Hydraulic bore interaction with a column
A comparison between the solution of the shallow equation and experimental results

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Outline

Introduction

Comparison between Solution of the shallow water equations solved with CLAWPACK and experimental results.

- Experiment by Halldór Árnason
- Comaprison of water level at various locations
- 3 Cases
  1. Dam Break
  2. Dam Break with square column
  3. Dam Break with cylindrical column

The Model equations

2D depth averaged shallow water equations

$$
\begin{aligned}
h_t + (uh)_x + (vh)_y &= 0 \\
(hu)_t + (huv)_y + (hu^2 + \frac{1}{2} g h^2)_x &= -ghB_x - Du \\
(hv)_t + (huv)_x + (hv^2 + \frac{1}{2} g h^2)_y &= -ghB_y - Dv
\end{aligned}
$$

where B is the topography and D the drag coefficient. g stands for the gravitational acceleration.

CLAWPACK/GeoClaw

GeoClaw

- Geophysical flow problems
- Specialized version of CLAPACK and AMRClaw

Requirements for solver

- Flow over topography
- Handle non-trivial steady state
  - Ocean at rest
- Dry state handling
- Multiple scales in space and time

Riemann solver

Approaches

- f-wave approach
  - guarantees numerical conservation
  - source terms simply included in solver
  - not clear how to prevent depth negativity
- HLLE solver
  - guarantees depth non-negativity
  - fails to capture large transonic rarefactions
  - not well balanced for steady states

Augmented Riemann solver

- Split into more than two waves
  - Add 1 wave to have two waves to represent large transonic rarefactions
  - Add 1 wave to incorporate topography - balance for steady state

Wave propagation algorithm

- 2D hyperbolic system

$$q_t + A(q,x,y)q_x + B(q,x,y)q_y = 0$$

- Approximation of the state

$$\begin{aligned} Q_{ij}^{n+1} = Q_{ij}^n \quad & - \quad \frac{\Delta t}{\Delta x}\left(\mathcal{A}^+\Delta Q_{i-1/2,j}^n + \mathcal{A}^-\Delta Q_{i+1/2,j}^n\right) \\ & - \quad \frac{\Delta t}{\Delta y}\left(\mathcal{B}^+\Delta Q_{i,j-1/2}^n + \mathcal{B}^-\Delta Q_{i,j+1/2}^n\right) \\ & - \quad \frac{\Delta t}{\Delta x}\left(\tilde{F}_{i+1/2,j}^n - \tilde{F}_{i-1/2,j}^n\right) - \frac{\Delta t}{\Delta y}\left(\tilde{G}_{i,j+1/2}^n - \tilde{G}_{i,j-1/2}^n\right) \end{aligned}$$

- Fluctuations $\mathcal{A}^\pm\Delta Q_{i\mp1/2,j}^n$ and $\mathcal{B}^\pm\Delta Q_{i,j\mp1/2,j}^n$
- Second order correction terms $\tilde{F}_{i\pm1/2,j}^n$ and $\tilde{G}_{i,j\pm1/2}^n$

Approach in 1D

f-wave approach

$$f(Q_i) - f(Q_{i-1}) = \sum_{p=1}^{m} \mathcal{Z}^p_{i-1/2} = \sum_{p=1}^{m} \beta^p_{i-1/2} r^p_{i-1/2}$$

- jumps in the flux instead of state

Fluctuations

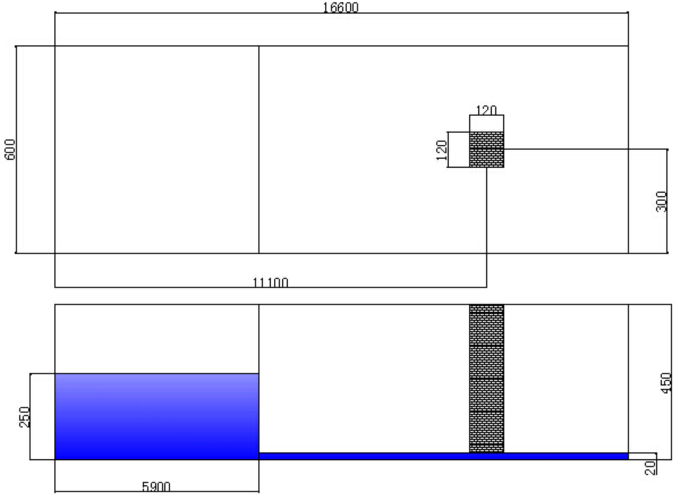
$$\mathcal{A}^- \Delta Q_{i-1/2} = \sum_{p:s^p_{i-1/2}<0} \mathcal{Z}^p_{i-1/2}$$

$$\mathcal{A}^+ \Delta Q_{i-1/2} = \sum_{p:s^p_{i-1/2}>0} \mathcal{Z}^p_{i-1/2}$$

Decomposition into waves

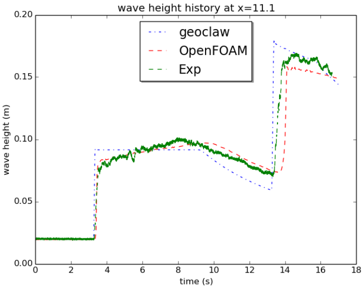
$$\begin{bmatrix} H_i - H_{i-1} \\ HU_i - HU_{i-1} \\ \varphi(Q_i) - \varphi(Q_{i-1}) \\ B_i - B_{i-1} \end{bmatrix} = \sum_{p=0}^{3} \alpha^p_{i-1/2} w^p_{i-1/2}$$

Setup of the test case



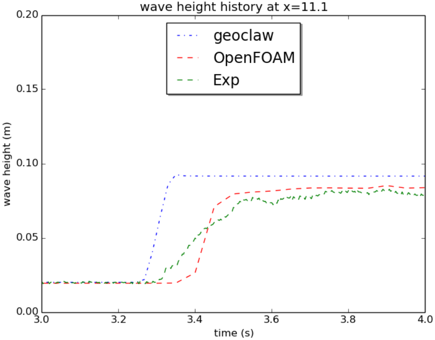
Case01 - Dam Break

$0 \leq t \leq 18$



- Wave arrives at t=3.8s
- Second jump from reflection from right wall

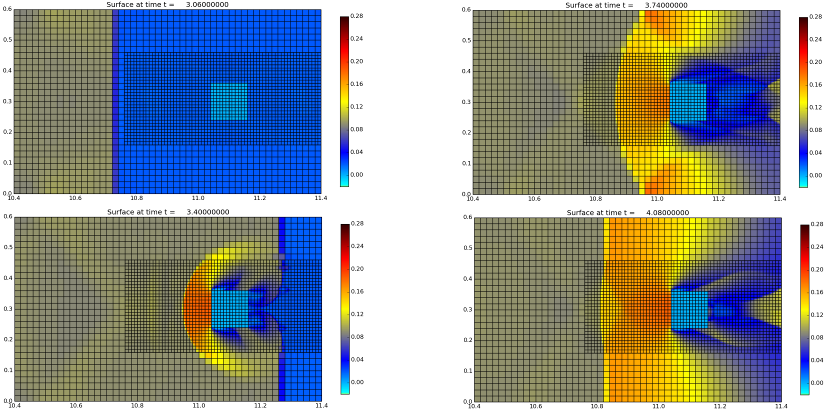
$3 \leq t \leq 4$



- GeoClaw 0.02s ahead
- Peak value overestimated

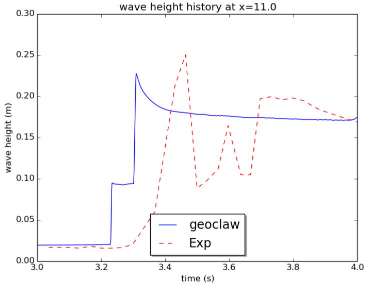
movie

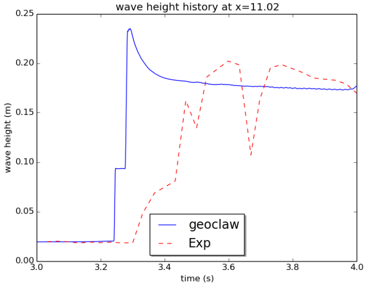
Case02 - Dam Break with Square Column



movie

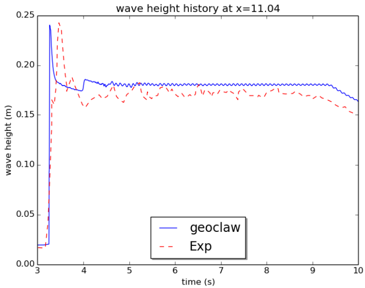
Case02 - Dam Break with Square Column

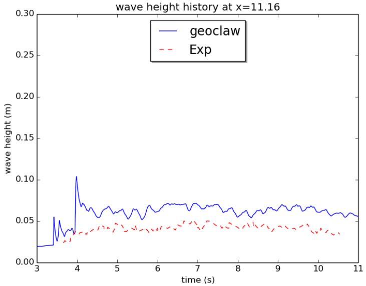




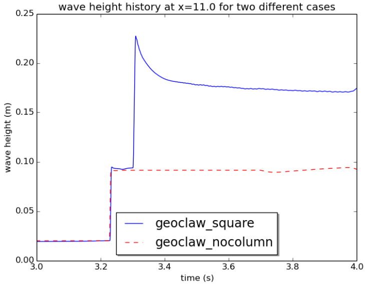
- Two jumps in GeoClaw result
  - One originates in dam break
  - Another results from wave reflected by column
- Time interval becomes shorter
- Difference between GeoClaw result and experiment result:
  - Shock shape
  - Arrival time of the shock

Case02 - Dam Break with Square Column

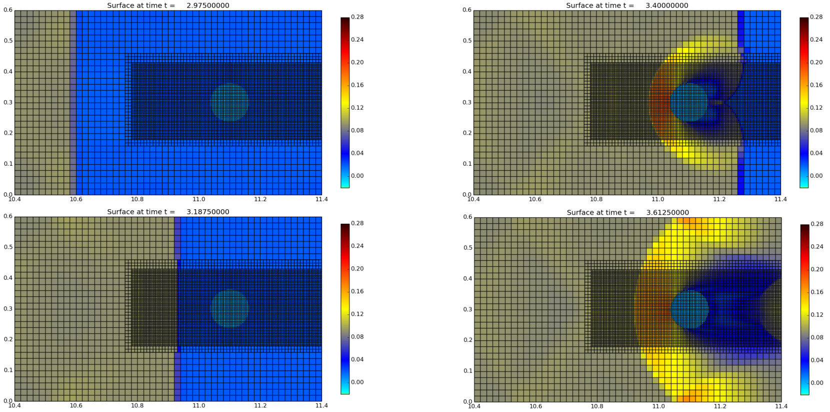




- Wave height at front face
- Wave height at back face
- Comparison between cases with and without column

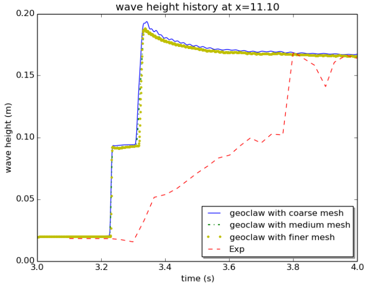


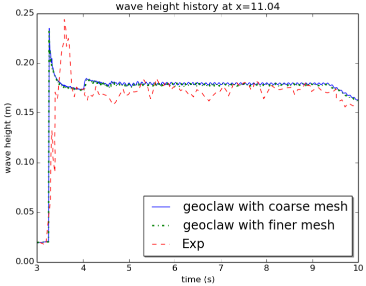
Case03 - Dam Break with Cylindrical Column



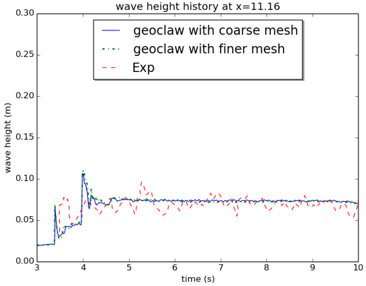
movie

Case03 - Dam Break with Cylindrical Column





- Mesh refinement
- History of wave height at front face and back of the column



Conclusion and Outlook

- For the dam break problem, GeoClaw underestimates the arrival time of the shock by only 0.02s
- In both cases with cylinder and square column, GeoClaw overestimates the arrival time of the shock
- In general GeoClaw agrees well with the measured water levels at various locations for the square as well as the cylindrical column

Future work

- A mapped grid could be used to improve the solution in cylindrical column case
- Comparing the forces on the structure by using $F = \frac{1}{2} C_D \rho d h u^2$