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Analytical solution of Gas Flow Through a Micro-Nano Porous Media by Homotopy Perturbation method

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Abstract—In this paper, we have applied the homotopy perturbation method (HPM) for obtaining the analytical solution of unsteady flow of gas through a porous medium and we have also compared the findings of this research with some other analytical results. Results showed a very good agreement between results of HPM and the numerical solutions of the problem rather than other analytical solutions which have previously been applied. The results of homotopy perturbation method are of high accuracy and the method is very effective and succinct.

Keywords—Unsteady gas equation, Homotopy perturbation method(HPM), Porous medium, Nonlinear ODE

I. INTRODUCTION

THE study of analytical solutions of differential equations (DEs) plays an important role in mathematical physics, engineering and the other sciences. In the past several decades, various methods for obtaining solutions of DEs have been presented, such as, Adomian decomposition method [1], [2], Homotopy perturbation method [3], variational iteration method [4], exp-function method [5], [6], [7] and so on.

Homotopy perturbation method (HPM) was established by Ji-Huan He in 1999 [3] and was further developed and improved by He [8], [9], [10], [11]. In this method, the solution is considered as the sum of an infinite series, which converges rapidly to accurate solutions. Using the homotopy technique in topology, a homotopy is constructed with an embedding parameter $0 \leq p \leq 1$, which is considered as a small parameter. The method has been used by many authors to handle a wide variety of scientific and engineering applications to solve various functional equations. Considerable research work has been recently conducted on applying this method to a class of linear and non-linear equations. This method was then used for different problems by many others. For example, Abbasbandy [12], [13] used it for Laplace transform, Siddiqui et al. [14], [15] applied this method for solving non-linear problems involving non-Newtonian fuids, Cveticanin [16] applied this method on pure non-linear differential equations, Ariel et al. [17] employed this method for axisymmetric flow over a stretching sheet, Ganji et al. [18] to applied this method for non-linear systems of reaction-diffusion equations. It can be said that He's homotopy perturbation method is a universal approach and is able to solve various kinds of nonlinear functional equations. For example, it was applied to nonlinear Schrödinger equations [19], to nonlinear equations arising in heat transfer [20],to the quadratic Riccati differential equation [21], asymptotology [22] and to other equations [21], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32].This method was applied to nonlinear oscillators with discontinuities [33], nonlinear wave equations [34], limit cycle and bifurcation of nonlinear problems [35], and many other subjects [15], [36], [37], [38], [39].

This paper is arranged as follows:

In section II, we describe Unsteady gas equation. In section III, we describe the Homotopy perturbation method (HPM). In section IV we apply HPM for Unsteady gas equation and then compare our solutions with some well-known results, comparisons show that the present solutions are highly accurate. The conclusions are described in the end.

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II. UNSTEADY GAS EQUATION

In the study of the unsteady flow of gas through a semi-infinite porous medium [40] initially filled with gas at a uniform pressure $P_0 \geq 0$, at time $t = 0$, the pressure at the outflow face is suddenly reduced from P_0 to $P_1 \geq 0$ ($P_1 = 0$ is the case of diffusion into a vacuum) and is, thereafter, maintained at this lower pressure. The unsteady isothermal flow of gas is described by a nonlinear partial differential equation

$$\nabla^2(P^2) = 2A\frac{\partial P}{\partial t}, \tag{1}$$

where the constant A is given by the properties of the medium. In the one dimensional medium extending from $z = 0$ to $z = \infty$, this reduces to

$$\frac{\partial}{\partial z}\left(P\frac{\partial P}{\partial z}\right) = A\frac{\partial P}{\partial t}, \tag{2}$$

with the boundary conditions

$$\begin{aligned} &P(z, 0) = P_0, \quad 0 < z < \infty; \\ &P(0, t) = P_1(< P_0), \quad 0 \leq t < \infty. \end{aligned} \tag{3}$$

To obtain a similarity solution, Authors[42] introduced the new independent variable

$$x = \frac{z}{\sqrt{t}}\left(\frac{A}{4P_0}\right)^{1/2}, \tag{4}$$

and the dimension-free dependent variable y, defined by

$$y(x) = \alpha^{-1}\left(1 - \frac{P^2(z)}{P_0^2}\right) , \tag{5}$$

where $\alpha = 1 - \frac{P_1^2}{P_0^2}$. In terms of the new variable, the problem takes the form (unsteady gas equation)

$$\begin{aligned} y''(x) + \frac{2x}{\sqrt{(1-\alpha\ y(x))}} y'(x) = 0, \\ x > 0, \qquad 0 \le \alpha \le 1, \end{aligned} \tag{6}$$

The typical boundary conditions imposed by the physical properties are

$$y(0) = 1, \qquad y(\infty) = 0. \tag{7}$$

A substantial amount of numerical and analytical work has been invested so far [40], [45] on this model. The main reason of this interest is that the approximation can be used for many engineering purposes. As stated before, the problem (6) was handled by Kidder [40] where a perturbation technique is carried out to include terms of the second order. Recently wazwaz [46] solved this equation nonlinearly by modifying the decomposition method and Padé approximation.Also, Parand et al. [47], [48] also applied the Lagrangian method, generalized Laguerre polynomials and Rational Chebyshev collocation method for solving unsteady gas equation. Aslam Noor [49] applied the Variational iteration method (VIM) for solving nonlinear this equation.

III. He's homotopy perturbation method

To illustrate the homotopy perturbation method (HPM), consider the following general nonlinear differential equation:

$$A(u) = f(r), \qquad r \in \Omega \tag{8}$$

with boundary conditions

$$B(u, \partial u/\partial n) = 0, \qquad r \in \Gamma \tag{9}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function, Γ is the boundary of the domain Ω. The operator A can be decomposed into a linear part and a nonlinear one, designated as L and N respectively. Therefore Eq. (8) can be rewritten as follows:

$$L(u) + N(u) = f(r). \tag{10}$$

He [10], [11] constructed a homotopy $v(r,p) : \Omega \times [0,1] \to R$ which satisfies

$$\begin{aligned} H(v,p) = (1-p)(L(v) - L(y_0)) \\ + p(A(v) - f(r)) = 0, \end{aligned} \tag{11}$$

or

$$\begin{aligned} H(v,p) = L(v) - L(y_0) \\ + pL(y_0) + p(N(v) - f(r)) = 0, \end{aligned} \tag{12}$$

where $r \in \Omega$ and $p \in [0,1]$ is an imbedding parameter, y_0 is an initial approximation of Eq. (8). Clearly , we have

$$\begin{aligned} H(v,0) = L(v) - L(y_0) = 0, \\ H(v,1) = A(v) - f(r) = 0, \end{aligned}$$

and the changing process of p from 0 to 1, is just that of $A(v,p)$ from $L(v) - L(y_0)$ to $A(v) - f(r)$. In topology, this is called deformation, $L(v) - L(y_0)$ and $A(v) - f(r)$ are called homotopic. If, the embedding parameter p, $(0 \le p \le 1)$ is considered as a small parameter, applying the classical perturbation method [50], we can naturally assume that the solution of Eqs. (11) and (12) can be given as a power series in p, i.e.

$$v = v_0 + pv_1 + p^2 v_2 + \ldots , \tag{13}$$

Setting $p = 1$ results in the approximate solution of Eq. (8):

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \ldots . \tag{14}$$

The convergence of the series Eq. (14) has been proved in He [10].

IV. Numerical application

In this section, we apply the homotopy perturbation method for finding the analytical solution of the unsteady flow of gas through a porous medium. We consider the Unsteady gas equation

$$\begin{aligned} y''(x) + \frac{2x}{\sqrt{(1-\alpha\ y(x))}} y'(x) = 0, \\ x > 0, \qquad 0 \le \alpha \le 1, \end{aligned} \tag{15}$$

with the typical boundary conditions

$$y(0) = 1 , \qquad \lim_{x \to \infty} y(x) = 0 . \tag{16}$$

In Eq. (15), we suppose:

$$\begin{aligned} \sqrt{1-\alpha\ y(x)} \approx\ & 1 + \frac{1}{2}\alpha y(x) + \frac{3}{8}\alpha^2 y^2(x) \\ & + \frac{5}{16}\alpha^3 y^3(x) . \end{aligned} \tag{17}$$

So, we have:

$$\begin{aligned} & y''(x) + 2xy'(x)\Bigg(1 + \frac{1}{2}\alpha y(x) + \frac{3}{8}\alpha^2 y^2(x) \\ & + \frac{5}{16}\alpha^3 y^3(x)\Bigg) = 0, \end{aligned} \tag{18}$$

To solve Eq. (18) with initial condition Eq. (16), according to the homotopy perturbation technique [3], we construct the following convex homotopy:

$$\begin{aligned} & (1-p)\left(\frac{d^2}{dx^2}v(x) + 2x\frac{d}{dx}v(x) - \frac{d^2}{dx^2}y_0(x) \right. \\ & \left. -2x\frac{d}{dx}y_0(x)\right) + p\left(\frac{d^2}{dx^2}v(x) + 2x\frac{d}{dx}v(x) \right. \\ & +\alpha x v(x)\frac{d}{dx}v(x) + \frac{3}{4}\alpha^2 x v^2(x)\frac{d}{dx}v(x) \\ & \left. +\frac{5}{8}\alpha^3 x v^3(x)\frac{d}{dx}v(x)\right) = 0 . \end{aligned} \tag{19}$$

Suppose that the solution of Eq. (6) has the form:

$$v(x) = v_0(x) + pv_1(x) + p^2 v_2(x) + \ldots , \tag{20}$$

where the $v_i(x)$, $i = 0, 1, 2, \ldots$ are functions yet to be determined. The substitution of Eq. (20) into Eq. (19), and comparing coefficients of terms with identical powers of p, leads to:

$$p^0 \ : \ \frac{d^2}{dx^2}v_0(x) + 2x\frac{d}{dx}v_0(x) - \frac{d^2}{dx^2}y_0(x) - 2x\frac{d}{dx}y_0(x) = 0 \ , \tag{21}$$

$$\begin{aligned} p^1 \ : \ & \frac{d^2}{dx^2}v_1(x) + 2x\frac{d}{dx}v_1(x) + \frac{d^2}{dx^2}y_0(x) \\ & + 2x\frac{d}{dx}y_0(x) + \alpha \ x \ v_0(x)\frac{d}{dx}v_0(x) \\ & + \frac{3}{4}\alpha^2 \ x \ (v_0(x))^2\frac{d}{dx}v_0(x) \\ & + \frac{5}{8}\alpha^3 \ x \ (v_0(x))^3\frac{d}{dx}v_0(x) = 0 \ , \end{aligned} \tag{22}$$

$$\begin{aligned} p^2 \ : \ & \frac{d^2}{dx^2}v_2(x) + 2x\frac{d}{dx}v_2(x) + \frac{3}{2}\alpha^2 x v_0(x) \\ & v_1(x)\frac{d}{dx}v_0(x) + \frac{3}{4}\alpha^2 x \ (v_0(x))^2\frac{d}{dx}v_1(x) \\ & + \alpha x \ v_1(x)\frac{d}{dx}v_0(x) + \alpha x \ v_0(x)\frac{d}{dx}v_1(x) \\ & + \frac{15}{8}\alpha^3 x \ (v_0(x))^2 \ v_1(x)\frac{d}{dx}v_0(x) \\ & + \frac{5}{8}\alpha^3 x \ (v_0(x))^3\frac{d}{dx}v_1(x) = 0 \ , \end{aligned} \tag{23}$$

$$\begin{aligned} p^3 \ : \ & \frac{3}{2}\alpha^2 x \ v_0(x) \ v_1(x)\frac{d}{dx}v_1(x) + \frac{3}{4}\alpha^2 x \ (v_1(x))^2 \\ & \frac{d}{dx}v_0(x) + \frac{3}{2}\alpha^2 x \ v_0(x) \ v_2(x)\frac{d}{dx}v_0(x) \\ & + \frac{15}{8}\alpha^3 x \ (v_0(x))^2 \ v_1(x)\frac{d}{dx}v_1(x) \\ & + \frac{15}{8}\alpha^3 x \ (v_0(x))^2 \ v_2(x)\frac{d}{dx}v_0(x) \\ & + \alpha x \ v_0(x)\frac{d}{dx}v_2(x) + \alpha x \ v_1(x)\frac{d}{dx}v_1(x) \\ & + \alpha x \ v_2(x)\frac{d}{dx}v_0(x) + \frac{3}{4}\alpha^2 x \ (v_0(x))^2\frac{d}{dx}v_2(x) \\ & + \frac{15}{8}\alpha^3 x \ v_0(x) \ (v_1(x))^2\frac{d}{dx}v_0(x) \\ & + \frac{d^2}{dx^2}v_3(x) + 2x\frac{d}{dx}v_3(x) \\ & + \frac{5}{8}\alpha^3 x \ (v_0(x))^3 \ \frac{d}{dx}v_2(x) = 0 \ , \end{aligned} \tag{24}$$

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The initial approximation $v_0(x)$ or $y_0(x)$ can be freely chosen, for simplicity we take (the solution of Eq. (21))

$$v_0(x) = y_0(x) = 1 \ , \quad \forall \ x > 0 \ . \tag{25}$$

According to the Eq. (16), we have $v(0) = 1$ and according to the Eq. (20), we have

$$y(x) = \lim_{p\to 1} v(x) = v_0(x) + v_1(x) + v_2(x) + ...,$$

So

$$\begin{aligned} y(0) &= v_0(0) + v_1(0) + v_2(0) + ..., \\ 1 &= 1 + v_1(0) + v_2(0) + ..., \\ 0 &= v_1(0) + v_2(0) + ..., \end{aligned} \tag{26}$$

for simplicity we take $v_i(0) = 0 \quad \forall \ i \geq 1$. According to the Eq. (16), we have $v(\infty) = 0$, from Eq. (25),we have $v_0(\infty) = 1$, and according to the Eq. (20), we have

$$y(x) = \lim_{p\to 1} v(x) = v_0(x) + v_1(x) + v_2(x) + ... \ .$$

So

$$\begin{aligned} y(\infty) &= v_0(\infty) + v_1(\infty) + v_2(\infty) + ..., \\ 0 &= 1 + v_1(\infty) + v_2(\infty) + ..., \\ -1 &= v_1(\infty) + v_2(\infty) + ..., \end{aligned} \tag{27}$$

for simplicity we take

$$\begin{aligned} & v_1(\infty) = -1 \ , \\ & v_i(\infty) = 0 \ , \qquad \forall \ i \geq 2 \ . \end{aligned} \tag{28}$$

The substitution of $v_0(x)$ into Eq. (22) yields

$$\frac{d^2}{dx^2}v_1(x) + 2x\frac{d}{dx}v_1(x) = 0, \quad v_1(0) = 0, \quad v_1(\infty) = -1.$$

Therefore

$$v_1(x) = -erf(x) \ , \tag{29}$$

where

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}dt \ .$$

The substitution of Eq. (29) into Eq. (23) yields

$$\begin{aligned} & \frac{d^2}{dx^2}v_2(x) + 2x\frac{d}{dx}v_2(x) - \frac{2}{\sqrt{\pi}}\alpha x e^{-x^2} \\ & -\frac{3}{2\sqrt{\pi}}\alpha^2 x e^{-x^2} - \frac{5}{4\sqrt{\pi}}\alpha^3 x e^{-x^2} = 0 \ , \\ & v_2(0) = 0, \qquad v_2(\infty) = 0. \end{aligned}$$

therefore

$$v_2(x) = -\frac{1}{16} \ \frac{5x \ \alpha^3 + 8x \ \alpha + 6x \ \alpha^2}{\sqrt{\pi} \ e^{x^2}} \ . \tag{30}$$

The substitution of Eq. (30) into Eq. (24) yields

$$
\begin{aligned}
&\left(\frac{-1}{8\sqrt{\pi}}\alpha(5\alpha^2+6\alpha+8)e^{-x^2}\right.\\
&\left.+\frac{1}{4\sqrt{\pi}}x^2\alpha(5\alpha^2+6\alpha+8)e^{-x^2}\right)\\
&\left(\frac{1}{2}\alpha x+\frac{3}{8}\alpha^2 x+\frac{5}{16}\alpha^3 x\right)\\
&+\frac{2}{\sqrt{\pi}}\alpha x e^{-x^2} erf(x)+\frac{3}{\sqrt{\pi}}\alpha^2 x e^{-x^2} erf(x)\\
&+\frac{15}{4\sqrt{\pi}}\alpha^3 x e^{-x^2} erf(x)+\frac{d^2}{dx^2}v_3(x)+2x\frac{d}{dx}v_3(x)=0\ ,
\end{aligned}
$$

$$v_3(0)=0, \qquad v_3(\infty)=0. \tag{31}$$

Therefore

$$
\begin{aligned}
v_3(x)=&\frac{1}{64\pi^{3/2}\ e^{2x^2}}\left(2\alpha(5\alpha^2+6\alpha+8)\right.\\
&(\frac{1}{2}\alpha+\frac{3}{8}\alpha^2+\frac{5}{16}\alpha^3)\ x^3\ e^{x^2}\ \pi(\frac{1}{2}\alpha+\frac{3}{8}\alpha^2+\frac{5}{16}\alpha^3)\\
&x\ \alpha(5\alpha^2+6\alpha+8)\ e^{x^2}\ \pi+16(2\ \alpha+3\ \alpha^2+\frac{15}{4}\alpha^3)\\
&erf(x)\ x\ e^{x^2}\ \pi+16(2\ \alpha+3\ \alpha^2+\frac{15}{4}\alpha^3)\\
&erf(x)\ \sqrt{\pi}\ e^{2x^2}+16(2\ \alpha+3\ \alpha^2+\frac{15}{4}\alpha^3)\ \sqrt{\pi}\\
&\left.-16(2\ \alpha+3\ \alpha^2+\frac{15}{4}\ \alpha^3)\ \sqrt{\pi}\ e^{2x^2}\right)\ .
\end{aligned}
$$

Therefore, the approximate solution of Eq. (6) can be readily obtained by

$$y(x)=v_0(x)+v_1(x)+v_2(x)+v_3(x)\ ,$$

or

$$
\begin{aligned}
&y(x)=1-erf(x)\\
&-\frac{1}{16}\ \frac{5x\ \alpha^3+8x\ \alpha+6x\ \alpha^2}{\sqrt{\pi}\ e^{x^2}}+\frac{1}{64\pi^{3/2}\ e^{2x^2}}\\
&\left(2\alpha(5\alpha^2+6\alpha+8)(\frac{1}{2}\alpha+\frac{3}{8}\alpha^2+\frac{5}{16}\alpha^3)\ x^3\ e^{x^2}\right.\\
&\pi(\frac{1}{2}\alpha+\frac{3}{8}\alpha^2+\frac{5}{16}\alpha^3)\ x\ \alpha(5\alpha^2+6\alpha+8)\ e^{x^2}\ \pi\\
&+16(2\ \alpha+3\ \alpha^2+\frac{15}{4}\alpha^3)\ erf(x)\ x\ e^{x^2}\ \pi\\
&+16(2\ \alpha+3\ \alpha^2+\frac{15}{4}\alpha^3)\ erf(x)\ \sqrt{\pi}\ e^{2x^2}\\
&+16(2\ \alpha+3\ \alpha^2+\frac{15}{4}\alpha^3)\ \sqrt{\pi}\\
&\left.-16(2\ \alpha+3\ \alpha^2+\frac{15}{4}\ \alpha^3)\ \sqrt{\pi}\ e^{2x^2}\right)\ .
\end{aligned}
$$

Table 1 shows the initial slope $y'(0)$ by HPM and by using the padé[2,2] and padé[3,3] by wazwaz [46] approximants for specific value of $\alpha=0.5$.
Table 2 shows the approximations of $y(x)$ for standard unsteady gas with $\alpha=0.5$ obtained by homotopy perturbation method(HPM) with 4th order approximation, Perturbation technique [40] and padé[2,2] and padé[3,3] by wazwaz [46] approximants.
Also, Figure 1 shows Unsteady gas equation graph obtained by homotopy perturbation method(HPM) with 4th order approximation and Perturbation method by Kidder [40].

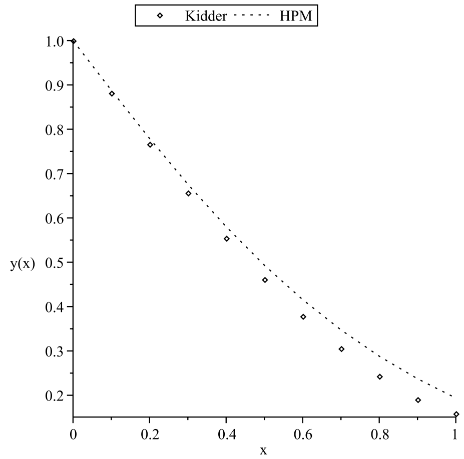

Fig. 1. Unsteady gas equation graph obtained by homotopy perturbation method(HPM) (dash) with 4th order approximation and Perturbation method by Kidder [40](points) .

TABLE I
HPM SOLUTION WITH 4TH ORDER APPROXIMATION OF INITIAL SLOPE $y'(0)$ FOR $\alpha=0.5$.

		Wazwaz[46]	
α	HPM	Padé[2,2]	Padé[3,3]
0.5	−1.124458680	−1.373178096	−1.025529704

V. Conclusion

In this work, an explicit analytical solution is obtained for the unsteady gas equation by means of the homotopy perturbation method(HPM) , which is a powerfull mathematical tool in dealing with nonlinear equations. Using the homotopy perturbation method, it is possible to find the exact solution or an approximate solution of the problem. The numerical results show that the present method is accurate.

TABLE II
HPM SOLUTION WITH 4TH ORDER APPROXIMATION OF $y(x)$ FOR $\alpha=0.5$.

			Wazwaz[46]	
x	HPM	Kidder	Padé[2,2]	Padé[3,3]
0.1	0.88808651	0.88165883	0.86330606	0.89791670
0.2	0.77922351	0.76630768	0.73012623	0.79852282
0.3	0.67597925	0.65653800	0.60330541	0.70411297
0.4	0.58027292	0.55440240	0.48488987	0.61650379
0.5	0.49332936	0.46136503	0.37616039	0.53705338
0.6	0.41572078	0.37831093	0.27773116	0.46656257
0.7	0.34747118	0.30559765	0.18968434	0.40624260
0.8	0.28819396	0.24313255	0.11171052	0.35608017
0.9	0.23723457	0.19046237	0.04323673	0.31799666
1.0	0.19379708	0.15876898	0.01646751	0.29002550

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