Comparison of the solution of the shallow equation with experimental

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Abstract

In this paper we compare the solution of the shallow water equations with experimental results. The experiment simulates the interaction between an incident bore and a free-standing coastal structure. The shallow water equations are solved using the CLAWPACK software.

Three different cases are simulated and compared. In the first case, the column is removed. This is a simple dambreak problem and history of wave height and velocity at where column center should be are measured and compared. In the second case, a square column is set and wave height at some locations ahead the column are measured and compared. For the third case, the square column is replaced with a cylinder with the same data measured and compared. The discrepancy between geoclaw simulation and experiment results are also discussed and analyzed in a deep manner.

1   Introduction

The aim of this work is to compare computational results using CLAWPACK with experimental results. This is done for the interaction of a bore and a free-standing coastal structure. The experiment results are given by Halldór Árnason [1]. We model the water using the shallow water equations which will be introduced in Section 2. We solve these equations using CLAW-PACK's GeoClaw solver which is specialized for geophysical flow problems. In Section 3 we show how GeoClaw solves the shallow water equations and how it handles the topography. In Section 4 we describe the setup of the test case. We focus on three cases. First we compare wave height and velocity for the pure dambreak without structure in the domain in Section 5. Second we add a square column and compare wave height upstream as well as downstream of the column in 6. Lastly in Section 7 we compare the wave height at the same location but for a cylinder column. We close this paper with a conclusion in Section 8.

2   The shallow water equations

We write the 2D depth averaged shallow water equations as

$$h_t + (uh)_x + (vh)_y = 0 \tag{1}$$

$$(hu)_t + (huv)_y + (hu^2 + \frac{1}{2}gh^2)_x = -ghB_x - Du \tag{2}$$

$$(hv)_t + (huv)_x + (hv^2 + \frac{1}{2}gh^2)_y = -ghB_y - Dv \tag{3}$$

where u, v are the horizontal depth averaged velocities, B the topography and D the drag coefficient. g denotes the gravitational acceleration. In the present case the topography will be flat except the column located downstream. The drag coefficient D is given by

$$D = \frac{gM^2\sqrt{(u^2 + v^2)}}{h^{5/3}} \tag{4}$$

where M is the Manning coefficient which is $M = 0.015$. The Manning coefficient is an empirical coefficient that includes the roughness of the surface. The initial condition is given by a jump in the water depth h. The water is initially at rest.

3   The Algorithm in CLAWPACK/GeoClaw

We solve the shallow water equations with CLAWPACK. Specifically we used the GeoClaw solver which used a special version of the f-wave approach described in [2]. This solver combines the qualities of the Roe solver, HLLE-type solver and the f-wave approach. From the Roe solver it has the quality of giving a exact solution for a single-shock Riemann problem. Like the HLLE solver this method is depth positive semidefinite. The method has also a natural entropy-fix and gives a better approximation for problems with large rarefactions. The solver also maintains large class of steady states. The topography and drag term are incorporated in the Riemann problem. Stability and accuracy are improved by solving a transverse Riemann problem, where the waves moving normal to the cell are split into traverse direction.

In the GeoClaw solver adaptive mesh refinement (AMR) can be used. Multiple levels can be use where for each level the domain is specified. The topography for the problem is defined in a file. The topography goes into the shallow water equations through the right hand side B. The value from the file are converted into cell averages using a picewise bilinear function that interpolates the pointwise values. B is then obtained by evaluating the exact integral of this function over the cell.

4   The setup of the test case

The benchmark experiment was conducted in a 16.6 m long, 0.6m wide and 0.45 m deep wave tank. When a gate is lifted, it generates a single bore to impinge the column located downstream. We are going to simulate 2 cases in the experiment. The 1st case purely generated the single bore, the 2nd case generated the bore to impinge the column with one side facing the flow.

In the 1st case, wave height and streamwise velocity, U, at different depth, at where the center of column should be located are measured. In the 2nd case, the wave height in front and behind the column are measured.
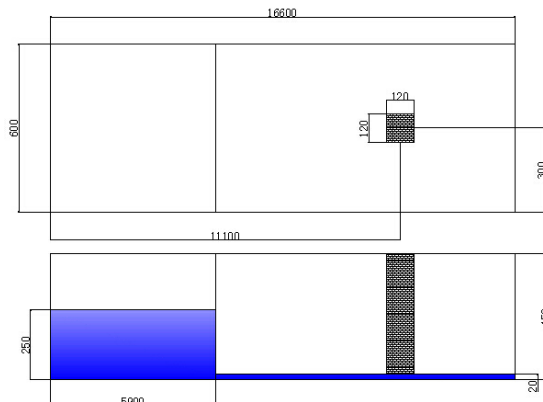

Figure 1: Diagram of experiment apparatus

In all cases below, boundary conditions are all set to reflection wall.

5 Dam Break

The initial condition for this and the following cases is given by

$$h_0(x,y) = \begin{cases} 0.25 & x < 5.9 \\ 0.02 & \text{otherwise} \end{cases} \tag{5}$$

$$u_0(x,y) = \qquad 0 \tag{6}$$

In this comparison we will include results from OpenFOAM, an open source CFD software modeling air and water using the Navier-Stokes equations with turbulence model. Figure 2 shows the time history of the wave height measured at the center location where the column will be mounted. Result from GeoClaw agree quite well in general(Show zoom in plot). At around t=3.8, GeoClaw accurately predicted the arrival of the shock. GeoClaw overestimates the peak water level a little, while OpenFOAM, gives a better value. After a later time, the wave gets reflected at the right wall of the tank, which can be seen by the second jump in wave height. GeoClaw also predicts the arrival time of this peak quite well by overestimating the peak value by a small amount.

Figure 3 gives the time history of the streamwise velocity u from GeoClaw and experiment. Note that this velocity u in both GeoClaw and experiment are depth-averaged. GeoClaw accurately predicted the time of the velocity peak as well as the value of the peak. After the peak arrived, GeoClaw overestimated the value of velocity. The reason why GeoClaw gives a higher velocity may be attributed to its depth-averaged property: Water at all depth are propagating with the same speed. In the experiment, the velocity is getting smaller the closer we get to the bottom.
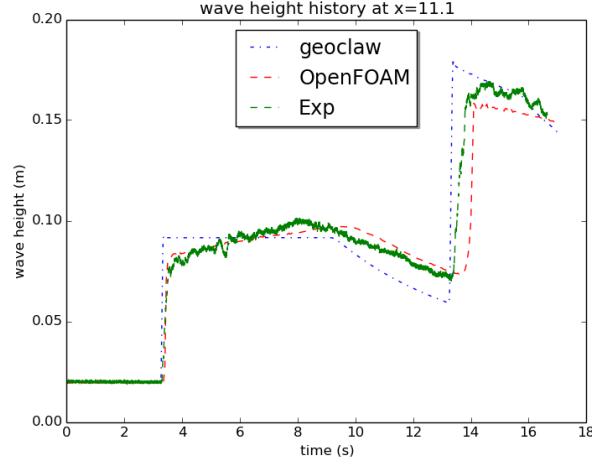
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Figure 2: time history of wave height for case without a column.
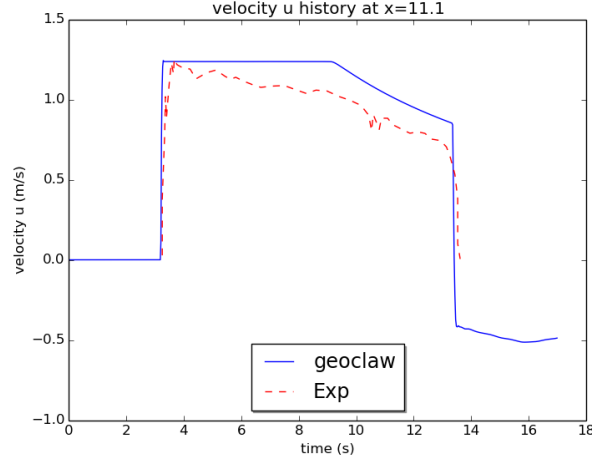

Figure 3: time history of streamwise velocity U for case without a column

6 Dam Break with square column

Figure 4 gives a comparison of the time history of the wave height measured at 0.04m ahead of the leading edge of the square column.(0.1m ahead of center of the column) Note that this figure is from t=3.0 to t=4.0, which is quite a short range compared to the previous figures. There are two jumps in result given by geoclaw. First jump change water level from around 0.02 to 0.1, second one jump from 0.1 to a higher level of around 0.22. If we recall that the wave height is around 0.1 in figure 2, we can confirm this first jump is cause by the arrival of shock wave caused by dambreak. The second jump is induced by wave reflected by the column. If we compare the time interval between first jump and second jump with that in figure 7, we can see that as the gauge gets closer to column, this time interval also gets shorter.

There are two distinct differences between geoclaw result and experiment result. First, in experiment, the jump is more like a smooth increase. Second, arrival time of the peak predicted

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by geoclaw is about 0.08 second earlier than the time measured from experiment.

The difference in shock shape can be explained by the difference between the shallow water equations that geoclaw solves and N-S equations that better describe the physics. In shallow water equations, fluid viscosity is not taken into account, while it's significant in real physics. This causes the shock remain its shape as it propagates in shallow water equation while smooth out in experiment. Thus we see a jump in result from shallow water equation while a smooth increase in experiment result.

If we zoom in figure 2 for smaller time range from 3s to 4s, we get figure 5. In figure 5, arrival time of peak predicted by geoclaw is almost the same as that from experiment (only about 0.02 second earlier). This time interval is much smaller than that in figure 4 though data are not measured at exactly the same point for this two plots. This difference indicates that the column gives more resistance to shock waves in experiment than it does in shallow water equation model.

A simple analysis to this phenomenon is draw as below. Shallow water equation is essentially a hyperbolic PDE, whose solution depends on initial condition along characteristics in spatio-temporal domain. Thus before the propagating shock arrive, all water downstream the shock stays still. In other words, it even does not know there is a column before the shock hits the column. This can be verified by checking history of wave height measured at exactly the same place, 0.04m ahead of leading edge of the square column, but in different cases, one with a square column and one without. This comparison is showed in figure 6. As we expected, there is no difference in arrival time of the shock wave. In contrast, N-S equation is a elliptic PDE. In real physics governed by N-S equation, when front of the shock wave gets close to the column, water downstream it and upstream column are affected and resist the comming of wave to some extent. This slows down the propagating speed of shock wave when it comes close to the column in real world and thus shifts the peak toward later time in the plots.
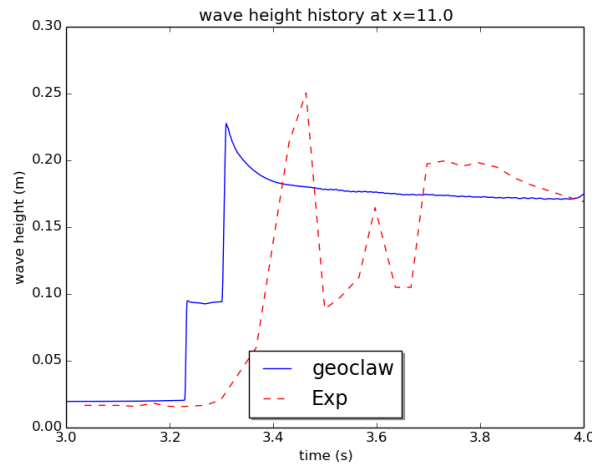

Figure 4: time history of wave height for case with square column

Figure 8 shows time history of wave height measured at front face of the square column. Difference between arrival time of peak still exists as expected. After the peak arrive, the wave gradually becomes stable as indicated by a horizontal line in the plots, in which region the
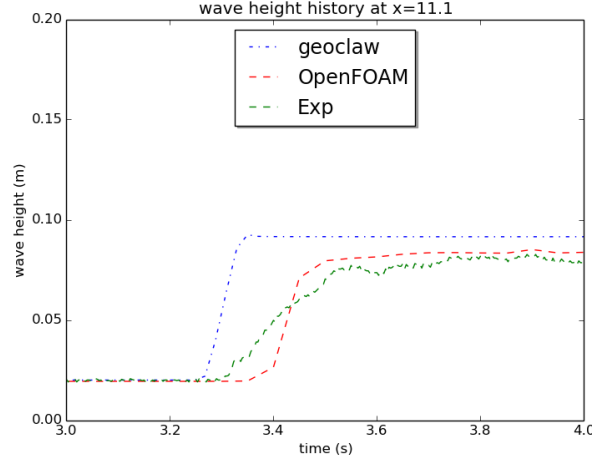
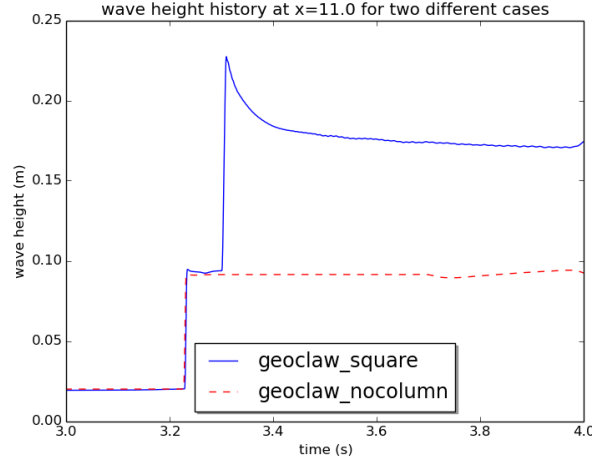Figure 5: time history of wave height for case without column (zoom in)

Figure 6: comparison of time history of wave height for case with a square column and case without one

differences between geoclaw and experiment are quite small.

7   Dam Break with cylinder column

Figure 9 is time history of the wave height measured at 0.04m ahead of the leading edge of cylinder column.(0.1m ahead of the center of the column) Similar feature as what we see in case with a square column can be found in the figure. However, the wave height at second peak in this case is almost the same as that in square-column case (around 0.22m in both case), while this feature is quite different for experiment results between two cases.

Note that diameter of the cylinder is equal to length of the transection for square column. Thus the square column can block the flow in in a more efficient way than cylinder does since cylinder has a smoother shape for the fluid to pass by. This explains why we see a lower
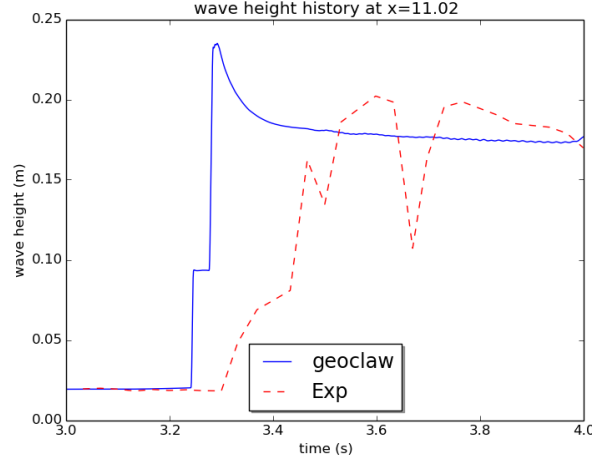
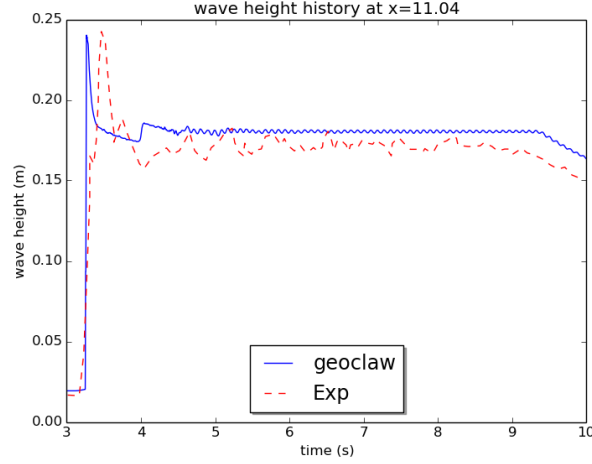Figure 7: time history of wave height for case with square column, at x = 11.02

Figure 8: time history of wave height for case with square column, at x = 11.04

peak of wave height as well as less steep curve for history of wave height in case with cylinder in experiment. In geoclaw, a geometry is represented with many quadrangular cells. In our cylinder case, the cylinder is represented with many square cells since our background mesh is simple structured quadragular mesh with square cells. Based on this, we can deduce in geoclaw, when the shock wave hits the cylinder, it is hitting onto many small faces vertical to flow direction at different locations. Thus it is similar to hitting onto one big face vertical to flow direction, which is the case with square column. This is why my get similar result in geoclaw for case with square column and case with cylinder.

Figure 10 is time history of wave height for cylinder case. As we expected, peak predicted by geoclaw appears earlier but the peak values agree quite well. After the peak, geoclaw gives a satisfied prediction of wave height.
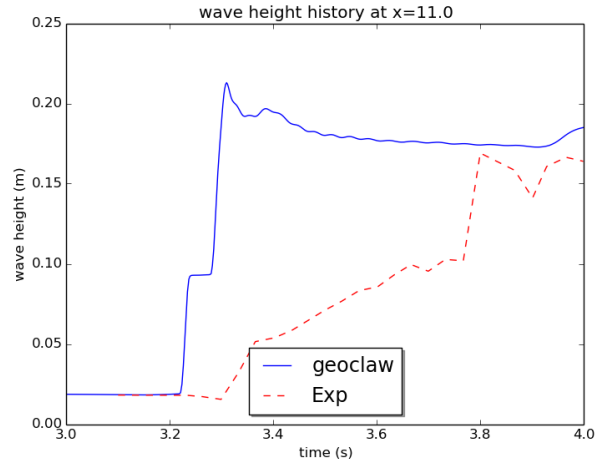
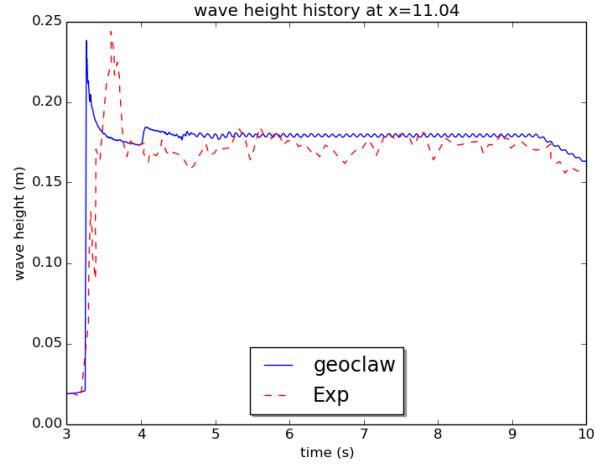Figure 9: time history of wave height for case with cylinder column

Figure 10: time history of wave height for case with cylinder column, at x = 11.04

8   Conclusion

References

[1] Halldór Árnason (2005). Thesis - Interactions between an incident bore and a free-standing coastal structure. http://faculty.washington.edu/cpetroff/Halldor%20dissertation.pdf

[2] David L. George (2008) Journal of Computational Physics - Augmented Riemann solvers for the shallow water equations over variable topography with steady states and inundation.

[3] Marsha J. Berger and David L. George and Randall J. LeVeque and Kyle T. Mandli (2011) Advances in Water Resources - The GeoClaw software for depth-averaged flows with adaptive refinement.