hw6

March 14, 2025

1 CSCE 633 :: Machine Learning :: Texas A&M University :: Spring 2022

2 Programming Assignment 3 (PA 3)

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3 Support Vector Machines

- **100 points**
- **Due Tuesday, *April* 12, 11:59 pm**

In this assignment, you'll be training support vector machines for classification.

3.0.1 Instructions

- You are allowed to use machine learning libraries such as `scikit-learn` for this assignment. A few of the basic library methods have been already imported for you. Feel free to import any additional methods that you need.
- You are required to complete the functions defined in the code blocks following each question. Fill out sections of the code marked `"YOUR CODE HERE"`.
- You are free to add any number of additional code blocks that you deem necessary.
- Once you've filled out your solutions, submit the notebook on Canvas following the instructions here.
- Do **NOT** forget to type in your name and UIN at the beginning of the notebook.
- Do **NOT** remove any code provided.

[6]:
```
# importing libraries
import sys
import pickle

import pandas as pd
import numpy as np

from sklearn.preprocessing import MinMaxScaler
from sklearn.svm import SVC
from sklearn.model_selection import StratifiedKFold
from sklearn.metrics import mean_absolute_error
```

```python
import matplotlib.pyplot as plt
%matplotlib inline
```

3.1 Question 1 (10 points)

3.2 Data Preprocessing

For this assignment, we will use the Statlog dataset. This database consists of the multi-spectral values of pixels in 3x3 neighbourhoods in a satellite image, and the classification associated with the central pixel in each neighbourhood. The aim is to predict this classification, given the multi-spectral values. In the sample database, the class of a pixel is coded as a number. The attributes are numerical, in the range 0 to 255. More information about the database can be found here.

[7]:
```python
# Read the data
df_train = pd.read_csv('./satimage_train.csv')
df_test = pd.read_csv('./satimage_test.csv')
```

[8]:
```python
df_test.head().T
```

[8]:

	0	1	2	3	4
Class	1.000000	1.000000	1.000000	1.000000	1.000000
X1	-0.031250	-0.031250	-0.031250	-0.031250	-0.031250
X2	0.236364	0.309091	0.309091	0.381818	0.381818
X3	0.142857	0.142857	0.261905	0.238095	0.357143
X4	-0.107438	-0.107438	-0.107438	-0.090909	0.041322
X5	0.129032	-0.129032	NaN	-0.225806	NaN
X6	0.163636	0.381818	0.236364	0.309091	0.381818
X7	0.242105	0.242105	0.136842	0.221053	0.326316
X8	-0.015625	-0.093750	-0.093750	-0.015625	0.046875
X9	0.093750	-0.281250	-0.156250	-0.250000	-0.031250
X10	0.320388	0.242718	0.320388	0.475728	0.553398
X11	0.136842	0.242105	0.052632	0.431579	0.536842
X12	-0.093750	-0.093750	-0.156250	0.046875	0.109375
X13	-0.161290	-0.387097	-0.258065	-0.258065	-0.032258
X14	0.076923	0.153846	0.576923	0.230769	0.480769
X15	0.021739	-0.065217	0.391304	0.108696	0.282609
X16	-0.186441	-0.186441	NaN	-0.084746	0.067797
X17	-0.187500	-0.187500	-0.187500	-0.406250	-0.062500
X18	0.087379	0.417476	0.417476	0.242718	0.495146
X19	-0.036145	0.253012	0.253012	0.060241	0.349398
X20	-0.235772	-0.105691	-0.105691	-0.138211	0.008130
X21	-0.169231	-0.169231	-0.169231	-0.384615	-0.046154
X22	0.165049	0.339806	0.339806	0.165049	0.417476
X23	0.200000	0.288889	0.022222	0.200000	0.377778
X24	-0.040000	0.008000	-0.104000	-0.072000	0.008000
X25	-0.156250	-0.500000	-0.156250	-0.281250	0.093750
X26	0.096154	0.038462	0.634615	0.153846	0.557692

```
X27   -0.080460  0.011494  0.517241  0.172414  0.402299
X28   -0.183333 -0.250000  0.066667 -0.150000       NaN
X29   -0.138462 -0.384615 -0.138462 -0.476923 -0.015385
X30    0.128713  0.128713  0.504950  0.267327  0.603960
X31   -0.010526 -0.094737  0.347368  0.221053  0.347368
X32   -0.156250 -0.218750 -0.046875 -0.062500  0.015625
X33   -0.406250 -0.406250 -0.281250 -0.500000  0.093750
X34   -0.029126  0.184466  0.242718  0.165049  0.572816
X35   -0.157895  0.157895  0.157895  0.136842  0.452632
X36   -0.281250 -0.109375 -0.156250 -0.125000  0.078125
```

3.2.1 To-do steps

1. Remove rows with `NaN` values from `df_train` and `df_test`.
2. Create `X_train` and `X_test` by selecting columns `X1` through `X36`. Create `y_train` and `y_test` by selecting column `Class`.
3. Normalize `X_train` using `MinMaxScaler` from scikit-learn. Then normalize `X_test` on the normalization parameters derived from `X_train`.

[9]:
```
# Step 1: Drop NaN valuesc
df_train = df_train.dropna()
df_test = df_test.dropna()

# Step 2: Create train and test data
# Extract features (X) and labels (y) from training and test data
X_train = df_train.iloc[:, 1:37]  # Select columns X1 through X36
y_train = df_train['Class']

X_test = df_test.iloc[:, 1:37]    # Select columns X1 through X36
y_test = df_test['Class']

# Step 3: Normalize data
scaler = MinMaxScaler()
X_train = scaler.fit_transform(X_train)
X_test = scaler.transform(X_test)
```

[10]:
```
# Convert to binary classification
y_train[y_train != 6] = 0
y_train[y_train == 6] = 1

y_test[y_test != 6] = 0
y_test[y_test == 6] = 1
```

```
/var/folders/0d/lzd4_bln5456tqbpclpkwljc0000gn/T/ipykernel_11422/717146307.py:3:
SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-
```

```
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  y_train[y_train == 6] = 1
/var/folders/0d/lzd4_bln5456tqbpclpkwljc0000gn/T/ipykernel_11422/717146307.py:6:
SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  y_test[y_test == 6] = 1
```

3.3 Question 2 (30 points)

3.4 Hyperparameter Tuning

Consider the binary classification that consists of distinguishing class 6 from the rest of the data points. Use SVMs combined with polynomial kernels to solve this classification problem. For each value of the polynomial degree, $d = 1, 2, 3, 4$, plot the average 5-fold cross-validation error plus or minus one standard deviation as a function of C (let the other parameters of the polynomial kernels be equal to their default values) on the training data.

Choose a minimum of 5 C values spread across a wide range

Report the best value of the trade-off constant C measured on the training internal cross-validation.

[11]:
```python
def cross_validation_score(X, y, c_vals, n_folds, d_vals):
    """
    Calculates the cross validation error and returns its mean and standard␣
    ↪deviation.

    Args:
        X: features
        y: labels
        c_vals: list of C values
        n_folds: number of cross-validation folds
        d_vals: list of degrees of the polynomial kernel

    Returns:
        Tuple of (list of error_mean, list of error_std)
    """

    error_mean = np.zeros((len(c_vals), len(d_vals)))
    error_std = np.zeros((len(c_vals), len(d_vals)))

    skf = StratifiedKFold(n_splits=n_folds)

    for i, c in enumerate(c_vals):
        for j, d in enumerate(d_vals):
            errors = []
```

```
            for train_index, val_index in skf.split(X, y):
                X_train_fold, X_val_fold = X[train_index], X[val_index]
                y_train_fold, y_val_fold = y[train_index], y[val_index]

                model = SVC(C=c, kernel='poly', degree=d)
                model.fit(X_train_fold, y_train_fold)
                y_pred = model.predict(X_val_fold)
                error = mean_absolute_error(y_val_fold, y_pred)
                errors.append(error)

            error_mean[i, j] = np.mean(errors)
            error_std[i, j] = np.std(errors)

    return error_mean, error_std
```

[12]:
```
######################
#   YOUR CODE HERE   #
######################
n_folds = 5
d_vals = [1, 2, 3, 4]
c_vals = [0.01, 0.1, 1, 10, 100]  # Provide a list of C values
```

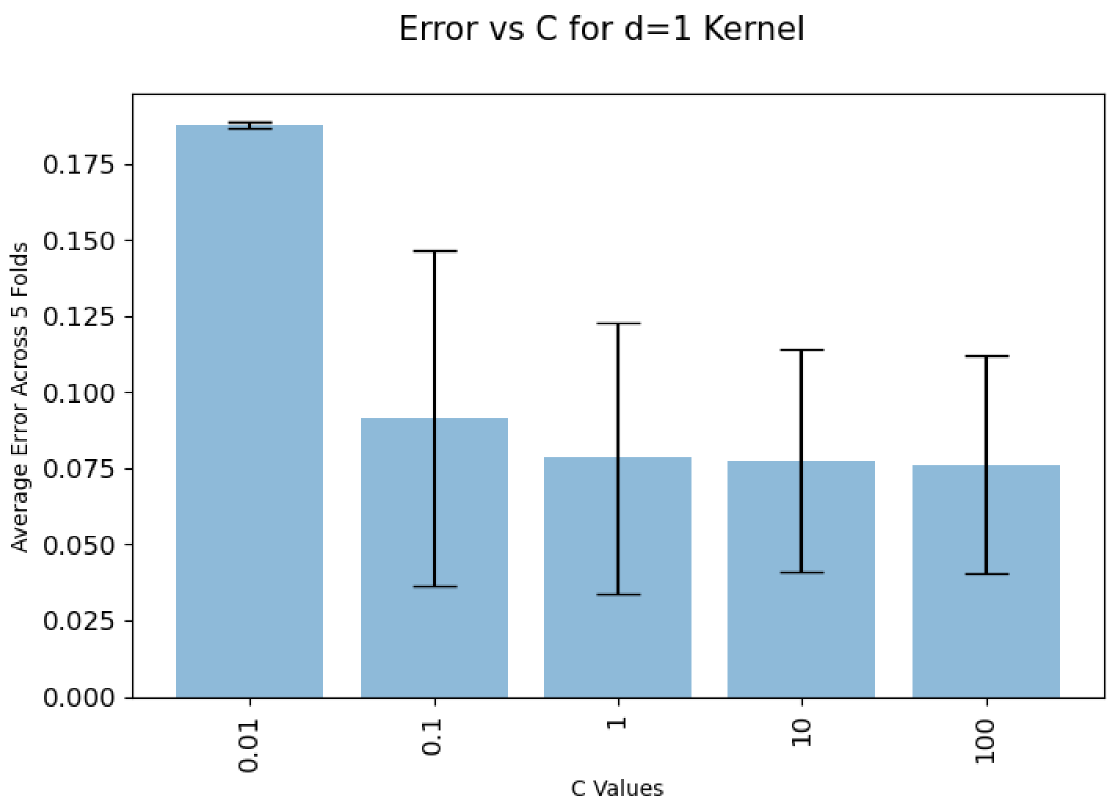
[13]:
```
# Run cross validation to find the best hyperparameters
X_train_array = np.array(X_train)
y_train_array = np.array(y_train)
error_mean, error_std = cross_validation_score(X_train_array, y_train_array,␣
 ↪c_vals, n_folds, d_vals)

# Find the best C value for each degree
best_c_indices = np.argmin(error_mean, axis=0)
best_c_values = [c_vals[i] for i in best_c_indices]

print(f"Cross-validation completed for {len(c_vals)} C values and {len(d_vals)}␣
 ↪polynomial degrees")
print(f"Best C values for each degree: {list(zip(d_vals, best_c_values))}")
```

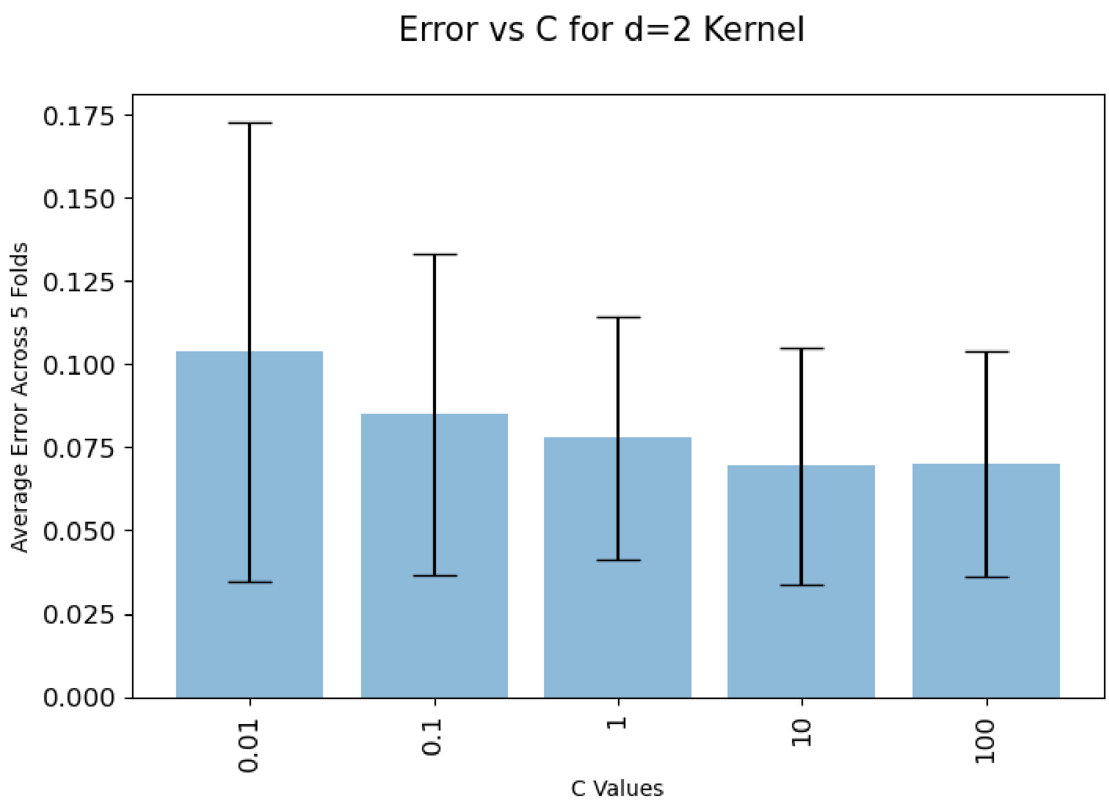
```
Cross-validation completed for 5 C values and 4 polynomial degrees
Best C values for each degree: [(1, 100), (2, 10), (3, 10), (4, 1)]
```

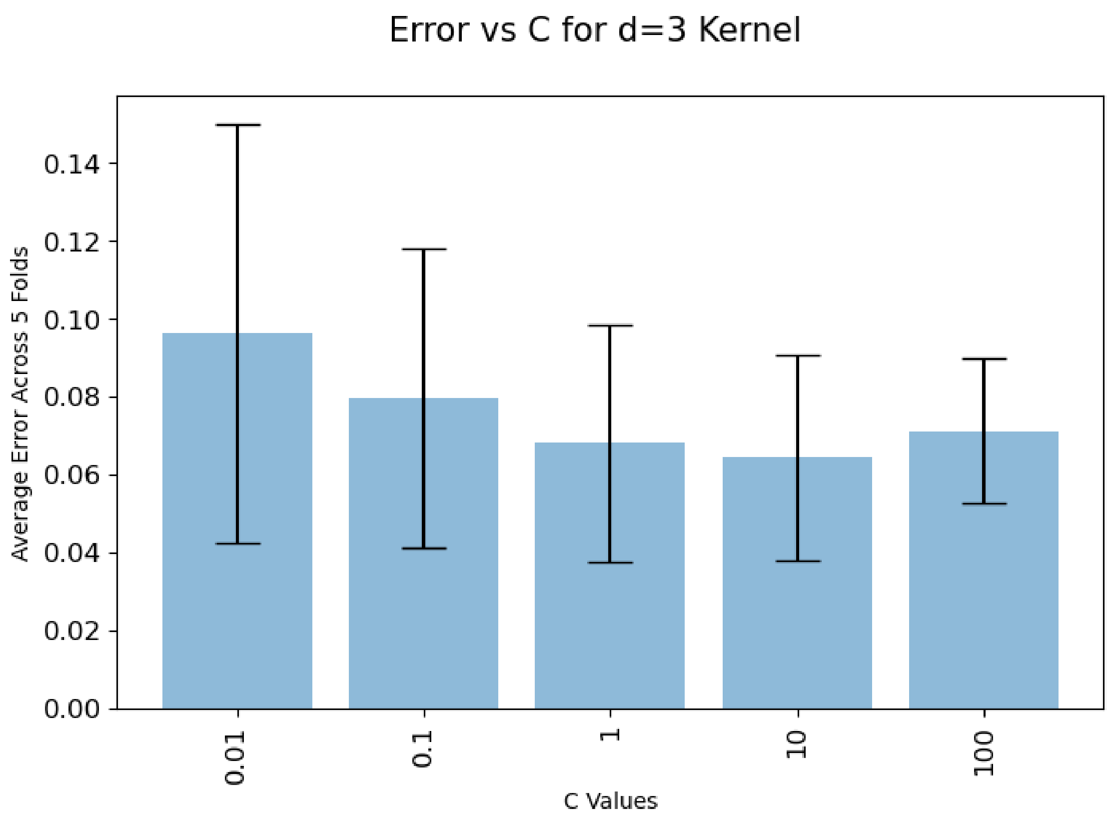
Plot the average cross validation error.

[14]:
```
for i,d_val in enumerate(d_vals):
    plt.rcParams.update({'font.size': 12})
    plt.figure(figsize = (8,5))
    plt.bar(range(len(c_vals)), error_mean[:,i],
            yerr=error_std[:,i],
            align='center',
            alpha=0.5,
```

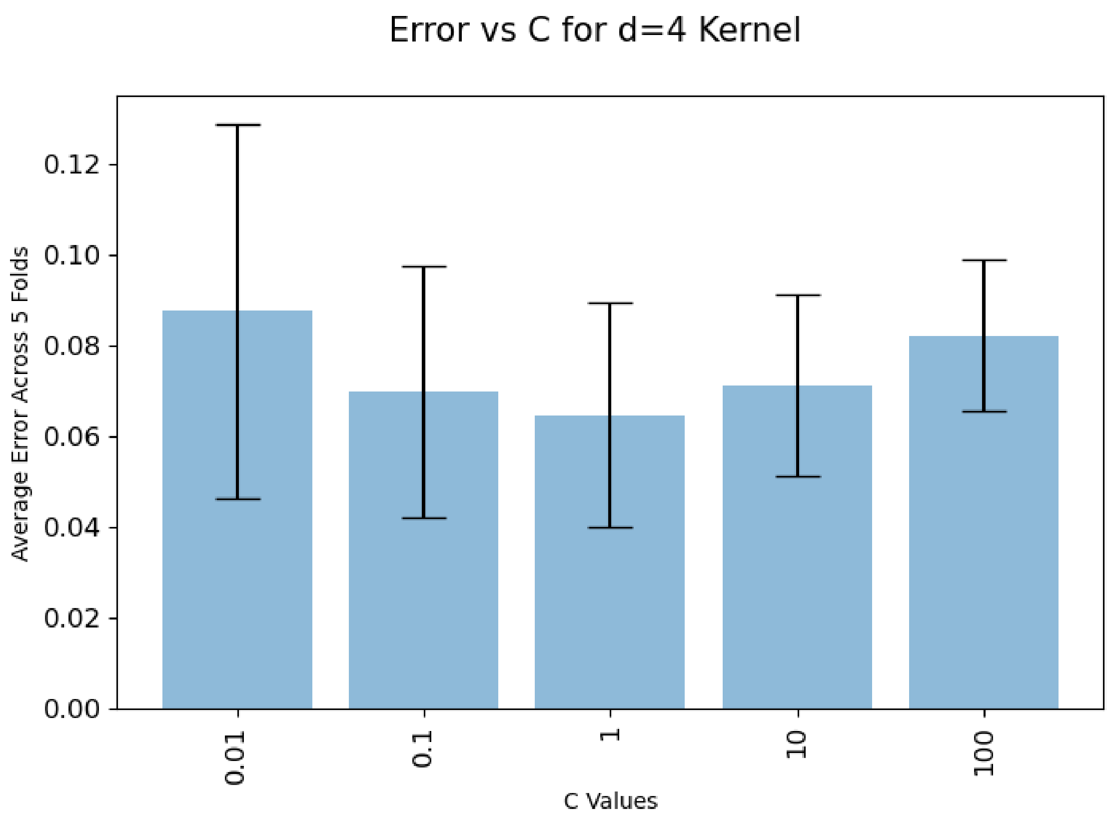
```
            ecolor='k',
            capsize=10,
            label = "Average Error")
    plt.suptitle('Error vs C for d={} Kernel'.format(d_val), fontsize=15)
    plt.xlabel('C Values', fontsize=10)
    plt.xticks(range(len(c_vals)), c_vals, rotation='vertical')
    plt.ylabel('Average Error Across {} Folds'.format(n_folds), fontsize=10)
```




Error vs C for d=2 Kernel
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
Average Error Across 5 Folds
0.01
0.1
1
10
100
C Values



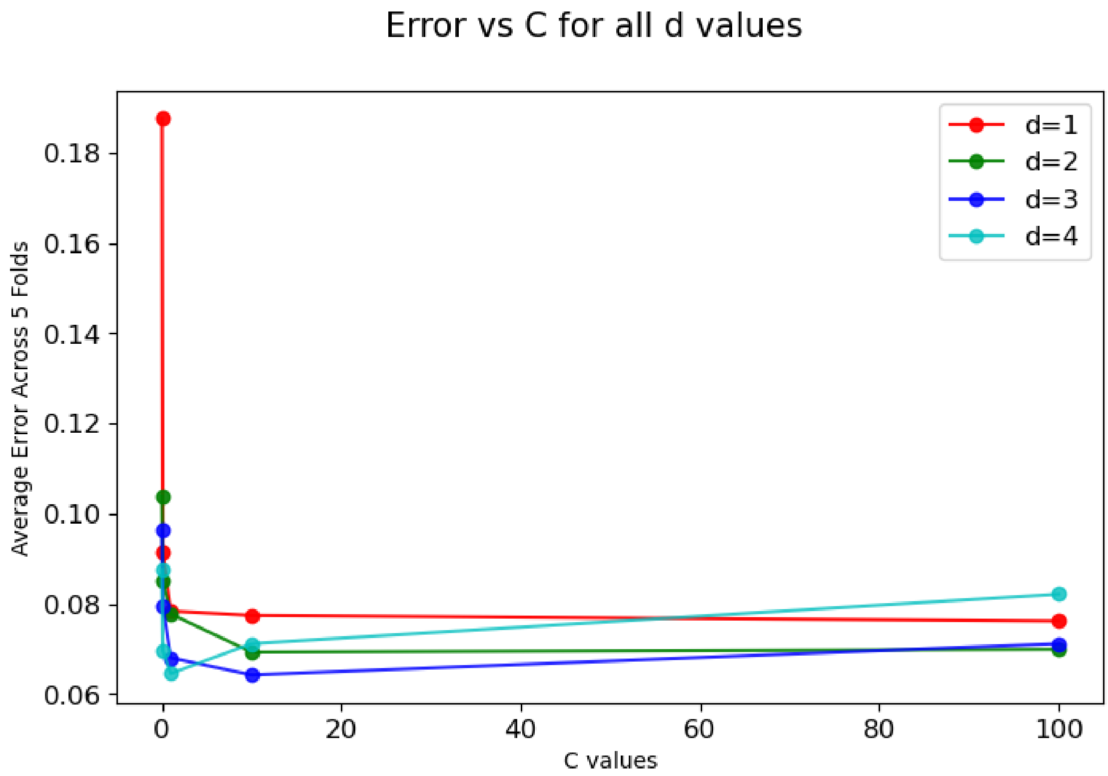
Error vs C for d=3 Kernel
Average Error Across 5 Folds
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.01
0.1
1
10
100
C Values



Plot $(C; d)$ pairs with their corresponding cross validation errors.

[15]:
```
plt.rcParams.update({'font.size': 12})
plt.figure(figsize = (8,5))
colors = ['r', 'g', 'b', 'c', 'y']
for i,d_val in enumerate(d_vals):
    plt.plot(c_vals, error_mean[:,i],
             marker='o',
             color=colors[i%5],
             alpha=1 - 0.2 * d_val/len(d_vals),
             label = "d={}".format(d_val))
plt.suptitle('Error vs C for all d values', fontsize=15)
plt.xlabel('C values', fontsize=10)
plt.ylabel('Average Error Across {} Folds'.format(n_folds), fontsize=10)
plt.legend()
```

[15]:
```
<matplotlib.legend.Legend at 0x133729600>
```



3.5 Question 3 (30 points)

3.6 Model Training and Testing

Build models on the full training data on the best C value you found previously for each d value using the 5-fold cross validation.

You need to return test error, number of support vectors, number of margin violations, and margin size

A data point (i.e., x) is said to violate margin if **distance of data point from hyperplane < margin size**. Therefore, number of total margin violations is count of such data points present in dataset.

Hint: Margin Size: Use the dual coefficients (alpha values) of SVM along with support vectors to calculcate margin. Please refer to slide 17 in slide deck 13 for more details.

[16]:
```
def build_model(X_train, y_train, X_test, y_test, c_vals, d_vals):
    """
    Trains model on a dataset for given values of C and d. Returns the error on␣
↪the test data,
    the number of support vectors, the number of margin violations, and the␣
↪margin size.
```

```
    Args:
        X_train: features in training data
        y_train: train labels
        X_test: features in test data
        y_test: test labels
        c_vals: list of C values
        d_vals: list of degrees of the polynomial kernel

    Returns:
        Tuple of (error_test, support_vectors, margin_violations, margin_size) ␣
↪
    """
    error_test = np.zeros(len(d_vals))
    support_vectors = np.zeros(len(d_vals))
    margin_violations = np.zeros(len(d_vals))
    margin_size = np.zeros(len(d_vals))

    for i, (d, c) in enumerate(zip(d_vals, c_vals)):
        # Train SVM model with the given C and d values
        model = SVC(C=c, kernel='poly', degree=d)
        model.fit(X_train, y_train)

        # Calculate test error
        y_pred = model.predict(X_test)
        error_test[i] = mean_absolute_error(y_test, y_pred)

        # Count number of support vectors
        support_vectors[i] = len(model.support_)

        # Calculate margin size
        # For a linear SVM, margin = 1/||w||, for kernel SVMs we use dual␣
↪formulation
        # We extract the dual coefficients (alpha values)
        dual_coef = model.dual_coef_
        support_vectors_data = model.support_vectors_

        # Calculating the margin size (1/||w||)
        # For kernel SVM, ||w||² = sum_i sum_j alpha_i * alpha_j * K(x_i, x_j)
        margin_size[i] = 1.0 / np.sqrt(np.sum(np.square(dual_coef)))

        # Calculate margin violations
        # A data point violates margin if its distance from hyperplane < margin␣
↪size
        decision_values = np.abs(model.decision_function(X_train))
        violations = np.sum(decision_values < margin_size[i])
        margin_violations[i] = violations
```

```
    return error_test, support_vectors, margin_violations, margin_size
```

[17]:
```
######################
#   YOUR CODE HERE   #
######################
d_vals= [1,2,3,4] # List of degrees
c_vals = [0.01, 0.1, 1, 10, 100]# Provide a list of corresponding best C values
```

[18]:
```
error_test, support_vectors, margin_violations, margin_size =␣
 ↪build_model(X_train, y_train,
                                                                        ␣
 ↪X_test, y_test,
                                                                        ␣
 ↪c_vals, d_vals)
```

Plot the test errors for each model, as a function of d.

[26]:
```
plt.rcParams.update({'font.size': 12})
plt.figure(figsize = (8,5))
plt.plot(d_vals, error_test ,marker='o', color='b')
plt.suptitle('Test Error vs d values', fontsize=20)
plt.xlabel('d values', fontsize=10)
plt.ylabel('Test Error', fontsize=10)
```

[26]:
```
Text(0, 0.5, 'Test Error')
```

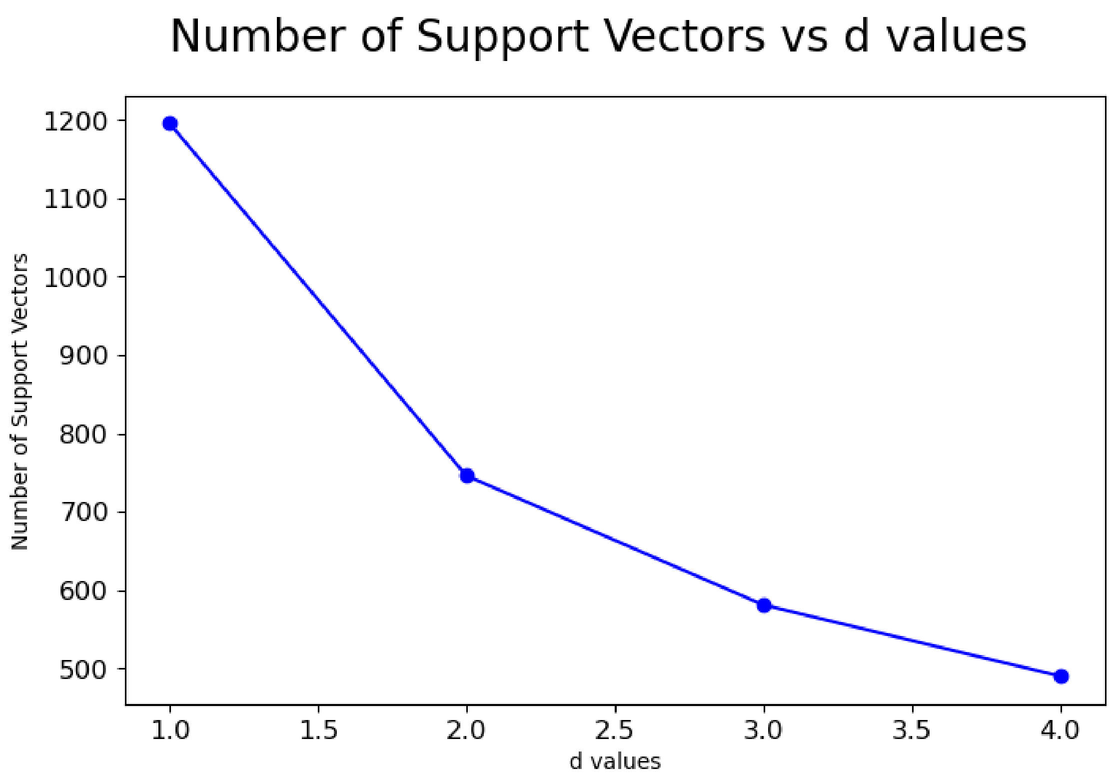
3.7 Question 4 (10 points)

3.8 Number of support vectors

Plot the number of support vectors obtained as a function of d.

[25]:
```
plt.rcParams.update({'font.size': 12})
plt.figure(figsize = (8,5))
plt.plot(d_vals, support_vectors, marker='o', color='b')
plt.suptitle('Number of Support Vectors vs d values', fontsize=20)
plt.xlabel('d values', fontsize=10)
plt.ylabel('Number of Support Vectors', fontsize=10)
```

[25]:
```
Text(0, 0.5, 'Number of Support Vectors')
```



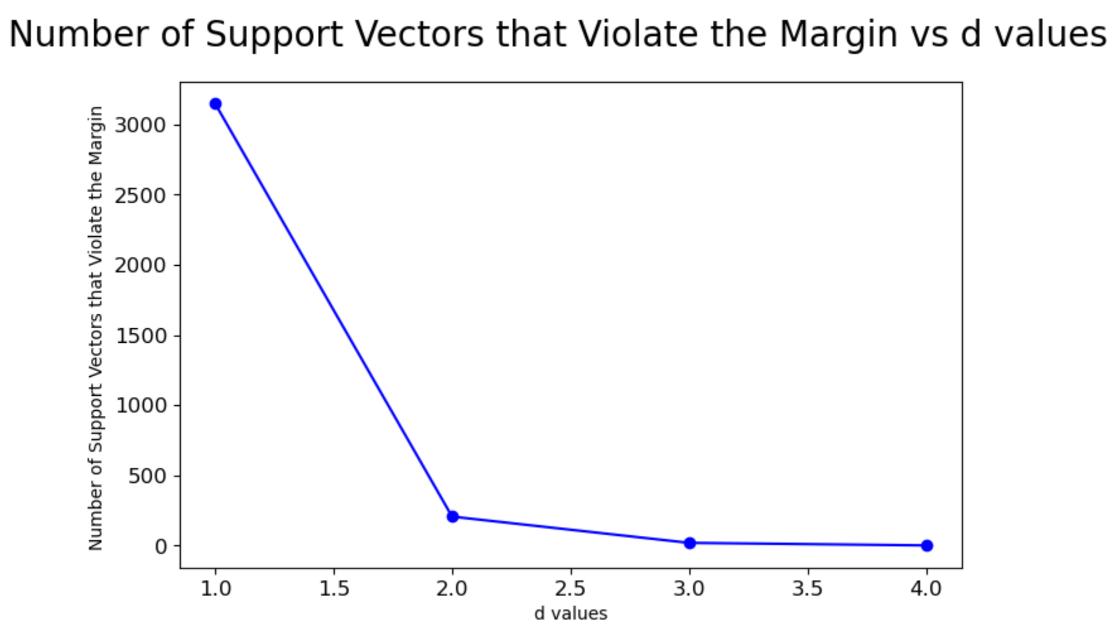
3.9 Question 5 (10 points)

3.10 Number of Margin Violations

Plot the number of support vectors that violate the margin hyperplanes as a function of $d.

```
[24]: plt.rcParams.update({'font.size': 12})
plt.figure(figsize = (8,5))
plt.plot(d_vals, margin_violations, marker='o', color='b')
plt.suptitle('Number of Support Vectors that Violate the Margin vs d values',␣
  ↪fontsize=20)
plt.xlabel('d values', fontsize=10)
plt.ylabel('Number of Support Vectors that Violate the Margin ', fontsize=10)
```

```
[24]: Text(0, 0.5, 'Number of Support Vectors that Violate the Margin ')
```



3.11 Question 6 (10 points)

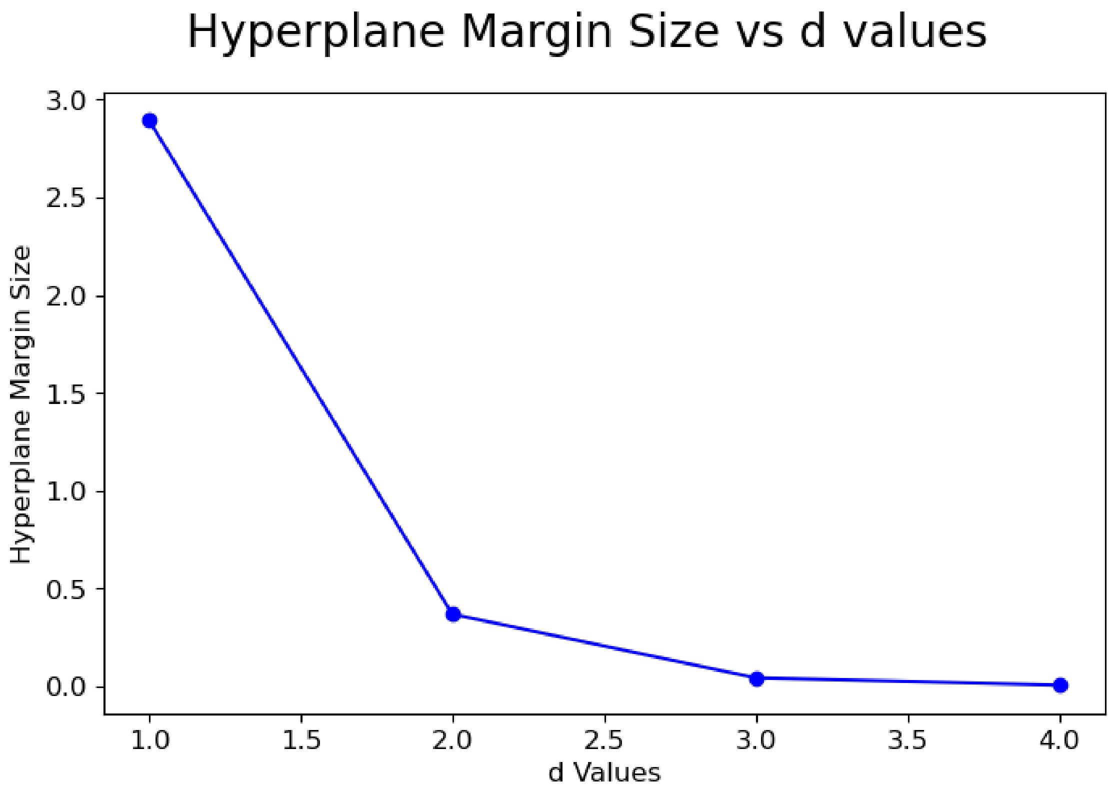
3.12 Margin Size vs Support Vectors

Explain how the parameter d infuences the model fit (plot the margin size as a function of $d).

Make sure to plot **AND** explain.

```
[23]: plt.rcParams.update({'font.size': 12})
plt.figure(figsize = (8,5))
plt.plot(d_vals, margin_size, marker='o', color='b')
plt.suptitle('Hyperplane Margin Size vs d values', fontsize=20)
plt.xlabel('d Values', fontsize=12)
plt.ylabel('Hyperplane Margin Size', fontsize=12)
```

```
[23]: Text(0, 0.5, 'Hyperplane Margin Size')
```



[27]:
```
## TYPE YOUR ANSWER BELOW
# From the plot, we can observe that margin size decreases dramatically as the
 ↪polynomial degree d increases.
# Using higher degree polynomials allow more flexibile decision boundaries to
 ↪help fit data with complex patterns.
# As the degrees go up the number of support vectors decrease, which means we
 ↪need fewer points to define the boundary, but this comes
# at the cost of a very tight margin.
```