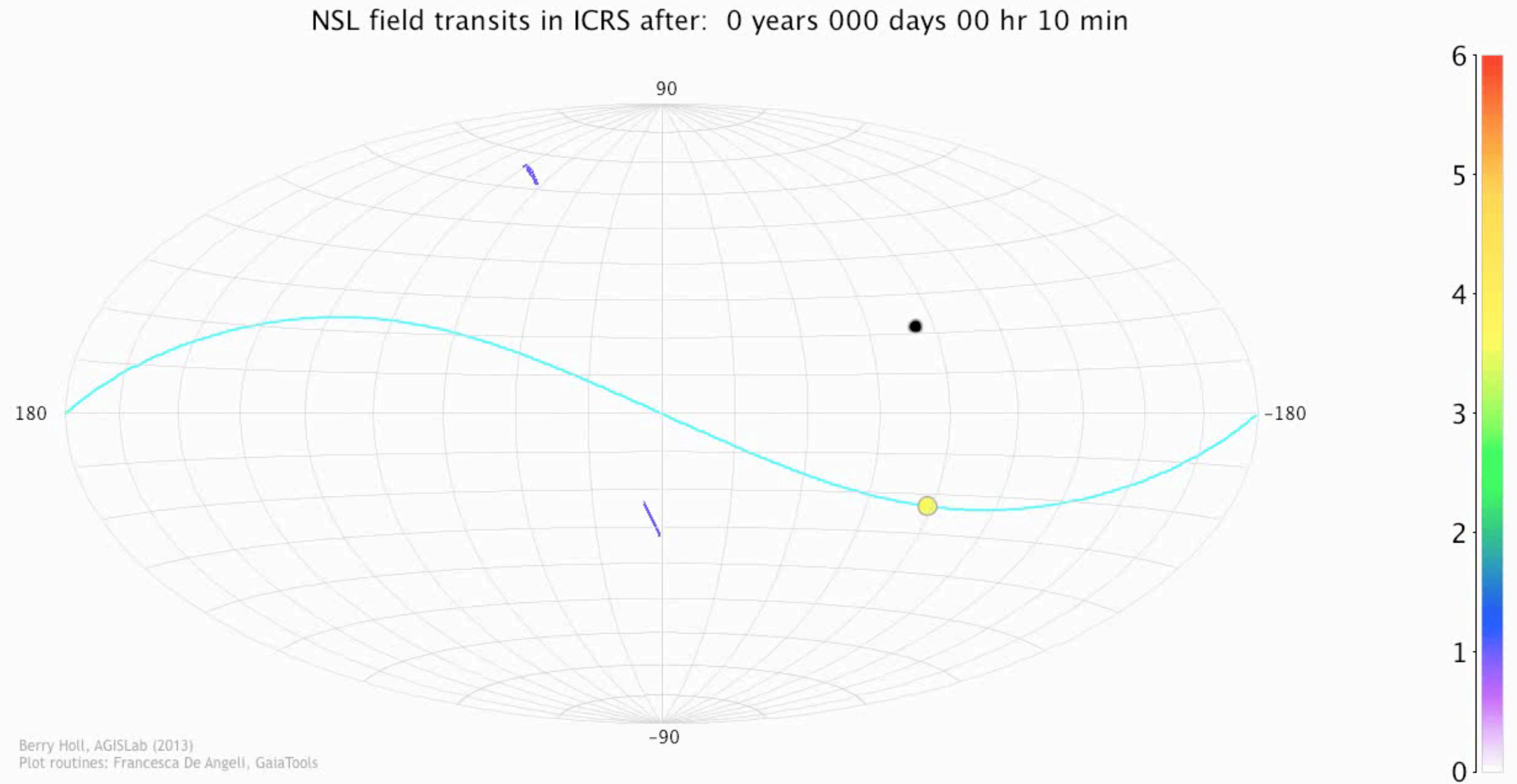
- To define the location of an object relative to a coordinate system, it is necessary to choose a **reference epoch**.

- It often makes sense to choose a reference epoch near the middle of your measurement timespan. This can mean that your epoch can change with time.

  - The reference epoch is different for each Gaia data release: DR3 uses J2016.0, DR2 uses J2015.5, and DR1 uses J2015.0.

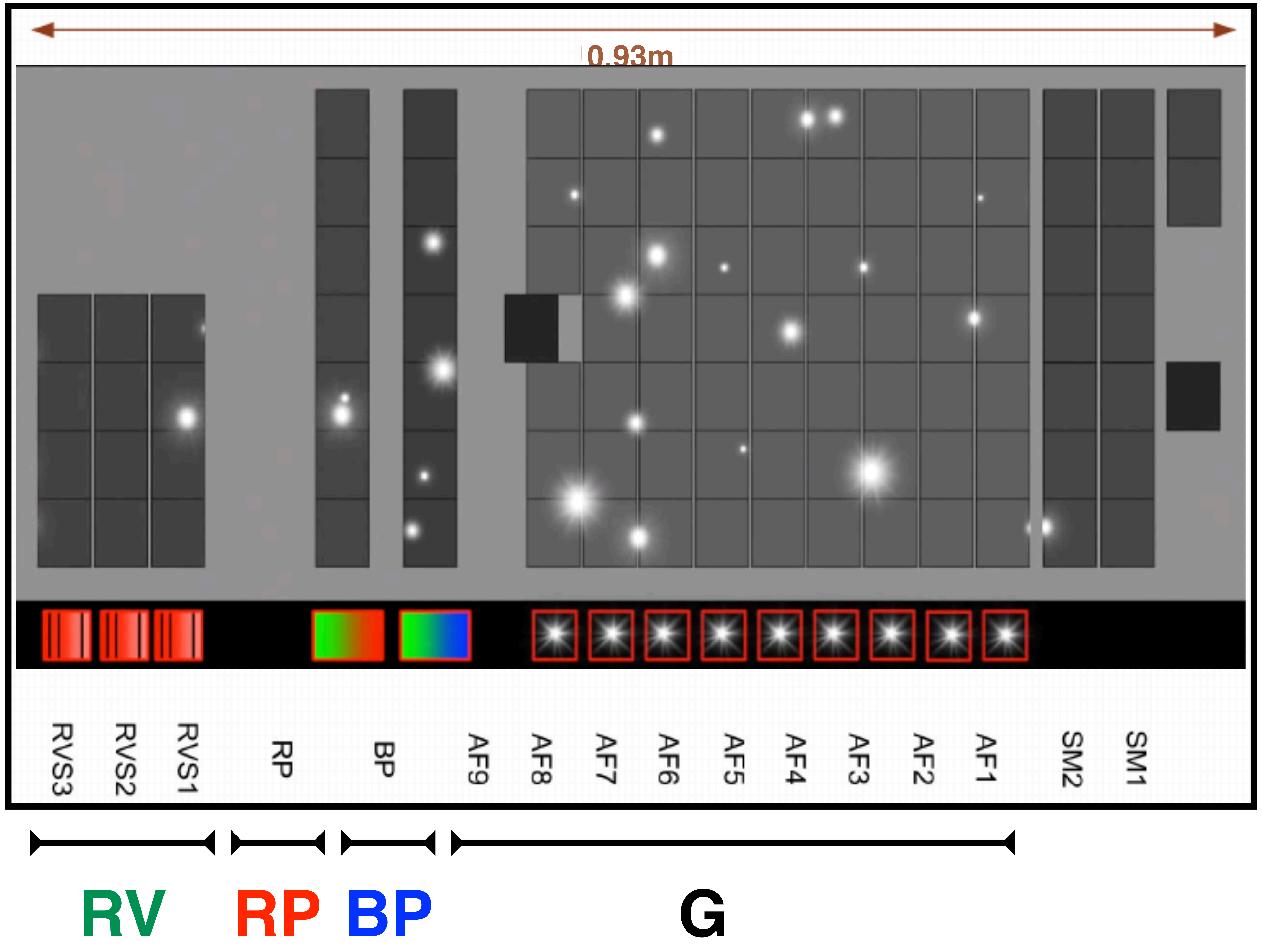
Gaia Satellite

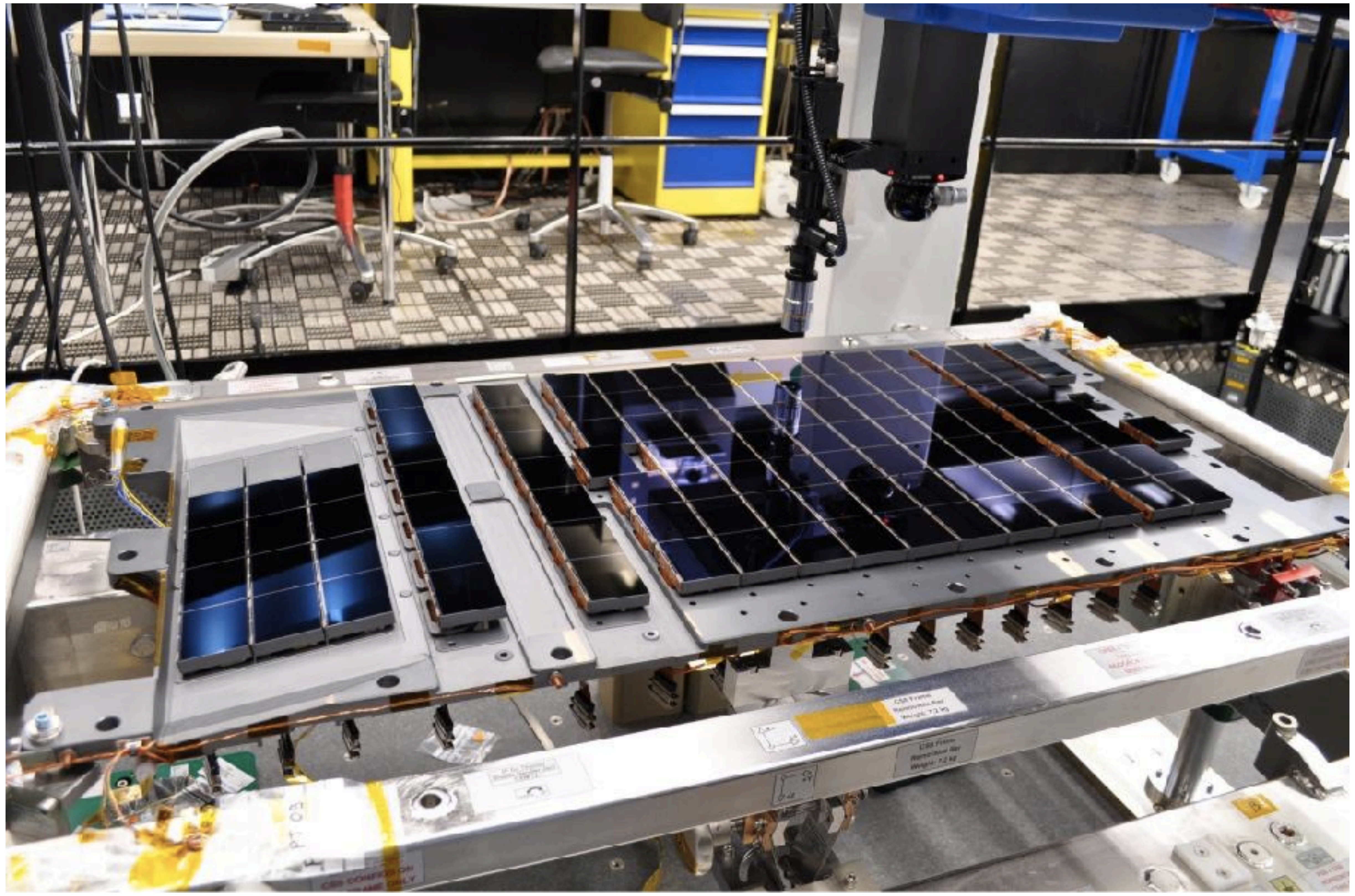
https://www.youtube.com/watch?v=bbfb8VhH7L0



https://www.youtube.com/watch?v=lRhe2grA9wE

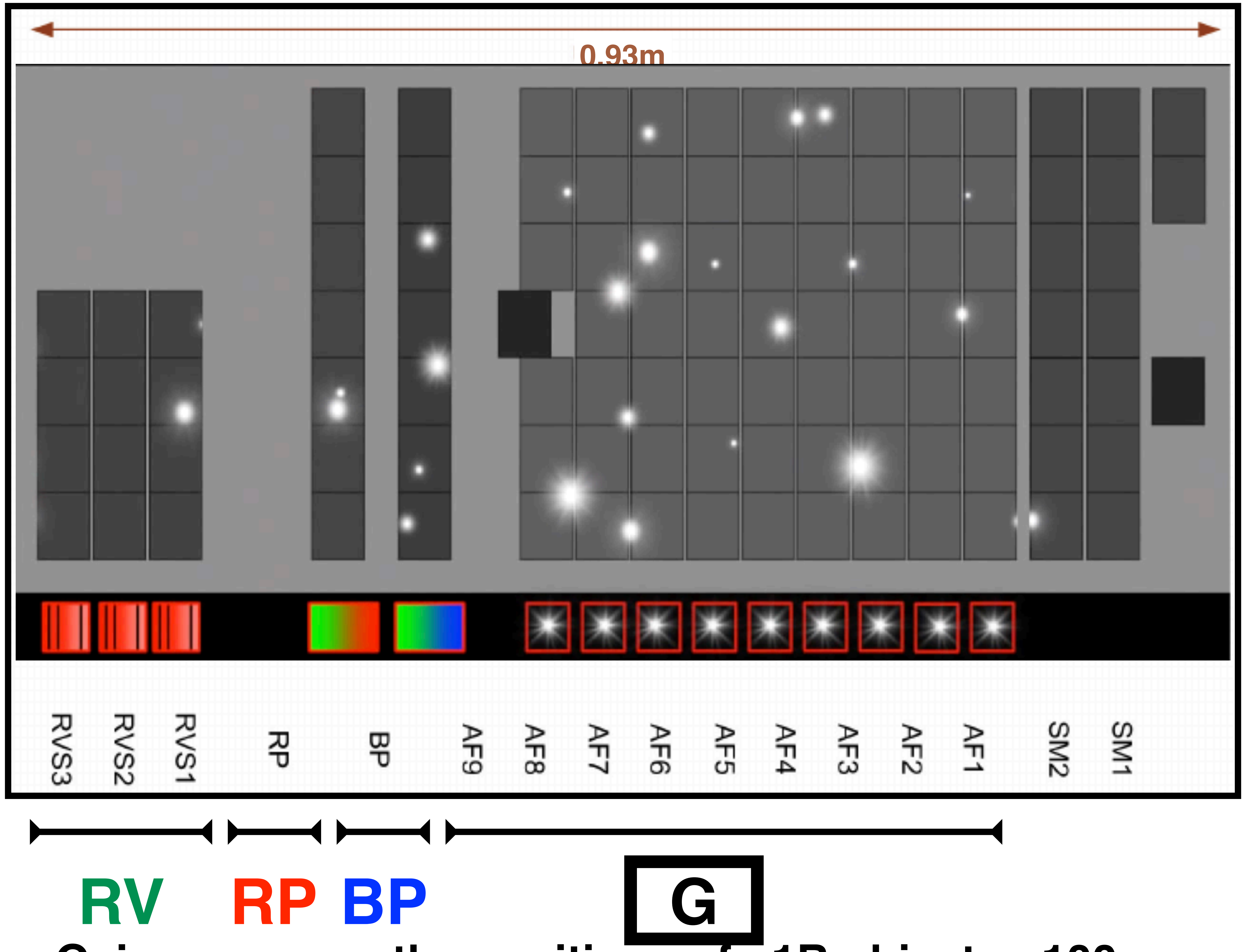
Gaia Focal Plane



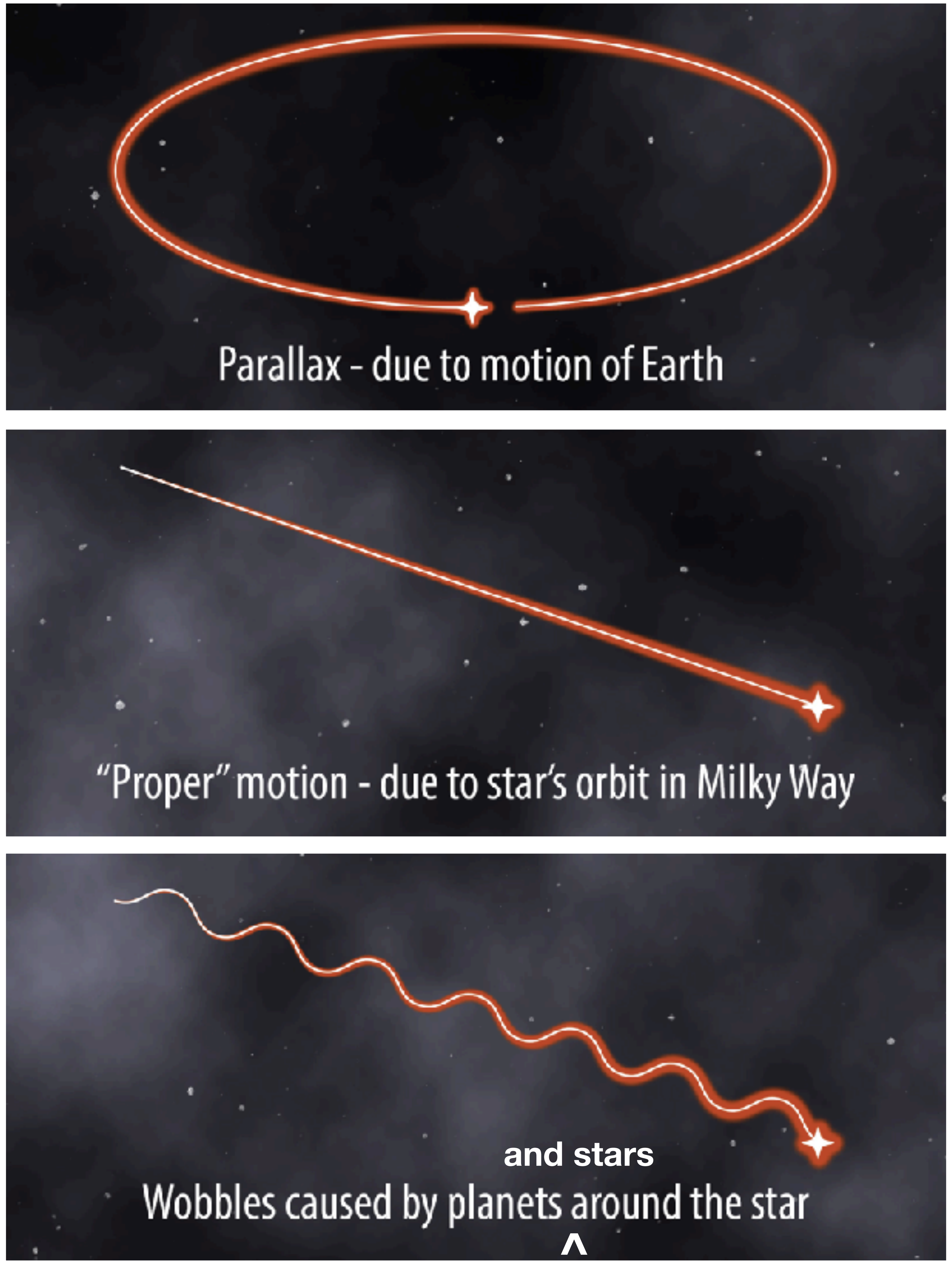


Radial Velocity
Spectrophotometry
Astrometry

Gaia Astrometry



Gaia measures the positions of >1B objects ~100 times each.



Moment	Point Sources	Resolved Sources
0: Flux $f = \int dx\,dy\, I(x,y)$	**Point Source Photometry**	**Galaxy Photometry**
1: Centroid $\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{1}{f}\int dx\,dy\,\begin{pmatrix}x\\ y\end{pmatrix} I(x,y)$	**Astrometry** minor planet orbits, etc.	*Nobody cares**
2: Size/Shape $\begin{pmatrix}M_r\\ M_+\\ M_-\end{pmatrix} = \int dx\,dy\,\begin{pmatrix}x^2+y^2\\ x^2-y^2\\ 2xy\end{pmatrix} I(x,y)$	*Nobody cares**	Weak gravitational lensing

*except as needed to facilitate galaxy shapes

Centroiding: Simplest Case

- Consider the case of a Gaussian random variable with unknown mean (μ) and standard deviation ($\boldsymbol{\sigma}$): $\mathcal{N}(\mu, \boldsymbol{\sigma})$

- From simple statistics we know that the uncertainty on the mean will be proportional to the standard deviation and will decrease as the square root of the number of samples.

$$\sigma_x = \frac{\sigma}{\sqrt{N}} \propto \frac{1}{\mathrm{SNR}}$$

- Since the signal-to-noise ration also goes as $\sqrt{N}$

Centroiding: More Realism

- We have a 2d, pixelated image…
  - The intensity of counts in a pixel: $I(x, y) = I_{xy}$
  - The total flux is: $f = \int dx\, dy\, I(x, y) \rightarrow \sum_{xy} I_{xy}$
  - Our centroid estimator is now: $\bar{x} = \frac{1}{f} \int dx\, dy\, x\, I(x, y) \rightarrow \frac{\sum_{xy} x I_{xy}}{\sum_{xy} I_{xy}}$

Centroiding: More Realism

- Everything continues to work as statistically expected as long as we are in the **source dominated** regime.
- However, once we become **background limited**, things break down.
- In the background limited cas, the centroiding becomes "confused" by the sky background and can **wander** far from the source. (See the excellent mathematical derivation in Gary Bernstein's DSFP slides from Session 11).
- For a **dark night on Rubin LSST**, this transition occurs for sources with $20 \lesssim r \lesssim 21$ mag.

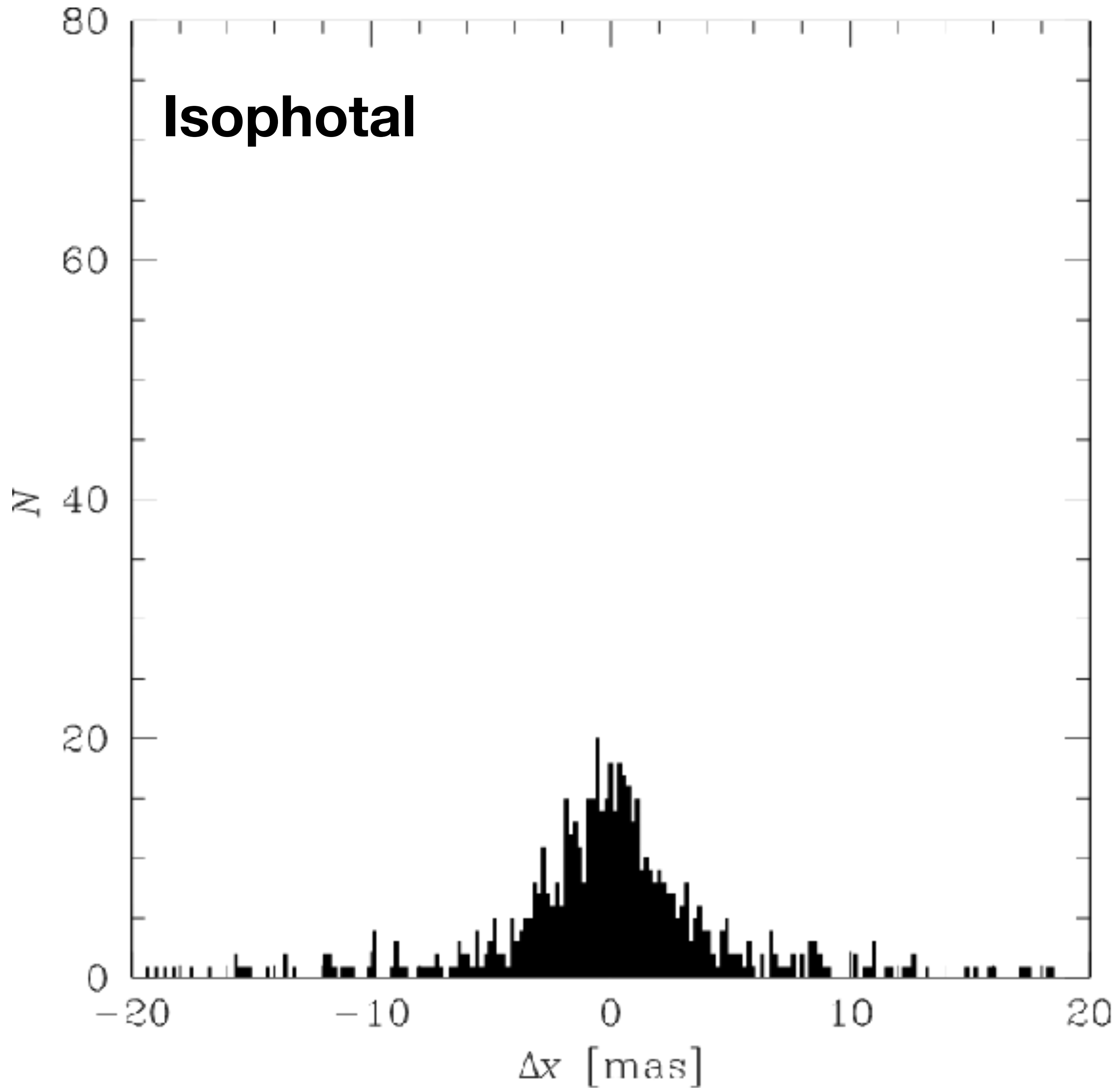
Centroiding: More Realism

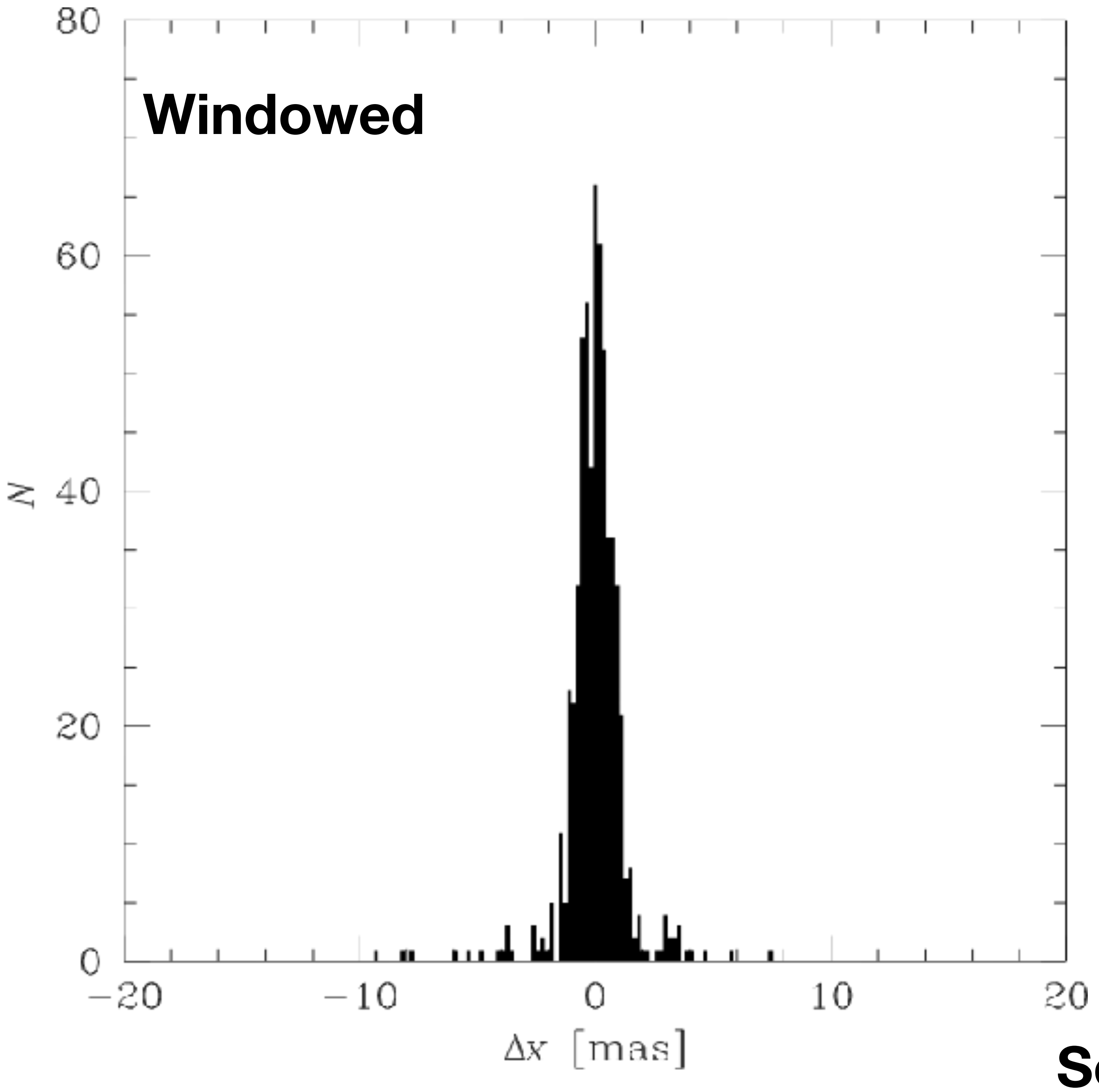
- The common solution is to apply an “aperture weight” or “window function”.
  - This can be a simple top-hat function, but generally a Gaussian works better.
  - Optimally use the PSF of the image as the weight (but usually a Gaussian is fine and fast).
- So long as the aperture can be centered fairly well initially, you can iteratively solve for the centroid position (updating the window at each step).

Centroiding: More Realism

- If you are a user of SourceExtractor, you may be familiar with the “WIN” parameters (XWIN, YWIN, AWIN, BWIN, etc.)

- These are properties derived following this windowing procedure.





SourceExtractor Manual

World Coordinate Systems

- Once you have a centroid in pixel coordinates, you would like to translate that into a location on the sky (generally in ICRS). This is commonly done by matching to an external reference catalog.

- Once you have the pixel and sky coordinates for multiple sources in the image, you can derive a World Coordinate System (WCS) for the image. This allows you to map any pixel to its sky coordinates (whether it contains a source or not).

- Many of the common distortions that you need to handle for this mapping from pixel to sky coordinates can be handled with a fairly low order polynomial.

- There is a FITS standard for encoding the WCS equations as key/value pairs in the image header. ***Note: The FITS standard does not work for Rubin***

https://community.lsst.org/t/how-to-use-wcss-in-dp1-and-commissioning-processing/10769

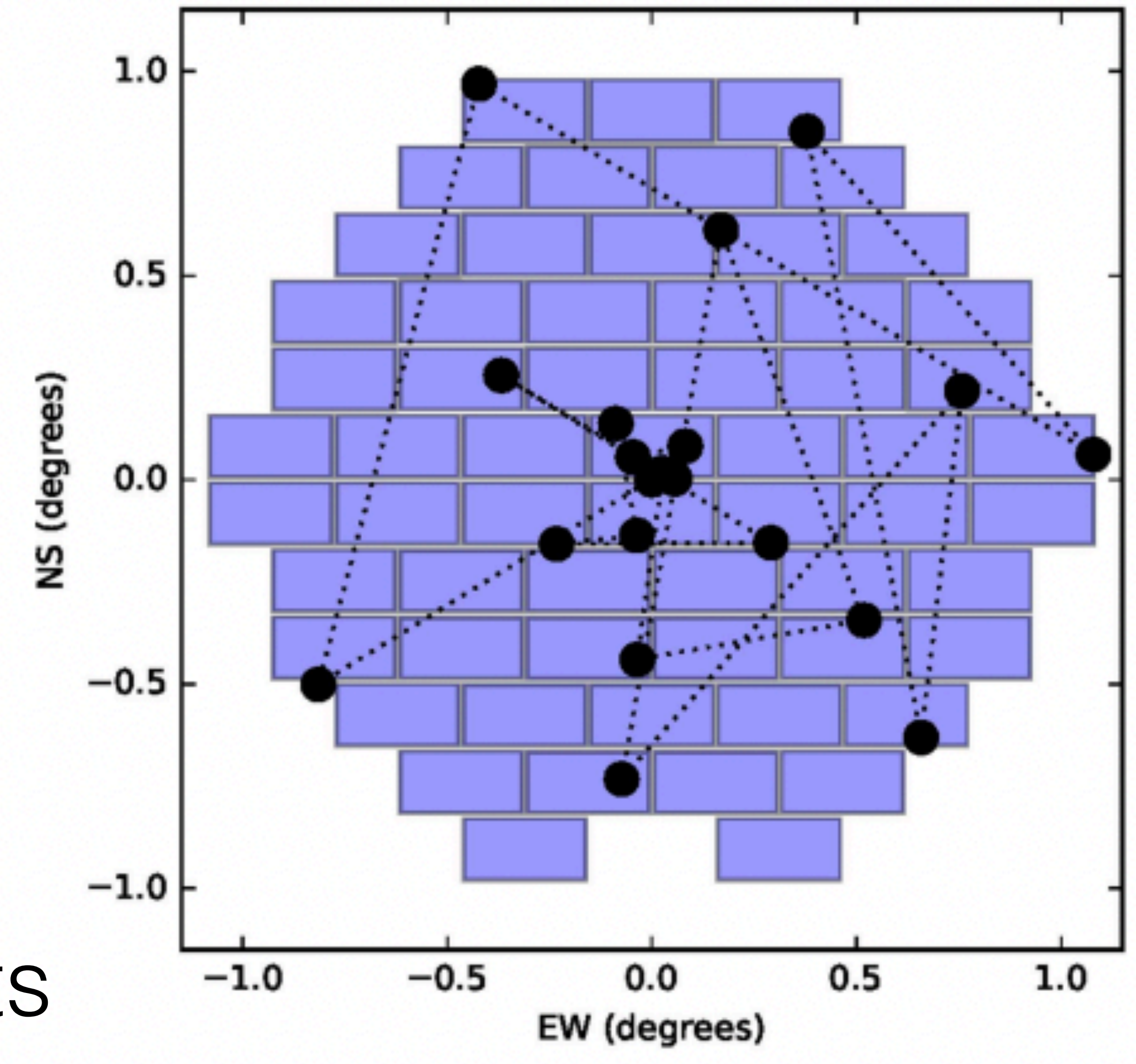
From Pixels to the Sky

- Our job is to translate (x,y) pixel coordinates to (alpha, delta) sky coordinates (typically in the ICRS). We have to retrace the photon's journey through the solar system to electron on the CCD.
  - Gravitational deflection by solar system members
  - Stellar aberration due to Earth's motion
  - Refraction toward the vertical by $n>1$ in Earth's atmosphere
    - **wavelength-dependent** = "Differential chromatic refraction, DCR"
  - Stochastic deflections by turbulence in the atmosphere
  - Projections of spherical sky onto the flat focal plane.
  - Radial or other distortions by the telescope optics
    - **wavelength-dependent** = "lateral color" if there are refracting elements
  - Locations/orientations of the CCDs in the focal plane
  - Deflection of photo-electrons (or holes) by lateral electric fields in CCDs
  - Distortion of the CCD pixels from a perfect square grid.

From Pixels to the Sky

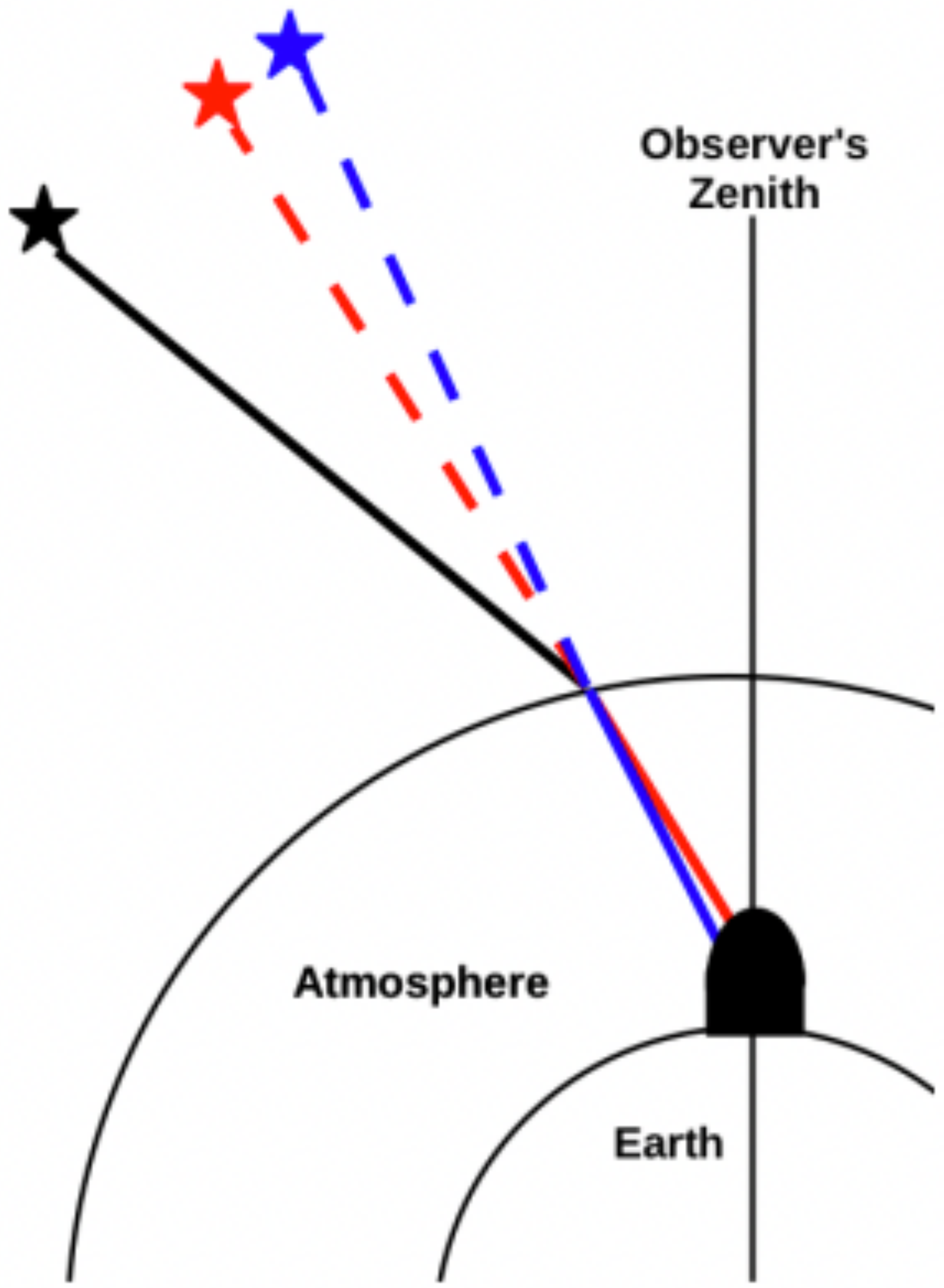
Most sources of astrometric distortion are smooth, and can be modeled by low-order polynomials (typically 3rd to 4th order across a CCD). These polynomials can be constrained by using a least-squares fit to reference stars and internal agreement.

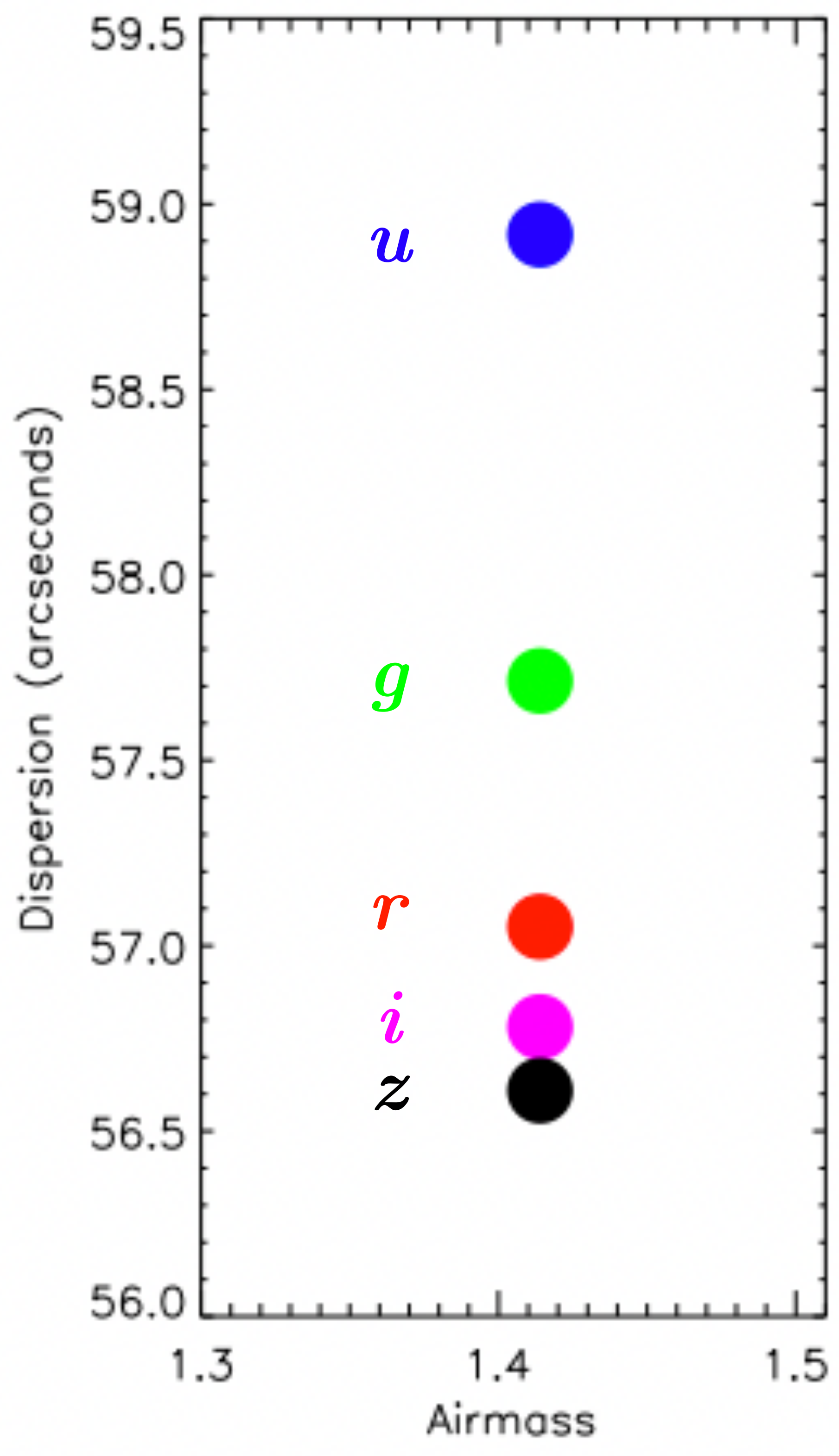
- ~~Gravitational deflection by solar system members~~
- ~~Stellar aberration due to Earth's motion~~
- ~~Refraction toward the vertical by n>1 in Earth's atmosphere~~
    - **wavelength-dependent** = "Differential chromatic refraction, DCR"
- Stochastic deflections by turbulence in the atmosphere
- ~~Projections of spherical sky onto the flat focal plane.~~
- ~~Radial or other distortions by the telescope optics~~
    - **wavelength-dependent** = "lateral color" if there are refracting elements
- ~~Locations/orientations of the CCDs in the focal plane~~
- Deflection of photo-electrons (or holes) by lateral electric fields in CCDs
- Distortion of the CCD pixels from a perfect square grid.



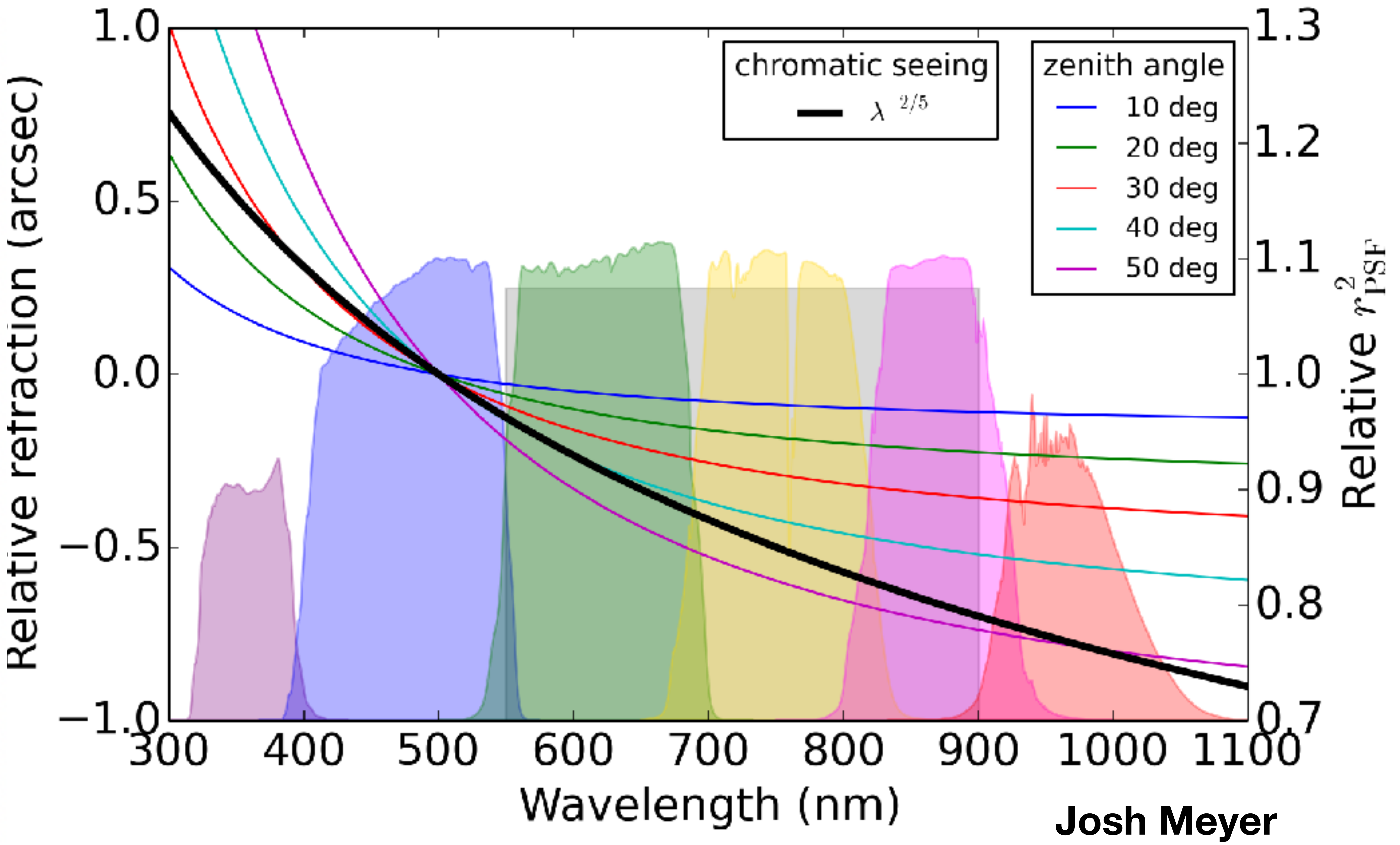
Differential Chromatic Refraction

NB: LSSTCam (and DECam) does *not* have an Atmospheric Dispersion Corrector (ADC)





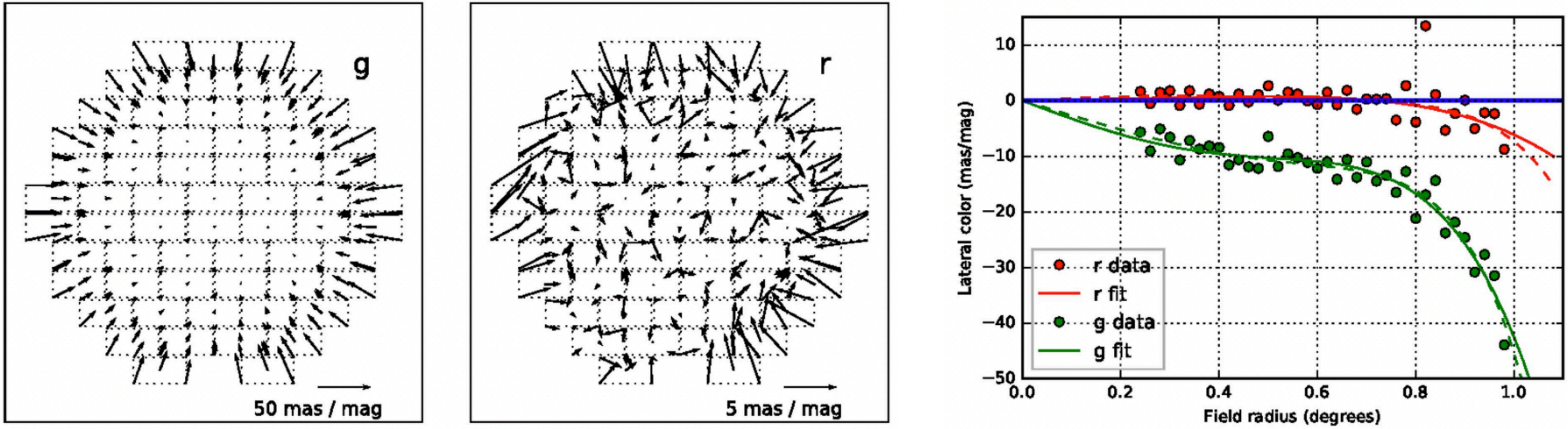
Kaczmarxczik et al. (2009)



Josh Meyer

Lateral Color Shifts from Optics

DECam

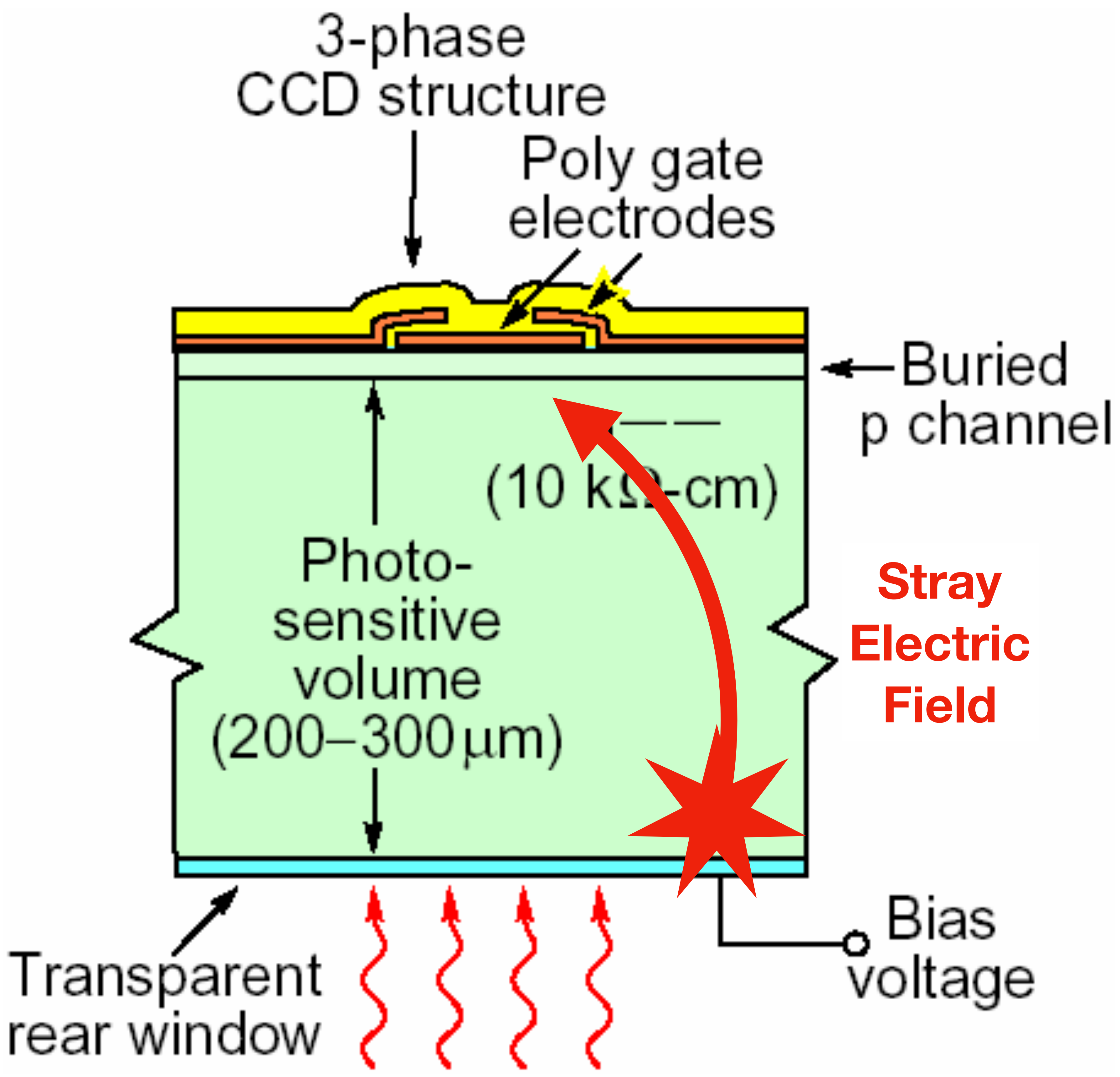


Bernstein et al. (2017)

- Smooth and can be accurately modeled/corrected.
- However, the magnitude of the correction is very color dependent and will depend on the SED of a source ***within*** a filter bandpass

Pixel Distortions

- What your CCD produces is a time series of “voltages” in pixels.
- Due to the quality of our photolithographic processes, we generally assume those pixels are a uniform, rectilinear grid. However, this is not necessarily true.
- Storing the *shape* of each pixel along with it’s position would be *heavy*



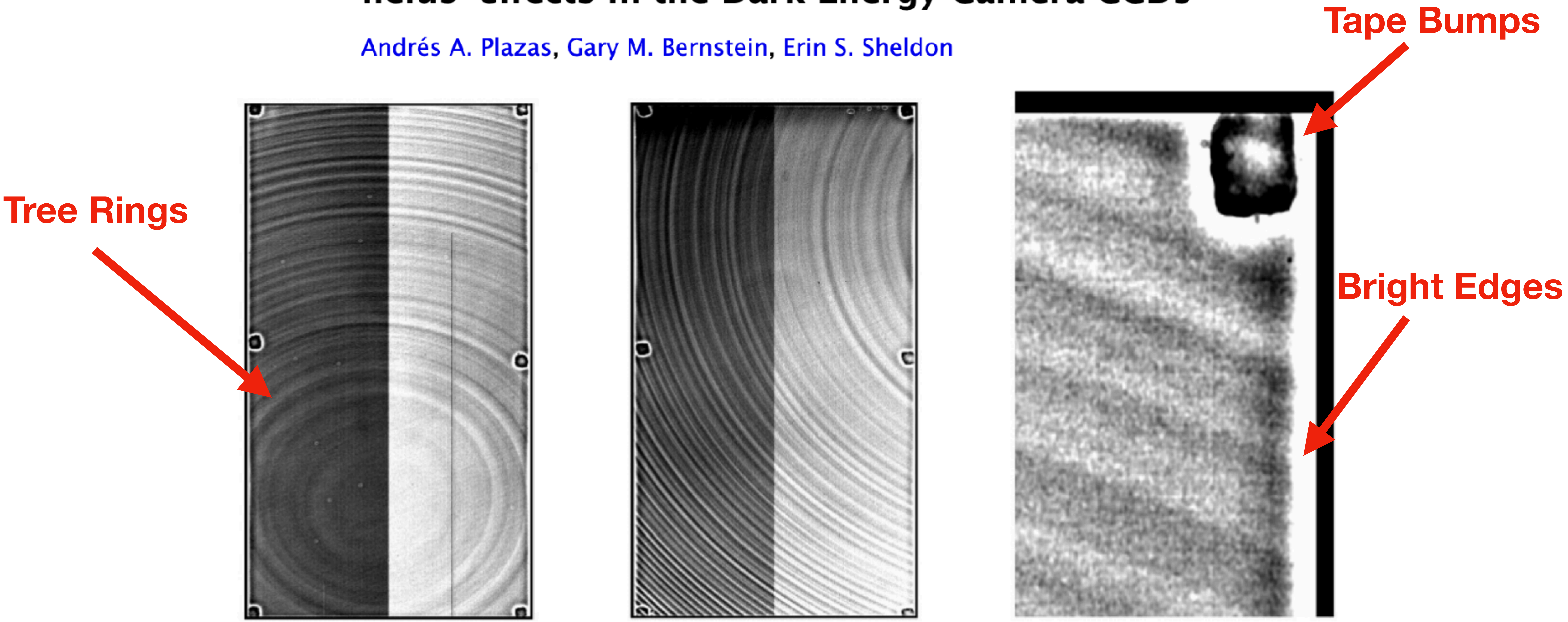
arXiv > astro-ph > arXiv:1403.6127

Astrophysics > Instrumentation and Methods for Astrophysics

[Submitted on 24 Mar 2014 (v1), last revised 11 Jul 2014 (this version, v3)]

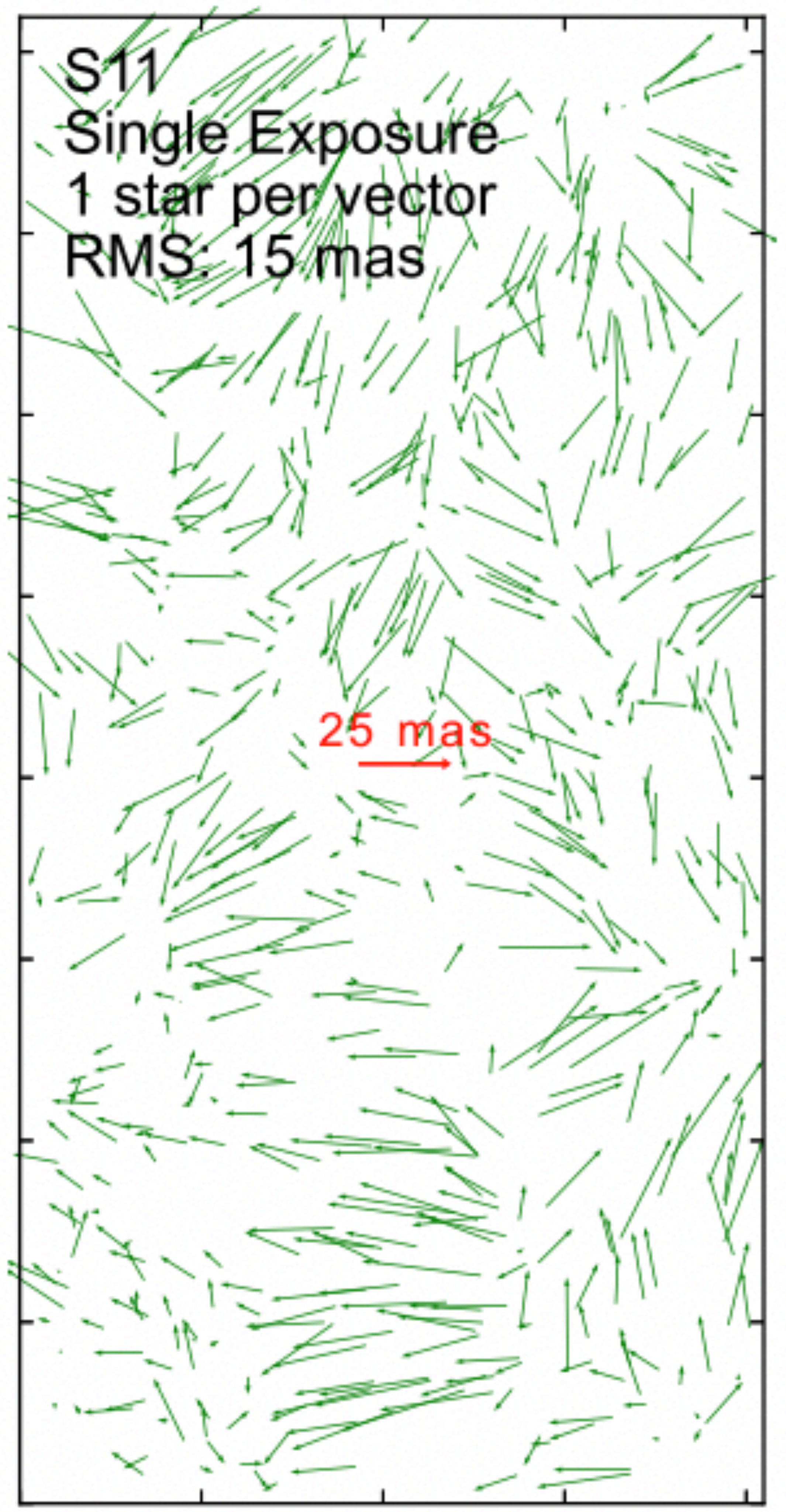
On-sky measurements of the transverse electric fields' effects in the Dark Energy Camera CCDs

Andrés A. Plazas, Gary M. Bernstein, Erin S. Sheldon



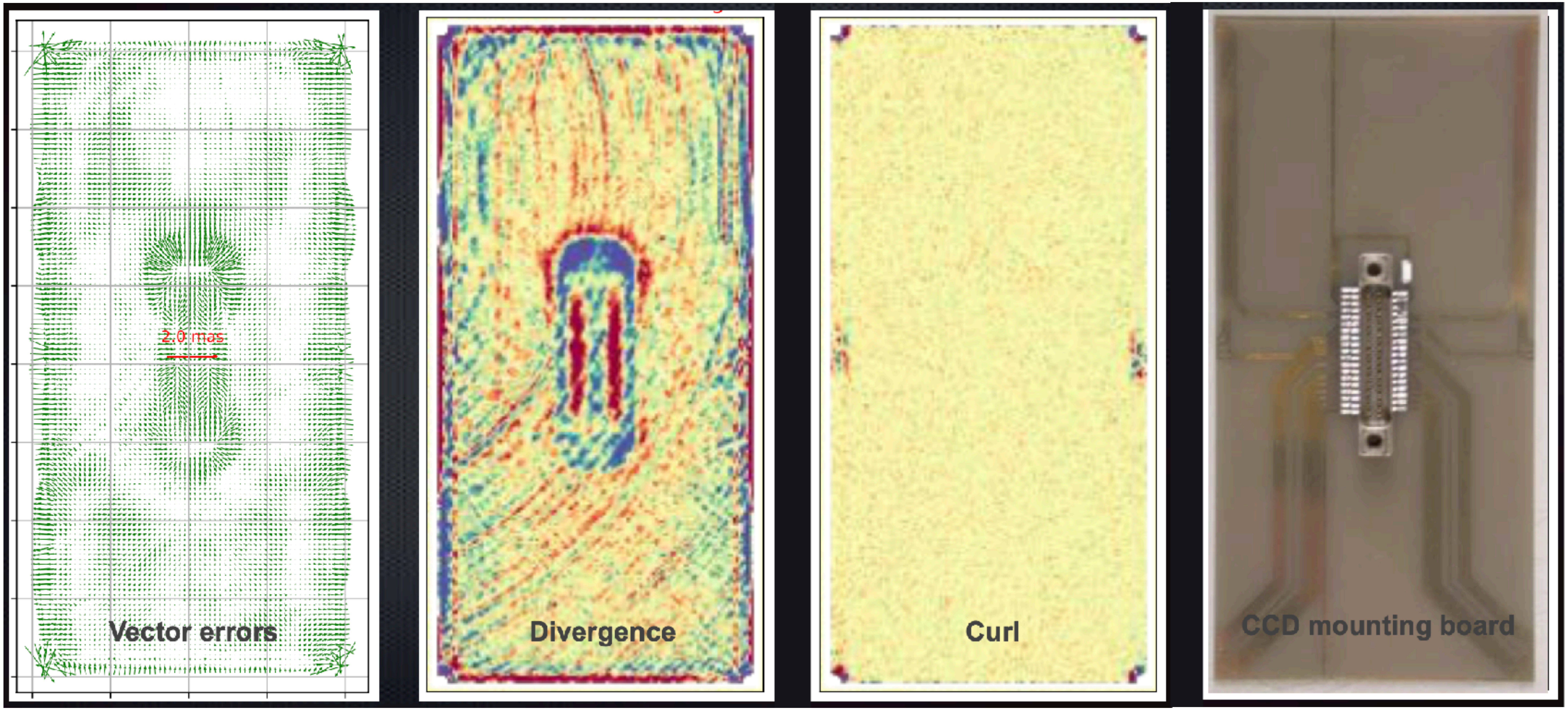
Pixel Distortions

Note change in scale

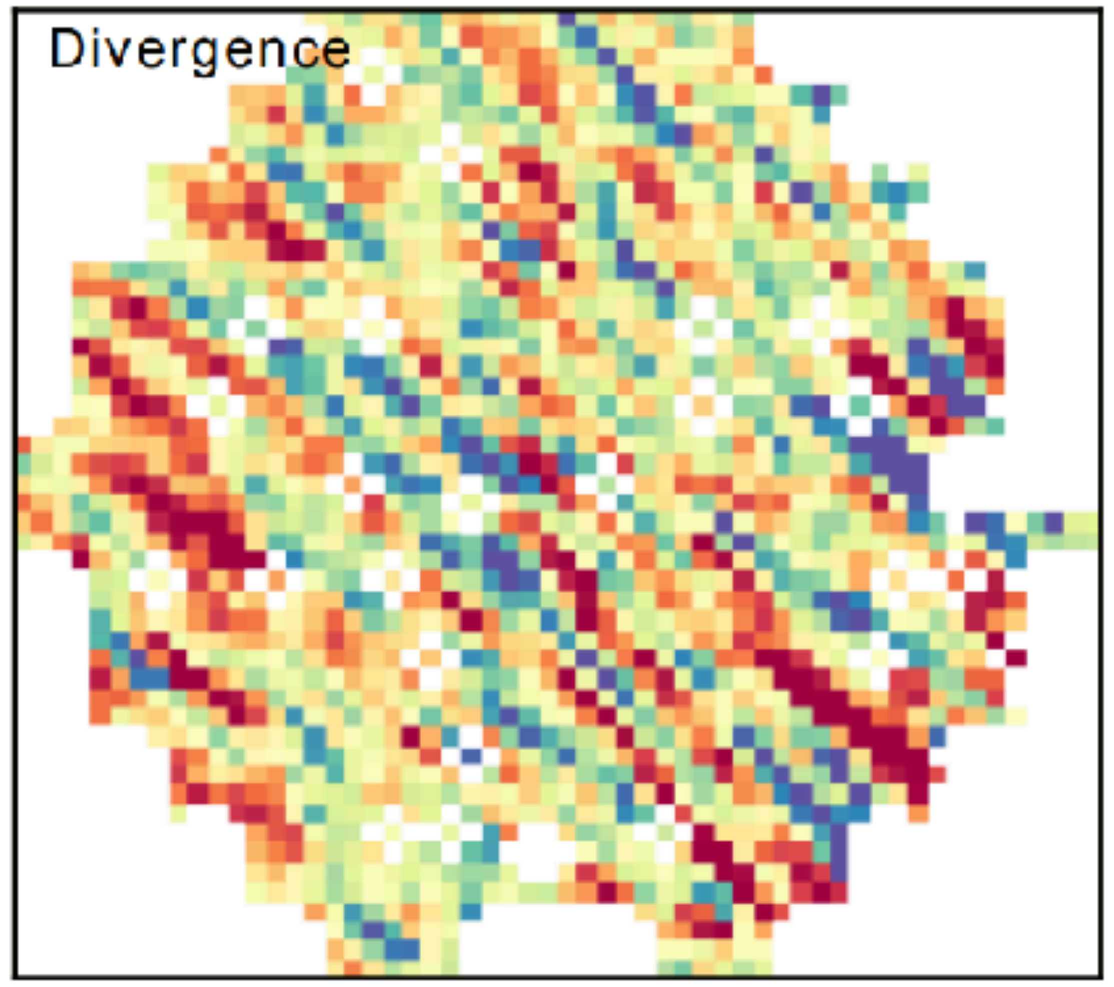


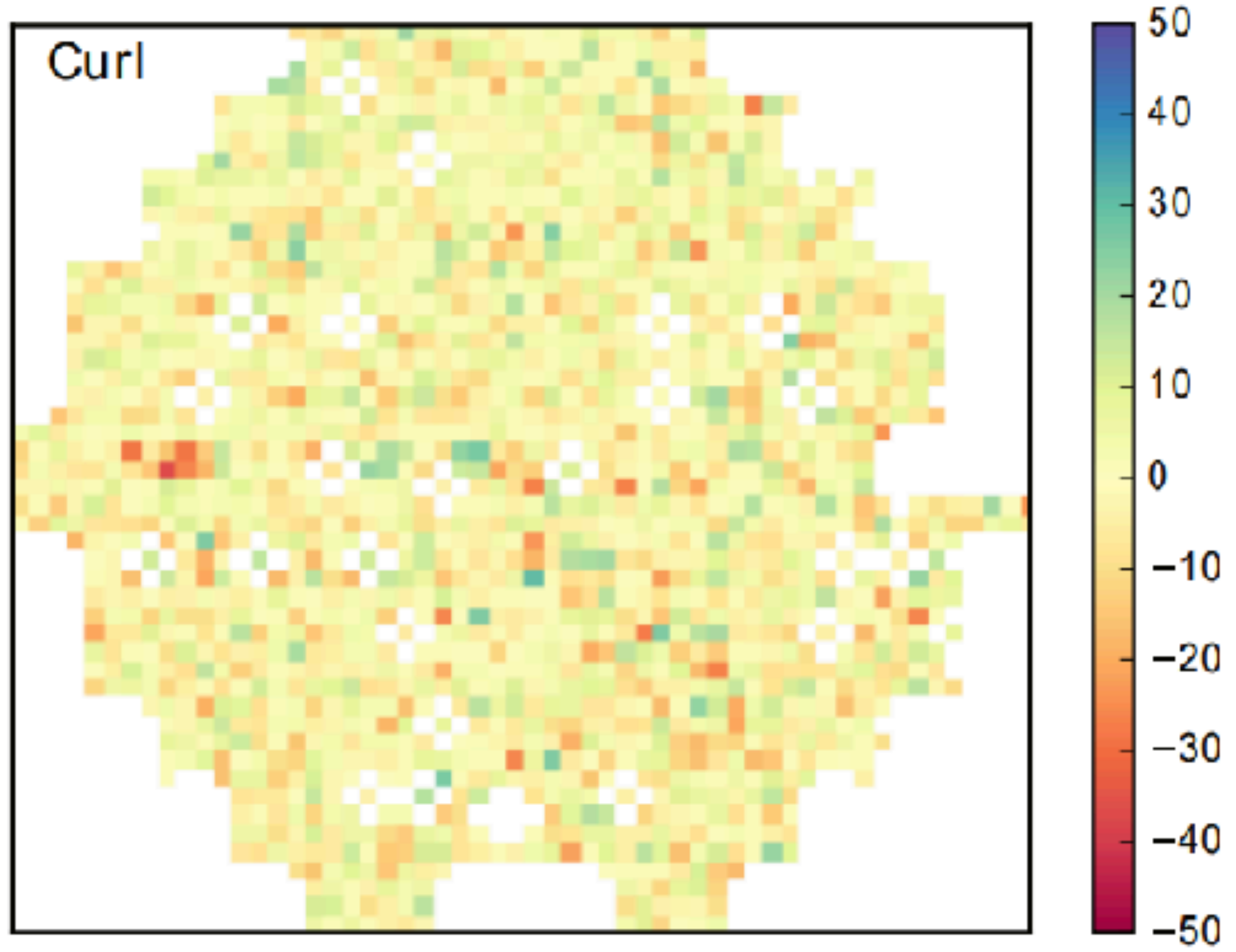
Pixel Distortions

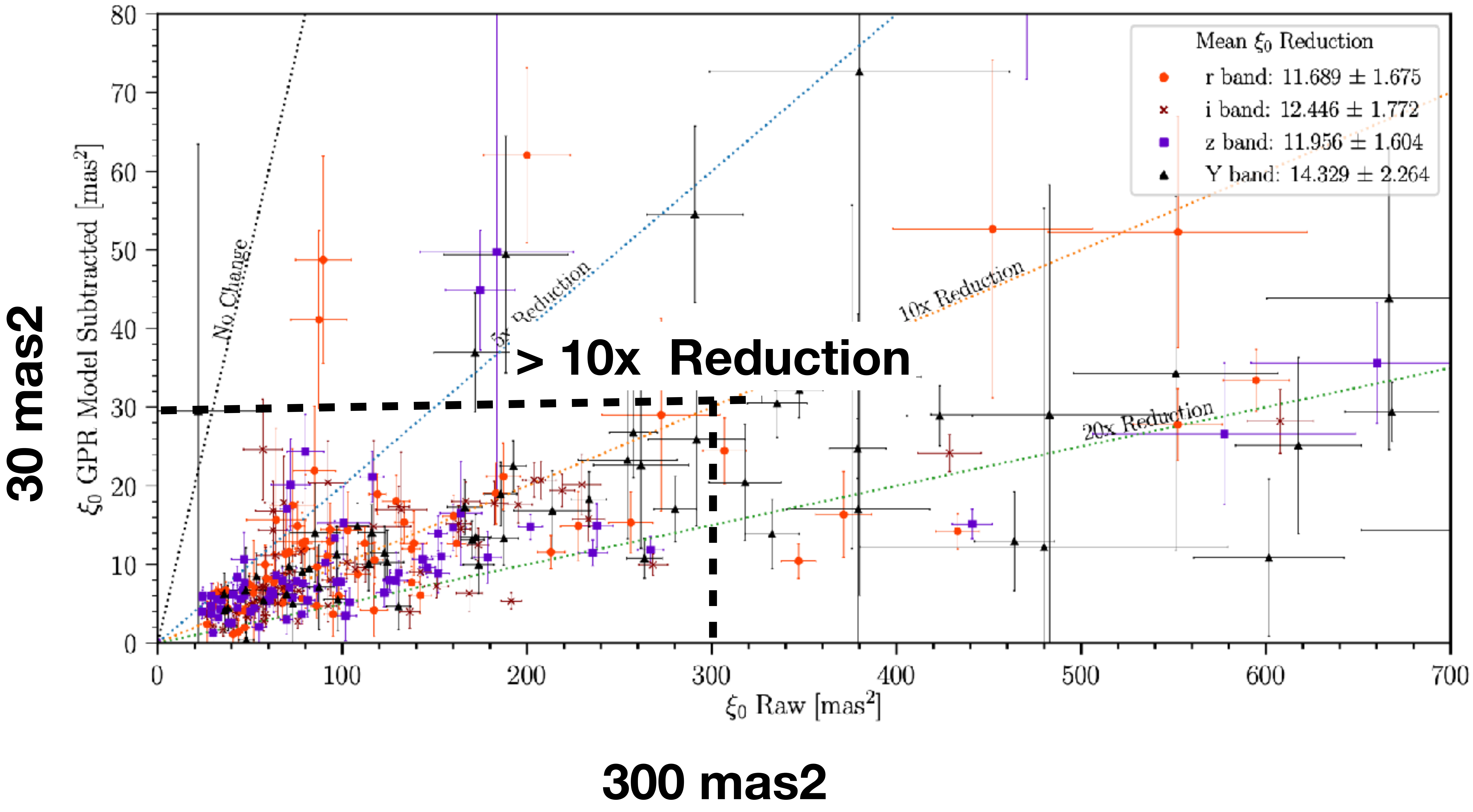
Note change in scale



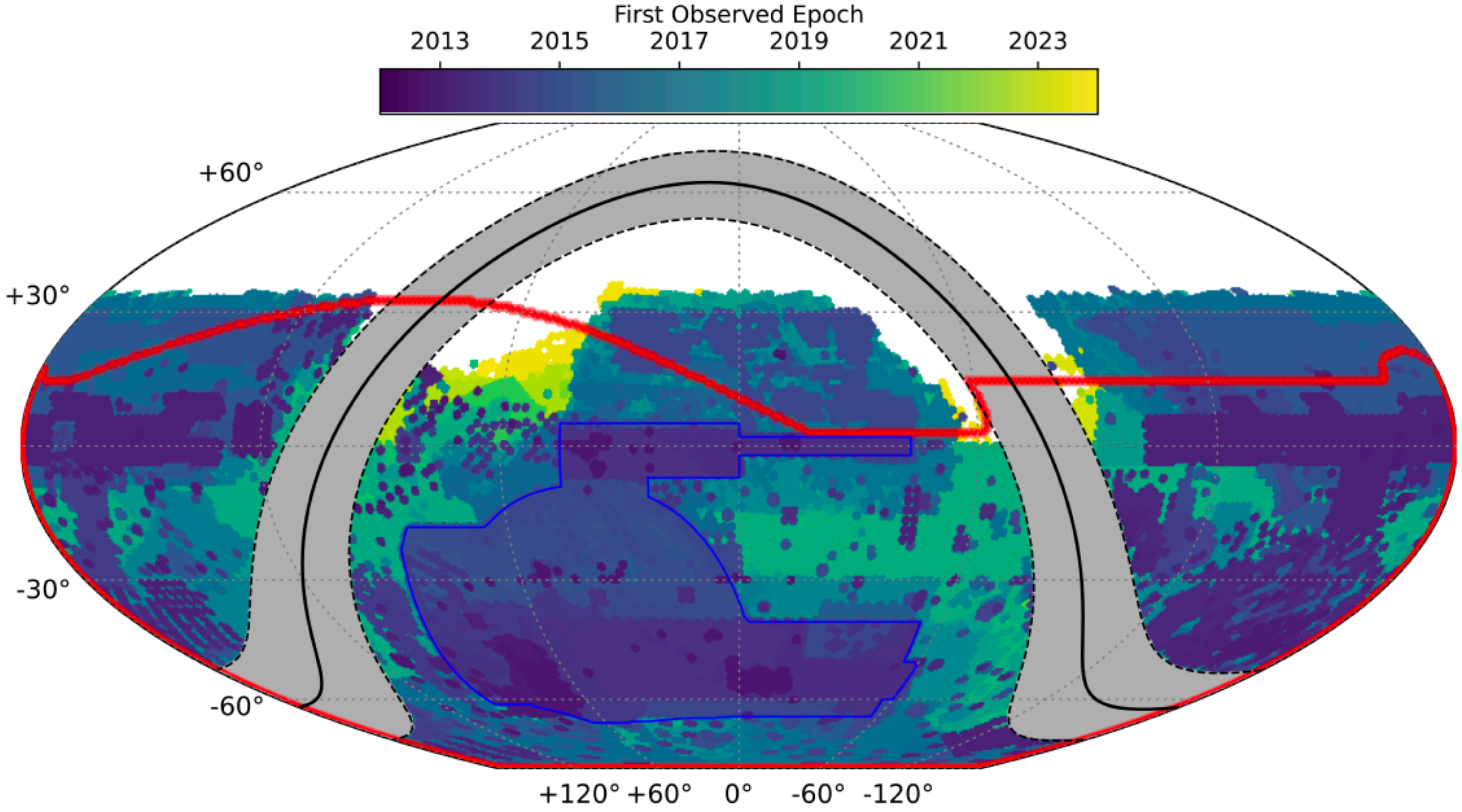
Atmospheric Turbulence

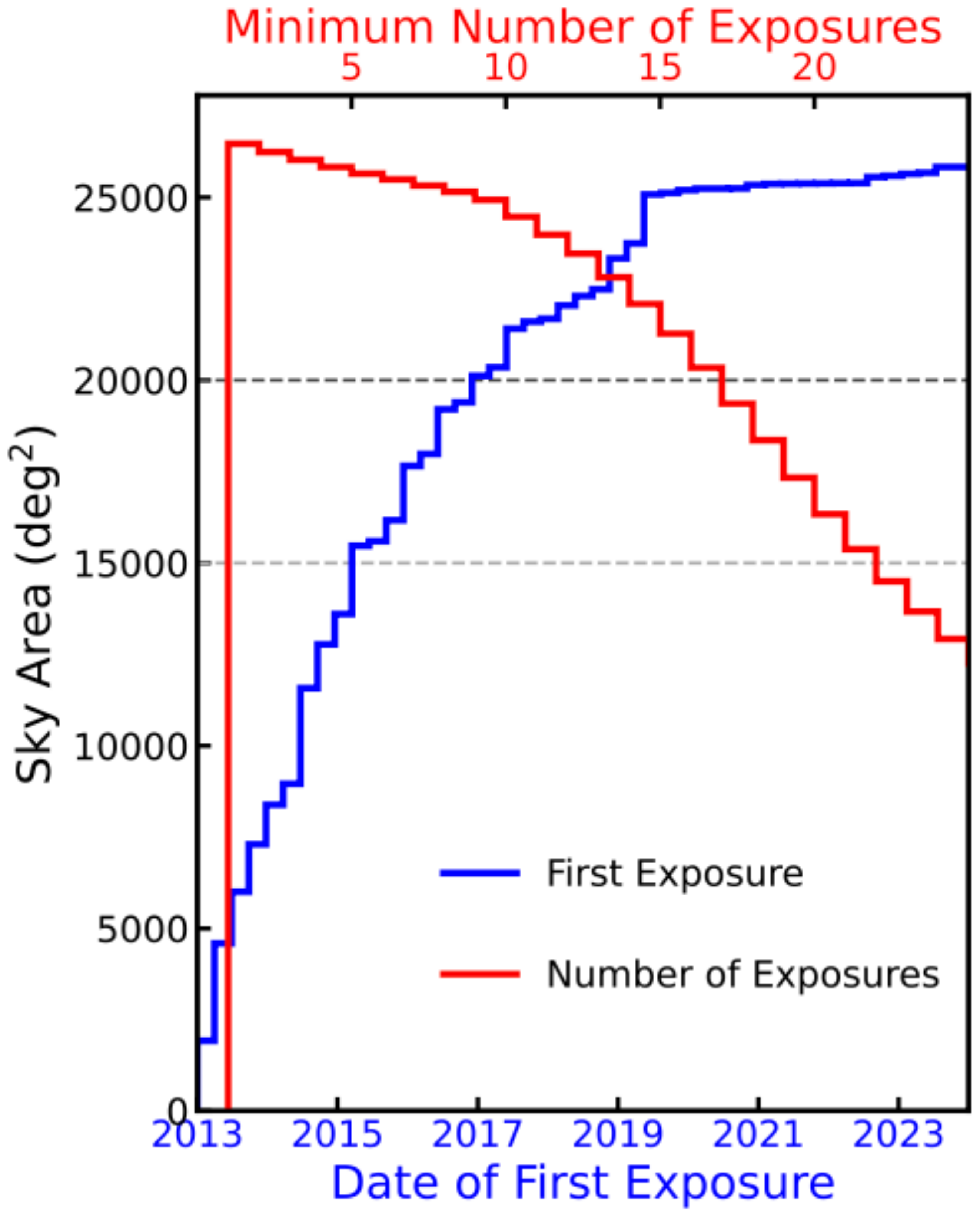






Combining DECam and LSST





Combining DECam and LSST