Calculus and Linear Algebra:
Gaussian Elimination

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Problem 1.1 *Using Cramer's rule solve the following systems of linear equations when possible*

a) $\begin{cases} -2x + 4y = 4 \\ -2x - 3y = -10 \end{cases}$

b) $\begin{cases} x + 3y = 0 \\ 4x + y = 6 \end{cases}$

c) $\begin{cases} x - 3y = 1 \\ 2x + y = 6 \end{cases}$

d) $\begin{cases} x - 4y + 2z = 0 \\ 2x + 5y + z = 1 \\ 5x - 2z = 0 \end{cases}$

e) $\begin{cases} x - y + 3z = -2 \\ 5x - z = 0 \\ y - 3z = 1 \end{cases}$

Problem 1.2 *Do the following systems of linear equations have a solution? Find it when they do.*

a) $\begin{cases} -3x - y = -14 \\ 3x + 4y = 11 \\ 5x + y = 24 \end{cases}$

b) $\begin{cases} -2x - 3z = -6 \\ x - 2y + 3z = -7 \\ 3x - y - 2z = 4 \end{cases}$

c) $\begin{cases} -x + y - 2z = 2 \\ -2x + y + z = -4 \\ -y + 3z = -4 \end{cases}$

d) $\begin{cases} -3x + 2y + z = 8 \\ -3x - 3y + 2z = -10 \\ -9x + y + 4z = 7 \end{cases}$

e) $\begin{cases} -2x - y = 5 \\ 3x - y + 3z = -7 \\ -x - 2z = 1 \\ 2x + 2y - 3z = -7 \\ 3x + 2y + z = -6 \end{cases}$

$$f)\ \begin{cases} \phantom{4x_1 +{}} 2x_2 - 2x_3 + x_4 = 4 \\ 4x_1 + 3x_2 + 2x_3 + 2x_4 = -11 \\ x_1 + x_2 - x_3 + x_4 = 4 \\ 5x_1 + x_2 + 9x_3 \phantom{{}+2x_4} = 8 \end{cases}$$

$$g)\ \begin{cases} - 3x_2 + 3x_3 + 2x_4 = 4 \\ -4x_1 \phantom{{}-3x_2} - 3x_3 - 3x_4 = -8 \\ -3x_1 - x_2 \phantom{{}+3x_3} + 3x_4 = 7 \\ -7x_1 - 4x_2 \phantom{{}+3x_3} + 2x_4 = 3 \end{cases}$$

Problem 1.3 *Given a set of 'm' vectors of dimension 'n' we can test whether they are linearly independent by building a matrix 'A' of size $m \times n$ (the vectors are the rows of the matrix) and find the number of non-zero rows after applying Gaussian elimination. That number will tell how many linearly independent vectors are in the set.*

Given the following sets of vectors find whether they are linearly independent.

a) $\mathbf{u}_1 = [2, 1]$ *and* $\mathbf{u}_2 = [-2, 1]$

b) $\mathbf{u}_1 = [-2, 5]$ *and* $\mathbf{u}_2 = [\frac{1}{5}, -2]$

c) $\mathbf{u}_1 = [-4, 8, 2]$, $\mathbf{u}_2 = [2, 12, 6]$ *and* $\mathbf{u}_3 = [0, -16, -7]$

d) $\mathbf{u}_1 = [-1, 6, 3]$, $\mathbf{u}_2 = [2, 16, 8]$ *and* $\mathbf{u}_3 = [3, 4, 0]$

e) $\mathbf{u}_1 = [0, 2, 1, 6]$, $\mathbf{u}_2 = [-4, 6, -10, 2]$ *and* $\mathbf{u}_3 = [2, 0, 6, 0]$

f) $\mathbf{u}_1 = [0, 1, 2, 3]$, $\mathbf{u}_2 = [2, -10, -4, 6]$ *and* $\mathbf{u}_3 = [3, -12, 0, 18]$