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REQUIEM FOR SECOND-ORDER FLUID APPROXIMATIONS OF TRAFFIC FLOW

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Abstract—Although the "first order" continuum theory of highway traffic proposed by Lighthill and Whitham (1955) and Richards (1956)—the LWR model—can predict some things rather well, it is also known to have some deficiencies. In an attempt to correct some of these, "higher order" theories have been proposed starting in the early 70s. Unfortunately, the usefulness of these improvements can be questioned. This note describes the logical flaws in the arguments that have been advanced to derive higher order continuum models, and shows that the proposed high order modifications lead to a fundamentally flawed model structure. The modifications can actually make things worse.

As an *illustration* of this, it is shown that any continuum model of traffic flow that smooths out all discontinuities in density will predict negative flows and negative speeds (i.e., "wrong way travel") under certain conditions. Such unreasonable predictions are made by all existing models formulated as a quasilinear system of partial differential equations in speed, density, and (sometimes) other variables but not by the LWR model.

The note discusses the available empirical evidence and ends with a (hopefully positive) commentary on what can be accomplished with first-order models.

1. BACKGROUND

The starting point of our discussion is the first order fluid approximation of traffic flow dynamics proposed by Lighthill and Whitham (1955) and Richards (1956)—the LWR model. This model provides a coarse description of traffic behavior for a single one-way road using three variables that vary in time and space: flow, q, density, k, and speed, v.

For a highway without entrances or exits, any model must conserve vehicles between any two locations x_1 and x_2 at any time t. If traffic flows in the direction of increasing x and we take $x_2 > x_1$, then any model must satisfy:

$$\frac{\partial}{\partial t}\int_{x_1}^{x_2} k(x,t)dx + q(x_2,t) - q(x_1,t) = 0. \tag{1}$$

In addition to (1), the LWR model assumes that the relation between flow and density observed under steady state conditions also holds when flow and density vary with x and/or t; i.e., for a homogenous highway:

$$q(x,t) = Q(k(x,t)) \tag{2}$$

where Q is a differentiable nonnegative, function that is zero for $k = 0$ and $k = k_j$ (the "jam" density). Equations (1) and (2) completely define the model. (The third variable, v, is, by definition, $v = q/k$.)

In the rest of this presentation the independent variables (x and t) are omitted from the notation and, as is customary, partial derivatives are denoted by subscripts. If k is differentiable, Eq. (1) can then be expressed in the equivalent form:

$$k_t + q_x = 0, \tag{3}$$

if we let $x_2 \to x_1$ in (1). If k has a jump discontinuity at (x,t), then (3) does not hold, but the conservation principle still applies; it specifies that the velocity of the jump, u, is:

$$u = [q]/[k] = [Q(k)]/[k], \tag{4}$$

where brackets denote the change in the enclosed variable across the discontinuity.

On substituting (2) into (3) we obtain a single quasilinear partial differential equation in k:

$$k_t + Q'k_x = 0, \qquad Q'(k) = dQ(k)/dk \tag{5}$$

which taken together with (4) defines the evolution of traffic over a specified road section, given a suitable set of initial/boundary conditions.

Except for the specific form of Q adopted for traffic flow, this model is identical to first-order fluid dynamics models of water flow in rivers and gas through pipes. The solutions and methods used to obtain them are quite similar; see e.g., Whitham (1974). For highway traffic, the density profile becomes piece-wise smooth as $t \to \infty$, with jumps in density ("shocks") separating the smooth pieces. This means that traffic is predicted to be stable, with transitions between stable regions approximated by discontinuous shocks.

The LWR theory is flawed for light traffic (with passing allowed) because it does not recognize that there is a distribution of desired velocities across vehicles, in addition to a variation of the desired velocity for each vehicle. These effects would cause a platoon to disentangle and disperse itself from the front and back in a way that is not predicted by the LWR theory. The velocity distribution across vehicles tends to spread a platoon linearly with time, and the variation within each vehicle with the square root of time. We note for future reference that the latter effect is much less important. We also note that the theory of light highway traffic (with weak passing interactions) has been quite complete for some time. For further reference the reader can consult Newell (1955), which includes a comprehensive summary with an extensive bibliography.

The LWR theory performs better when passing is restricted except: (1) it does not describe properly the detailed motion of a vehicle as it passes through a shock (it predicts that the vehicle speed changes instantaneously) and (2) it does not predict instabilities of the stop-and-go type. On the other hand, the macroscopic LWR predictions have been shown to be in agreement with those of car-following models with and without a reaction time (Newell, 1961).

It seems logical then to see if models that can correct these deficiencies can be developed. A kinetic theory of traffic that seemed aimed at capturing the dispersion effects by explicitly incorporating a velocity distribution was proposed in the late 1950s (Prigogine, 1959), and a higher-order refinement of the LWR model that attempted to describe the happenings inside a shock (and hopefully improve the other LWR deficiencies as well) more than a decade later. It is the latter theory that is the subject of this paper.

As in fluid mechanics, it is tempting to investigate the structure of shocks (e.g., their physical width) and stability phenomena by introducing "higher-order" relations, analogous to the conservation of momentum in fluids, that would hopefully lead to a more detailed level of description. [These laws would then be used to replace (5) by a suitable pair—or more numerous system—of partial differential equations.] Lighthill and Whitham (1955) end their classic article with such a suggestion, although they conclude that the effort would be "premature".

Payne (1971) seems to be the first publication following up on that suggestion, although a very similar theory appears in the book by Whitham (1974) (with no reference to Payne's work). These seminal works, which will be called here "the PW model", sparked a great deal of effort over the last 20 years resulting in numerous publications discussing analytical properties of the model, introducing variations and extensions and proposing numerical schemes. A detailed review of this literature would not serve a useful purpose here and it is therefore not given. Instead, this article shows that these representations of traffic as a fluid in the pursuit of high-order phenomena are not reasonable and lead to unrealistic results.

The main causes of the phenomena we are about to illustrate are three essential differences between traffic and fluids:

1. A fluid particle responds to stimuli from the front and from behind, but a car is an anisotropic particle that mostly responds to frontal stimuli,
2. The width of a traffic shock only encompasses a few vehicles, and
3. Unlike molecules, vehicles have personalities (e.g., aggressive and timid) that remain unchanged by motion.

Before proceeding with our critique of high-order models, we should mention that some undesirable features of the PW model are already known. First of all, the PW model does not remove all the shocks, and to fit the remaining ones a second conservation law (akin to the conservation of momentum law of fluids) is needed. This is recognized in Whitham (1974), where it is described that such a law is unavailable for traffic flow. (Whitham himself appears to consider the PW model primarily as an "amusing" scheme for describing wave phenomena.) Second, del Castillo et al. (1993) have noted that in the PW model vehicles could adjust their speed in response to disturbances reaching them from behind. (They propose an alternative model that does not exhibit this undesirable feature, but the flaw is only removed in the special cases that can by analyzed through linearization.)

This discussion complements the arguments of these authors by presenting additional evidence against the "high-order" modeling approach. Section 2 exposes the flaws in the various justifications that have been provided for high-order fluid models. Section 3 illustrates some of the consequences with an example that can be understood without specialized expertise in fluid mechanics or partial differential equations, and Section 4 discusses what the existing empirical evidence really says about these models.

2. CRITIQUE

Both Payne (1971) and Whitham (1974) derive the PW model as an approximation to the difference–differential equations of car-following, but the logic is flawed. They assume that the spacings and speeds for a line of cars vary slowly with position and time so that they can be closely approximated by smooth functions of space and time: $s(x,t)$ and $v(x,t)$. (The approximation is made to convert differences into differentials.) Under these conditions, they show that the PW equations are second order approximations of the car-following equations in s and v. Although this may give credence to the theory on first sight, it should be noted that the neglected terms include 2nd, 3rd, and higher derivatives of s and v, which will not be small if s and v are not slowly-varying. They cannot be neglected. This argument might have been successful if, on solving the PW model and/or the exact car-following model it purports to approximate, one had found that speeds and spacings varied only slightly from one car to the next and during a reaction time. However, this is not the case.

Newell (1961) started with a car-following model very similar to the one from which the PW theory was derived and obtained both, an exact solution with zero reaction time and an approximate solution in general. He demonstrated that nonlinear car-following leads to shocks with a particular width and profile; i.e., to rapid variations in $s(x,t)$ and $v(x,t)$ with respect to x that are invariant to time when viewed from a moving coordinate system. (Newell's theory, like car-following, does not violate conditions (1)–(3), listed in Sec. 1.)

In another (macroscopic) school of thought, it is argued that car-following phenomena is unimportant and that high-order models should be evaluated on their own merits; traffic is then described macroscopically as if it was a compressible fluid with the cars as molecules. Terms borrowed from the kinetic theory of gases such as *relaxation time*, and from gas dynamics such as *viscosity effects*, are often introduced in this literature as self-evident properties of traffic that high order models should capture (see, e.g., Kühne

& Beckschulte, 1993). This, however, ignores the special nature of traffic particles, and it leads to strange predictions, as we shall see.

Conceivably, one could derive realistic macroscopic models with a systematic application of statistical mechanics/kinetic theory principles—where macroscopic laws are derived from the integration of molecular properties such as positions, collisions, and velocities—but one would have to make sure that the laws assumed to govern the microscopic world accurately represent the behavior of real traffic vehicles. Among other things, these laws would have to prescribe that a slow car should be virtually unaffected by its interaction with faster cars passing it (or queueing behind it), that negative particle velocities cannot arise, and that interactions do not change the "personality" (aggressive/timid) of any car. Such a systematic approach has not yet been taken, although Prigogine's kinetic theory was a valiant attempt (Prigogine, 1959). This theory is briefly examined because it has been linked (albeit weakly) to the PW model (Phillips, 1977; Payne, 1979a,b) and this relationship may be viewed by some as one form of justification for the latter.

The kinetic theory was intended to improve the LWR model by considering the vehicular velocity distribution at each point in time-space. This theory, however, is derived from unreasonable microscopic laws. In particular, it includes a "relaxation" mechanism for speed adjustments that assumes:

1. That a distribution of desired velocities can be defined exogenously at every point in time and space independent of the drivers who happen to be there, and
2. That the actual distribution tends ("relaxes") toward this desired distribution as time passes.

The theory also includes questionable assumptions regarding the interactions among vehicles (Newell, 1995) but these are not germane to our discussion.

Clearly, the relaxation mechanism is unrealistic because (1) implies that the desired speed distribution is a property of the road and not the drivers, as noted by Paveri-Fontana (1975).[1] One must conclude that in ignoring differences in personality among traffic molecules (item 3 mentioned earlier) the kinetic theory neglects the very cause of the phenomena it intends to capture. It can not be used to justify higher-order fluid models.

A justification based on their own merits is not possible either. High-order models, as we shall see, only consider an average speed that varies with location and time. By ignoring the distribution of desired speeds, they cannot be expected to improve the LWR model for light traffic in the proper way. And for heavy traffic they invariably exhibit properties of gas-like fluids that have little to do with real traffic. The properties can not be missed when one attempts to solve the system of partial differential equations defining a high order model.

The first step in any such attempt must be the "classification" of the system, because without proper classification one cannot specify the kind of initial data required to establish the existence and uniqueness of a solution. Reasonably, the PW model and its variants are found to be "hyperbolic"—meaning that defining the system's conditions at time zero is sufficient to establish its future evolution. This is desirable, of course, but it also implies a particular form of cause and effect relation between the past and the future that must be understood. More specifically, it means that the current traffic conditions at any point in time-space are only affected by the conditions between two locations an instant before (the conditions at all other locations can be changed without an immediate effect to our location). The displacement of these two locations relative to the point of interest and the (short) elapsed time between the prior and current instants define segments in the (t,x) plane that are called *characteristics* or *waves*; their slopes are called

[1]As an illustration of the contradictory nature of assumption (1), consider the evolution of traffic conditions downstream of a blockage, after its release. If, as recommended in Prigogine and Herman (1971), we assume that the desired velocity distribution is independent of time and space, then the first vehicles to reach the observer would paradoxically include some that want to go . . . SLOW! (By changing the location of the blockage, and the timing of its release, this paradox can be seen also to occur in the general case.)

characteristic or *wave velocities*. The solution of a hyperbolic system is in fact defined by wave properties which play a central role in solution methods. (The specific details of these roles are not needed for our discussion.)

On carrying out the above-mentioned classification step it is found that high-order models always exhibit one characteristic speed greater than the macroscopic fluid velocity. (The formulas are explicitly given in Whitham, 1974.) This is highly undesirable because it means that the future conditions of a traffic element are, in part, determined by what is happening . . . BEHIND IT![2] This occurs even under heavy (no passing) traffic. Principle (1) of Sec. 1 is violated. Note that this is not just a minor annoyance that will go away in practice; it is a manifestation of the erroneous cause and effect relationship between current and future variables that is at the heart of all high-order models. As shown in Sec. 3, pathological consequences always result.

3. EXAMPLE

Any theory should pass any simple mental experiment whose outcome is known from accumulated experience, in addition to tests with formal data. In our case, experience would prescribe that a model of one-way travel should not predict negative flows and speeds, and other driver behavior that is known not to occur. Such mental experiments are especially important in a field like traffic theory where controlled experiments are difficult to conduct and the data are very noisy. This section describes that any high-order model that is successful in smoothing shocks will fail a fundamental mental test. This means that the "success" of a high-order fluid approximation also becomes its downfall.

3.1 The model without reaction times

From the fluid dynamics point of view, the simplest model that eliminates all shocks is obtained by introducing a diffusion term in the equation of state (2); i.e.,:

$$q = Q(k) - \mu k_x \tag{6a}$$

or equivalently:

$$v = V(k) - \frac{\mu}{k} k_x, \tag{6b}$$

where μ is a positive constant and $V(k)$ is the relationship between speed and distance: $V(k) = Q(k)/k$. With this modification, (5) becomes[3]

$$k_t + Q' k_x = \mu k_{xx}. \tag{7}$$

The diffusion term, μk_{xx}, is supposed to capture in a rough way the driver's awareness of conditions ahead (Whitham, 1974). Unfortunately, Eq. (7) also implies that drivers are aware of conditions behind, which is not reasonable for our *anisotropic* traffic fluid.

The anisotropy of the traffic fluid causes problems that are best revealed when k increases rapidly with x. As shown by (6a) and (6b) negative flows and velocities are then predicted. This is dramatically illustrated by considering the time evolution of the rear of a stopped queue without any arriving traffic.

The situation is formally described by the following initial/boundary conditions:

[2]del Castillo et al. (1993) seem to be the first to have challenged the existence of fast waves. Rather than using this evidence to dismiss the models, they propose a "fix" for the special case when the traffic density and speed are nearly constant (and consistent with the LWR relation: $v = Q(k)/k$). This fix, however, does not salvage the general case.

[3]Model (7) is a precursor of the PW model and Payne (1971) credits it to Lighthill and Whitham (1955). The LW paper does not propose it for traffic flow, though.

$$v = 0 \text{ and } k = k_j H(x) \qquad \text{for } x \le A \text{ and } t = 0 \ (A > 0) \tag{8a}$$

$$v = 0 \qquad \text{for } x = A \text{ and } t > 0 \tag{8b}$$

where $H(x)$ is the Heavyside unit step function.

In the correct solution to this problem nothing would happen, and this would also be the prediction of both car-following theory and the LWR model.

This result, however, is not obtained from (7). Because the equilibrium solution of our problem for $t \to \infty$ satisfies $q = 0$ everywhere, it can be found by solving the ordinary differential equation implied by (6a):

$$\mu k_x = Q(k) \tag{9}$$

with the condition

$$\int_{-\infty}^{A} k dx = k_j A \tag{10}$$

which states that the number of vehicles to the left of $x = A$ must remain constant.

Qualitatively, Eq. (9) indicates that the slope of the equilibrium density profile, $k(x,\infty)$, is proportional to the equilibrium flow, $Q(k)$, corresponding to the specific value of k. For a $Q(k)$ relation, the equilibrium profile is increasing and S-shaped. Furthermore, (10) indicates that the "S" is centered around $x = 0$, as shown in Fig. 1. Because $k < k_j$, the number of vehicles to the right of any location, x, declines from $t = 0$ to $t = \infty$, as should be apparent from the figure. If the order of the vehicles in the queue does not change, this means that everyone moves back.

This example illustrates that any model that smooths all the discontinuities must sometimes predict negative velocities. As such, it establishes that negative velocities observed in computer models cannot be removed by convergent numerical approximation methods.

More important, note that the smoothness of the shock is inherently unreasonable and would not be the prediction of car-following. In any theory where vehicles only respond to frontal stimuli, the vehicular spacings (i.e., the density) within a platoon must be independent of what is behind. Thus, spacings and density must change abruptly whenever the road behind is empty and the proper mathematical representation is a discontinuity. Note, as well, that initial conditions (8) are not unusual—they are the final conditions one would expect for a finite number of vehicles approaching a stationary obstruction—and that similar unreasonable results are obtained if the platoon is moving.

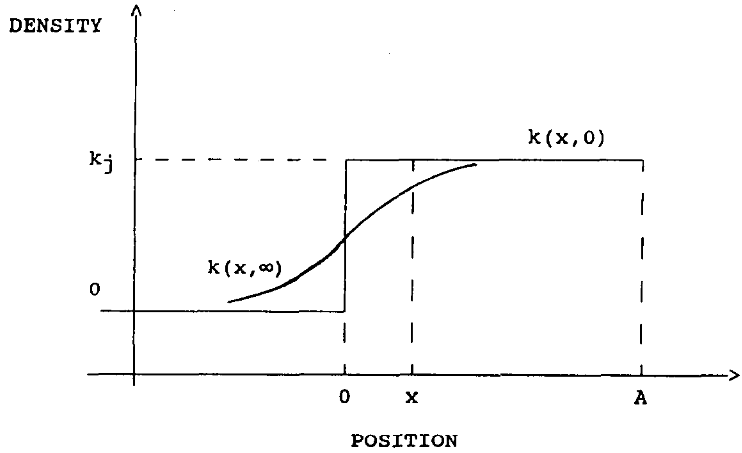

Fig. 1. Initial and final density profiles for the fixed obstruction problem.

In that case, an even smoother transition in density from the platoon to the empty road behind is predicted by the high order model.

If one now adds some vehicles to the end of an equilibrium queue, the new equilibrium will be one where the S-shaped profile of Fig. 1 has been shifted back by the distance needed to hold the arriving vehicles at jam density. This increases the density of the original queue, meaning that the queue is compressed from behind as a result of the arrivals. This compression occurs whether the queue is moving or still, and whether it is at equilibrium or not. It should be clear that any instabilities that are generated by this type of mechanism have little to do with how people drive and should not be used as evidence that the theory works.

3.2 Other higher order models

The same asymmetry argument can be made for all other existing higher-order models and the consequences can still be illustrated by the solution to the problem defined by Eqs. (8). For example, in the PW model, (6b) is replaced by the following relation for acceleration:

$$v_t + vv_x = \frac{1}{\tau}\left[V(k) - v - \frac{\mu}{k}k_x\right] \tag{11}$$

where (6b) is viewed as a desired speed that the drivers accelerate toward (τ is the parameter that converts speed differences into vehicular acceleration).[4] In terms of speed and density, the conservation equation (3) becomes:

$$k_t + kv_x + vk_x = 0 \tag{12}$$

which, together with (11), is a hyperbolic system of 2 quasilinear partial differential equations in k and v.

The model is not complete without a subsidiary jump condition that would allow (second-order) shocks to be placed appropriately when the shock profile "turns on itself", as is done in the analysis of monoclinal flood waves with an inner discontinuity. The lack of a jump condition does not pose a problem for the solution of (8) because no discontinuities develop in the evolution of the system as $t \to \infty$.

Analysis of problem (8), (11), (12) in the standard way (e.g., by iteratively integrating along the characteristics an initial density profile that is monotonic, smooth and arbitrarily close to the initial step function, as explained in Whitham, 1974, and Garabedian, 1986) reveals that the initial solution is not stable: the cars at the end of the queue move back and the behavior spreads to the remaining vehicles in the queue . . . from the back to the front! The solution then tends to a smooth equilibrium that is still given by (9) and (10). This can be verified easily by setting v and its derivatives equal to zero in (11) and solving the resulting ordinary differential equation. Physically, this means that fresh arrivals continue to compress a queue from behind. As in Sec. 3.1 the effect persists even if the queue is moving.

The test can be easily applied to other high-order models. For example, similar results are obtained if ν and τ are allowed to depend on k as is recommended in some of the variants or if *viscosity terms* involving the second derivative v_{xx} are introduced (as in the variant espoused by Kühne, 1993). To my knowledge all high-order models exhibit undesirable queue-end behavior.

4. EMPIRICAL EVIDENCE

This article would not be complete without some mention of the empirical evidence available.

Several comparisons of high-order models with actual data have been performed

[4] In the original derivation τ was intended to approximate the reaction time of car-following models.

(e.g., Cremer & May, 1985; Michalopoulos et al., 1987; Papageorgiou et al., 1989; Leo & Pretty, 1992). The typical procedure uses a computer model in which the road is discretized into sections and where an algorithm—e.g., Cremer (1979) or some other numerical approximation method—is used to manipulate the speed and density prevailing in each section as time passes. The algorithms use finite-difference equations that resemble the continuous partial differential equations of a high-order model but well-established schemes are not always used,[5] and in some of the studies (Cremer & May, 1985) the freeway segments seem rather long.

Most notably, Michalopoulos et al. (1987) and Leo and Pretty (1992) conclude, using well-established techniques, that high-order models (despite their added complexity and additional parameters) do not improve the LWR model.

Cremer and May (1985) find that the results of high-order models are unacceptable without numerous "engineering fixes". That is, the unadjusted models cannot produce acceptable results for simple situations, such as the onset and dissipation of congestion near a lane drop.

Both of these conclusions are consistent with our theoretical critique. Although reasonable fits to specific data-sets can be achieved with adjusted models that include many parameters (e.g., Cremer & May, 1985; Papageorgiou et al., 1989) these limited successes do not constitute a validation. [After all, given enough degrees of freedom—15 in the case of Cremer and May (1985)—a good fit should be expected.]

On the contrary, it is my opinion that the evidence points to improper numerical approximations. Comments made in Cremer and May (1985) lead one to believe that the results of these calibration exercises are very sensitive to the data and to the discretization used. This casts serious doubts about the relationship between the numerical results and the continuum model. [Clearly, unless a numerical model has been tested with a discretization so fine that its results no longer change if it is made finer still, one cannot claim that the continuum theory is being validated; a similar comment is made in Leo and Pretty (1992) in their evaluation of other studies.]

In view of all this, it should not be surprising to see that the estimates of the relaxation time and the anticipation coefficient obtained from various empirical studies (see del Castillo et al., 1993, for a review) are too large to admit a reasonable physical interpretation; they probably depend on the discretization used.

Perhaps there is a chance that, given enough parameters and testing, a numerical model of the form discussed in this section could fit real data slightly better than the LWR model. It should be clear, however, that without a link to an improved theory, the improved fit cannot be expected to continue when conditions change drastically from those in the calibration; e.g., in an application to incident detection and/or traffic control.

5. CLOSURE

In the evolution of human knowledge, new improved theories replace established theories every once in a while. A new theory (e.g., relativity) is deemed successful if it explains previously unexplained phenomena, plus everything that was correctly explained with the theory it intends to replace (e.g., Newtonian physics). In the case of traffic flow, the new theories (high-order fluid models) fail to explain the behavior of traffic at the end of a queue (unlike the simpler, older LWR model; which explains it perfectly).

In view of points 1, 2, 3, in Section 1, it seems that an improved description of the shock structure should be based on a correct analysis of a suitable car-following model (or other microscopic model) and not on conjectures borrowed from other fields (especially continuous fluid models).

On a positive note, it should be emphasized that the shock structure is irrelevant for some applications, and in those cases, the LWR model should suffice. A case in point is

[5]This is rather dangerous and the FREFLO computer program (Payne, 1979) is a case in point. FREFLO is known not to converge and to produce undesirable behavior (Cremer & May, 1985; Newell, 1988). It has not been used in any of the studies mentioned in this section.

given by current efforts to model freeway traffic networks numerically using finite difference approximations of fluid models. To be practical, these models must discretize freeway links into sections that are large compared with the width of a shock. Thus, even if a numerical model could be constructed that was consistent with the LWR theory and could capture the shock structure too, the improved accuracy would be lost in the subsequent aggregation of the data within each freeway section. The additional computational effort would have been wasted.[6]

Besides a coarse representation of shocks, other deficiencies of the LWR theory include its failure to describe *platoon diffusion* properly (a phenomenon that takes place over long distances and times when traffic is light; see Newell (1960) for an analysis) and its inability to explain the instability of heavy traffic, which exhibits oscillatory phenomena on the order of minutes.

We have already explained that higher order refinements of the LWR model do not correct these deficiencies in the proper way. Additional evidence in this respect has been presented recently by Cassidy and Windover (1995) while addressing another problem. Using a high resolution graphical technique for analyzing freeway data, they show that disturbances of higher flow/density move with traffic without diffusing, presumably because the traffic stream is composed of heterogeneous drivers who like to follow at different headways for the same speed. Although an extension of the LWR model with heterogeneous drivers can capture the phenomena, the relaxation and diffusion terms of higher order models preclude them from doing so.

Fortunately, applications involving short term predictions, such as incident detection with on-line data from closely spaced detectors, are unlikely to benefit from enhancements of the LWR theory aimed at correcting its deficiencies. One could speculate, however, that an improved theory (including the correct cause for instability) might be a useful basis for developing dynamic control schemes to improve the flow of traffic through some bottlenecks.

It is my opinion that the car-following mechanism causing instability—in view of the comprehensive evidence presented in Newell (1962)—could be one where drivers do nothing most of the time (i.e., coast) and respond to changes in spacing only when they are about to leave a range of acceptable spacings with their lead car. Unfortunately, Newell's theory needs further work as it does not describe the mechanism for generation of the disturbances.

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