Comparison of the solution of the shallow equation with experimental

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Abstract

In this paper we compare the solution of the shallow water equations with experimental results. The experiment simulates the interaction between an incident bore and a free-standing coastal structure. The shallow water equations are solved using the CLAWPACK software.

1 Introduction

The aim of this work is to compare computational results using CLAWPACK with experimental results. This is done for the interaction of a bore and a free-standing coastal structure. The experiment results are given by Halldór Árnason [1]. We model the water using the shallow water equations which will be introduced in Section 2, followed by a description how CLAWPACK solves the system and handles the mesh in Section 3. In Section 4 we describe the setup of the test case. We compare wave height and velocity for the case of a pure dambreak without structure in the domain in Section 5. In Section 6 we add a square column and compare wave height upstream as well as downstream of the column. In Section 7 we compare the wave height at the same location but for a cylinder column. We close this paper with a conclusion in Section 8.

2 The shallow water equations

We write the shallow water equations as

$$h_t + (uh)_x + (vh)_y = 0 \tag{1}$$

$$(hu)_t + (huv)_y + (hu^2 + \frac{1}{2}gh^2)_x = -ghB_x - Du \tag{2}$$

$$(hv)_t + (huv)_x + (hv^2 + \frac{1}{2}gh^2)_y = -ghB_y - Dv \tag{3}$$

where u, v are the horizontal depth averaged velocities, B the topography and D the drag coefficient. g denotes the gravitational acceleration. In the present case the topography will be flat except the column located downstream. The drag coefficient D is given by

$$D = \frac{gM^2\sqrt{(u^2+v^2)}}{h^{5/3}} \tag{4}$$

where M is the Manning coefficient which is $M = 0.015$. The Manning coefficient is an empirical coefficient that includes the roughness of the surface. The initial condition is given by a jump in the water depth h. The water is initially at rest.

3 The Algorithm in CLAWPACK/GeoClaw

We solve the shallow water equations with CLAWPACK. Specifically we used the GeoClaw solver which used the f-wave approach described in [2]. There the difference in flux normal to the cell interface is separated into propagating waves. The topography and drag term are incorporated in the Riemann problem. Stability and accuracy are improved by solving a transverse Riemann problem, where the waves moving normal to the cell are split into traverse direction.

4 The setup of the test case

The benchmark experiment was conducted in a 16.6 m long, 0.6m wide and 0.45 m deep wave tank. When a gate is lifted, it generates a single bore to impinge the column located downstream. We are going to simulate 2 cases in the experiment. The 1st case purely generated the single bore, the 2nd case generated the bore to impinge the column with one side facing the flow.

In the 1st case, wave height and streamwise velocity, U, at different depth, at where the center of column should be located are measured. In the 2nd case, the wave height in front and behind the column are measured.

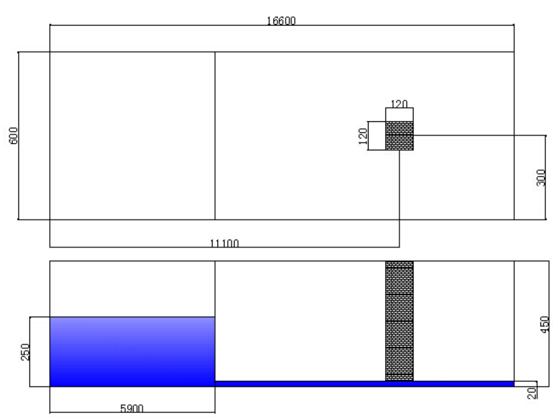

Figure 1: Diagram of experiment apparatus

In all cases below, boundary conditions are all set to reflection wall.

5 Dam Break

The initial condition for this and the following cases is given by

$$h_0(x, y) = \begin{cases} 0.25 & x < 5.9 \\ 0.02 & \text{else} \end{cases} \tag{5}$$

$$u_0(x, y) = 0 \tag{6}$$

Figure 2 shows the time history of the wave height measured at the center location where the column will be mounted. Result from geoclaw agree quite well in general. At around t=3.8, geoclaw accurately predicted the arrival of the shock. As for the peak value of this shock, geoclaw overestimated it a little, while OpenFOAM, another open source computational fluid dynamics software gave a better value by solving the Navier-Stokes equations with turbulent model. After a longer time, the wave gets reflected at the right wall of the tank, which can be seen by the second jump in wave height in the figure. Geoclaw also predicted the arrival time of this peak quite well and got only a little overestimation of the peak value.

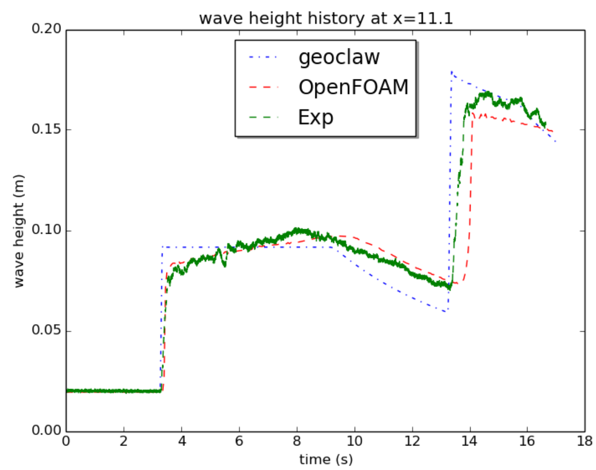

Figure 2: time history of wave height for case without a column

Figure 3 gives the time history of the streamwise velocity u from geoclaw and experiment. Note that this velocity u in both geoclaw and experiment are depth-averaged. Geoclaw accurately predicted the time of the velocity peak as well as the value of the peak. After the peak arrived, geoclaw overestimated the value of velocity. (The reason why geoclaw gives a higher velocity may be attributed to its neglection of fraction force from the tank bottom). In the experiment, the velocity gets smaller as it comes closer to the bottom. In geoclawa, however, there is only one u in depth direction and it does not take fraction into account.

6 Dam Break with square column

Figure 4 gives a comparison of the time history of the wave height measured at 0.04m ahead of the leading edge of the square column.(0.1m ahead of center of the column) Note that this

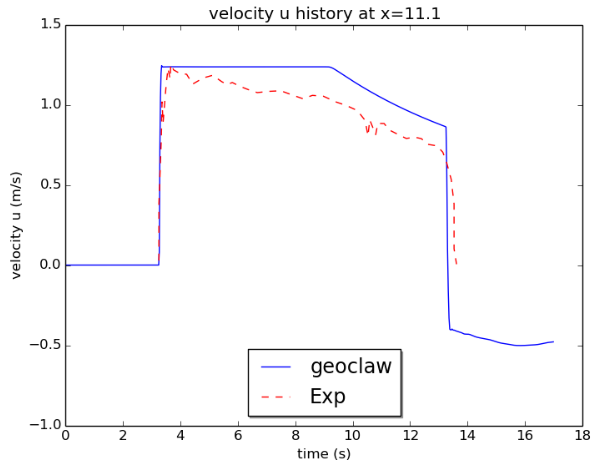

Figure 3: time history of streamwise velocity U for case without a column

figure is from t=3.0 to t=4.0, which is quite a short range compared to the previous figures. The arrival time of the peak predicted by geoclaw is about 0.2 second earlier than the time measured from experiment.

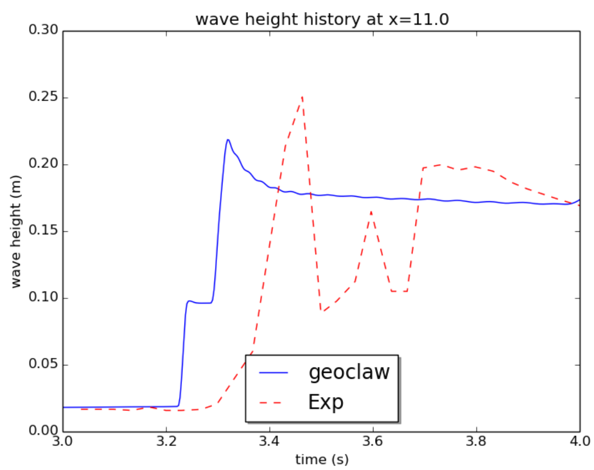

Figure 4: time history of wave height for case with square column

7 Dam Break with cylinder column

Figure 5 is the time history of the wave height measured at 0.04m ahead of the leading edge of cylinder column.(0.1m ahead of the center of the column) Since the way geoclaw treat a cell partly belong to cylinder and partly belong to tank bottom is to give that cell an average value of topography from cylinder and tank bottom based on area ratio, we get a quasi-circle actually instead of circle. Thus the result in this case does not agree well with the experiment result. Future test can be conducted by setting a higher level for mesh refinement to get a better circle.

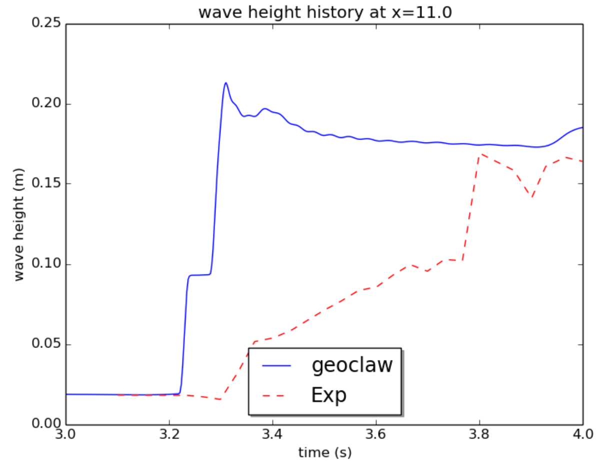

Figure 5: time history of wave height for case with cylinder column

8 Conclusion

References

[1] Halldór Árnason (2005). Thesis - Interactions between an incident bore and a free-standing coastal structure. `http://faculty.washington.edu/cpetroff/Halldor%20dissertation.pdf`

[2] David L. George (2008) Journal of Computational Physics - Augmented Riemann solvers for the shallow water equations over variable topography with steady states and inundation.

[3] Marsha J. Berger and David L. George and Randall J. LeVeque and Kyle T. Mandli (2011) Advances in Water Resources - The GeoClaw software for depth-averaged flows with adaptive refinement.