

# Report on CSP (Constraint Satisfaction Problem)

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## 1. CSP Formulation of Sudoku

- **Variables:** The variables are each cell on the grid. ( $9 \times 9 = 81$  cells: variables)
- **Domain:** The domain is any digit from 1 to N (here,  $N=9$ )
- **Constraints:** The constraints are:
  - Same digit can't appear twice (or more) in the same row.
  - Same digit can't appear twice (or more) in the same column.
  - Same digit can't appear twice (or more) in the same region.
  - There are 9 rows, 9 columns, 9 boxes and each have an AllDiff constraint, so there are 27 such constraints.

## 2. Backtracking Search with Constraint Propagation

(a) Forward checking

(b) Maintaining-arc consistency(MAC)

(c) Powerful constraint propagators

### **Singleton:**

For a given cell there is only one value that can go into the cell, because all other values occur in row, column or sub-grid of the cell (any other number would lead to a direct violation of rules).

### **Twins:**

A hidden pair (twins) occurs when a pair of numbers appears in exactly two squares in a row, column, or block, but those two numbers aren't the only ones in their squares.

1	3	1	3	2	4	<del>5</del>	<del>8</del>	<del>5</del>	<del>8</del>	6	1	3	1
7	8	7	8			<del>7</del>	<del>9</del>	<del>8</del>	<del>9</del>				8

In the above example, the 5 and 9 in red are a hidden pair. The 5 and 9 only occur in those two squares and nowhere else in the row, so those two squares can only contain 5 and 9 and no other numbers. Because of that, you can get rid of the other candidates in those squares, as shown crossed out in red.

### Triplets:

A hidden triple occurs when three cells in a row, column, or block contain the same three numbers, or a subset of those three. The three cells also contain other candidates.

3	8	<del>4</del>	<del>5</del>		4	6	4	6	<del>4</del>	<del>5</del>	<del>6</del>	<del>4</del>	<del>5</del>	<del>6</del>
		7	9	7	9	7	9	7	9	7	9	7	9	7

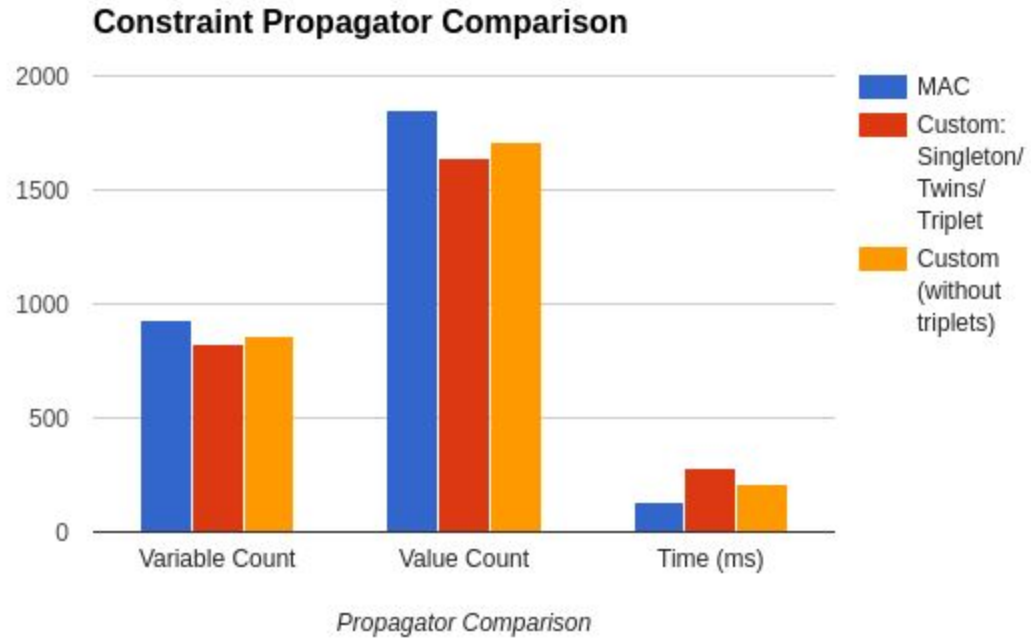
In the above example, 1, 2, and 5 (shown in red) are a hidden triple. Those three numbers are only seen in three squares in the row. Since they only appear there, those three squares MUST contain 1 or 2 or 5 and no other numbers. For that reason, the other candidates can be removed from those squares as shown crossed out in red.

### Quads:

Hidden quads are pretty rare, and they can be difficult to spot unless you are specifically looking for them.

<del>1</del>	<del>2</del>	<del>3</del>	<del>4</del>	<del>5</del>	<del>6</del>	<del>7</del>	<del>8</del>	<del>9</del>	<del>1</del>	<del>2</del>	<del>3</del>	<del>4</del>	<del>5</del>	<del>6</del>
7	8	9	1	2	3	4	5	6	7	8	9	1	2	3

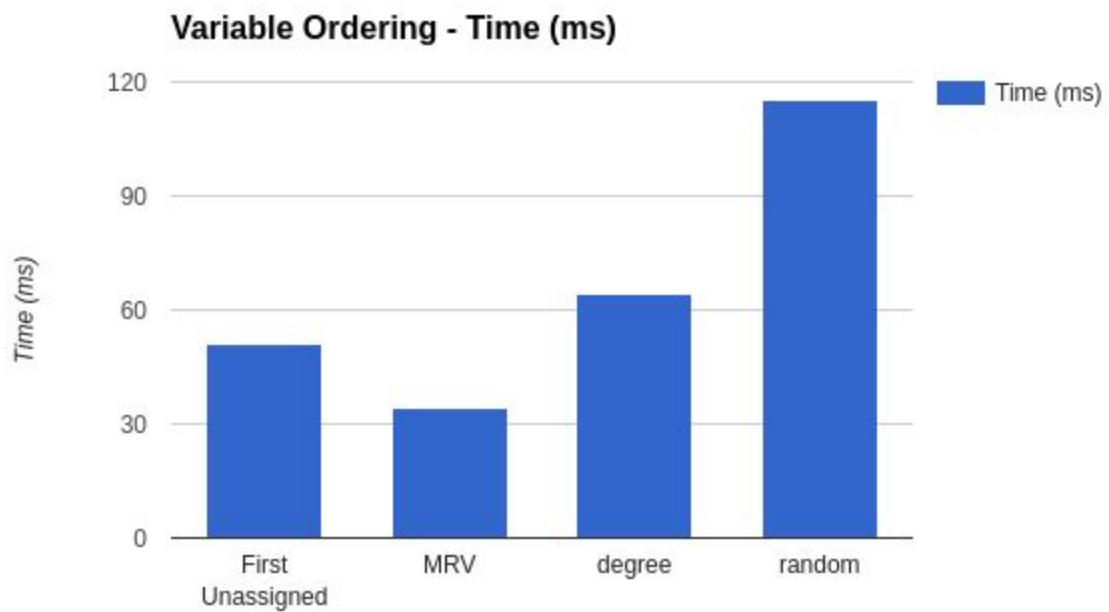
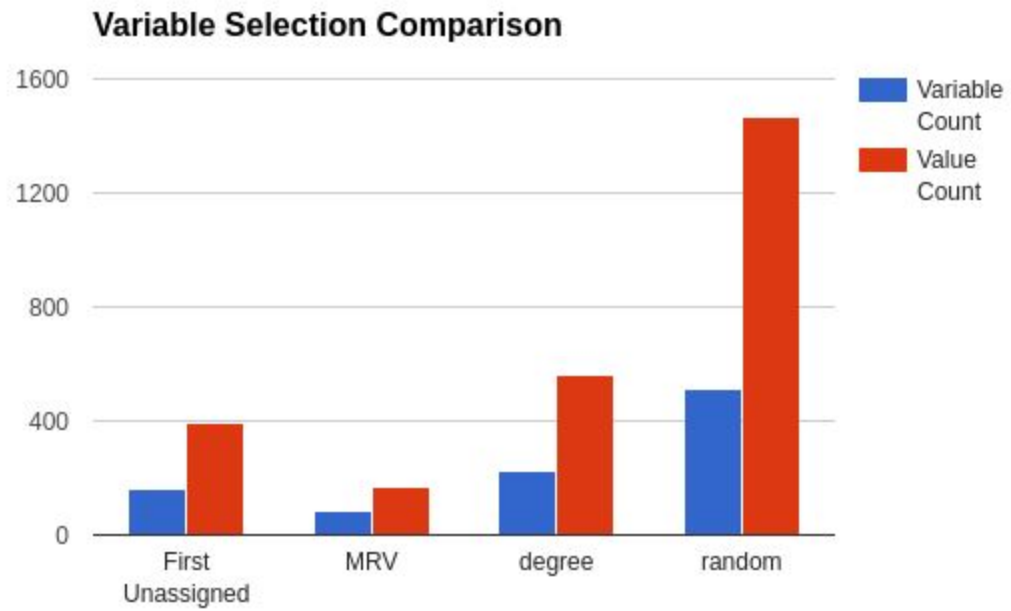
In the above example, since the four numbers 1, 6, 7, and 8 (shown in red) appear only in four squares, they are a quad. Those four numbers MUST occur in those four squares. For that reason, the other candidates can be eliminated from those squares, as shown crossed out in red.



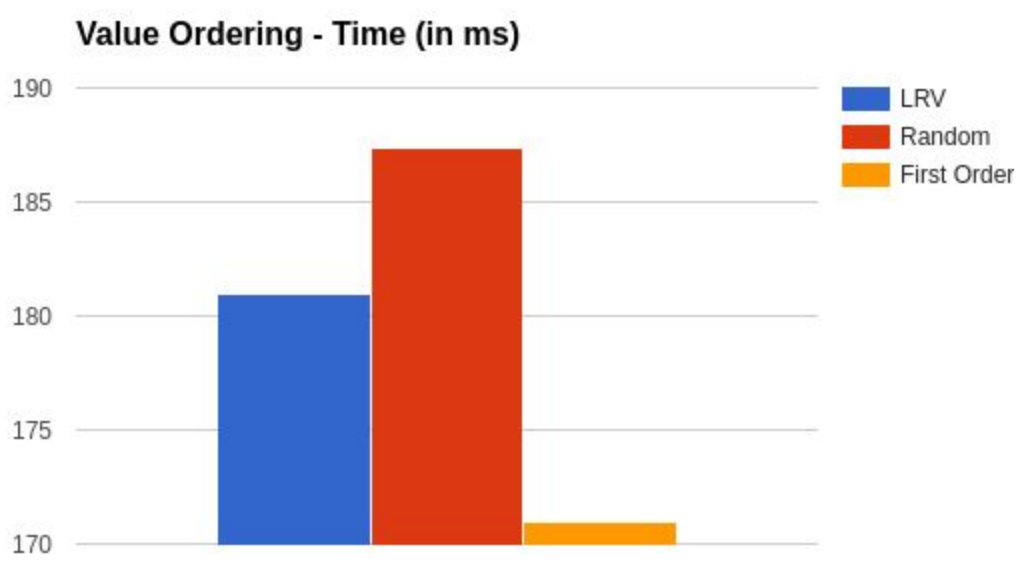
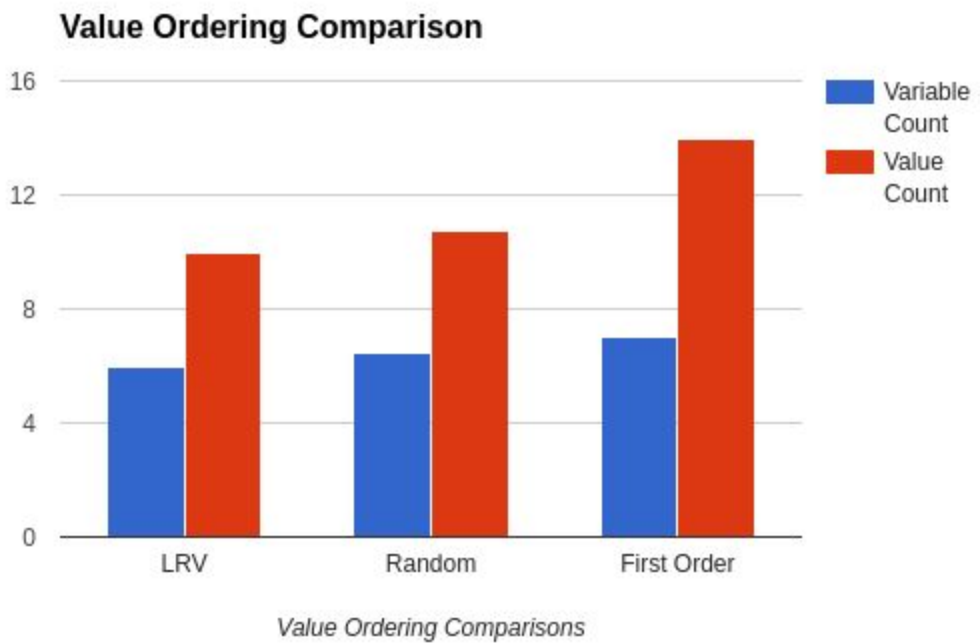
	MAC	Custom Propagator	Forward Elimination
Variable Count	928	859	47846
Value Count	1850	1712	95671
Time (ms)	134	209	2379

**Problem Name:** worldhard.txt

### 3. (i) Variable Selection



### 3. (ii) Value Selection



**Problem Name:** *hard3.txt*