Capitolo 1

Studio Numerico

1.1 Kuramoto-Shinomoto-Sakaguchi MV-SDE

Scriviamo, in primis, la MV-SDE relativa al modello che andremo ad analizzare, ovvero il modello di Kuramoto-Shinomoto-Sakaguchi:

$$dX_t = (\mathbb{E}[\sin(X_t)]\cos(X_t) - \mathbb{E}[\cos(X_t)]\sin(X_t))\,dt + \sigma dW_t, \quad X_0 = x_0,$$

dove nel nostro caso $\sigma = 0.5$ e $x_0 = 0.5$. Da questa equazione differenziale si evince facilmente che:

- K = 3, d = 1 e q = 1,
- $\varphi(x) = (1, \sin x, \cos x)$,
- $\alpha(t, x) = (0, \cos x, -\sin x)^T$,
- $\beta(t, x) = (\sigma, 0, 0)^T$.

Il nostro studio, i risultati numerici e le osservazioni che emergono sono ottenuti dal lavoro di due macro funzioni, entrambe volte al calcolo approssimato dei valori di $\mathbb{E}[\sin(X_t)]$ e $\mathbb{E}[\cos(X_t)]$.

- La prima, che usiamo per ottenere la soluzione che svolgerà per noi il ruolo di benchmark, è l'ormai noto metodo Eulero - Monte Carlo. Esso consiste nell'approssimare i due valori attesi in questione mediante una media diretta di $M_1 = 10^6$ simulazioni di processi $(X_t)_{t=0,\cdots T}$ ottenuti dividendo l'intervallo $[0, T]$ in N_1 step temporali e applicando in ciascuno di essi il metodo di Eulero. Nello specifico il Metodo di Eulero calcola gli $N + 1$ valori di un processo discretizzato ottenendo, ad ogni passo, il valore al tempo t dal valore del processo al tempo precedente e dal valore dello step temporale h, secondo la formula:

$$X_{t+1} = X_t + \text{drift}(t) \cdot h + \text{diffusione}(t) \cdot \sqrt{h} \cdot W,$$

dove W è il valore della realizzazione di una Normale Standard. Nel nostro caso gli step temporali sono tutti equivalenti in quanto suddividiamo l'intervallo $[0, T]$ in punti equispaziati. Mettiamo in evidenza che la funzione in questione è strutturata in modo da poter essere applicata anche alla categoria di SDEs di McKean-Vlasov, per le quali il metodo di Eulero al passo $t+1$ richiede che siano noti, all'interno delle parti di drift e diffusione, i valori attesi cercati, al tempo t. Per ottenerli sfruttiamo anche in questo caso una media sulle M_1 simulazioni delle realizzazioni dei processi al tempo t.

- La seconda è il metodo di Discesa Stocastica del Gradiente descritto nel capitolo precedente. Questa produrrà i polinomi $(\mathcal{L}(a))_1(t)$ e $(\mathcal{L}(a))_2(t)$ che saranno rispettivamente le approssimazioni di $\mathbb{E}[\sin(X_t)]$ di $\mathbb{E}[\cos(X_t)]$, risolvendo il nostro problema di ottimizzazione. Per quanto concerne la scelta dello spazio dei polinomi, i due output sono calcolati nella base di quelli ortogonali di Lagrange. La dimensione n dello spazio dei polinomi verrà fatta variare, mentre ciascun elemento della base è della forma:

$$g_i(t) := \prod_{j \leq n \ e \ j \neq n} \left(\frac{t - t_j}{t_i - t_j} \right), \text{ con nodi di Chebyshev } \frac{T}{2} + \frac{T}{2} \cos\left(\frac{2k+1}{2n+2} \pi \right).$$

Anche in questo caso sarà necessario ricorre agli N step di Eulero per trovare la soluzione delle SDEs. Questa volta il metodo di Eulero per la simulazione di $Z(\xi, W)$ e di $\left(Z^a(\tilde{\xi}, \tilde{W}), \partial_{a_{h,j}} Z^a(\tilde{\xi}, \tilde{W}) \right)$ dovrà portare avanti 4 processi simultaneamente: $(X_t)_{t=0,\cdots T}$ e $(Z_t)_{t=0,\cdots T}$ monodimensionali e le due $(Y_t)^{(1,2)}_{t=0,\cdots T}$ $n+1$ dimensionali. Inoltre sarà necessario usare a ogni passo il valore ottenuto per il processo X per poter calcolare le due $Y^{(1,2)}$. Notiamo che X e Z implementano lo step di Eulero al medesimo modo della funzione precedente, ma con due realizzazioni differenti del Browniano. In questo algoritmo semplificato le mappe $\mathbf{h}$ e H sono prese rispettivamente come l'identità e la funzione nulla. Riprendendo i valori delle funzioni dei coefficienti per la MV-SDE relativa al modello di Kuramoto-Shninomoto-Sakaguchi si ottiene che nello specifico le equazioni diventano:

$$dZ_t = ((\mathcal{L}a)_1(t) \cos(Z_t) - (\mathcal{L}a)_2(t) \sin(Z_t))\, dt + \sigma dW_t,$$

$$dY^{j,1}_t = \left(g_j(t) \cos(Z_t) - Y^{j,1}_t \left((\mathcal{L}a)_1(t) \sin(Z_t) + (\mathcal{L}a)_2(t) \cos(Z_t) \right) \right) dt,$$

$$dY^{j,2}_t = \left(-g_j(t) \sin(Z_t) - Y^{j,2}_t \left((\mathcal{L}a)_1(t) \sin(Z_t) + (\mathcal{L}a)_2(t) \cos(Z_t) \right) \right) dt,$$

con $Z_0 = X_0$, $Y^{j,1}_0 = 0$ e $Y^{j,2}_0 = 0$ e per $j = 0, \cdots, n$. Questi processi sono necessari per calcolare la realizzazione del gradiente per la Discesa

Stocastica, ovvero la variabile aleatoria v. Avendo suddiviso il tempo in N step temporali e approssimato l'integrale con una sommatoria la scrittura della v della nostra funzione, componente per componente, è la seguente:

$$v_{j,1}(a;W;\tilde{W}) = \\ 2h\sum_{t=0}^{N}\Big[(\sin(Z_t^a(W)) - (\mathcal{L}a)_1(t))\cdot\Big(\cos(Z_t^a(\tilde{W}))Y_t^{a;j,1}(\tilde{W}) - g_j(t)\Big) \\ +(\cos(Z_t^a(W)) - (\mathcal{L}a)_2(t))\cdot\Big(-\sin(Z_t^a(\tilde{W}))Y_t^{a;j,1}(\tilde{W})\Big)\Big],$$

$$v_{j,2}(a;W;\tilde{W}) = \\ 2h\sum_{t=0}^{N}\Big[(\sin(Z_t^a(W)) - (\mathcal{L}a)_1(t))\cdot\Big(\cos(Z_t^a(\tilde{W}))Y_t^{a;j,2}(\tilde{W})\Big) \\ +(\cos(Z_t^a(W)) - (\mathcal{L}a)_2(t))\cdot\Big(-\sin(Z_t^a(\tilde{W}))Y_t^{a;j,2}(\tilde{W}) - g_j(t)\Big)\Big],$$

con $j = 0,\cdots,n$. Concludiamo mettendo in evidenza che prima di restituire il valore v, questa funzione fa una media di M realizzazioni ottenute corrispondenti ad altrettante simulazioni indipendenti di moti Browniani. Se tale parametro è lasciato a 1 il metodo è un classico metodo SGD, ma se portato a ∞ porta a un metodo GD, ovvero di discesa deterministica. Questo strategia, chiamata Mini Batch, è al centro delle analisi numeriche che abbiamo eseguito. Infine, per quanto concerne la scelta dei *learing rates*, necessari per il calcolo di v ad ogni iterazione, scegliamo $\eta_m = \dfrac{r_0}{(m+1)^\rho}$, basandoci anche sui risultati di [**fehrman2020convergence**], dove i coefficienti $r_0 \in (0,+\infty)$ e $\dfrac{1}{2} < \rho \leq 1$ sono stati fatti variare nel corso dello studio numerico.

1.2 Tabelle e Grafici

Lo scopo della nostra analisi è quello di trovare il numero di iterazioni necessarie per portare a convergenza il metodo di Discesa del Gradiente. A tal proposito esplicitiamo il criterio d'arresto per l'iterazione di a_m: fissati $\gamma_{1,bench}$ e $\gamma_{2,bench}$ ottenuti dalla prima funzione, le iterazioni terminano quando

$$\frac{\|\sum_{i=0}^{n}(a_1)_i g_i - \gamma_{1,bench}\|_{L_2}}{\|\gamma_{1,bench}\|_{L_2}} < 1\% \quad \text{e} \quad \frac{\|\sum_{i=0}^{n}(a_2)_i g_i - \gamma_{2,bench}\|_{L_2}}{\|\gamma_{2,bench}\|_{L_2}} < 1\%.$$

Pertanto la seconda funzione, descritta nella sezione precedente, produrrà come output una soluzione avente errore relativo dell'1%, in norma L_2, ri-

spetto alla soluzione benchmark. Per rendere più generali e corretti possibile tali risultati, ripetiamo lo stesso test al variare di: istante finale $T = 0.5, 1, 2, 4$; dimensione dello spazio dei polinomi $n = 3, 4, 5, 6$; e valori dei parametri $\rho = 0.6, 0.7, 0.8, 0.9$ e $r_0 = 1, 5, 10$ dei *learning rates*. Specifichiamo che, al fine di ottenere lo stesso ordine di errore nel metodo di Eulero e pertanto mantenere costante in tutti i test il parametro h, suddividiamo l'intervallo $[0, T]$ in $N_1 = 50, 100, 200, 400$ step, rispettivamente ai valori di T appena elencati. In particolare ripetiamo per ciascuna casistica lo stesso test 10 volte e riportato: i tempi di convergenza medi, una tabella che per ogni combinazione di ρ e r_0 mostra il numero minore, massimo e medio di iterazioni di convergenza e i grafici delle soluzioni approssimate. I grafici che mostreremo saranno quelli relativi ai valori di ρ e r_0 che nelle tabelle presentano minor numero di iterazioni medie, per ogni combinazione dei parametri T, n e M. Il tutto è eseguito al variare del valore del Mini Batch $M = 1, 10, 10^2, 10^3, 10^4$ e del grado dei polinomi n. Raggruppiamo tali tabelle e grafici in sezioni in base al valore di T.

N.B. 1.2.1. *Quando nelle tabelle compaiono valori come "overflow" o* 49999 *significano rispettivamente che durante l'esecuzione del programma è avvenuto un overflow, ovvero si è raggiunto il limite della capacità di storage del valore (quindi un numero troppo grande); mentre il secondo significa che l'algoritmo non è arrivato a convergenza entro le* 50000 *iterazioni imposte come threshold.*

N.B. 1.2.2. *Per quanto concerne i primi tre valori di M, ovvero $M = 1, 10, 100$, il confronto col benchmark avviene ogni dieci iterazioni, così da contenere i tempi di esecuzione. Mentre per gli ultimi due casi, ovvero $M = 1000, 10000$, tale controllo avviene ogni step.*

N.B. 1.2.3. *Specifichiamo che gli algoritmi sono stati scritti nel linguaggio di programmazione Python, nello specifico nell'applicazione web Jupyter Notebook per la creazione e la condivisione di documenti computazionali. Per quanto concerne i tempi di esecuzione dell'algoritmo, scrivo di seguito le specifiche della macchina sulla quale ho eseguito i test:*

Processore Intel(R) Core(TM) i5-8250U CPU @ 1.60GHz 1.80 GHz
RAM installata 8,00 GB (7,90 GB utilizzabile)
Tipo sistema Sistema operativo a 64 bit, processore basato su x64

1.3 T = 0.5

Caso n = 3

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.95	1.48	4.34
$\rho = 0.7$	2.17	1.5	1.80
$\rho = 0.8$	2.82	1.23	1.96
$\rho = 0.9$	19.22	1.14	1.35

Tabella 1.1: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	80	490	278	170	720	438	630	2500	1281
$\rho = 0.7$	30	2380	639	140	730	442	80	1050	532
$\rho = 0.8$	40	2070	831	100	1240	362	210	1170	581
$\rho = 0.9$	400	20930	5668	160	600	338	150	1020	403

Tabella 1.2: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.41	0.32	0.74
$\rho = 0.7$	0.48	0.38	0.44
$\rho = 0.8$	1.25	0.34	0.40
$\rho = 0.9$	2.46	0.35	0.41

Tabella 1.3: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	20	230	105	10	150	82	60	510	192
$\rho = 0.7$	30	400	124	30	190	99	30	270	113
$\rho = 0.8$	30	1160	322	30	180	88	60	170	104
$\rho = 0.9$	20	2660	635	20	220	89	30	290	106

Tabella 1.4: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.11	0.14	0.19
$\rho = 0.7$	0.18	0.12	0.17
$\rho = 0.8$	0.36	0.15	0.17
$\rho = 0.9$	0.53	0.14	0.12

Tabella 1.5: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	10	30	17	10	50	23	10	80	30
$\rho = 0.7$	10	60	29	10	40	19	10	70	27
$\rho = 0.8$	20	270	58	10	70	24	10	50	27
$\rho = 0.9$	20	470	86	10	50	22	10	40	20

Tabella 1.6: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.34	0.09	0.26
$\rho = 0.7$	0.42	0.09	0.20
$\rho = 0.8$	0.52	0.10	0.18
$\rho = 0.9$	0.84	0.09	0.16

Tabella 1.7: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	7	13	9.1	2	4	2.5	6	11	7.1
$\rho = 0.7$	9	15	11.4	2	5	2.4	5	8	5.6
$\rho = 0.8$	11	27	14.1	2	6	2.7	4	7	5
$\rho = 0.9$	14	35	23	2	5	2.4	4	6	4.4

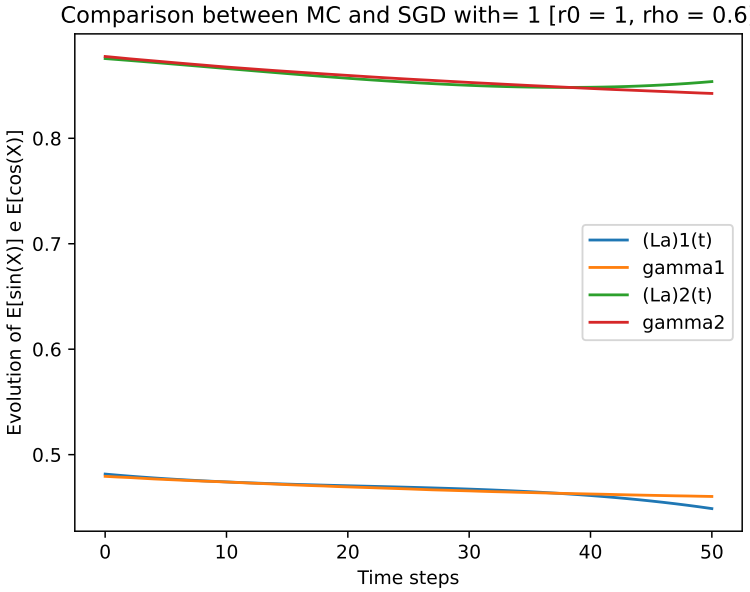
Tabella 1.8: Number of iterations m to achieve convergence with $M = 1000$

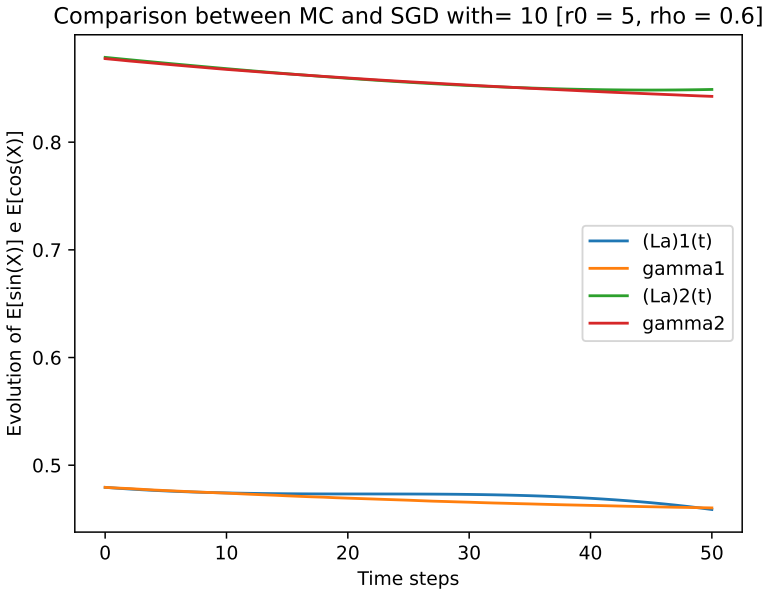
	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	3.48	0.90	2.69
$\rho = 0.7$	4.23	0.90	2.23
$\rho = 0.8$	5.14	0.90	1.79
$\rho = 0.9$	7.21	0.94	1.79

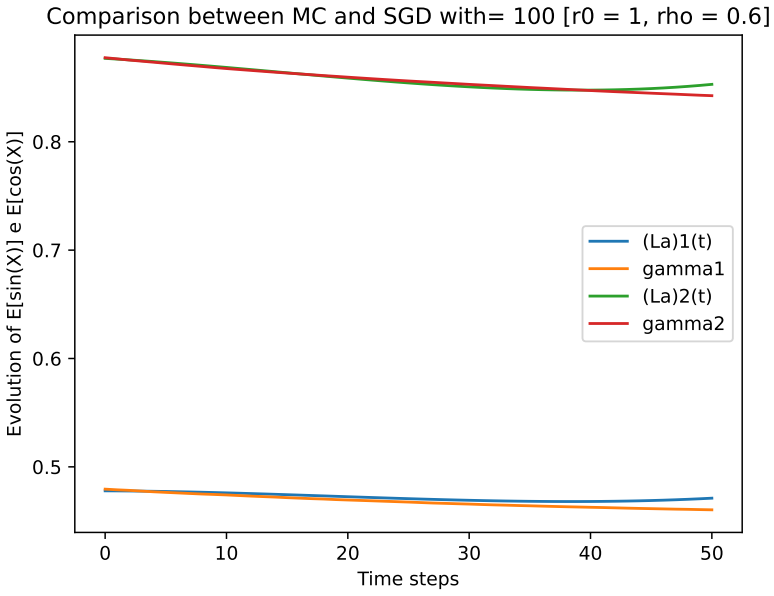
Tabella 1.9: Average execution times (in seconds s) with $M = 10000$

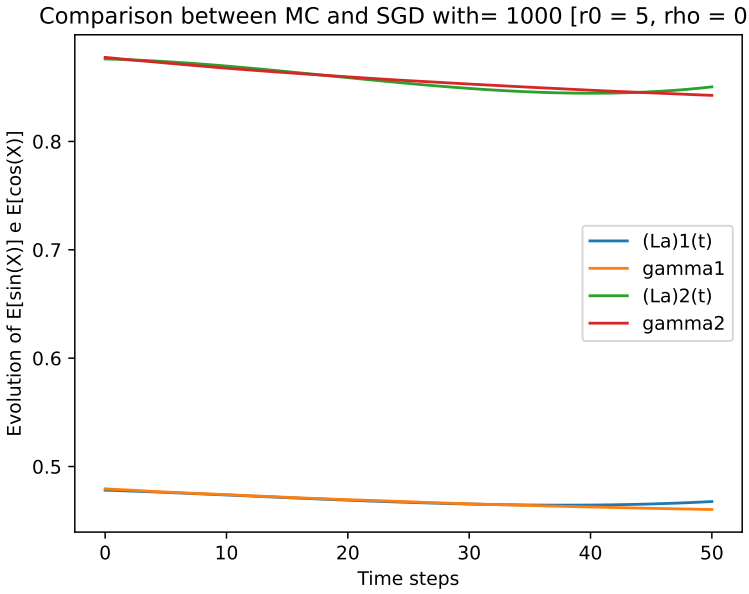
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	7	8	7.7	2	2	2	6	6	6
$\rho = 0.7$	9	11	9.4	2	2	2	5	5	5
$\rho = 0.8$	11	13	11.5	2	2	2	4	4	4
$\rho = 0.9$	15	18	16.1	2	3	2.1	4	4	4

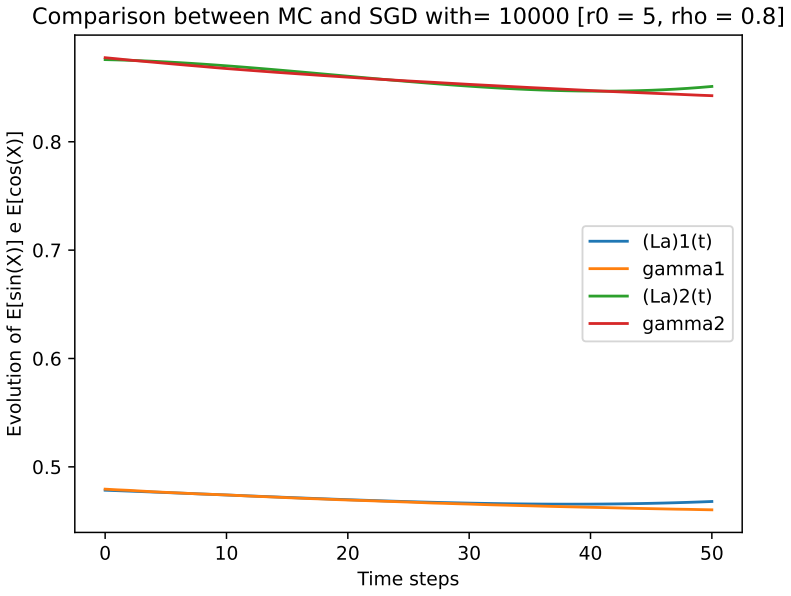
Tabella 1.10: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 5, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 100 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 1000 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 10000 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2

Caso n = 4

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	1.72	2.90	4.55
$\rho = 0.7$	1.45	1.78	2.92
$\rho = 0.8$	15.90	2.54	2.16
$\rho = 0.9$	56.10	1.97	1.70

Tabella 1.11: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	60	2080	484	270	1460	819	950	1680	1288
$\rho = 0.7$	120	1350	412	150	850	505	350	1620	828
$\rho = 0.8$	120	22300	4501	250	1480	720	230	1510	609
$\rho = 0.9$	190	49999	15893.9	60	1520	560	210	1180	481

Tabella 1.12: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.26	0.39	0.77
$\rho = 0.7$	0.57	0.33	0.57
$\rho = 0.8$	0.42	0.43	0.34
$\rho = 0.9$	6.07	0.22	0.31

Tabella 1.13: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	30	150	64	40	160	95	70	310	188
$\rho = 0.7$	30	450	139	20	150	80	40	220	140
$\rho = 0.8$	30	210	104	30	230	104	20	200	84
$\rho = 0.9$	20	6840	1486	10	120	54	30	180	76

Tabella 1.14: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.16	0.12	0.23
$\rho = 0.7$	0.21	0.13	0.26
$\rho = 0.8$	0.32	0.12	0.15
$\rho = 0.9$	0.55	0.13	0.17

Tabella 1.15: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	20	40	24	10	30	18	10	80	34
$\rho = 0.7$	20	90	31	10	50	19	20	100	38
$\rho = 0.8$	20	100	48	10	30	17	10	30	21
$\rho = 0.9$	40	180	82	10	30	20	10	40	25

Tabella 1.16: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.56	0.13	0.2
$\rho = 0.7$	0.66	0.13	0.20
$\rho = 0.8$	0.99	0.14	0.16
$\rho = 0.9$	1.53	0.15	0.14

Tabella 1.17: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	10	19	13.1	2	7	2.9	4	6	4.6
$\rho = 0.7$	13	20	15.4	2	5	2.9	4	6	4.6
$\rho = 0.8$	19	37	23.3	2	11	3.2	3	4	3.8
$\rho = 0.9$	30	44	35.9	2	16	3.6	3	4	3.3

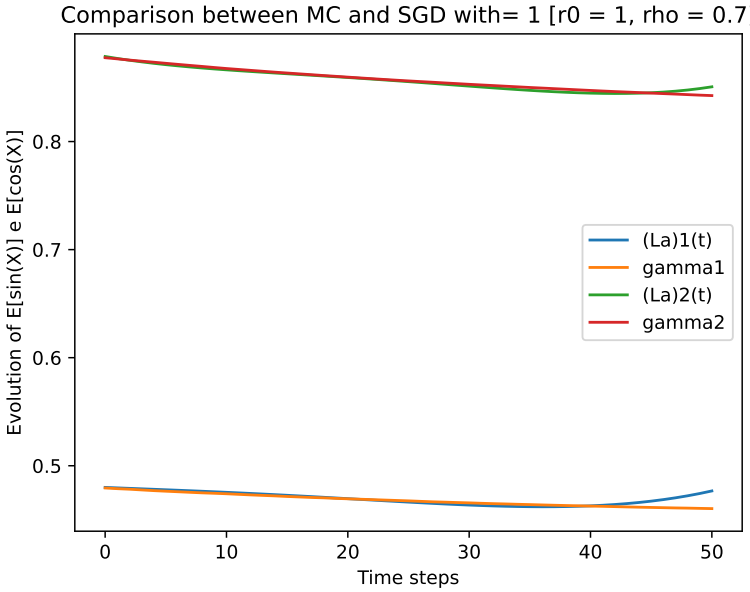
Tabella 1.18: Number of iterations m to achieve convergence with $M = 1000$

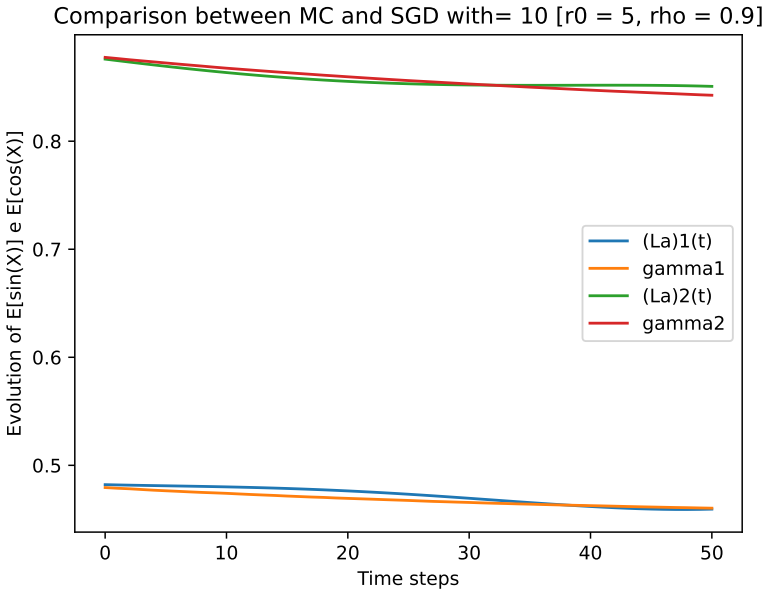
	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	6.26	1.07	2.12
$\rho = 0.7$	7.68	1.07	2.14
$\rho = 0.8$	10.63	1.06	1.71
$\rho = 0.9$	17.56	1.06	1.66

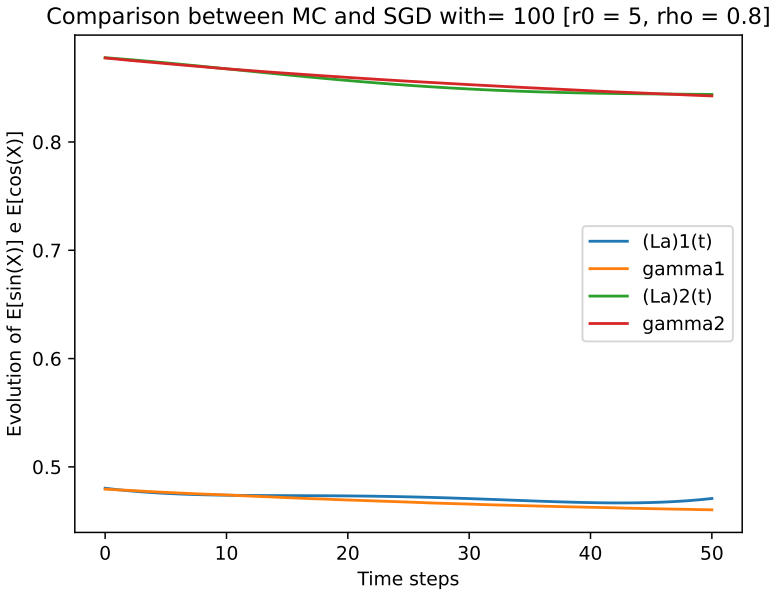
Tabella 1.19: Average execution times (in seconds s) with $M = 10000$

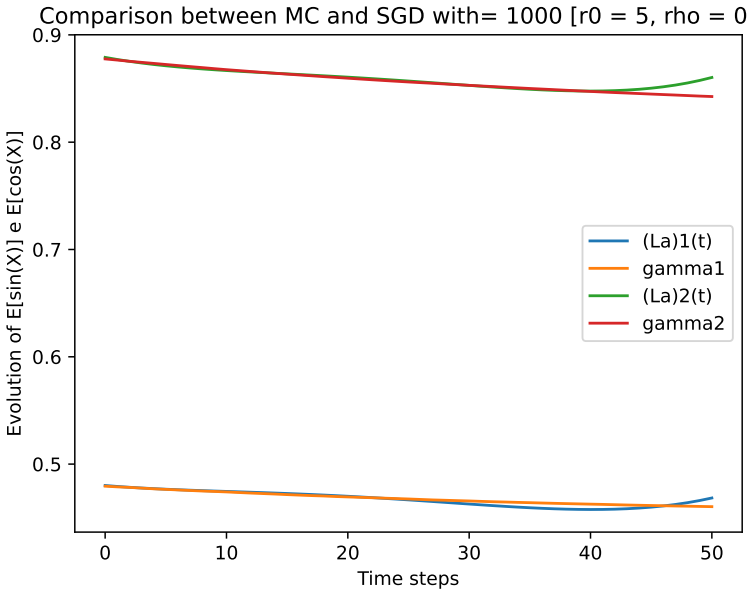
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	11	12	11.6	2	2	2	4	4	4
$\rho = 0.7$	14	15	14.3	2	2	2	4	4	4
$\rho = 0.8$	18	21	19.7	2	2	2	3	4	3.2
$\rho = 0.9$	29	35	32.7	2	2	2	3	4	3.1

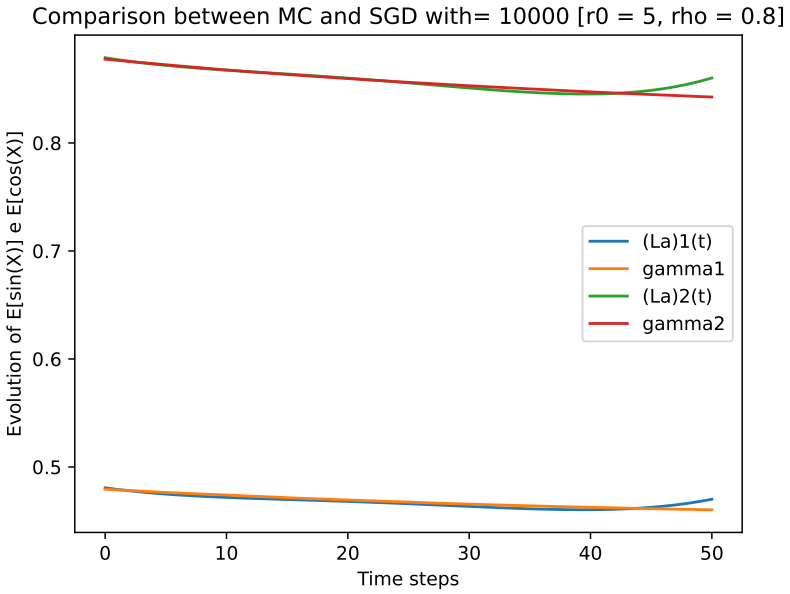
Tabella 1.20: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
0.5
0.6
0.7
0.8
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 100 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.5
0.6
0.7
0.8
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 1000 [r0 = 5, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
10
20
30
40
50


Comparison between MC and SGD with= 10000 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.8
0.7
0.6
0.5
0
10
20
30
40
50

Caso n = 5

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	1.36	4.12	8.9
$\rho = 0.7$	2.67	2.78	3.51
$\rho = 0.8$	4.66	2.35	2.92
$\rho = 0.9$	90.29	3.76	2.65

Tabella 1.21: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	130	890	371	370	1760	866	610	2870	1714
$\rho = 0.7$	110	2470	729	240	1070	589	420	1050	753
$\rho = 0.8$	90	4260	1271	130	1190	496	240	1100	613
$\rho = 0.9$	440	49999	22338.7	200	1330	661	160	1380	565

Tabella 1.22: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.85	0.64	1.25
$\rho = 0.7$	0.88	0.53	0.73
$\rho = 0.8$	2.96	0.61	0.37
$\rho = 0.9$	2.35	0.58	0.53

Tabella 1.23: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	20	400	148	30	290	110	80	340	217
$\rho = 0.7$	20	630	157	20	190	89	40	250	132
$\rho = 0.8$	60	2110	513	20	280	104	30	100	68
$\rho = 0.9$	110	1210	411	30	250	103	20	210	93

Tabella 1.24: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.29	0.28	0.25
$\rho = 0.7$	0.48	0.25	0.40
$\rho = 0.8$	0.98	0.18	0.38
$\rho = 0.9$	1.31	0.29	0.19

Tabella 1.25: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	20	40	27	10	60	21	10	40	22
$\rho = 0.7$	30	80	49	10	60	19	10	60	33
$\rho = 0.8$	40	180	85	10	30	16	10	50	30
$\rho = 0.9$	70	180	114	10	60	24	10	40	16

Tabella 1.26: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	1.41	0.22	0.30
$\rho = 0.7$	1.73	0.17	0.26
$\rho = 0.8$	2.85	0.15	0.23
$\rho = 0.9$	5.33	0.19	0.28

Tabella 1.27: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	15	25	18.6	1	8	2.6	3	6	3.8
$\rho = 0.7$	20	24	22.5	1	7	2.2	3	4	3.3
$\rho = 0.8$	32	45	37.5	1	4	1.9	3	4	3.2
$\rho = 0.9$	53	104	70.3	1	6	2.6	3	6	3.5

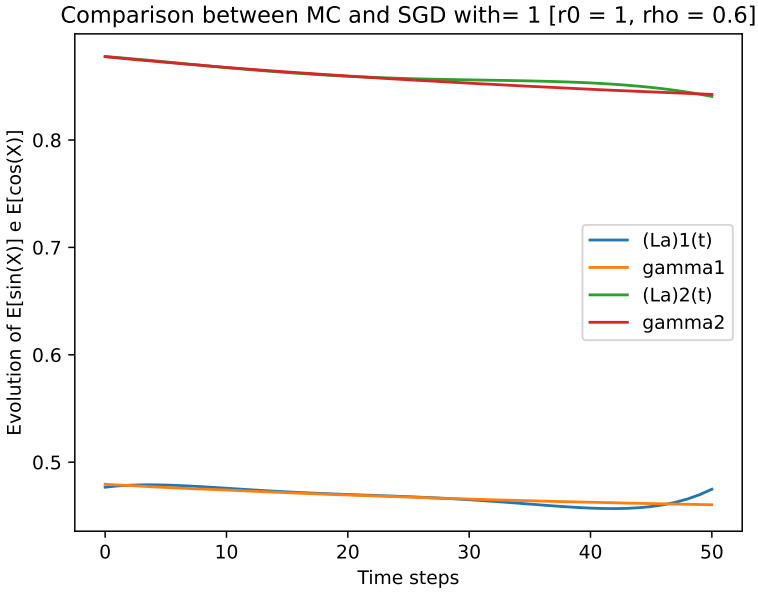
Tabella 1.28: Number of iterations m to achieve convergence with $M = 1000$

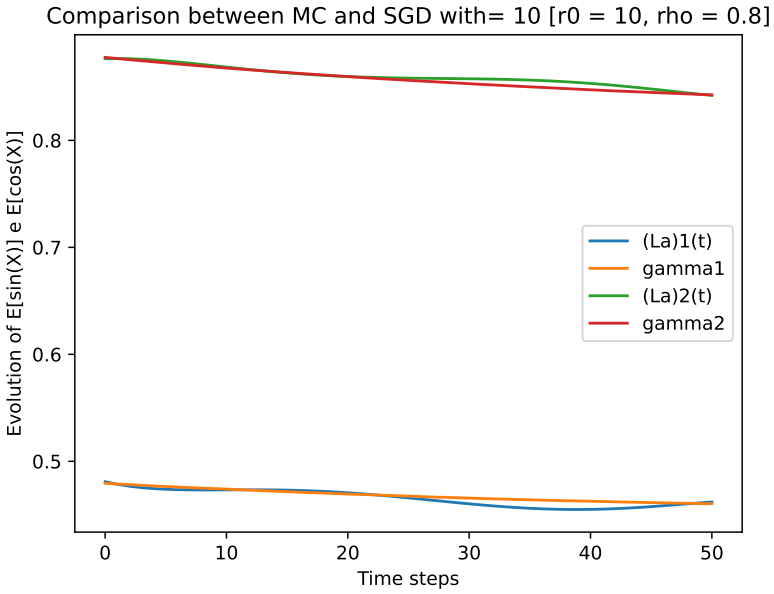
	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	16.95	1.42	3.38
$\rho = 0.7$	23.12	1.48	3.20
$\rho = 0.8$	36.69	1.3	3.22
$\rho = 0.9$	68.4	1.13	3.13

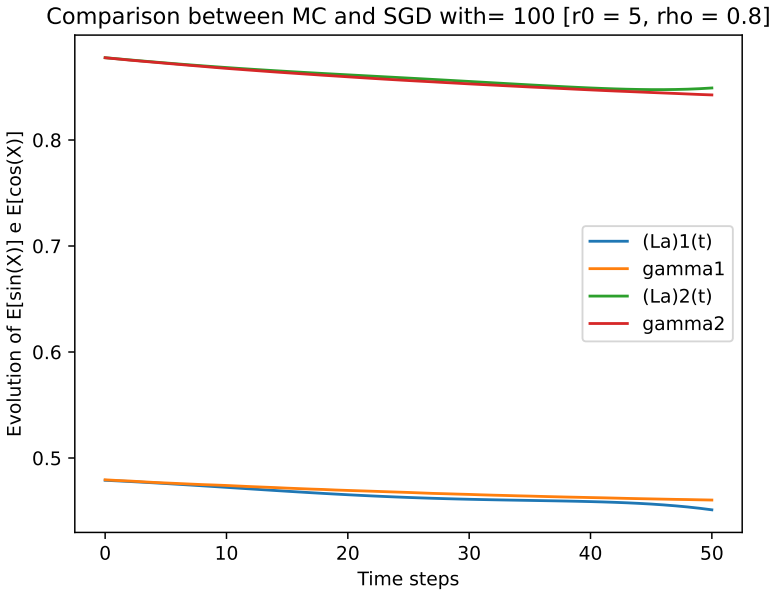
Tabella 1.29: Average execution times (in seconds s) with $M = 10000$

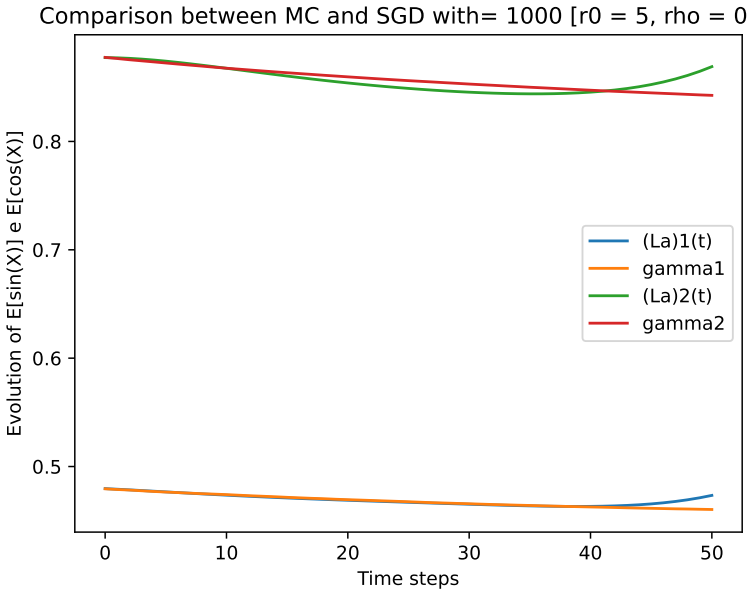
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	15	16	15.7	1	2	1.3	3	4	3.1
$\rho = 0.7$	21	23	21.5	1	2	1.4	3	3	3
$\rho = 0.8$	32	36	34.4	1	2	1.2	3	3	3
$\rho = 0.9$	59	66	63.4	1	1	1	2	3	2.9

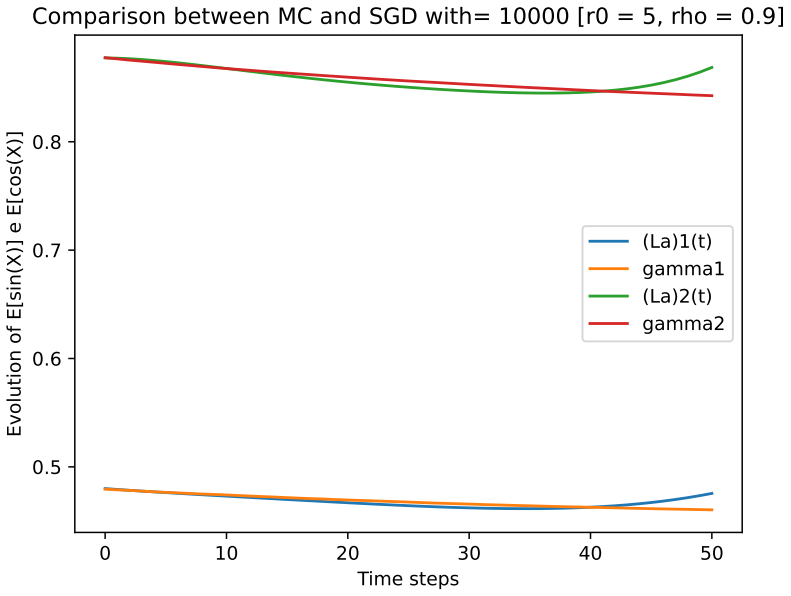
Tabella 1.30: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 10, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 100 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 1000 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 10000 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2

Caso n = 6

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	5.18	4.60	8.74
$\rho = 0.7$	4.49	4.45	5.47
$\rho = 0.8$	19.83	5.0	4.59
$\rho = 0.9$	147.74	9.4	4.57

Tabella 1.31: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	130	2650	864	260	1430	791	410	2140	1250
$\rho = 0.7$	80	2080	731	210	1060	782	260	1370	766
$\rho = 0.8$	100	8230	3184	200	1550	809	260	1720	663
$\rho = 0.9$	180	49999	25371.6	220	6350	1360	190	2440	765

Tabella 1.32: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.68	0.33	0.74
$\rho = 0.7$	0.56	0.44	0.49
$\rho = 0.8$	2.31	0.53	0.51
$\rho = 0.9$	24.46	0.48	0.39

Tabella 1.33: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	30	240	89	20	180	75	30	380	167
$\rho = 0.7$	40	160	77	40	210	99	40	160	110
$\rho = 0.8$	50	1080	329	40	300	119	40	290	116
$\rho = 0.9$	120	21270	4210	10	240	108	30	220	89

Tabella 1.34: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.32	0.19	0.19
$\rho = 0.7$	0.64	0.13	0.25
$\rho = 0.8$	0.64	0.14	0.18
$\rho = 0.9$	1.54	0.16	0.17

Tabella 1.35: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	30	130	41	10	40	24	10	50	24
$\rho = 0.7$	40	290	81	10	30	16	10	60	31
$\rho = 0.8$	40	120	81	10	60	18	10	40	23
$\rho = 0.9$	110	300	196	10	60	20	10	40	22

Tabella 1.36: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	1.28	0.13	0.20
$\rho = 0.7$	1.87	0.12	0.19
$\rho = 0.8$	2.90	0.16	0.13
$\rho = 0.9$	6.89	0.11	0.16

Tabella 1.37: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	22	29	23.7	1	4	2.5	3	8	3.8
$\rho = 0.7$	31	43	34.6	1	4	2.1	2	7	3.5
$\rho = 0.8$	50	56	53.4	1	7	2.9	2	4	2.5
$\rho = 0.9$	105	174	127.3	1	6	2	2	4	2.9

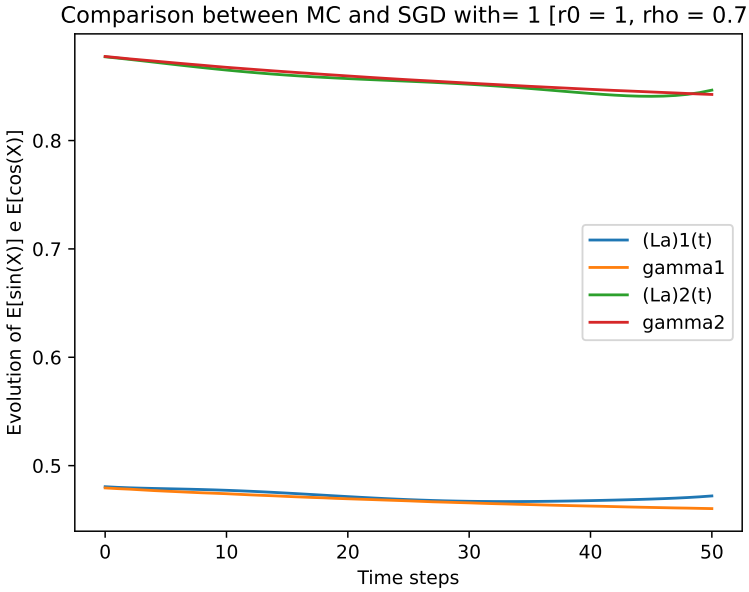
Tabella 1.38: Number of iterations m to achieve convergence with $M = 1000$

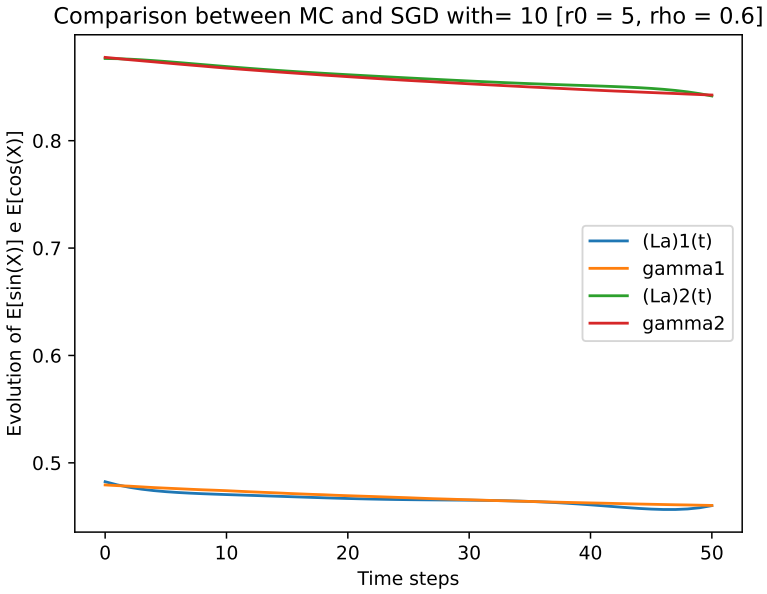
	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	16.77	0.77	1.92
$\rho = 0.7$	24.63	1.00	1.61
$\rho = 0.8$	41.76	0.77	1.61
$\rho = 0.9$	93.11	0.85	1.54

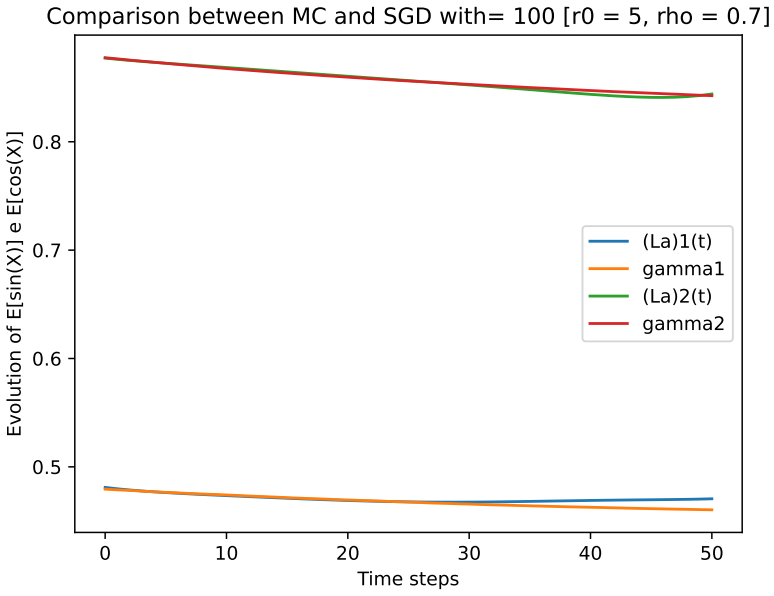
Tabella 1.39: Average execution times (in seconds s) with $M = 10000$

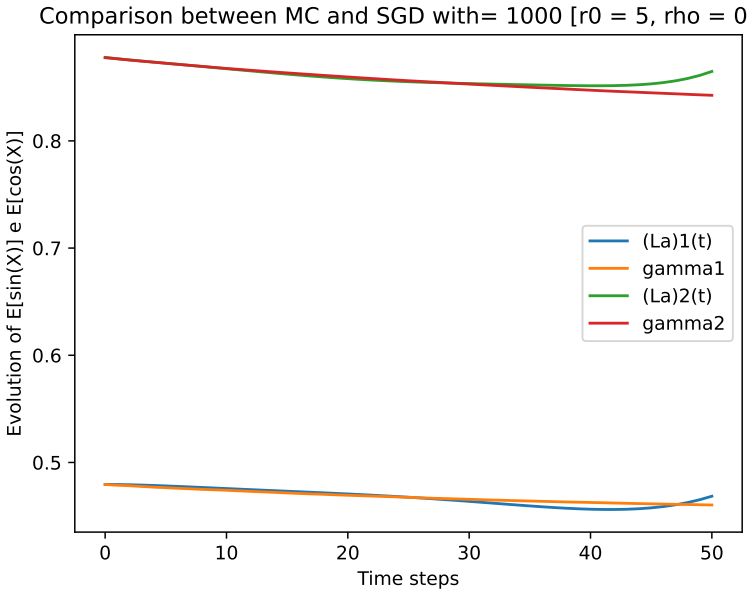
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	21	23	21.8	1	1	1	2	3	2.5
$\rho = 0.7$	31	34	31.9	1	2	1.3	2	3	2.1
$\rho = 0.8$	52	59	54.4	1	1	1	2	3	2.1
$\rho = 0.9$	114	128	121.3	1	2	1.1	2	2	2

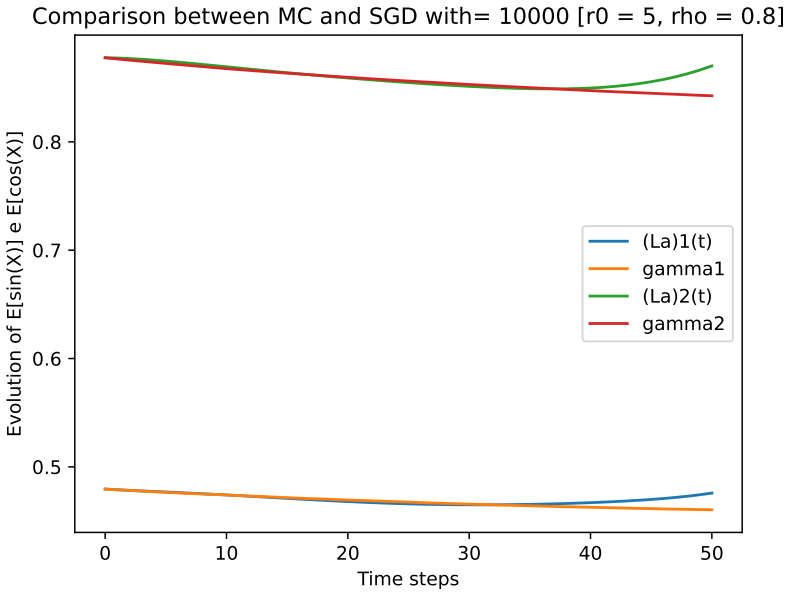
Tabella 1.40: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 5, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 100 [r0 = 5, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 1000 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 10000 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
10
20
30
40
50
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2

1.4 T = 1

Caso n = 3

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	4.60	14.19	30.59
$\rho = 0.7$	6.06	8.29	13.83
$\rho = 0.8$	17.74	6.40	9.32
$\rho = 0.9$	29.5	5.47	5.58

Tabella 1.41: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	220	1470	683	1090	3170	2090	2110	6360	4544
$\rho = 0.7$	170	2540	921	280	2290	1199	890	4200	2029
$\rho = 0.8$	160	13840	2754	390	1590	947	580	2140	1350
$\rho = 0.9$	690	10410	4305	330	1740	812	230	1860	836

Tabella 1.42: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	1.22	1.81	3.94
$\rho = 0.7$	1.23	1.34	2.58
$\rho = 0.8$	2.29	1.33	1.53
$\rho = 0.9$	6.92	0.93	1.95

Tabella 1.43: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	50	330	159	90	440	234	200	1060	496
$\rho = 0.7$	10	500	160	70	500	173	140	500	323
$\rho = 0.8$	30	1210	298	50	480	172	110	410	194
$\rho = 0.9$	50	2210	904	50	290	120	20	550	253

Tabella 1.44: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.27	0.68	0.88
$\rho = 0.7$	0.39	0.47	0.92
$\rho = 0.8$	0.32	0.45	0.47
$\rho = 0.9$	0.37	0.59	0.59

Tabella 1.45: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	10	40	21	10	100	54	40	130	69
$\rho = 0.7$	10	50	30	10	70	38	20	110	74
$\rho = 0.8$	10	40	25	20	70	36	20	90	38
$\rho = 0.9$	10	50	29	20	150	46	30	90	47

Tabella 1.46: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.49	0.62	1.82
$\rho = 0.7$	0.75	0.54	1.21
$\rho = 0.8$	0.66	0.44	1.03
$\rho = 0.9$	0.82	0.46	0.75

Tabella 1.47: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	5	7	5.6	6	12	6.9	19	24	20.8
$\rho = 0.7$	5	27	8.9	5	9	6.3	13	16	14.4
$\rho = 0.8$	6	10	7.9	4	10	5.1	10	14	11.8
$\rho = 0.9$	7	18	9.7	4	13	5.4	8	13	8.9

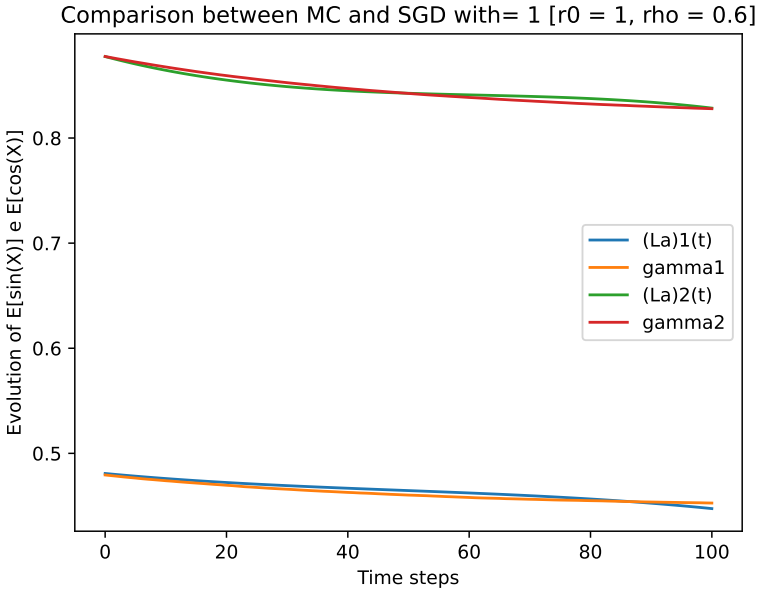
Tabella 1.48: Number of iterations m to achieve convergence with $M = 1000$

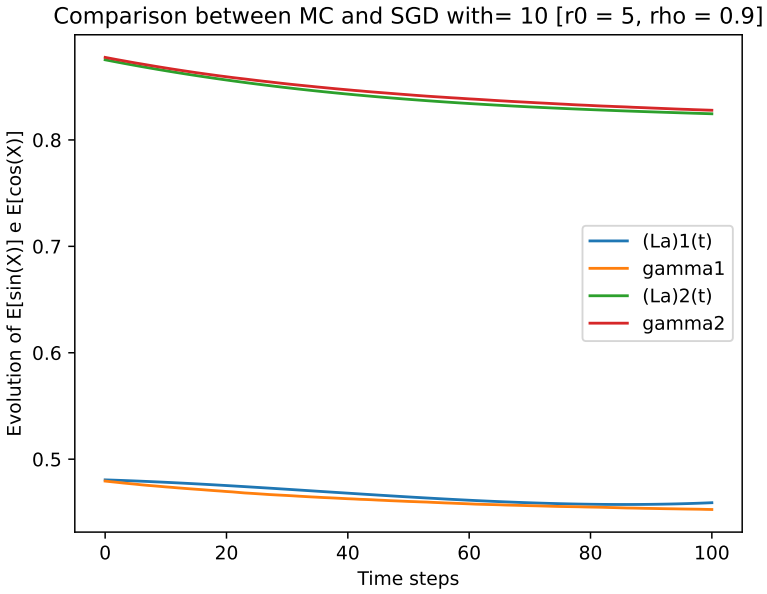
	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	4.71	5.83	18.25
$\rho = 0.7$	5.65	4.93	12.90
$\rho = 0.8$	6.19	3.93	9.96
$\rho = 0.9$	7.45	3.98	7.93

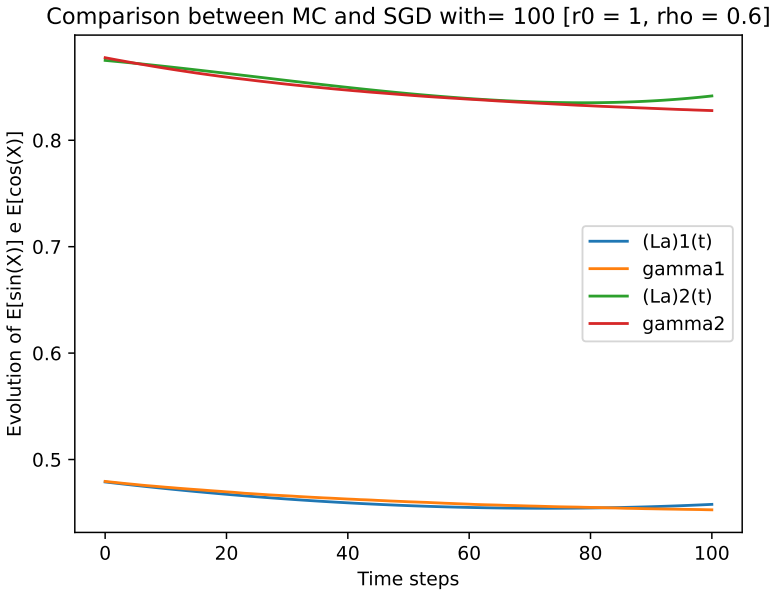
Tabella 1.49: Average execution times (in seconds s) with $M = 10000$

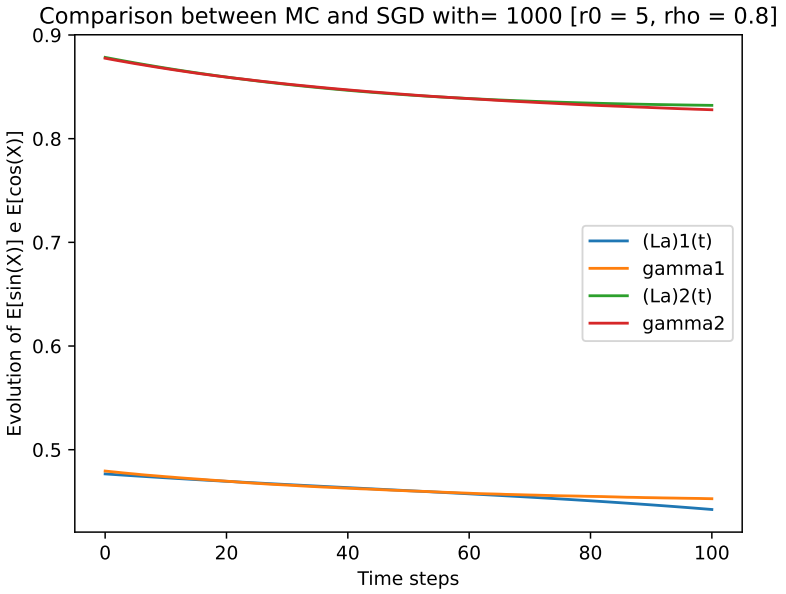
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	4	5	4.8	6	6	6	17	20	17.8
$\rho = 0.7$	5	7	5.5	5	5	5	13	13	13
$\rho = 0.8$	6	7	6.1	4	4	4	10	10	10
$\rho = 0.9$	7	11	7.7	4	4	4	8	8	8

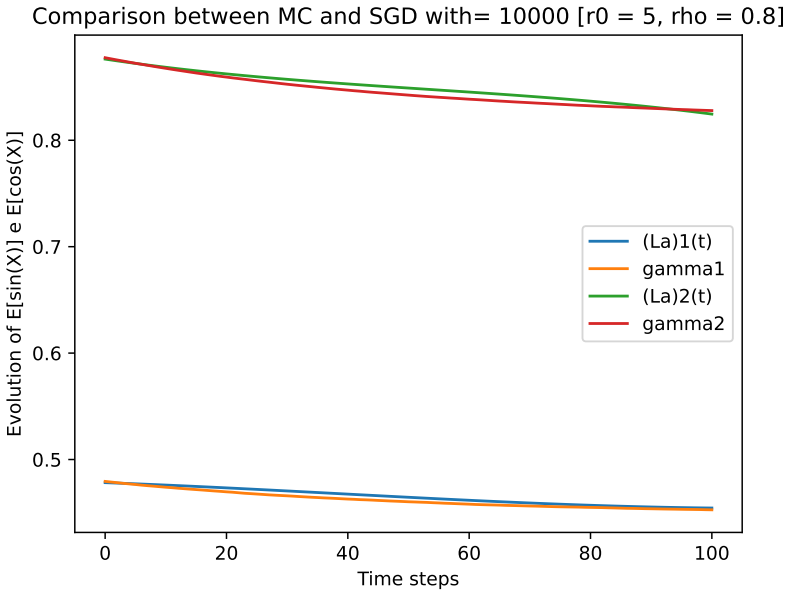
Tabella 1.50: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
20
40
60
80
100
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 100 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
20
40
60
80
100
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 1000 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
20
40
60
80
100


Comparison between MC and SGD with= 10000 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.8
0.7
0.6
0.5
0
20
40
60
80
100

Caso n = 4

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	3.72	15.20	37.20
$\rho = 0.7$	6.40	6.31	15.76
$\rho = 0.8$	10.64	5.83	9.13
$\rho = 0.9$	97.45	5.70	7.09

Tabella 1.51: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	120	1280	517	1080	3820	2261	1830	8590	5523
$\rho = 0.7$	290	3140	949	400	1500	942	540	4030	2334
$\rho = 0.8$	170	3520	1504	300	1270	869	440	2330	1346
$\rho = 0.9$	720	49999	14405.8	280	1960	848	380	1850	1047

Tabella 1.52: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.92	2.69	6.23
$\rho = 0.7$	0.96	1.72	7.68
$\rho = 0.8$	2.55	1.45	5.01
$\rho = 0.9$	17.99	1.39	3.59

Tabella 1.53: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	50	180	117	110	590	340	260	890	548
$\rho = 0.7$	10	300	123	100	660	219	190	530	369
$\rho = 0.8$	40	910	325	70	290	183	100	450	256
$\rho = 0.9$	40	11880	2280	20	520	177	60	310	180

Tabella 1.54: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	1.36	2.10	3.85
$\rho = 0.7$	1.81	1.36	2.15
$\rho = 0.8$	1.61	1.74	1.55
$\rho = 0.9$	1.33	1.11	1.45

Tabella 1.55: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	10	130	40	20	150	62	60	140	106
$\rho = 0.7$	20	260	55	10	100	39	20	150	61
$\rho = 0.8$	20	80	49	20	80	48	20	90	44
$\rho = 0.9$	10	120	41	10	60	31	20	60	41

Tabella 1.56: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	2.17	1.38	3.64
$\rho = 0.7$	2.24	1.33	2.73
$\rho = 0.8$	3.97	1.31	2.31
$\rho = 0.9$	5.90	1.08	1.85

Tabella 1.57: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	7	12	9.1	4	10	5.9	13	21	15.3
$\rho = 0.7$	8	16	9.5	4	13	5.6	10	15	11.5
$\rho = 0.8$	10	40	16.2	3	11	5.6	8	15	9.8
$\rho = 0.9$	12	58	24.7	3	11	4.6	7	11	7.9

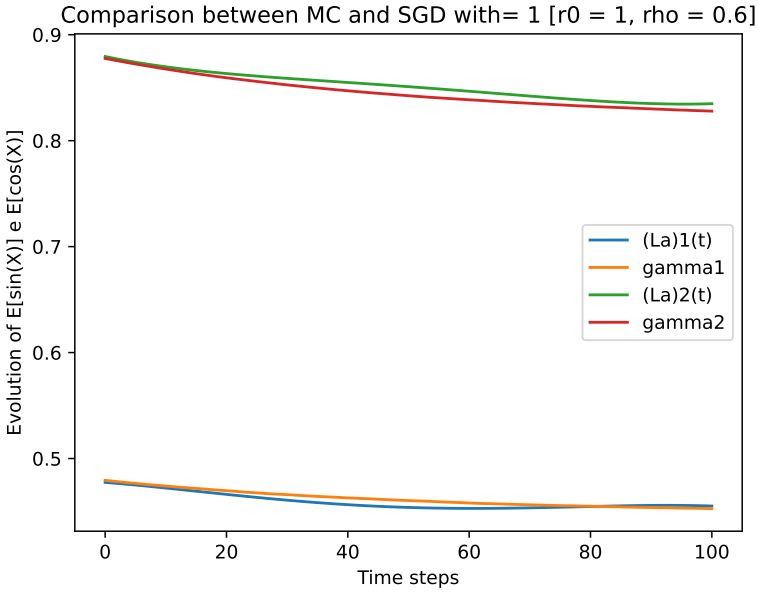
Tabella 1.58: Number of iterations m to achieve convergence with $M = 1000$

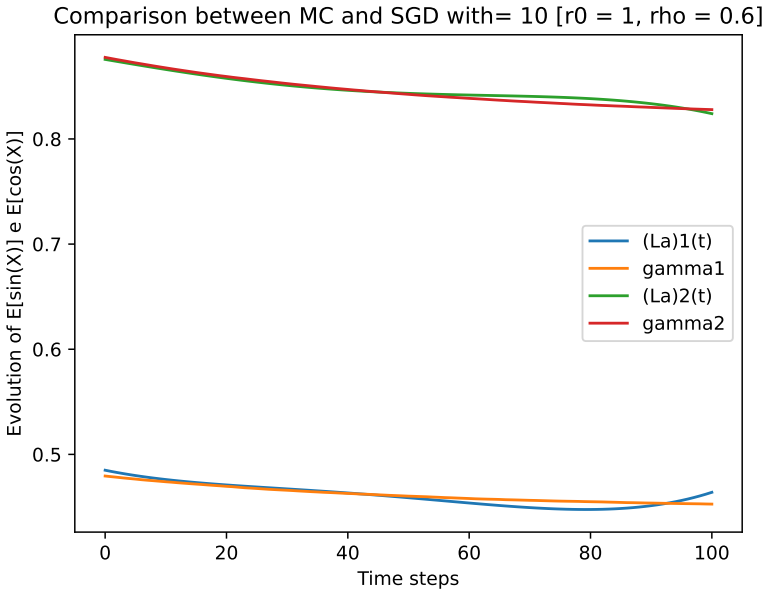
	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	17.09	10.90	33.12
$\rho = 0.7$	20.29	9.95	24.45
$\rho = 0.8$	25.10	9.10	16.95
$\rho = 0.9$	32.85	8.00	7.84

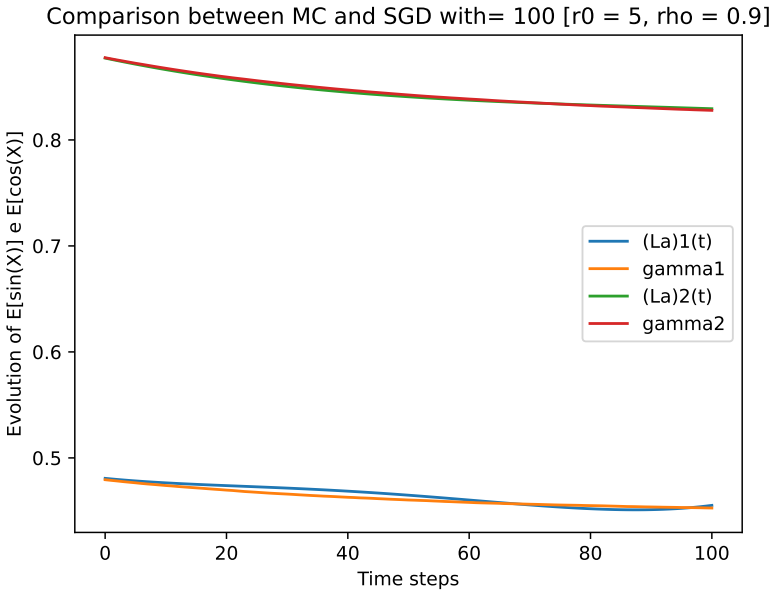
Tabella 1.59: Average execution times (in seconds s) with $M = 10000$

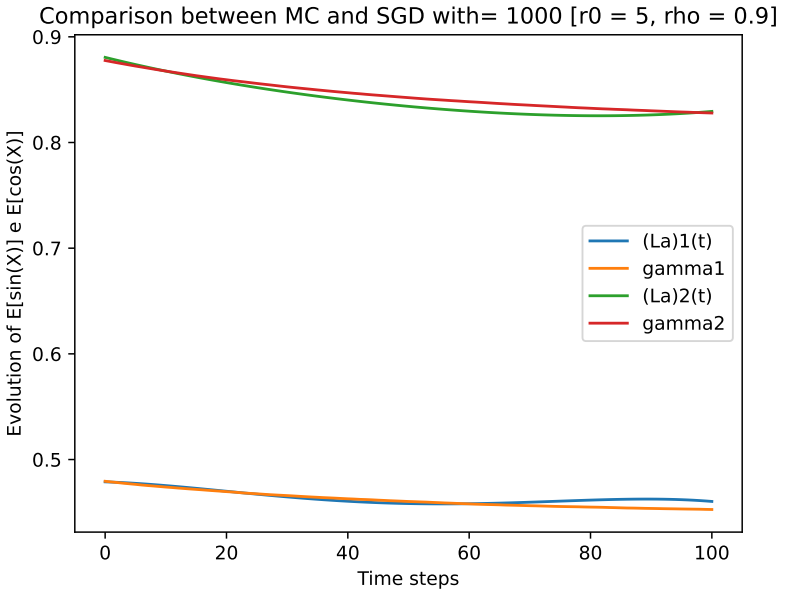
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	7	7	7	4	5	4.4	13	14	13.3
$\rho = 0.7$	7	9	8.3	4	4	4	10	10	10
$\rho = 0.8$	9	13	10.2	3	4	3.7	8	8	8
$\rho = 0.9$	12	15	13.3	3	3	3	7	7	7

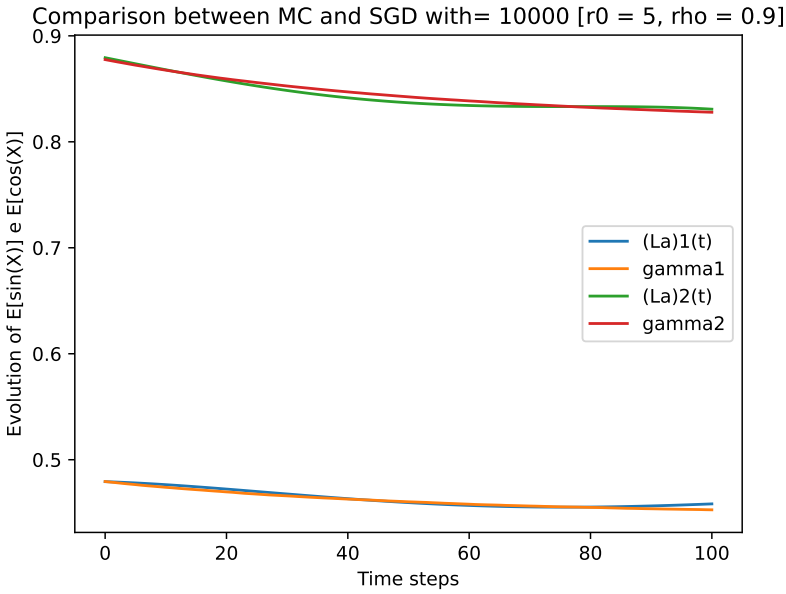
Tabella 1.60: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
20
40
60
80
100
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 100 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
20
40
60
80
100
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 1000 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
20
40
60
80
100


Comparison between MC and SGD with= 10000 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
20
40
60
80
100

Caso n = 5

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	4.81	14.76	31.03
$\rho = 0.7$	9.11	7.29	17.28
$\rho = 0.8$	31.30	5.37	10.54
$\rho = 0.9$	90.16	6.65	5.83

Tabella 1.61: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	340	1340	709	1670	3170	2166	1690	8820	4546
$\rho = 0.7$	410	2260	1323	160	2300	1071	770	4350	2533
$\rho = 0.8$	240	24910	4579	90	1650	789	820	2910	1543
$\rho = 0.9$	570	49999	13249.9	470	1860	978	380	1350	857

Tabella 1.62: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	1.42	1.65	4.35
$\rho = 0.7$	1.71	1.87	2.07
$\rho = 0.8$	3.27	1.42	1.88
$\rho = 0.9$	1.47	1.26	1.81

Tabella 1.63: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	50	580	179	80	420	206	120	870	540
$\rho = 0.7$	60	1180	216	80	430	235	120	570	257
$\rho = 0.8$	50	1460	411	140	330	178	70	390	235
$\rho = 0.9$	40	750	185	50	370	157	90	490	228

Tabella 1.64: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.66	0.47	1.23
$\rho = 0.7$	0.63	0.69	0.87
$\rho = 0.8$	0.73	0.39	0.95
$\rho = 0.9$	1.93	0.49	0.43

Tabella 1.65: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	20	100	46	10	60	33	30	150	86
$\rho = 0.7$	20	150	44	10	130	48	20	90	61
$\rho = 0.8$	20	180	51	10	40	27	30	110	66
$\rho = 0.9$	40	370	135	10	90	34	10	60	30

Tabella 1.66: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	1.35	0.67	1.37
$\rho = 0.7$	1.48	0.52	0.92
$\rho = 0.8$	2.33	0.37	0.87
$\rho = 0.9$	3.61	0.53	0.74

Tabella 1.67: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	10	18	12.3	3	14	6.1	10	15	12.5
$\rho = 0.7$	11	16	13.4	3	8	4.7	8	10	8.4
$\rho = 0.8$	17	27	21.1	3	4	3.3	6	12	7.9
$\rho = 0.9$	23	66	32.9	3	11	4.8	6	8	6.7

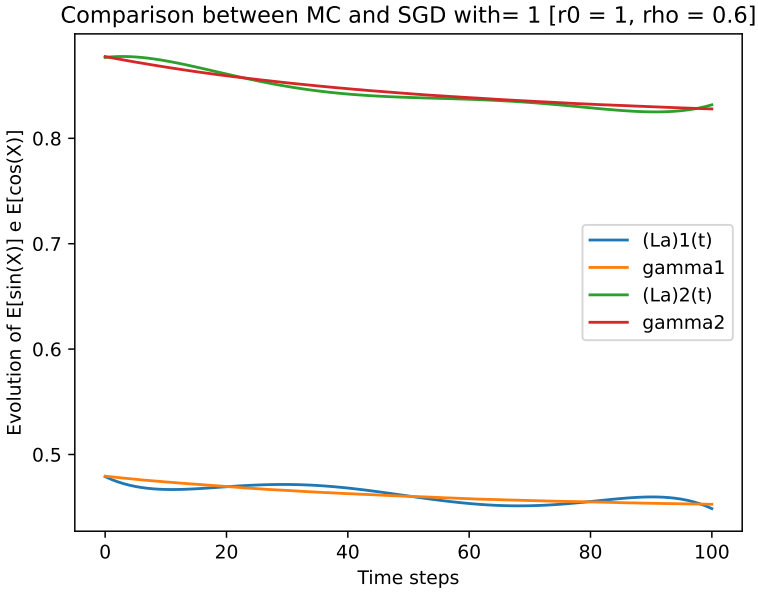
Tabella 1.68: Number of iterations m to achieve convergence with $M = 1000$

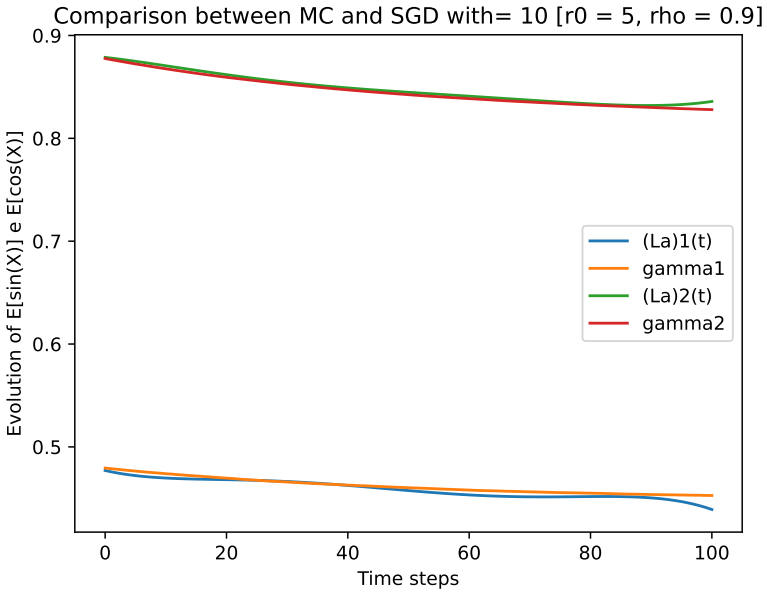
	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	13.12	4.45	13.18
$\rho = 0.7$	15.57	3.89	10.47
$\rho = 0.8$	20.77	3.93	8.27
$\rho = 0.9$	31.73	3.90	7.20

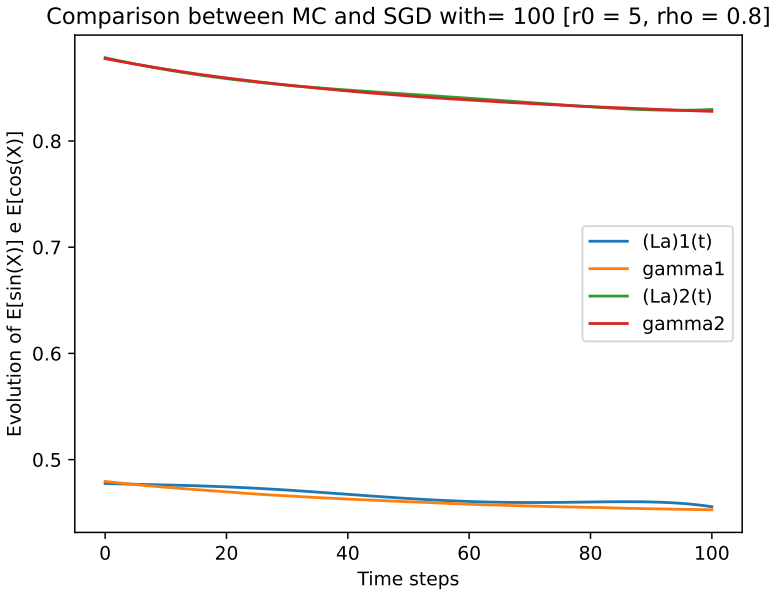
Tabella 1.69: Average execution times (in seconds s) with $M = 10000$

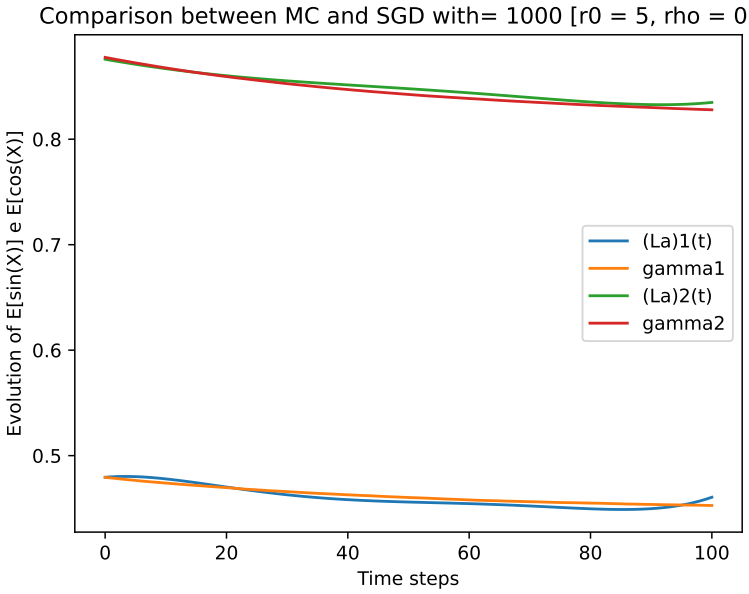
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	9	11	10	3	4	3.4	10	11	10.1
$\rho = 0.7$	11	13	11.9	3	3	3	8	8	8
$\rho = 0.8$	15	16	15.9	3	3	3	6	7	6.3
$\rho = 0.9$	22	31	24.3	3	3	3	5	6	5.5

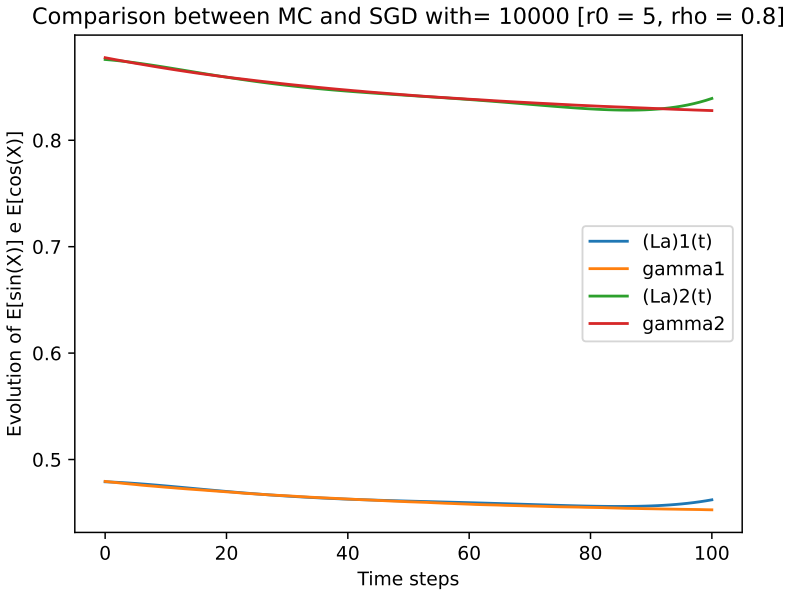
Tabella 1.70: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
20
40
60
80
100


Comparison between MC and SGD with= 100 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.8
0.7
0.6
0.5
0
20
40
60
80
100


Comparison between MC and SGD with= 1000 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
20
40
60
80
100
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 10000 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
20
40
60
80
100
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2

Caso n = 6

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	7.97	13.95	35.22
$\rho = 0.7$	11.89	9.05	18.96
$\rho = 0.8$	17.13	10.48	9.50
$\rho = 0.9$	157.41	10.32	6.82

Tabella 1.71: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	170	1920	1142	610	3310	1991	2510	9890	5037
$\rho = 0.7$	320	6730	1702	210	2550	1292	770	3790	2713
$\rho = 0.8$	390	10610	2453	500	3180	1501	540	2200	1359
$\rho = 0.9$	590	49999	22514.8	430	5640	1475	380	2510	976

Tabella 1.72: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	1.13	2.55	6.69
$\rho = 0.7$	2.00	2.10	4.14
$\rho = 0.8$	5.07	2.23	2.89
$\rho = 0.9$	19.58	1.94	2.00

Tabella 1.73: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	40	220	96	100	420	216	230	880	565
$\rho = 0.7$	30	430	171	70	330	178	130	580	351
$\rho = 0.8$	40	1560	431	70	550	187	70	360	246
$\rho = 0.9$	100	6200	1661	60	270	164	70	330	170

Tabella 1.74: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	0.62	0.72	1.11
$\rho = 0.7$	0.86	0.52	0.85
$\rho = 0.8$	1.57	0.34	0.73
$\rho = 0.9$	1.55	0.54	0.46

Tabella 1.75: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	20	110	40	10	110	47	40	110	72
$\rho = 0.7$	30	140	56	20	60	34	30	70	55
$\rho = 0.8$	40	300	102	10	40	22	10	70	47
$\rho = 0.9$	40	320	100	10	100	35	10	60	30

Tabella 1.76: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	2.18	0.55	1.05
$\rho = 0.7$	2.51	0.53	0.89
$\rho = 0.8$	3.59	0.52	1.00
$\rho = 0.9$	6.78	0.48	0.73

Tabella 1.77: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	14	23	17.7	3	7	4.5	8	11	8.6
$\rho = 0.7$	16	26	20.4	3	8	4.2	6	9	7.2
$\rho = 0.8$	24	45	29.2	2	8	4.2	5	17	7.9
$\rho = 0.9$	38	111	54.9	3	6	3.9	5	7	5.9

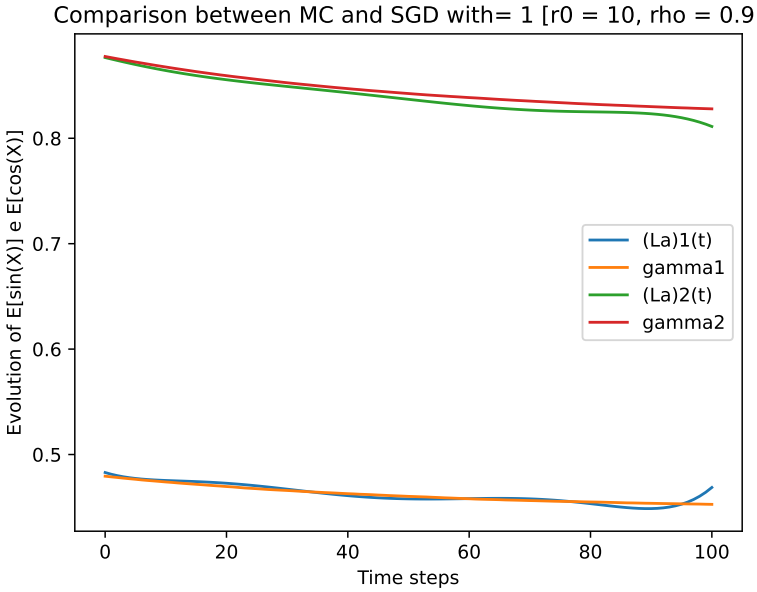
Tabella 1.78: Number of iterations m to achieve convergence with $M = 1000$

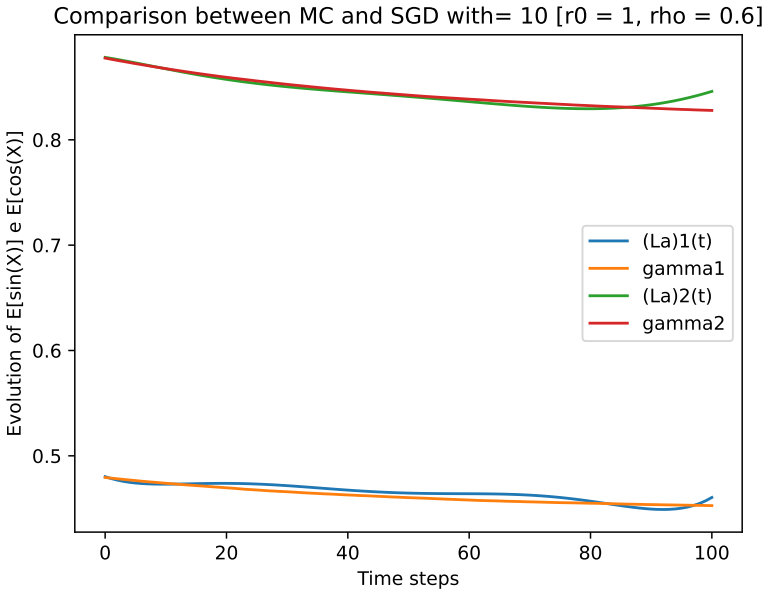
	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	20.21	4.72	12.51
$\rho = 0.7$	27.02	4.22	9.48
$\rho = 0.8$	39.52	3.61	7.80
$\rho = 0.9$	66.18	3.11	7.78

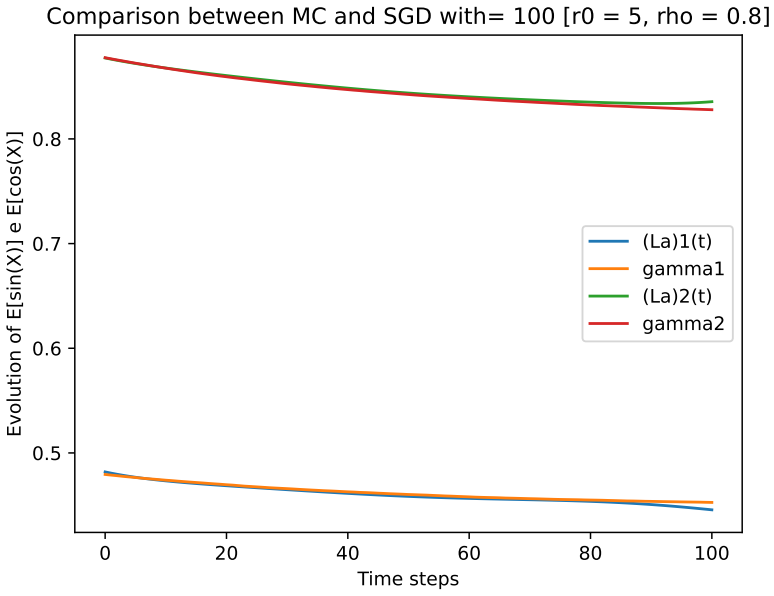
Tabella 1.79: Average execution times (in seconds s) with $M = 10000$

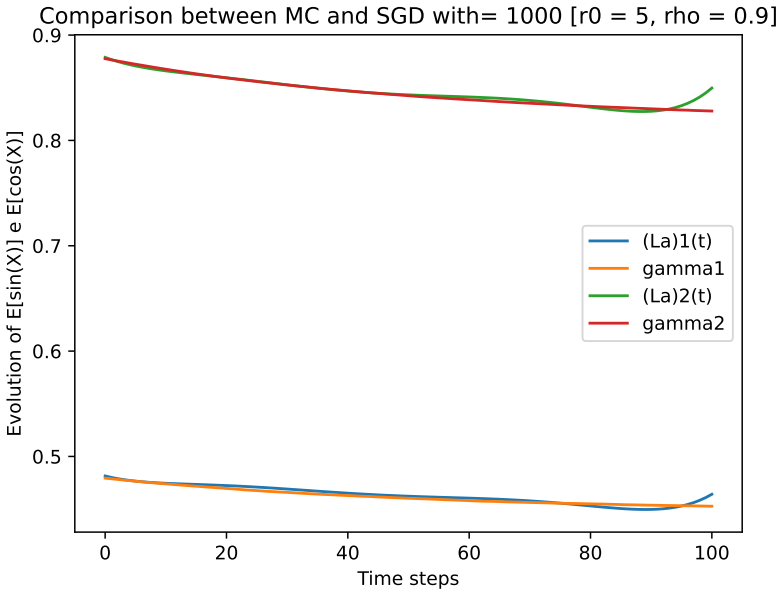
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	12	14	12.9	3	3	3	8	8	8
$\rho = 0.7$	16	19	17.2	2	3	2.7	6	7	6.1
$\rho = 0.8$	23	28	25.3	2	3	2.3	5	5	5
$\rho = 0.9$	41	45	42.3	2	2	2	5	5	5

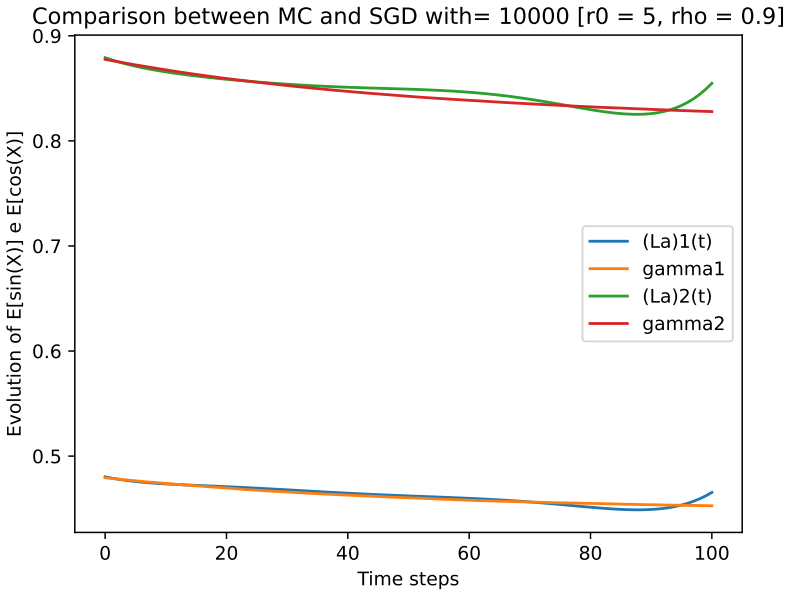
Tabella 1.80: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
20
40
60
80
100
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 100 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
20
40
60
80
100
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 1000 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
20
40
60
80
100


Comparison between MC and SGD with= 10000 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
20
40
60
80
100

1.5 T = 2

Caso n = 3

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	22.35	80.57
$\rho = 0.7$	12.28	42.48
$\rho = 0.8$	34.07	19.80
$\rho = 0.9$	94.34	23.10

Tabella 1.81: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	1070	3030	1798	3760	10900	6485		overflow	
$\rho = 0.7$	200	1460	992	1310	4700	3419		overflow	
$\rho = 0.8$	360	8220	2753	950	2600	1597		overflow	
$\rho = 0.9$	560	24050	7602	880	2870	1863		overflow	

Tabella 1.82: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	3.91	11.72
$\rho = 0.7$	1.98	7.75
$\rho = 0.8$	7.45	5.10
$\rho = 0.9$	5.94	4.78

Tabella 1.83: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	50	500	275	260	1360	819		overflow	
$\rho = 0.7$	20	440	139	190	1110	544		overflow	
$\rho = 0.8$	80	2130	492	140	640	357		overflow	
$\rho = 0.9$	40	1300	416	120	830	335		overflow	

Tabella 1.84: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	1.2	2.03
$\rho = 0.7$	0.78	2.05
$\rho = 0.8$	1.26	1.67
$\rho = 0.9$	0.98	1.60

Tabella 1.85: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	10	140	50	20	260	85		overflow	
$\rho = 0.7$	10	50	33	30	150	86		overflow	
$\rho = 0.8$	10	160	53	30	130	70		overflow	
$\rho = 0.9$	10	90	41	40	110	67		overflow	

Tabella 1.86: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	1.02	5.35
$\rho = 0.7$	0.88	3.42
$\rho = 0.8$	0.88	2.31
$\rho = 0.9$	1.64	2.29

Tabella 1.87: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	2	11	5.3	21	36	27.5		overflow	
$\rho = 0.7$	3	14	4.6	12	29	17.7		overflow	
$\rho = 0.8$	3	12	4.6	10	18	11.9		overflow	
$\rho = 0.9$	3	29	8.5	8	25	11.9		overflow	

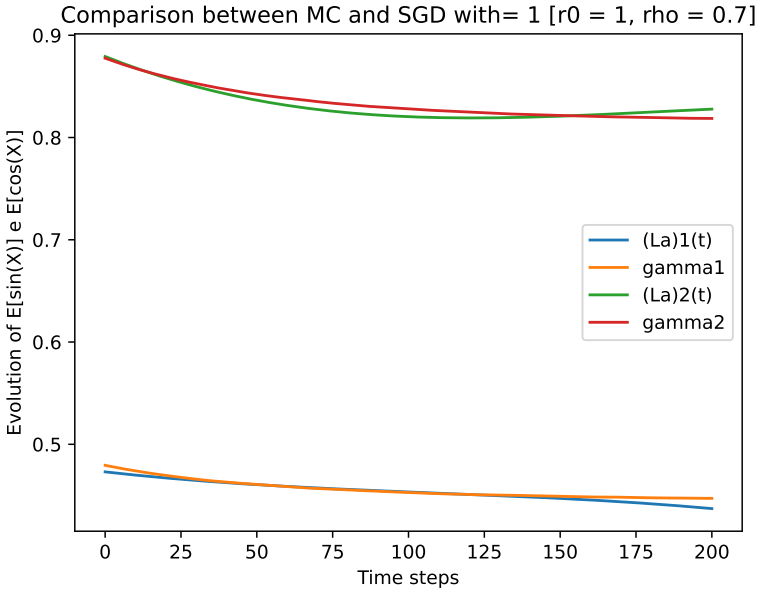
Tabella 1.88: Number of iterations m to achieve convergence with $M = 1000$

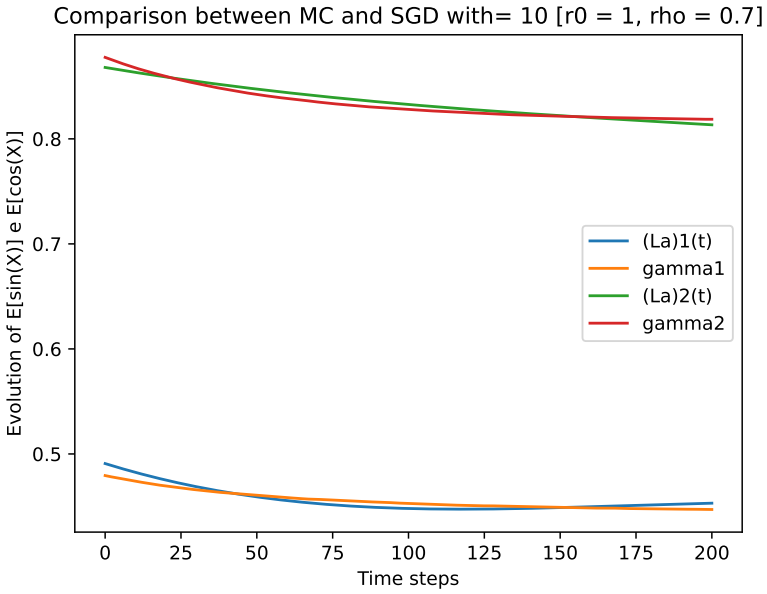
	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	4.67	38.12
$\rho = 0.7$	4.30	24.13
$\rho = 0.8$	5.42	19.99
$\rho = 0.9$	5.61	15.43

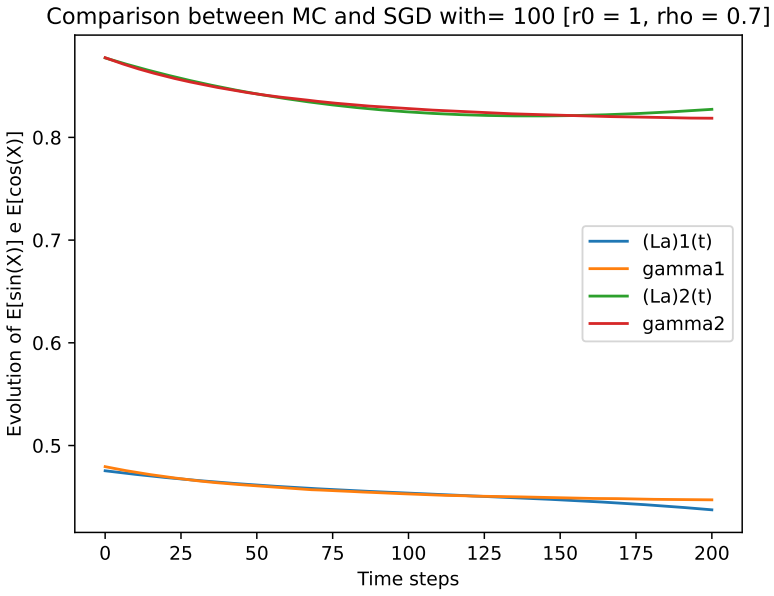
Tabella 1.89: Average execution times (in seconds s) with $M = 10000$

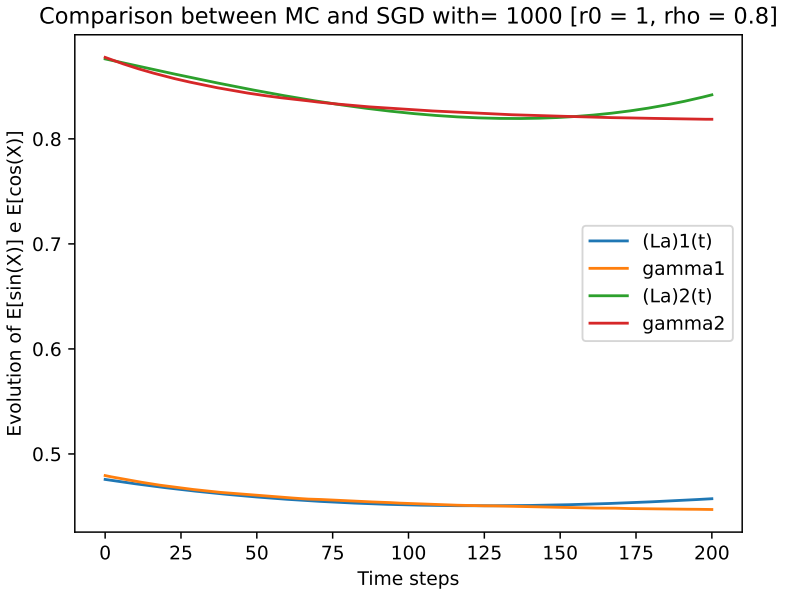
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	2	3	2.5	15	26	20		overflow	
$\rho = 0.7$	2	3	2.3	12	17	12.7		overflow	
$\rho = 0.8$	2	4	2.9	9	14	10.6		overflow	
$\rho = 0.9$	3	3	3	8	9	8.2		overflow	

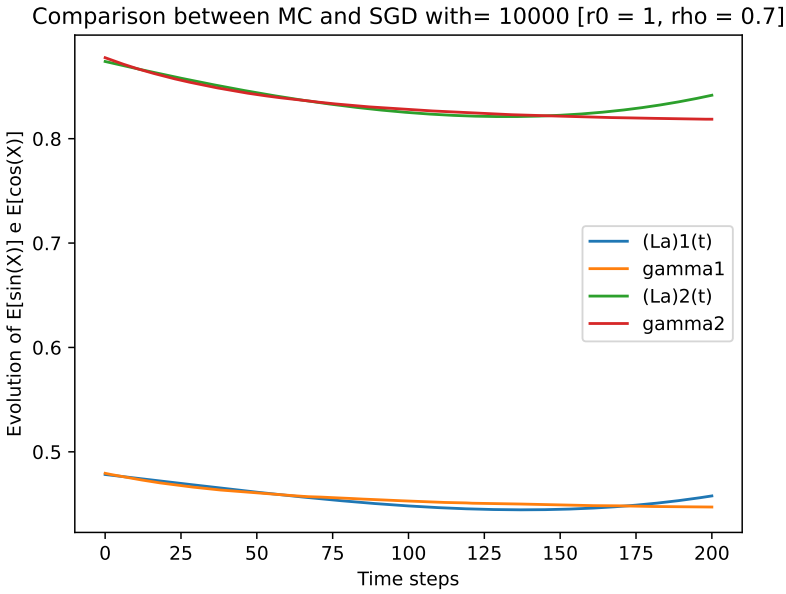
Tabella 1.90: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 100 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 1000 [r0 = 1, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 10000 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2

Caso n = 4

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	29.67	104.08
$\rho = 0.7$	19.5	58.47
$\rho = 0.8$	44.65	23.28
$\rho = 0.9$	271.35	17.65

Tabella 1.91: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	620	4050	2331	2890	11970	7908		overflow	
$\rho = 0.7$	670	3850	1529	1570	6540	4510		overflow	
$\rho = 0.8$	490	13550	3428	280	4230	1806		overflow	
$\rho = 0.9$	1560	49999	20370.8	670	2480	1312		overflow	

Tabella 1.92: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	4.56	13.23
$\rho = 0.7$	4.85	7.61
$\rho = 0.8$	6.83	5.09
$\rho = 0.9$	29.93	4.01

Tabella 1.93: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	80	620	310	340	1380	886		overflow	
$\rho = 0.7$	20	810	322	290	1090	519		overflow	
$\rho = 0.8$	40	2420	438	80	680	348		overflow	
$\rho = 0.9$	60	6660	1909	110	690	273		overflow	

Tabella 1.94: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	0.84	2.54
$\rho = 0.7$	1.13	1.82
$\rho = 0.8$	1.41	1.38
$\rho = 0.9$	4.74	1.27

Tabella 1.95: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	10	90	31	30	190	90		overflow	
$\rho = 0.7$	10	110	40	30	100	64		overflow	
$\rho = 0.8$	10	150	42	30	70	49		overflow	
$\rho = 0.9$	20	950	169	10	80	45		overflow	

Tabella 1.96: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	1.43	3.93
$\rho = 0.7$	1.27	3.09
$\rho = 0.8$	1.34	2.35
$\rho = 0.9$	1.92	2.17

Tabella 1.97: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	4	11	6	13	20	16.4		overflow	
$\rho = 0.7$	3	11	5.3	10	21	12.8		overflow	
$\rho = 0.8$	4	11	5.6	8	13	9.7		overflow	
$\rho = 0.9$	4	23	8	7	14	9.1		overflow	

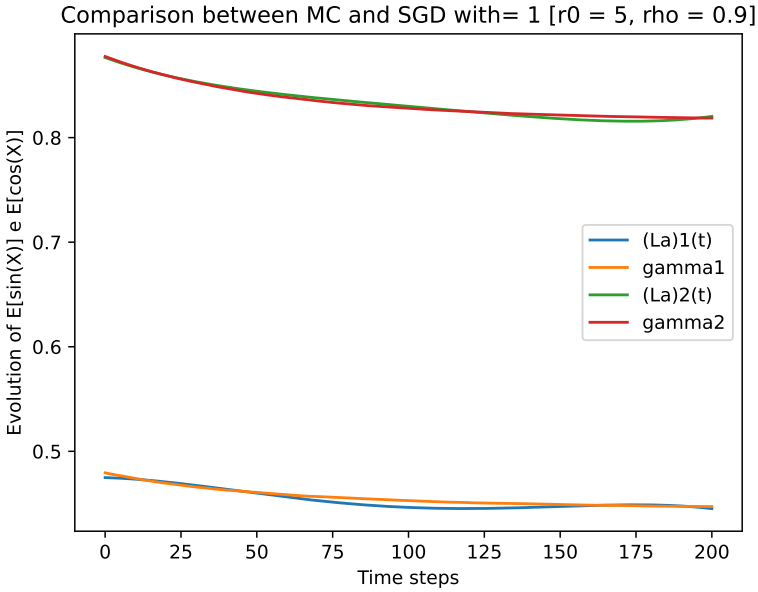
Tabella 1.98: Number of iterations m to achieve convergence with $M = 1000$

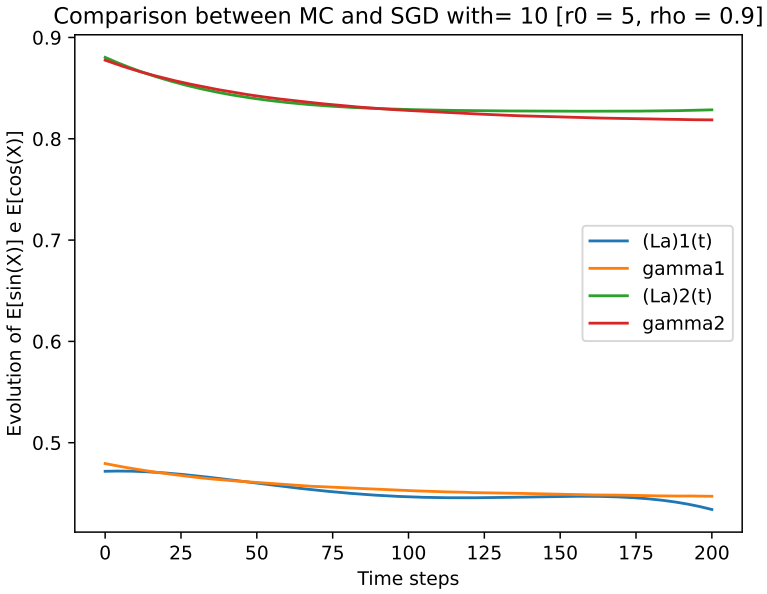
	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	7.24	30.59
$\rho = 0.7$	7.20	22.75
$\rho = 0.8$	7.77	18.19
$\rho = 0.9$	8.51	16.25

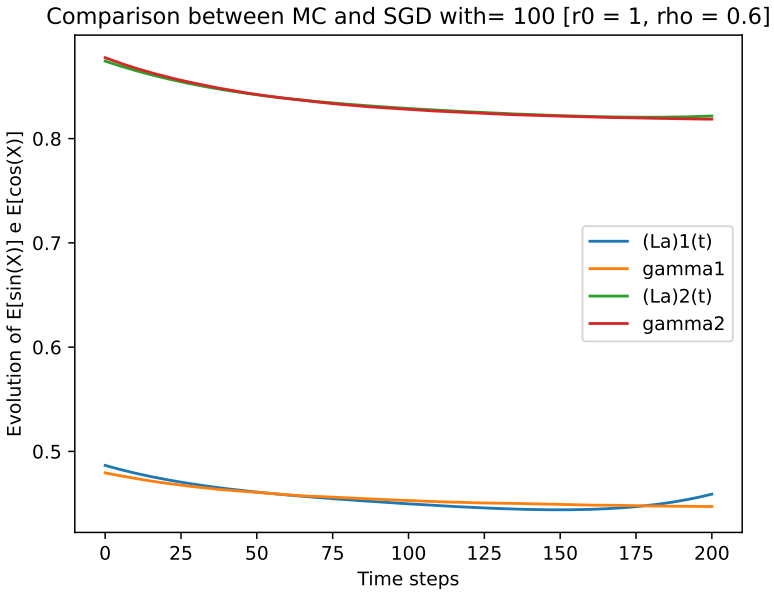
Tabella 1.99: Average execution times (in seconds s) with $M = 10000$

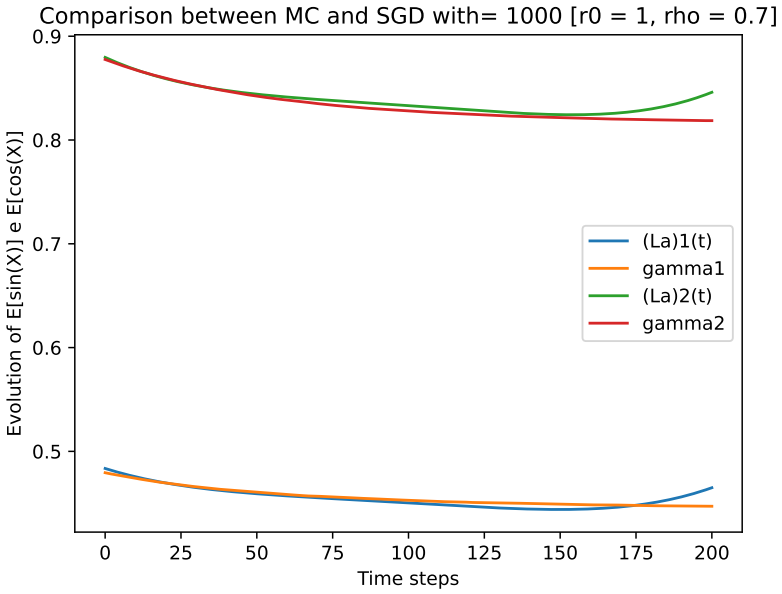
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	3	4	3.1	13	13	13		overflow	
$\rho = 0.7$	3	4	3.1	10	10	10		overflow	
$\rho = 0.8$	3	4	3.3	8	8	8		overflow	
$\rho = 0.9$	3	4	3.6	7	7	7		overflow	

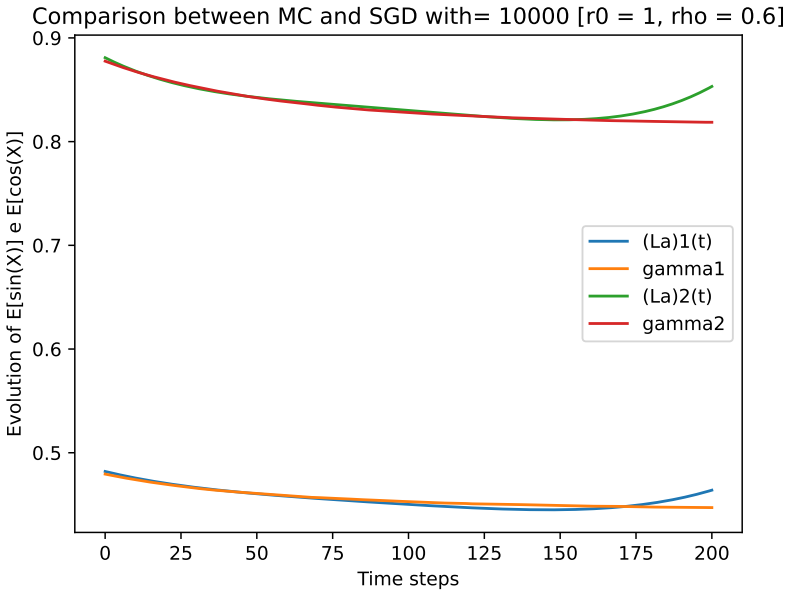
Tabella 1.100: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200


Comparison between MC and SGD with= 100 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200


Comparison between MC and SGD with= 1000 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200


Comparison between MC and SGD with= 10000 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200

Caso n = 5

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	28.57	133.75
$\rho = 0.7$	33.20	54.61
$\rho = 0.8$	59.02	42.73
$\rho = 0.9$	306.42	20.43

Tabella 1.101: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	820	3420	2167	4540	17080	10248		overflow	
$\rho = 0.7$	490	6630	2459	2280	8430	4194		overflow	
$\rho = 0.8$	470	9120	4432	1410	7030	3278		overflow	
$\rho = 0.9$	1160	49999	23456.7	400	4680	1570		overflow	

Tabella 1.102: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	4.13	13.39
$\rho = 0.7$	5.30	7.85
$\rho = 0.8$	18.94	3.82
$\rho = 0.9$	53.15	5.07

Tabella 1.103: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	90	490	272	380	1610	883		overflow	
$\rho = 0.7$	100	570	333	180	950	517		overflow	
$\rho = 0.8$	70	6300	1230	90	510	252		overflow	
$\rho = 0.9$	80	24990	3503	150	1130	335		overflow	

Tabella 1.104: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	1.99	2.05
$\rho = 0.7$	1.05	2.14
$\rho = 0.8$	1.84	2.11
$\rho = 0.9$	3.72	1.92

Tabella 1.105: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	20	190	68	40	130	70		overflow	
$\rho = 0.7$	10	80	36	20	170	73		overflow	
$\rho = 0.8$	10	180	63	30	150	72		overflow	
$\rho = 0.9$	10	680	127	30	120	65		overflow	

Tabella 1.106: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	1.59	3.27
$\rho = 0.7$	1.98	2.5
$\rho = 0.8$	1.86	2.56
$\rho = 0.9$	2.85	1.93

Tabella 1.107: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	4	10	6.1	10	17	12.4		overflow	
$\rho = 0.7$	4	16	7.6	8	13	9.6		overflow	
$\rho = 0.8$	5	12	7.1	7	14	9.8		overflow	
$\rho = 0.9$	7	18	11	5	12	7.4		overflow	

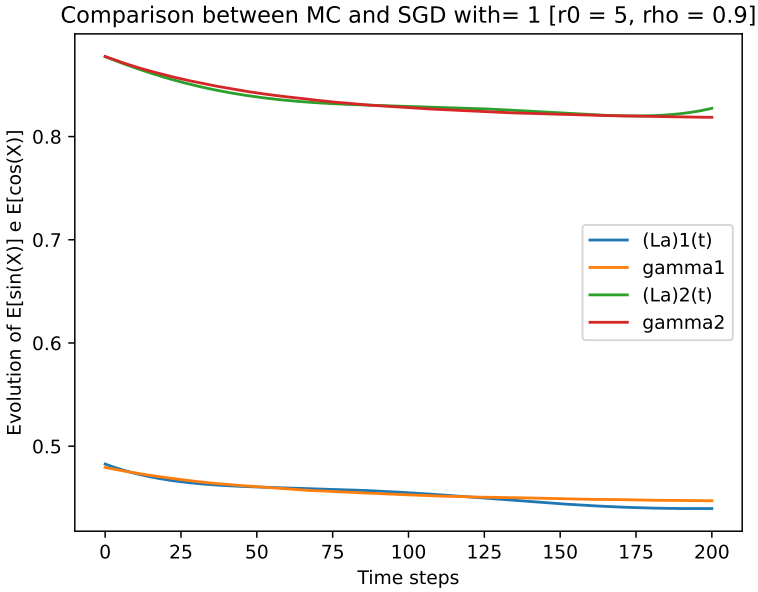
Tabella 1.108: Number of iterations m to achieve convergence with $M = 1000$

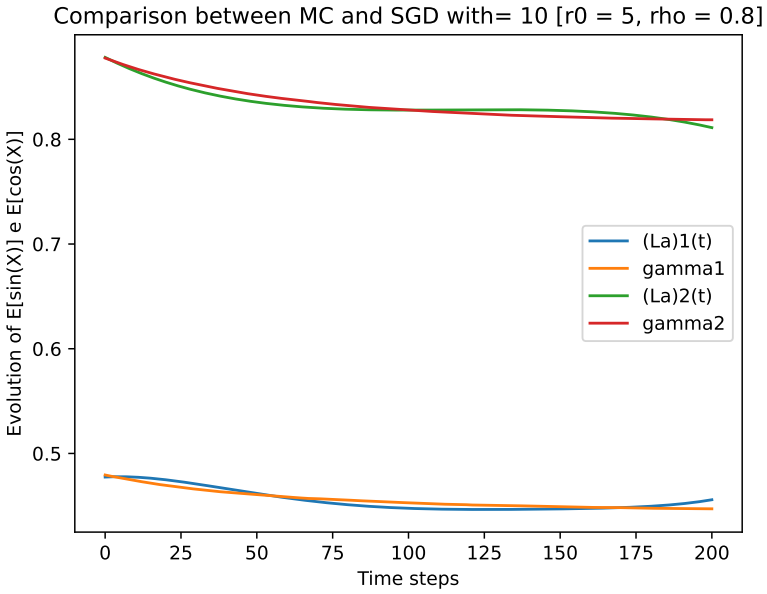
	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	10.72	26.45
$\rho = 0.7$	11.51	21.05
$\rho = 0.8$	15.16	15.97
$\rho = 0.9$	16.24	13.36

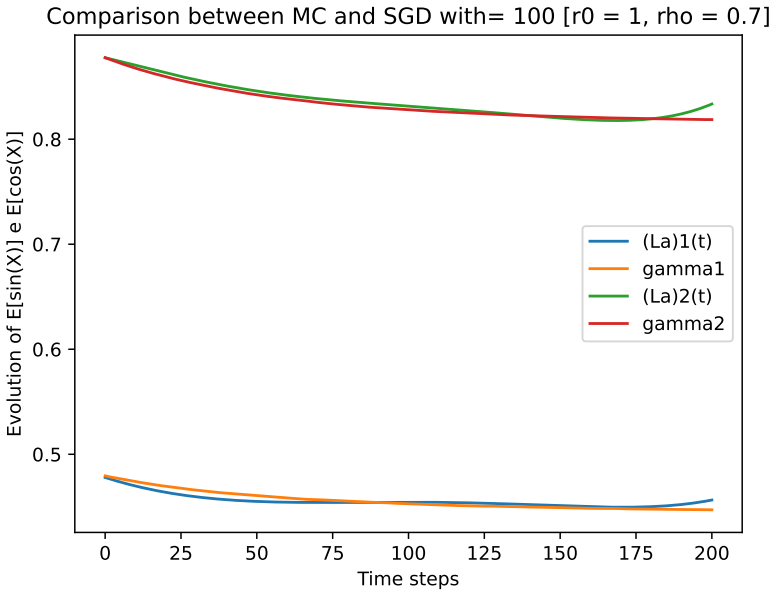
Tabella 1.109: Average execution times (in seconds s) with $M = 10000$

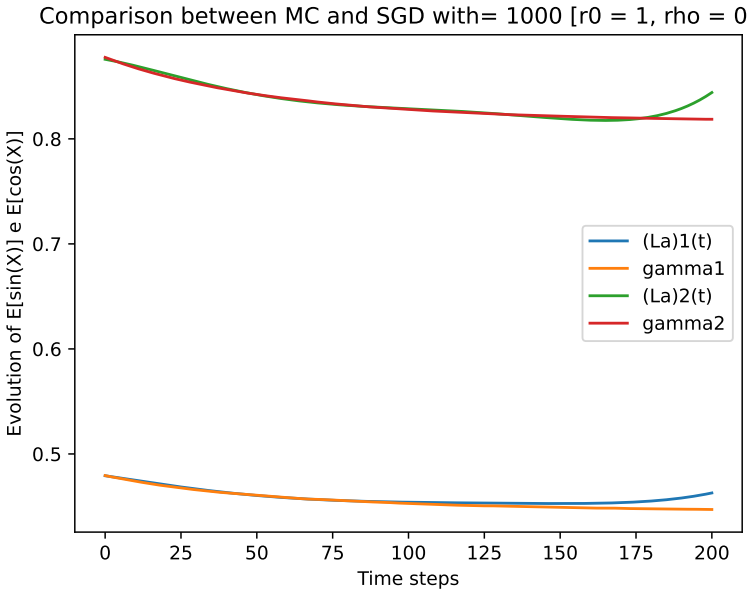
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	4	5	4.1	10	11	10.1		overflow	
$\rho = 0.7$	4	5	4.4	8	8	8		overflow	
$\rho = 0.8$	5	8	5.8	6	7	6.1		overflow	
$\rho = 0.9$	5	8	6.2	5	6	5.1		overflow	

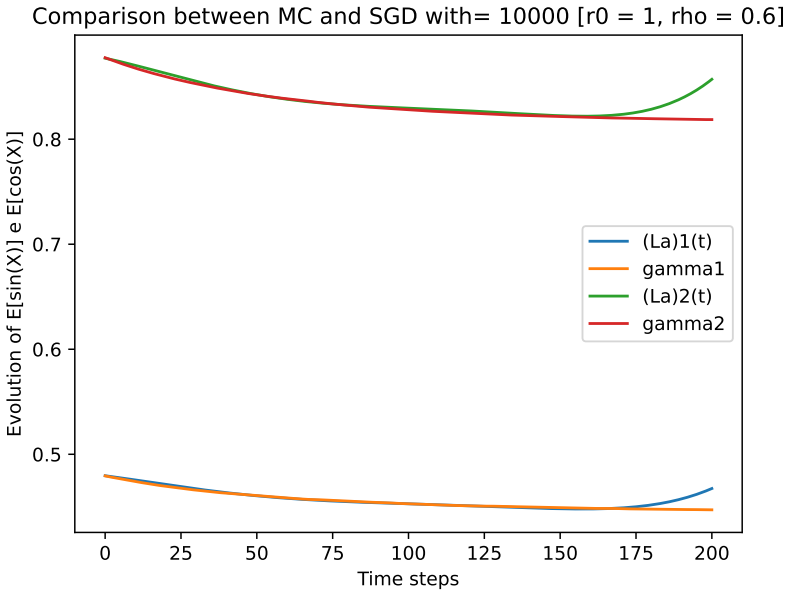
Tabella 1.110: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 5, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
(La)1(t)
gamma1
(La)2(t)
gamma2
0
25
50
75
100
125
150
175
200
Time steps


Comparison between MC and SGD with= 100 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
(La)1(t)
gamma1
(La)2(t)
gamma2
0
25
50
75
100
125
150
175
200
Time steps


Comparison between MC and SGD with= 1000 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 10000 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2

Caso n = 6

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	32.08	143.46	
$\rho = 0.7$	17.72	59.82	
$\rho = 0.8$	45.81	38.29	56.83
$\rho = 0.9$	415.62	20.29	33.01

Tabella 1.111: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	530	5630	2424	4710	21280	10855		overflow	
$\rho = 0.7$	380	3370	1336	990	8130	4511		overflow	
$\rho = 0.8$	900	12100	3456	1050	4840	2893	2490	6850	4171
$\rho = 0.9$	7640	49999	31384.5	440	4390	1532	1620	4050	2405

Tabella 1.112: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	3.39	14.24	
$\rho = 0.7$	5.58	6.85	
$\rho = 0.8$	19.75	7.34	8.91
$\rho = 0.9$	15.37	5.23	5.81

Tabella 1.113: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	50	640	219	410	1280	917		overflow	
$\rho = 0.7$	60	710	359	130	900	442		overflow	
$\rho = 0.8$	90	7070	1271	90	870	472	290	990	569
$\rho = 0.9$	130	2720	988	70	670	337	100	750	369

Tabella 1.114: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	1.07	3.43	
$\rho = 0.7$	0.80	2.40	
$\rho = 0.8$	3.97	2.23	4.02
$\rho = 0.9$	9.14	1.61	2.75

Tabella 1.115: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	10	50	35	60	170	111		overflow	
$\rho = 0.7$	10	60	26	40	160	78		overflow	
$\rho = 0.8$	10	520	129	30	190	72	70	260	127
$\rho = 0.9$	20	1540	295	20	80	52	50	140	88

Tabella 1.116: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	2.03	3.15	
$\rho = 0.7$	3.47	2.27	
$\rho = 0.8$	3.84	2.44	7.27
$\rho = 0.9$	3.87	1.91	7.67

Tabella 1.117: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	5	10	6.8	8	24	10.7		overflow	
$\rho = 0.7$	6	32	11.8	6	12	7.7		overflow	
$\rho = 0.8$	7	29	13	5	15	8.3	20	31	23.4
$\rho = 0.9$	8	32	13.1	5	10	6.5	13	50	24.7

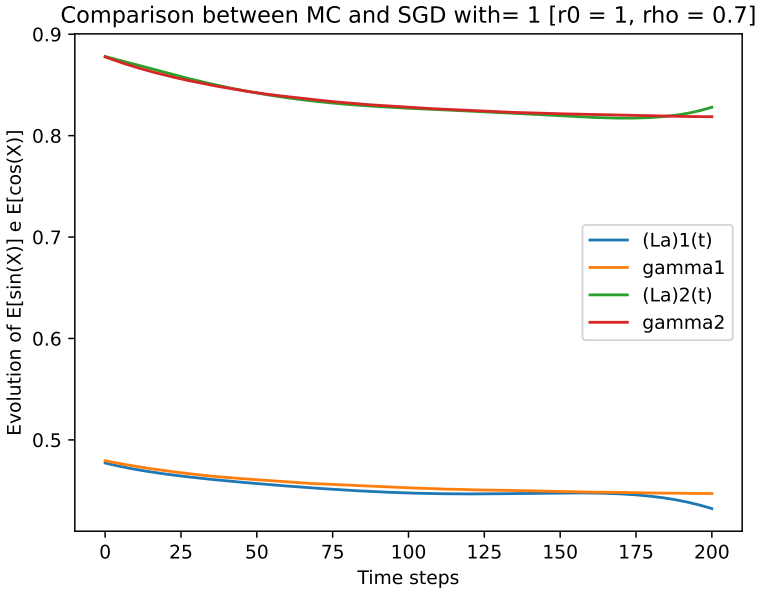
Tabella 1.118: Number of iterations m to achieve convergence with $M = 1000$

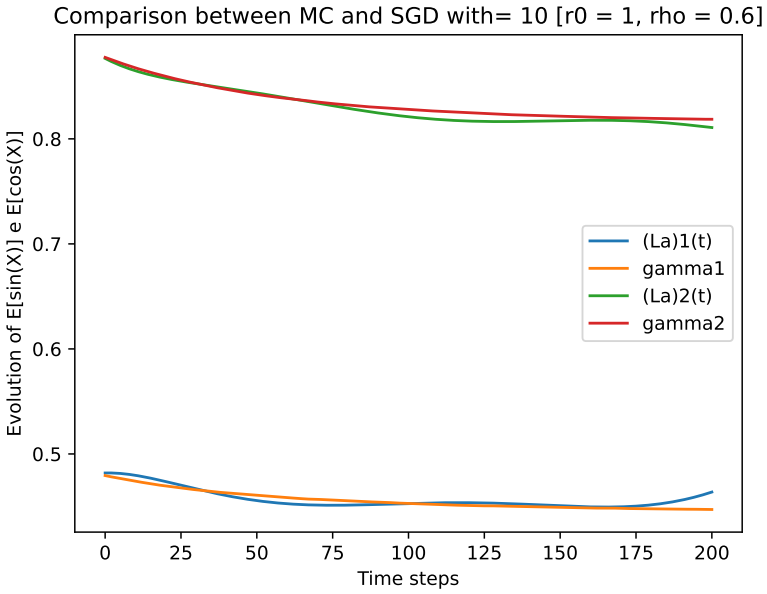
	$r_0 = 1$	$r_0 = 5$	$r_0 = 10$
$\rho = 0.6$	17.80	27.06	
$\rho = 0.7$	20.13	19.79	
$\rho = 0.8$	23.78	16.89	67.48
$\rho = 0.9$	27.38	16.50	45.50

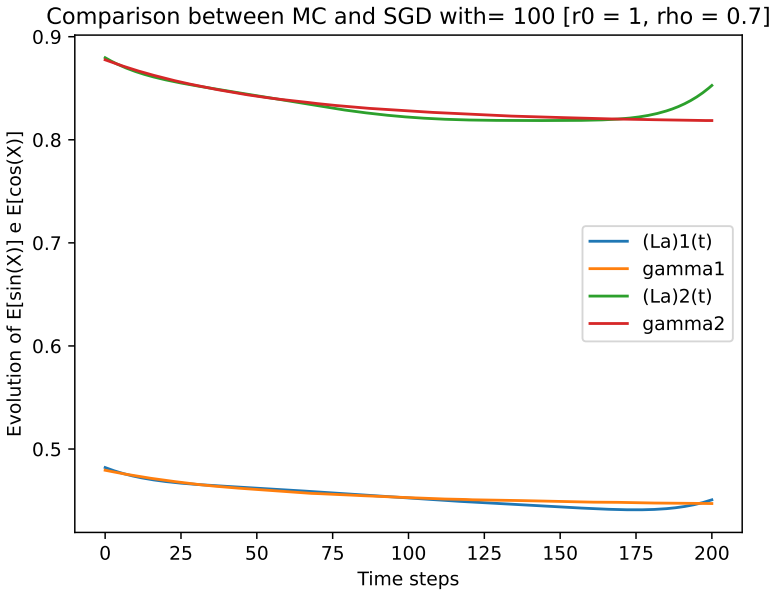
Tabella 1.119: Average execution times (in seconds s) with $M = 10000$

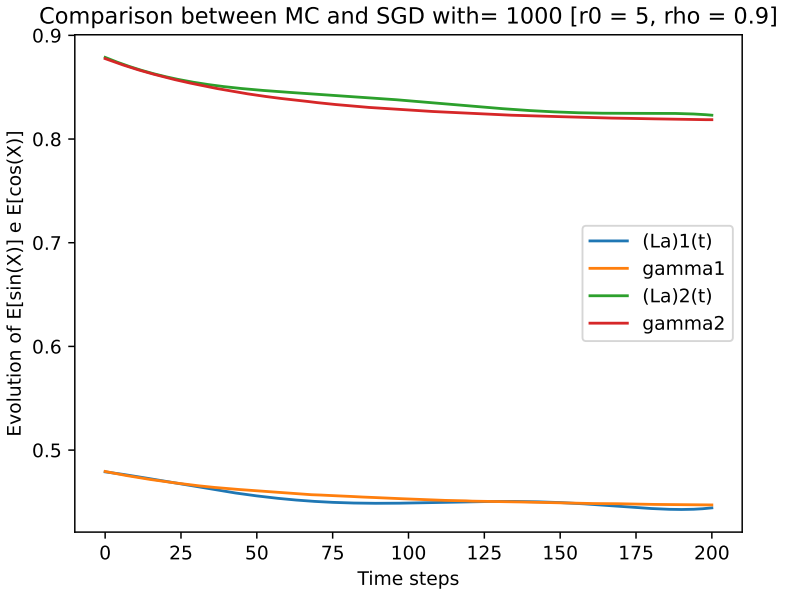
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	5	7	5.4	8	9	8.2		overflow	
$\rho = 0.7$	6	7	6.1	6	6	6		overflow	
$\rho = 0.8$	7	8	7.2	5	6	5.1	17	24	18.8
$\rho = 0.9$	7	9	8.3	5	5	5	12	17	13.6

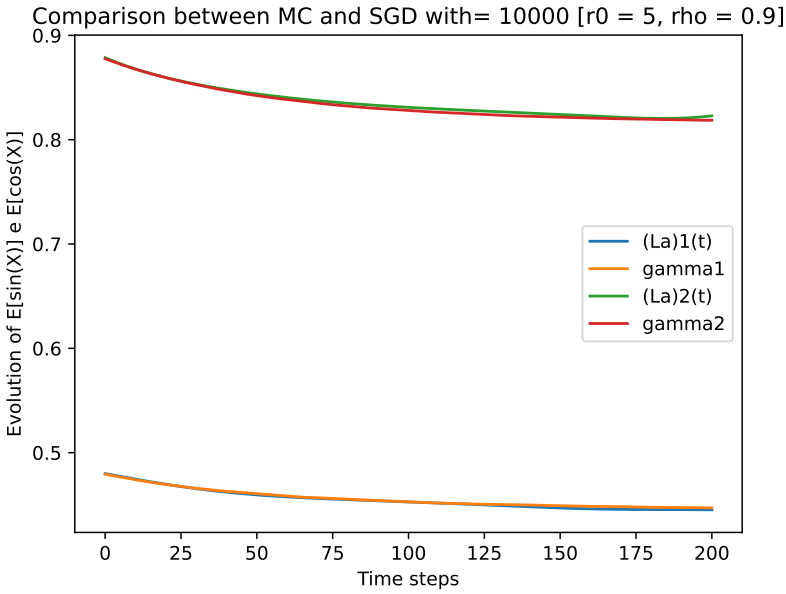
Tabella 1.120: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
(La)1(t)
gamma1
(La)2(t)
gamma2
0
25
50
75
100
125
150
175
200
Time steps


Comparison between MC and SGD with= 100 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.9
0.8
0.7
0.6
0.5
(La)1(t)
gamma1
(La)2(t)
gamma2
0
25
50
75
100
125
150
175
200
Time steps


Comparison between MC and SGD with= 1000 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200


Comparison between MC and SGD with= 10000 [r0 = 5, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
25
50
75
100
125
150
175
200

1.6 T = 4

Caso n = 3

	$r_0 = 1$
$\rho = 0.6$	148.41
$\rho = 0.7$	98.14
$\rho = 0.8$	99.71
$\rho = 0.9$	269.29

Tabella 1.121: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	3450	12910	6042		overflow			overflow	
$\rho = 0.7$	1150	7610	3999		overflow			overflow	
$\rho = 0.8$	1030	9770	4062		overflow			overflow	
$\rho = 0.9$	900	49999	10512.9		overflow			overflow	

Tabella 1.122: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$
$\rho = 0.6$	15.35
$\rho = 0.7$	10.90
$\rho = 0.8$	8.20
$\rho = 0.9$	72.03

Tabella 1.123: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	170	1180	521		overflow			overflow	
$\rho = 0.7$	110	740	370		overflow			overflow	
$\rho = 0.8$	110	690	280		overflow			overflow	
$\rho = 0.9$	50	12920	2475		overflow			overflow	

Tabella 1.124: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$
$\rho = 0.6$	3.48
$\rho = 0.7$	5.24
$\rho = 0.8$	5.58
$\rho = 0.9$	15.62

Tabella 1.125: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	30	140	68		overflow			overflow	
$\rho = 0.7$	20	310	101		overflow			overflow	
$\rho = 0.8$	30	330	108		overflow			overflow	
$\rho = 0.9$	20	1360	307		overflow			overflow	

Tabella 1.126: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$
$\rho = 0.6$	2.06
$\rho = 0.7$	3.32
$\rho = 0.8$	3.08
$\rho = 0.9$	2.61

Tabella 1.127: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	4	8	4.7		overflow			overflow	
$\rho = 0.7$	4	16	7.8		overflow			overflow	
$\rho = 0.8$	3	19	7.1		overflow			overflow	
$\rho = 0.9$	4	9	6		overflow			overflow	

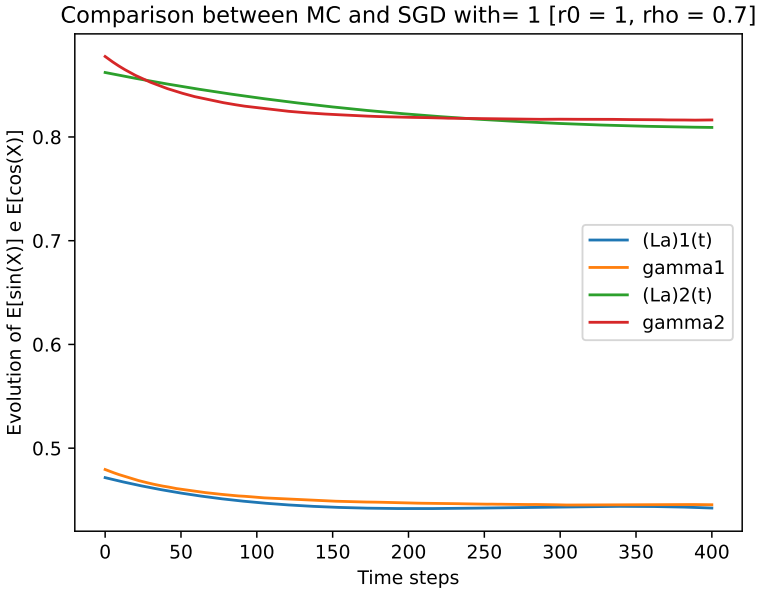
Tabella 1.128: Number of iterations m to achieve convergence with $M = 1000$

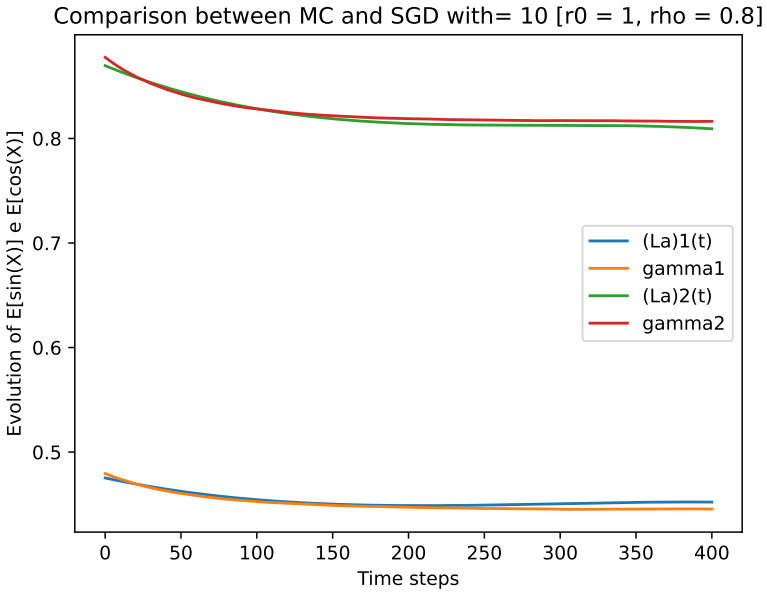
	$r_0 = 1$
$\rho = 0.6$	16.91
$\rho = 0.7$	16.04
$\rho = 0.8$	12.17
$\rho = 0.9$	12.15

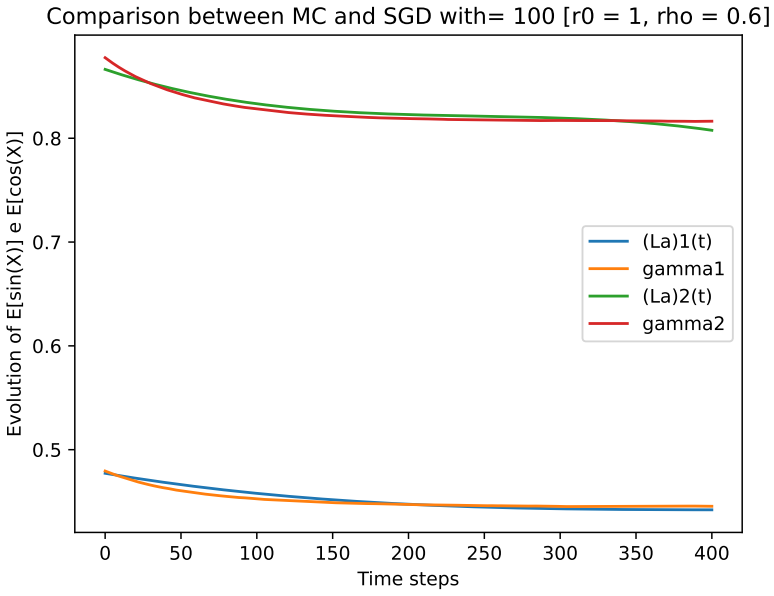
Tabella 1.129: Average execution times (in seconds s) with $M = 10000$

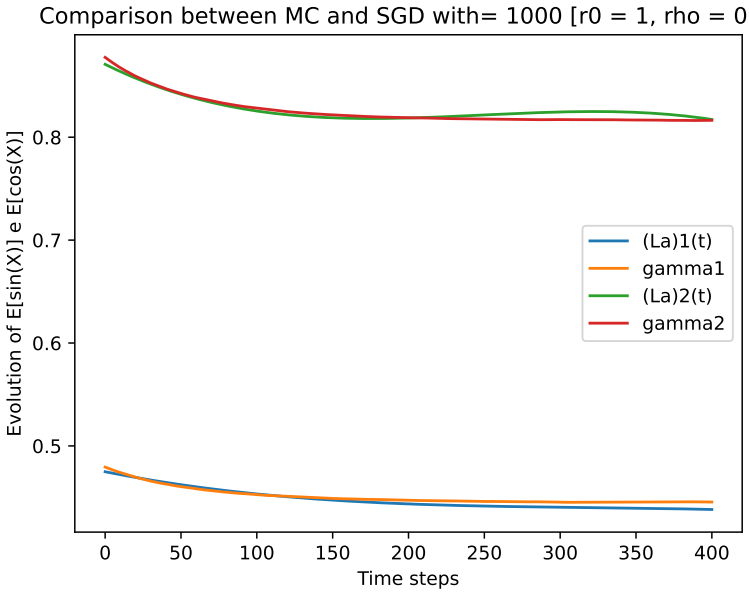
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	4	4	4		overflow			overflow	
$\rho = 0.7$	4	4	4		overflow			overflow	
$\rho = 0.8$	3	3	3		overflow			overflow	
$\rho = 0.9$	3	3	3		overflow			overflow	

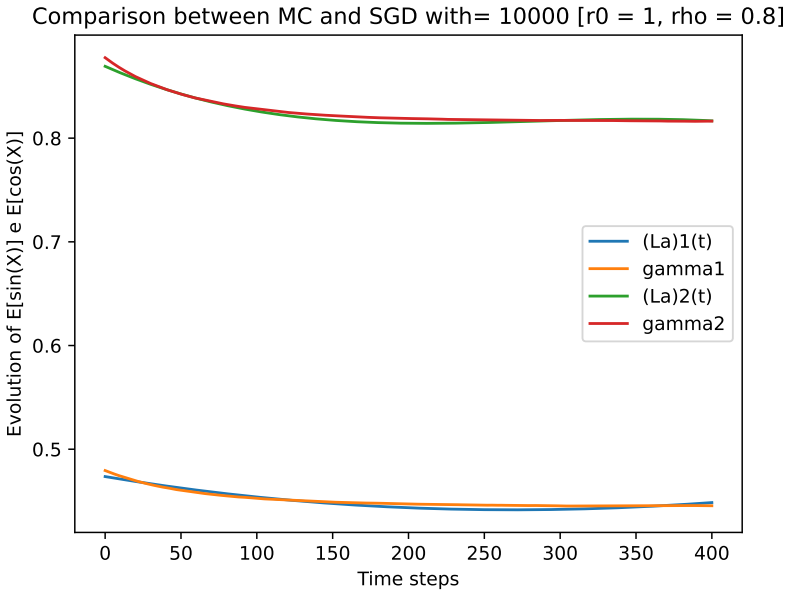
Tabella 1.130: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 1, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 100 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 1000 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
(La)1(t)
gamma1
(La)2(t)
gamma2
0
50
100
150
200
250
300
350
400
Time steps


Comparison between MC and SGD with= 10000 [r0 = 1, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
(La)1(t)
gamma1
(La)2(t)
gamma2
0
50
100
150
200
250
300
350
400
Time steps

Caso n = 4

	$r_0 = 1$
$\rho = 0.6$	145.11
$\rho = 0.7$	102.23
$\rho = 0.8$	177.10
$\rho = 0.9$	890.75

Tabella 1.131: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	3320	11980	5645		overflow			overflow	
$\rho = 0.7$	1020	12700	3974		overflow			overflow	
$\rho = 0.8$	740	25910	6650		overflow			overflow	
$\rho = 0.9$	2470	49999	34868.5		overflow			overflow	

Tabella 1.132: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$
$\rho = 0.6$	13.09
$\rho = 0.7$	13.76
$\rho = 0.8$	22.88
$\rho = 0.9$	313.63

Tabella 1.133: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	120	820	446		overflow			overflow	
$\rho = 0.7$	250	820	473		overflow			overflow	
$\rho = 0.8$	70	4380	778		overflow			overflow	
$\rho = 0.9$	170	34500	10453		overflow			overflow	

Tabella 1.134: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$
$\rho = 0.6$	4.71
$\rho = 0.7$	2.61
$\rho = 0.8$	5.88
$\rho = 0.9$	37.38

Tabella 1.135: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	40	160	88		overflow			overflow	
$\rho = 0.7$	30	90	49		overflow			overflow	
$\rho = 0.8$	10	390	108		overflow			overflow	
$\rho = 0.9$	10	2810	703		overflow			overflow	

Tabella 1.136: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$
$\rho = 0.6$	4.57
$\rho = 0.7$	3.25
$\rho = 0.8$	3.55
$\rho = 0.9$	4.56

Tabella 1.137: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	4	18	9.5		overflow			overflow	
$\rho = 0.7$	3	27	6.7		overflow			overflow	
$\rho = 0.8$	3	18	7.4		overflow			overflow	
$\rho = 0.9$	3	41	9.5		overflow			overflow	

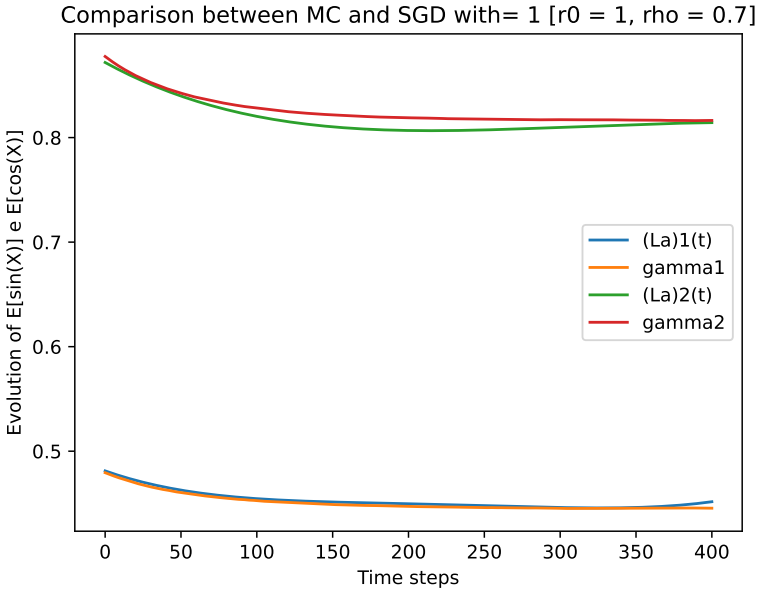
Tabella 1.138: Number of iterations m to achieve convergence with $M = 1000$

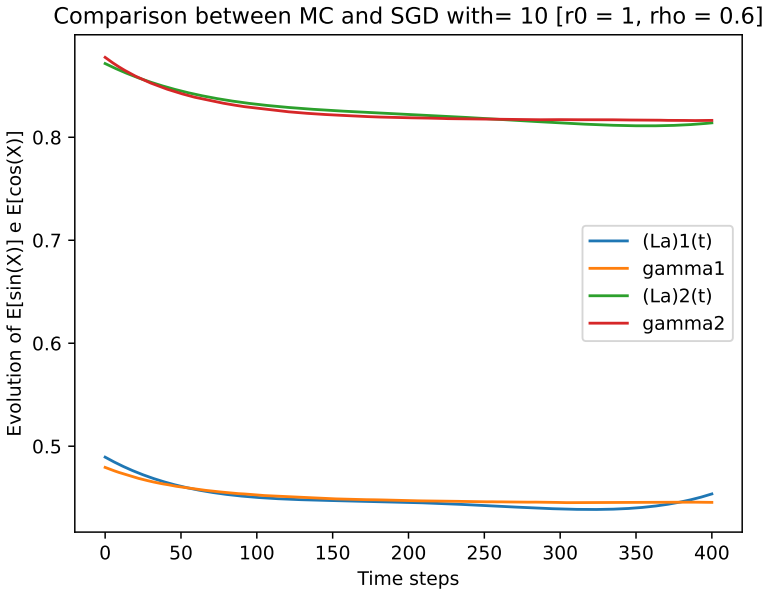
	$r_0 = 1$
$\rho = 0.6$	14.42
$\rho = 0.7$	14.35
$\rho = 0.8$	14.42
$\rho = 0.9$	14.42

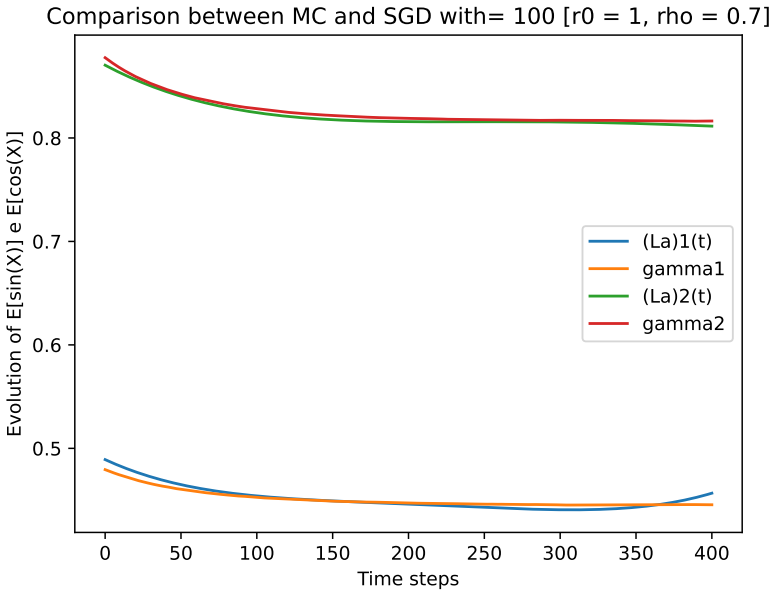
Tabella 1.139: Average execution times (in seconds s) with $M = 10000$

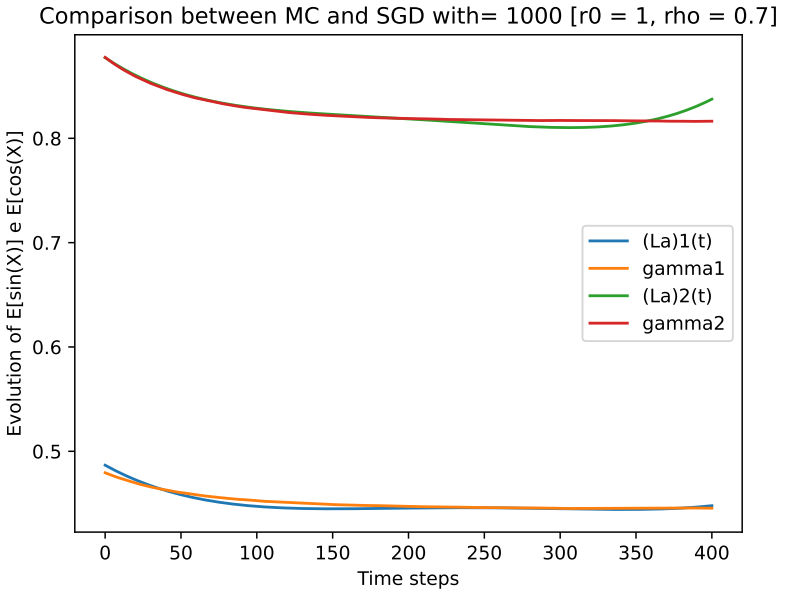
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	3	3	3		overflow			overflow	
$\rho = 0.7$	3	3	3		overflow			overflow	
$\rho = 0.8$	3	3	3		overflow			overflow	
$\rho = 0.9$	3	3	3		overflow			overflow	

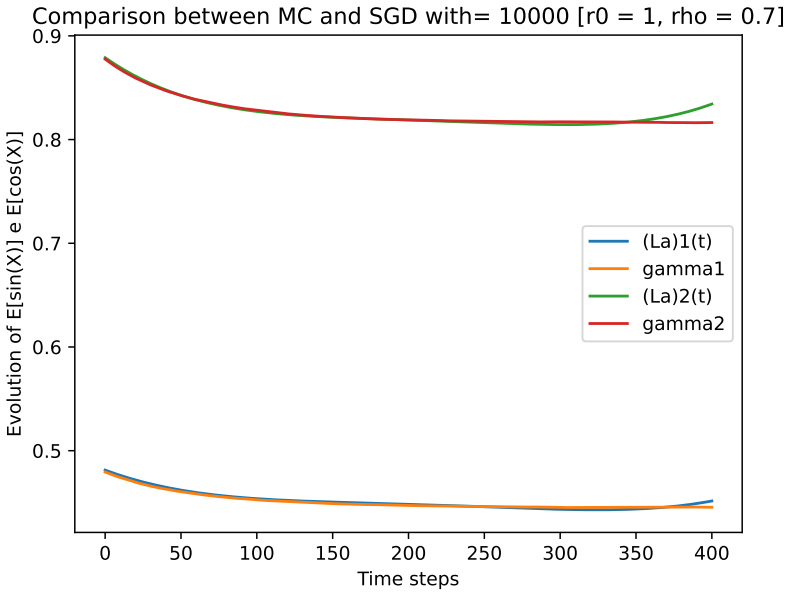
Tabella 1.140: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 100 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 1000 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.5
0.6
0.7
0.8
0
50
100
150
200
250
300
350
400
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2


Comparison between MC and SGD with= 10000 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.5
0.6
0.7
0.8
0.9
0
50
100
150
200
250
300
350
400
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2

Caso n = 5

	$r_0 = 1$
$\rho = 0.6$	187.19
$\rho = 0.7$	100.55
$\rho = 0.8$	343.00
$\rho = 0.9$	1090.67

Tabella 1.141: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	2410	17200	7261		overflow			overflow	
$\rho = 0.7$	1490	6980	3871		overflow			overflow	
$\rho = 0.8$	640	42740	13311		overflow			overflow	
$\rho = 0.9$	2840	49999	41910.3		overflow			overflow	

Tabella 1.142: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$
$\rho = 0.6$	30.49
$\rho = 0.7$	13.62
$\rho = 0.8$	22.37
$\rho = 0.9$	423.87

Tabella 1.143: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	120	3880	1020		overflow			overflow	
$\rho = 0.7$	180	860	455		overflow			overflow	
$\rho = 0.8$	120	2540	743		overflow			overflow	
$\rho = 0.9$	190	38600	14211		overflow			overflow	

Tabella 1.144: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$
$\rho = 0.6$	3.98
$\rho = 0.7$	3.58
$\rho = 0.8$	13.71
$\rho = 0.9$	108.12

Tabella 1.145: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	30	150	68		overflow			overflow	
$\rho = 0.7$	20	100	61		overflow			overflow	
$\rho = 0.8$	40	910	234		overflow			overflow	
$\rho = 0.9$	10	12040	1843		overflow			overflow	

Tabella 1.146: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$
$\rho = 0.6$	4.24
$\rho = 0.7$	7.74
$\rho = 0.8$	2.61
$\rho = 0.9$	2.29

Tabella 1.147: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	3	14	7.8		overflow			overflow	
$\rho = 0.7$	3	58	14.2		overflow			overflow	
$\rho = 0.8$	3	7	4.8		overflow			overflow	
$\rho = 0.9$	2	6	4.2		overflow			overflow	

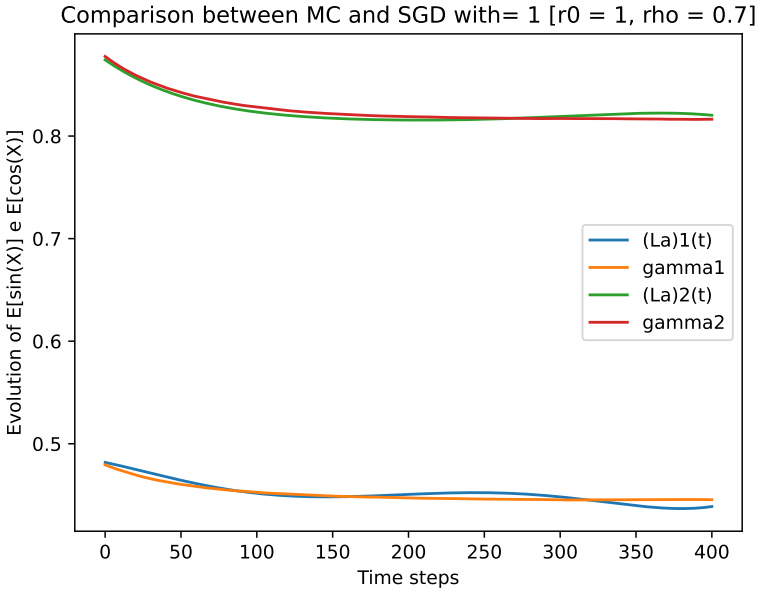
Tabella 1.148: Number of iterations m to achieve convergence with $M = 1000$

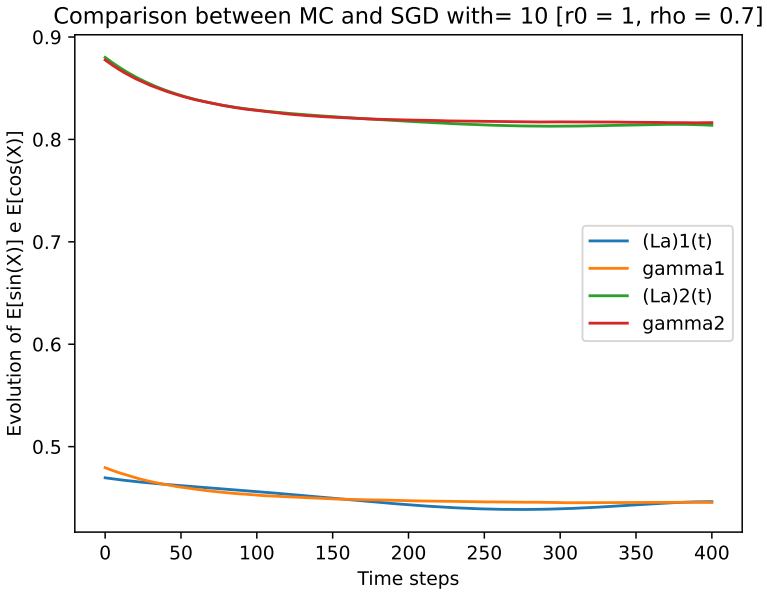
	$r_0 = 1$
$\rho = 0.6$	15.91
$\rho = 0.7$	12.07
$\rho = 0.8$	11.51
$\rho = 0.9$	13.16

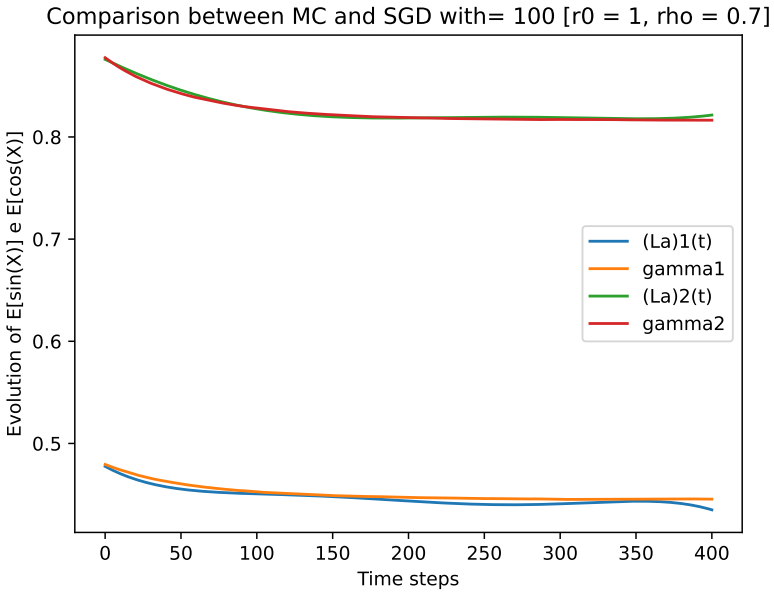
Tabella 1.149: Average execution times (in seconds s) with $M = 10000$

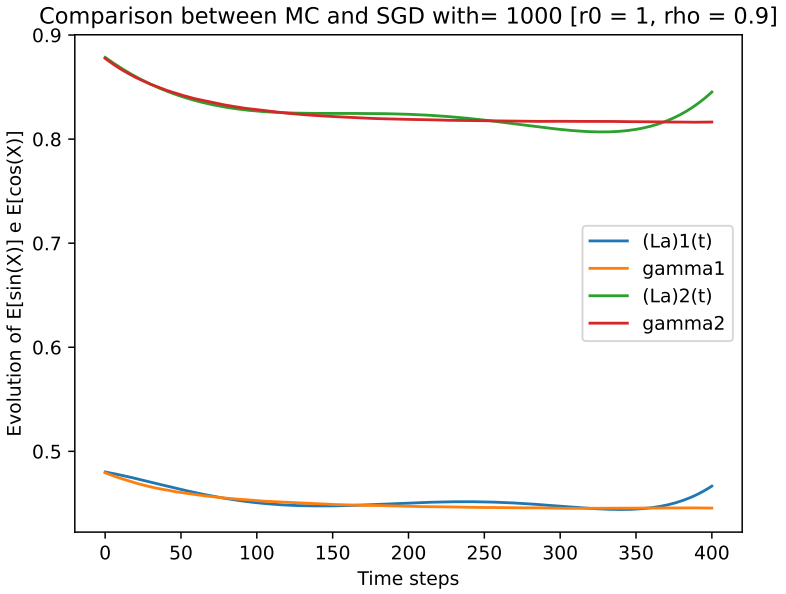
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	2	3	2.9		overflow			overflow	
$\rho = 0.7$	2	3	2.2		overflow			overflow	
$\rho = 0.8$	2	3	2.1		overflow			overflow	
$\rho = 0.9$	2	3	2.4		overflow			overflow	

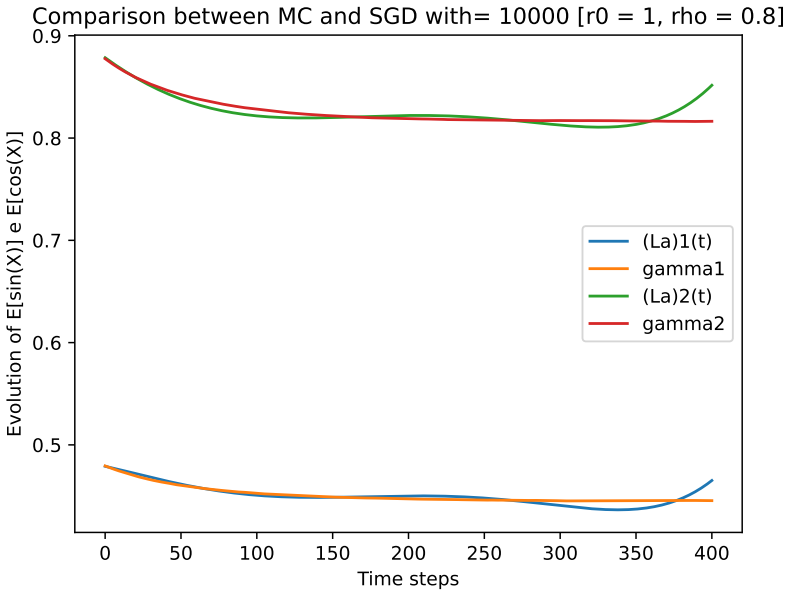
Tabella 1.150: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400


Comparison between MC and SGD with= 100 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400


Comparison between MC and SGD with= 1000 [r0 = 1, rho = 0.9]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400


Comparison between MC and SGD with= 10000 [r0 = 1, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400

Caso n = 6

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	146.81	
$\rho = 0.7$	104.65	
$\rho = 0.8$	412.79	241.22
$\rho = 0.9$	813.41	195.51

Tabella 1.151: Average execution times (in seconds s) with $M = 1$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	2280	10450	5647		overflow			overflow	
$\rho = 0.7$	1460	9530	4026		overflow			overflow	
$\rho = 0.8$	660	49999	15788.9	3320	17940	9134		overflow	
$\rho = 0.9$	820	49999	31095.4	2200	26720	7477		overflow	

Tabella 1.152: Number of iterations m to achieve convergence with $M = 1$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	26.03	
$\rho = 0.7$	9.16	
$\rho = 0.8$	67.37	32.54
$\rho = 0.9$	452.89	45.88

Tabella 1.153: Average execution times (in seconds s) with $M = 10$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	190	2900	848		overflow			overflow	
$\rho = 0.7$	120	620	296		overflow			overflow	
$\rho = 0.8$	170	7200	2200	310	2520	1056		overflow	
$\rho = 0.9$	50	49999	14881.8	260	4210	1503		overflow	

Tabella 1.154: Number of iterations m to achieve convergence with $M = 10$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	6.87	
$\rho = 0.7$	5.74	
$\rho = 0.8$	9.53	18.47
$\rho = 0.9$	34.43	36.04

Tabella 1.155: Average execution times (in seconds s) with $M = 100$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	50	230	111		overflow			overflow	
$\rho = 0.7$	20	240	93		overflow			overflow	
$\rho = 0.8$	20	650	154	80	390	269		overflow	
$\rho = 0.9$	20	3830	557	70	1110	528		overflow	

Tabella 1.156: Number of iterations m to achieve convergence with $M = 100$

	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	4.71	
$\rho = 0.7$	3.50	
$\rho = 0.8$	3.03	46.11
$\rho = 0.9$	7.87	131.98

Tabella 1.157: Average execution times (in seconds s) with $M = 1000$

	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	4	26	7.6		overflow			overflow	
$\rho = 0.7$	3	12	5.6		overflow			overflow	
$\rho = 0.8$	2	9	4.9	27	120	73.5		overflow	
$\rho = 0.9$	3	79	12.7	27	503	212.4		overflow	

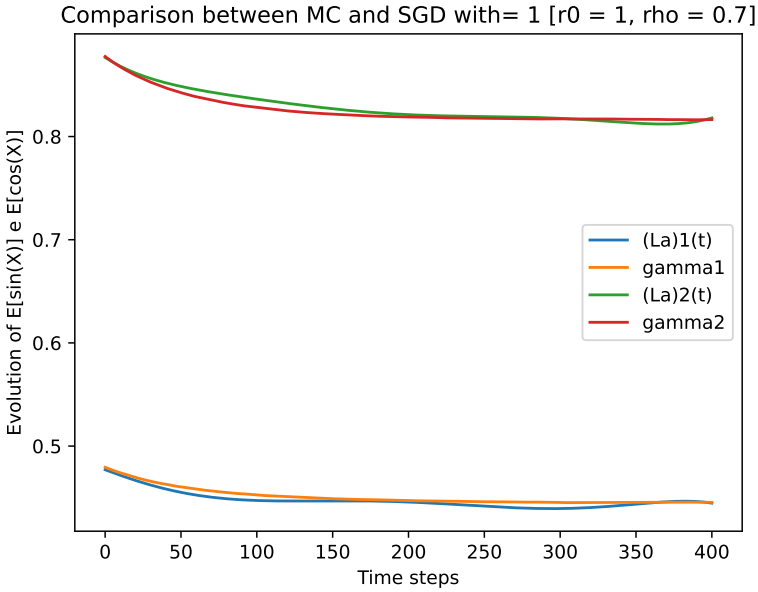
Tabella 1.158: Number of iterations m to achieve convergence with $M = 1000$

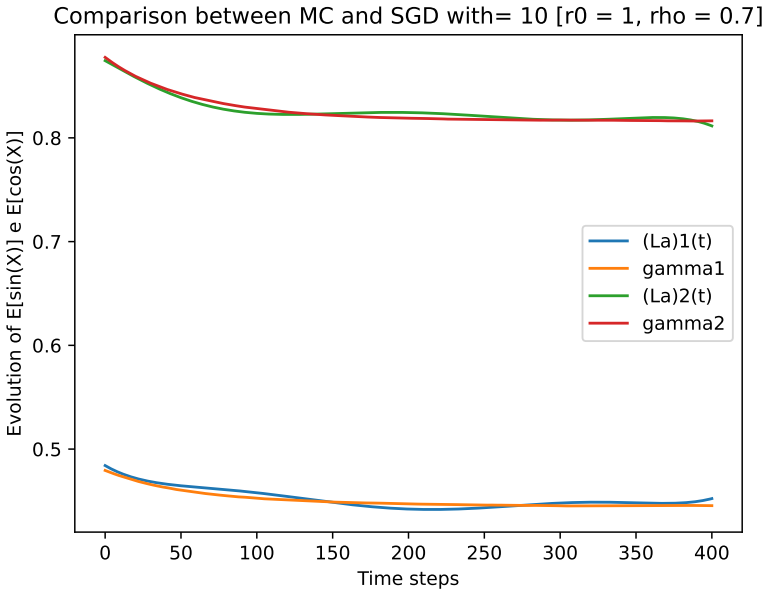
	$r_0 = 1$	$r_0 = 5$
$\rho = 0.6$	13.45	
$\rho = 0.7$	14.13	
$\rho = 0.8$	14.80	339.72
$\rho = 0.9$	20.18	402.01

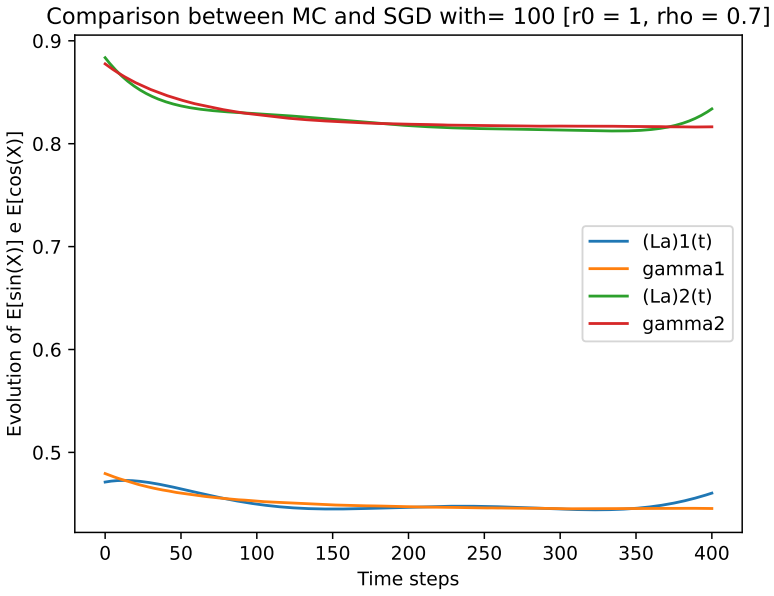
Tabella 1.159: Average execution times (in seconds s) with $M = 10000$

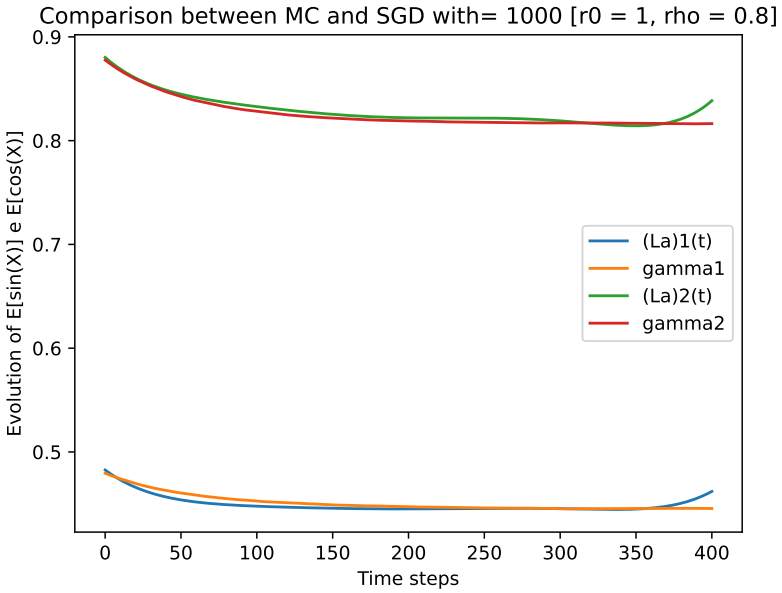
	$r_0 = 1$ min	$r_0 = 1$ max	$r_0 = 1$ average	$r_0 = 5$ min	$r_0 = 5$ max	$r_0 = 5$ average	$r_0 = 10$ min	$r_0 = 10$ max	$r_0 = 10$ average
$\rho = 0.6$	2	2	2		overflow			overflow	
$\rho = 0.7$	2	3	2.1		overflow			overflow	
$\rho = 0.8$	2	3	2.2	16	96	49.2		overflow	
$\rho = 0.9$	3	3	3	15	187	60.5		overflow	

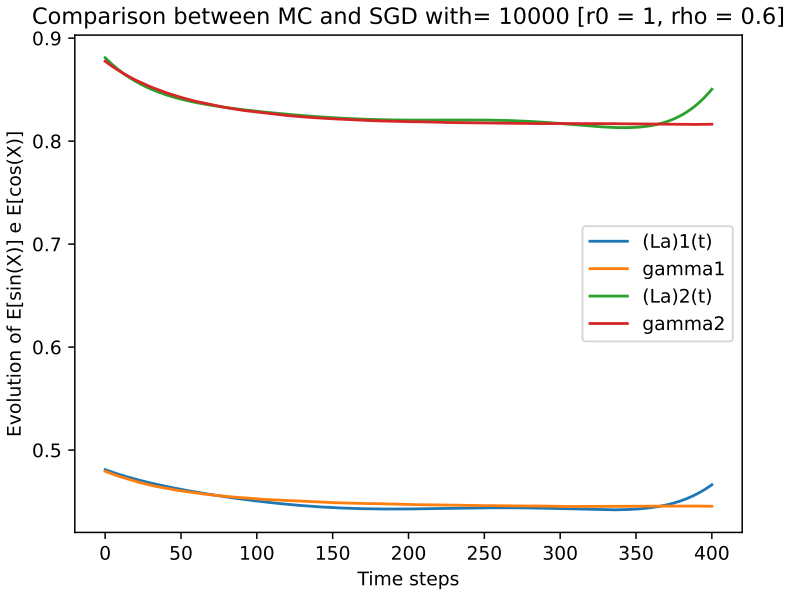
Tabella 1.160: Number of iterations m to achieve convergence with $M = 10000$




Comparison between MC and SGD with= 10 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.8
0.7
0.6
0.5
(La)1(t)
gamma1
(La)2(t)
gamma2
0
50
100
150
200
250
300
350
400
Time steps


Comparison between MC and SGD with= 100 [r0 = 1, rho = 0.7]
Evolution of E[sin(X)] e E[cos(X)]
0.9
0.8
0.7
0.6
0.5
(La)1(t)
gamma1
(La)2(t)
gamma2
0
50
100
150
200
250
300
350
400
Time steps


Comparison between MC and SGD with= 1000 [r0 = 1, rho = 0.8]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400


Comparison between MC and SGD with= 10000 [r0 = 1, rho = 0.6]
Evolution of E[sin(X)] e E[cos(X)]
Time steps
(La)1(t)
gamma1
(La)2(t)
gamma2
0.9
0.8
0.7
0.6
0.5
0
50
100
150
200
250
300
350
400

1.7 Osservazioni e Conclusioni

Lo studio condotto ha evidenziato come all'aumentare del parametro M (lasciando fissi gli altri) si riducano considerevolmente sia i tempi di esecuzione che il numero di iterazioni necessarie per arrivare a convergenza. Però nel caso $M = 10^4$ i tempi tornano ad aumentare rispetto ai casi precedenti. Pertanto ne consegue che la casistica $M = 10^3$ è la migliore sotto i due aspetti, tra quelle esposte. Questo è un risultato che si evince indipendentemente dalla scelta di T.

Per quanto concerne le sezioni $T = 0.5$ e $T = 1$, si riscontra come per valori piccoli di M le scelte di $r_0 = 1$ e $\rho = 0.6, 0.7$ generino spesso il caso più favorevole considerando tempi e numero di iterazioni. Mentre all'aumentare di M i valori che risultano più efficienti sono con $r_0 = 5, 10$ e $\rho = 0.8, 0.9$, a discapito di $r_0 = 1$ che presenta numeri di iterazioni relativamente più grandi.

Si riscontra, osservando i grafici, un ricorrente picco nell'istante finale della soluzione approssimante. Tale picco però scompare quando si osserva il grafico relativo alla medesima casistica appartenente alla sezione col tempo finale successivo. Ciò è prova del fatto che la causa di tale picco possa essere data dalla scelta della base dei polinomi e non dall'approssimazione stessa.

Alla luce dei risultati ottenuti si evince rallentamento della convergenza del metodo all'aumentare dell'istante finale T. Si verifica inoltre, sempre all'aumentare di T, una maggiore presenza di fenomeni di non convergenza del metodo, principalmente nei casi con $M = 1$.

Infine si nota come nei casi $T = 2$ e $T = 4$ si presenti il fenomeno di overflow. Nella sezione $T = 2$ esso riguarda esclusivamente il caso $r_0 = 10$, con qualsiasi ρ o n, fatta eccezione del caso $n = 6$ in cui si presenta solo per i valori $\rho = 0.6$ e 0.7. Per quanto riguarda la sezione $T = 4$ esso si verifica per tutti i casi con $r_0 = 10$. Si verifica anche per tutti i casi con $r_0 = 5$, fatta eccezione del caso $n = 6$ in cui si presenta solo per i valori $\rho = 0.6$ e 0.7.