Full Title of the Talk

Kelsey Maass and Brisa Davis

AMATH 574
Group 6

March 9, 2015

Introduction

Kelsey

LWR

Kelsey

PW

Intro, Kelsey. Eigenvectors and values

PW: Hugoniot Loci

From this system, the Rankine-Hugoniot condition

$$s(q_* - q) = f(q_*) - f(q)$$

gives us the two equations

$$\begin{aligned} s(\rho_* - \rho) &= \rho_* v_* - \rho v, \\ s(\rho_* v_* - \rho v) &= \rho_* v_*^2 + p(\rho) - \rho v^2 - p(\rho). \end{aligned}$$

From these two equations the equation for the Hugoniot loci can be found to be

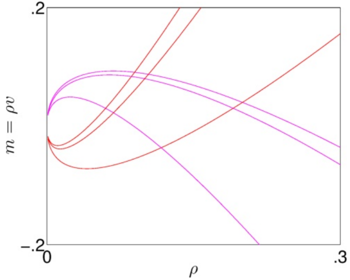
$$\rho v = \rho v_* \pm \rho \sqrt{(\rho - \rho_*)\left(p(\rho) - p(\rho_*)\right)/\rho_* \rho}.$$

PW: Hugoniot Loci

Plotting this equation

$$\rho v = \rho v_* \pm \rho \sqrt{(\rho - \rho_*)\left(p(\rho) - p(\rho_*)\right)/\rho_* \rho}.$$

gives



Here, λ_1 loci are shown in magenta and λ_2 loci are shown in red.

PW: Integral Curves

Using $p(\rho) = \rho^\gamma$ for $\gamma \neq 1$, since this is the value used in the AR model we will examine later, the equations for the integral curves can be found to be

$$\rho v = \rho v_* + \frac{2\rho}{\gamma - 1}\left(\sqrt{p'(\rho_*)} - \sqrt{p'(\rho)}\right)$$

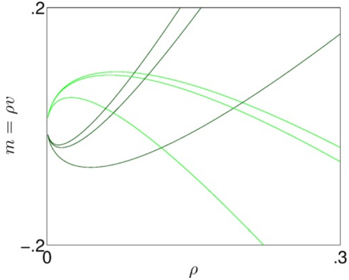
for λ_1, and

$$\rho v = \rho v_* + \frac{2\rho}{\gamma - 1}\left(\sqrt{p'(\rho)} - \sqrt{p'(\rho_*)}\right)$$

for λ_2.

PW: Integral Curves

Plotting these curves gives



Here, λ_1 integral curves are shown in light green and λ_2 integral curves are shown in dark green.

PW: Areas of Validity

Restrictions on the way that λ must vary across a wave allow us to identify the regions of the Hugoniot loci and integral curves that are valid for any given point.

Shocks:

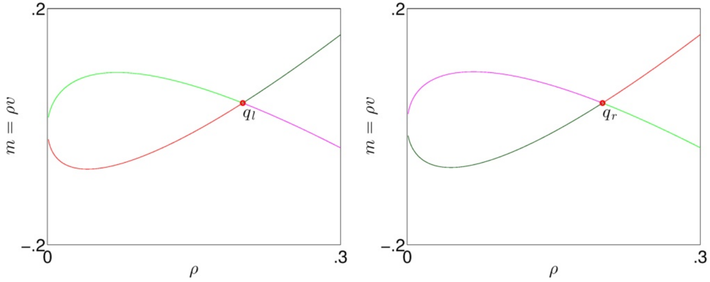
$$\begin{aligned} q_l \rightarrow q_r \quad &: \quad \lambda \text{ must decrease} \\ q_r \rightarrow q_l \quad &: \quad \lambda \text{ must increase} \end{aligned}$$

Rarefaction:

$$\begin{aligned} q_l \rightarrow q_r \quad &: \quad \lambda \text{ must increase} \\ q_r \rightarrow q_l \quad &: \quad \lambda \text{ must decrease} \end{aligned}$$

PW: Areas of Validity

Using this, we can determine the valid curves:



(a) Valid curves for a point q_l. (b) Valid curves for a point q_r.

Here, shocks are shown in red or magenta and rarefaction waves are shown in either light or dark green.

AR

Intro, Kelsey. Eigenvectors and values

AR: Hugoniot Loci

In order to consider the Hugoniot loci for this system we must first write the system in conservation form. With some manipulation, we can get the system

$$\begin{aligned} &\partial_t \rho + \partial_x(\rho v) = 0, \\ &\partial_t \left(\rho \left(v + p(\rho)\right)\right) + \partial_x \left(\rho v \left(v + p(\rho)\right)\right) = 0. \end{aligned}$$

This can be rewritten as $q_t + f(q)_x =$ by defining

$$q = \begin{bmatrix} \rho \\ y \end{bmatrix}, \qquad f(q) = \begin{bmatrix} v\rho \\ vy \end{bmatrix}.$$

where $y = \rho \left(v + p(\rho)\right)$.

AR: Hugoniot Loci

From the Rankine-Hugoniot condition

$$s(q_* - q) = f(q_*) - f(q).$$

we get the two equations

$$\begin{aligned} s(\rho_* - \rho) &= v_* \rho_* - v\rho, \\ s(y_* - y) &= v_* y_* - vy. \end{aligned}$$

From these equations, the equations for the Hugoniot loci can be found to be

$$\begin{aligned} \rho v &= \rho\left(v_* + p(\rho_*)\right) - \rho p(\rho)), \\ \rho v &= \rho_* v_*, \end{aligned}$$

corresponding to λ_1 and λ_2, respectively.

AR: Integral Curves

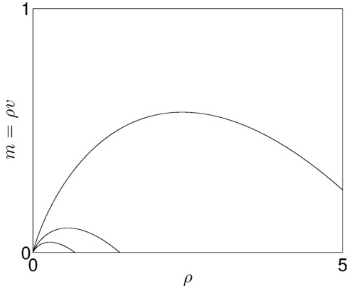
In order to consider the integral curves for this system [1] works from the system

$$\begin{aligned} &\partial_t \rho + \partial_x(\rho v) = 0, \\ &\partial_t v + \left(v - p'(\rho)\rho\right) \partial_x v = 0. \end{aligned}$$

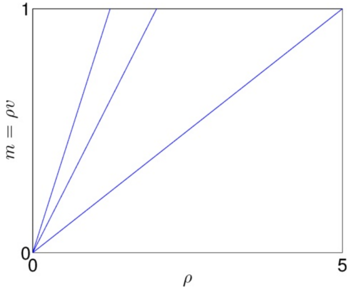
After working with these equations to find the integral curves, you find that they coincide with the Hugoniot loci.

AR: Curves

Plotting these curves gives:



(a) λ_1-curves in the M plane.



(b) λ_2-curves in the M plane.

AR: Areas of Validity

Restrictions on the way that λ must vary across a wave allow us to identify the regions of the Hugoniot loci and integral curves that are valid for any given point.

Shocks:

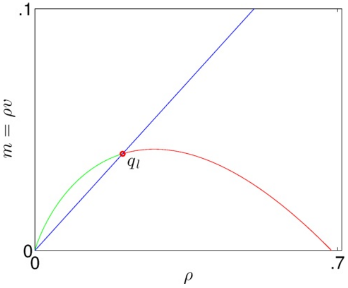
$$\begin{aligned} q_l \to q_r \quad &: \quad \lambda \text{ must decrease} \\ q_r \to q_l \quad &: \quad \lambda \text{ must increase} \end{aligned}$$

Rarefaction:

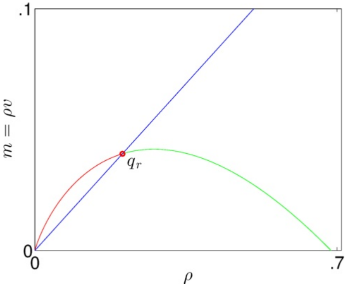
$$\begin{aligned} q_l \to q_r \quad &: \quad \lambda \text{ must increase} \\ q_r \to q_l \quad &: \quad \lambda \text{ must decrease} \end{aligned}$$

AR: Areas of Validity

Using this, we can determine the valid curves:



(a) Valid waves for the point q_l.



(b) Valid waves for the point q_r.

Here, shocks are shown in red, rarefaction waves are shown in green, and the contact discontinuity is shown in blue.

Examples

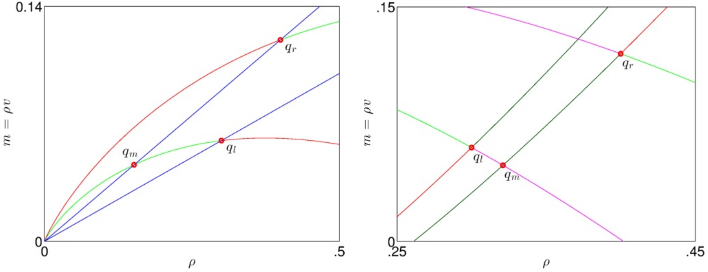
To illustrate a couple of examples where the AR model preforms better than the PW model, consider two different examples:

Table: Initial values used in examples 1 and 2.

	v_l	ρ_l	v_r	ρ_r
Example 1	0.2	.03	0.3	0.4
Example 2	0	0	0.3	0.4

Example 1

The solution to the Riemann problem is:



(a) Solution from the AR model. (b) Solution from the PW model.

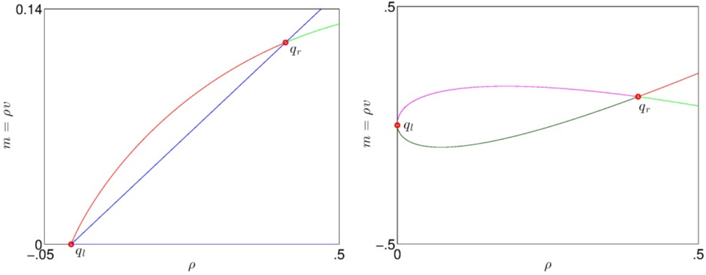
Example 1

AR model:

PW Model:

Example 2

The solution to the Riemann problem is:



(a) Solution from the AR model. (b) Solution from the PW model.

Example 2

AR model:

PW Model:

Questions?

A. Aw and M. Rascle.
Resurrection of “second order” models of traffic flow.
SIAM Journal of Applied Math, 60(3):916938, 2000.