Calculus and Linear Algebra:
Matrices

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Problem 1.1 *Given the matrices:*

$$A = \begin{bmatrix} 9 & -4 & 7 \\ -5 & 2 & -4 \\ 10 & -4 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 10 & 7 & -7 & 4 \\ -5 & -9 & 4 & -7 \\ -5 & 10 & -6 & -5 \end{bmatrix} \quad C = \begin{bmatrix} -3 & -9 & -6 \\ 7 & 1 & 7 \\ -4 & 5 & 1 \\ 3 & -4 & -5 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 6 & 7 \\ 3 & 10 & 1 \\ 0 & 5 & 0 \end{bmatrix}$$

Obtain the following matrices

a) A^T

b) C^T

c) $A + D$

d) $B + C^T$

e) $D - A$

f) AB

g) BC

h) CB

Problem 1.2 *Calculate the determinant of the following matrices*

a)

$$\begin{bmatrix} 1 & 2 & 0 \\ -3 & 5 & 0 \\ 7 & 4 & 1 \end{bmatrix}$$

b)

$$\begin{bmatrix} 5 & 1 & -8 \\ -3 & 7 & 2 \\ -4 & 1 & -1 \end{bmatrix}$$

c)

$$\begin{bmatrix} 3 & 9 & -3 \\ 9 & 1 & -1 \\ 6 & -8 & 2 \end{bmatrix}$$

d)

$$\begin{bmatrix} 2 & 0 & 7 & 0 \\ -1 & 3 & 6 & 1 \\ 2 & 0 & 1 & 3 \\ -2 & 4 & 3 & 1 \end{bmatrix}$$

e)

$$\begin{bmatrix} 0 & -4 & -4 & 0 \\ -1 & 2 & -3 & -4 \\ 0 & -4 & 1 & 4 \\ 0 & 2 & 3 & 0 \end{bmatrix}$$

f)

$$\begin{bmatrix} 0 & -2 & 0 & -5 & 3 \\ 0 & 1 & 0 & -2 & -1 \\ 0 & 5 & 0 & -1 & 0 \\ 7 & -1 & -1 & 2 & -9 \\ 1 & 0 & 0 & -10 & 0 \end{bmatrix}$$

Problem 1.3 *Given the following vectors* $\mathbf{u}$ *and* $\mathbf{v}$ *calculate their norm (*$|\mathbf{u}|$ *and* $|\mathbf{v}|$*), sum, difference (*$\mathbf{u}-\mathbf{v}$ *and* $\mathbf{v}-\mathbf{u}$*), dot/scalar product and the angle between them.*

a) $\mathbf{u} = [1, 3]$ *and* $\mathbf{v} = [6, -2]$

b) $\mathbf{u} = [\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}]$ *and* $\mathbf{v} = [0, -1]$

c) $\mathbf{u} = [-3, 1]$ *and* $\mathbf{v} = [-2, -1]$

d) $\mathbf{u} = [\sqrt{5}, \sqrt{3}, \sqrt{2}]$ *and* $\mathbf{v} = [0, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}]$

e) $\mathbf{u} = [2, 3, 6]$ *and* $\mathbf{v} = [-6, -9, -18]$

f) $\mathbf{u} = [6, 3\sqrt{2}, 3\sqrt{2}]$ *and* $\mathbf{v} = [1, -1 + \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}]$

g) $\mathbf{u} = [1, -1, 1, 1]$ *and* $\mathbf{v} = [3, 2, 4, -5]$

h) $\mathbf{u} = [2, 3, -1, 6]$ *and* $\mathbf{v} = [-4, -6, 2, -12]$