IMAGE REPRESENTATION OF WAVE PROPAGATION ON WIRES ABOVE, ON AND UNDER GROUND

Per Pettersson, Member IEEE

Vattenfall Utveckling AB
S-162 15 Vällingby, Sweden

Abstract: Approximate image type expressions for series impedance and shunt admittance are derived for self and mutual coupling of wires above, on, and under a lossy ground. The expressions are used for approximate evaluation of the propagation constant and the characteristic impedance of a single wire of general location relative to the surface of ground. Numerical comparisons with exact and quasi-TEM results are presented.

Key-words: Transmission line, counterpoise, frequency dependence, line parameters, travelling wave, propagation constant, characteristic impedance, image, quasi-TEM.

INTRODUCTION

Dimensioning of transmission line insulation requires modelling of propagation of the surges occurring at switching operations and lightning strikes. This necessitates knowledge of the self and mutual series (axial) and shunt (transversal) couplings of the phase-wires in the presence of a lossy earth for time-harmonic excitations of varying frequency. For the rather high-frequency lightning surges, the standard engineering simplification of the problem is to regard the ground as being perfectly conducting, which renders the problem virtually frequency-independent, since the series coupling will then be purely inductive and the shunt coupling purely capacitive. For the more moderately high-frequency switching surges, the ground is taken to be perfectly conducting for the shunt coupling assessment, but not for the series one, where the Carson correction is used. For a more accurate modelling, in both cases, correcting has to be done also on the shunt coupling. Proper inclusion of the shunt effects has also to be made in modelling parallel continuous counterpoises on or buried in the ground. A want would thus be a general theory working for a system of parallel wires above, at and under ground.

The equations of the rigorous theory of this problem are extremely complicated, even for the single wire case, and are not well suited for use in transients computation programs based on the extensive Fourier transform method, since this would require heavy computations for each frequency prior to inversion into time domain. This is where image representation comes in. Image representations are simple closed form approximate expressions for the potentials (electric and magnetic vector) defining the couplings, mimicking those of the static theory. One important feature of the dynamic theory is the concept of "complex depth" for the fictitious image sources. Besides to giving a great computational relief, image representations are appealing to intuition. The power-frequency counterpart of this problem was treated in Olsen et. al. [1].

93 SM 402-8 PWRD A paper recommended and approved by the IEEE Transmission and Distribution Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1993 Summer Meeting, Vancouver, B.C., Canada, July 18-22, 1993. Manuscript submitted August 31, 1992; made available for printing May 18, 1993.

PRINTED IN USA

The purpose of this paper is to propose a consistent approximate image theory for series and shunt self and mutual couplings of wires above at and under the ground surface. Only the one-wire case will be more thoroughly treated here. The propagation constant and the characteristic impedance of the wire will specifically be addressed.

A glossary of symbols appears at the end of the paper.

BASIC THEORY

A filamentary and infinitely long line-source is assumed to be situated in Medium 1 at distance $h \geq 0$ from a parallel plane interface to Medium 2. The media are both non-magnetic, permeability $\mu=\mu_0$, but are differing in relative permittivity to air ε_r and conductivity σ. Thus air can be either one of the media and ground the other, depending on the location of the wire. The line is excited by some distant time-harmonic source of angular frequency ω, so that current waves represented by $I=I_0\exp(-\gamma_w z+j\omega t)$ are travelling in the positive z-direction along the line. Here the quantity I_0 is current phasor, t time and γ_w the unknown axial propagation constant. This line-source gives rise to electric potentials V and magnetic vector potentials $A=(A_x, A_y, A_z)$ at field points on any other parallel line, see Figure 1.

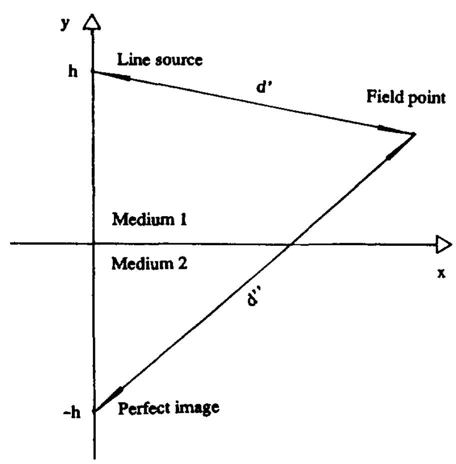


Figure 1 Current line-source near the interface of two media.

0885-8977/94/$04.00 © 1993 IEEE

The series and shunt coupling problems consist in assessing A_z and V, respectively on the second line. The potentials of a physical round wire of radius a>0 are considered to be created by a line-source located at the centre, and self coupling relates to the own potentials occurring on the surface of the wire. If the wire is thin enough, the potentials will be nearly constant over the cross-section perimeter, so that self coupling will be reasonably well defined. The mutual coupling between two wires is viewed analogously.

The potentials A_{zm} and V_m from the line-source, index m=1 or 2 denotes the medium of the field-point, can (see e.g. Kikuchi [2]) be written as

$$A_{zm}=I\mu_0(\Lambda_m+P_m)/(2\pi) \tag{1}$$

$$V_m=I[\gamma_w/(j\omega\varepsilon_0 n_1^2)](\Lambda_m+Q_m)/(2\pi)$$

$$\Lambda_1=K_0(\gamma_{\tau 1}d')-K_0(\gamma_{\tau 1}d'')$$
$$=\int_0^\infty u_1^{-1}\{\exp[-u_1|y-h|]-\exp[-u_1(y+h)]\}\cos(kx)dk$$

$$\Lambda_2=0$$

$$P_1=2\int_0^\infty(u_1+u_2)^{-1}\exp[-u_1(y+h)]\cos(kx)dk$$

$$P_2=2\int_0^\infty(u_1+u_2)^{-1}\exp(-u_1h+u_2y)\cos(kx)dk$$

$$Q_1=2\int_0^\infty(n^2u_1+u_2)^{-1}\exp[-u_1(y+h)]\cos(kx)dk$$

$$Q_2=2\int_0^\infty(n^2u_1+u_2)^{-1}\exp(-u_1h+u_2y)\cos(kx)dk$$

$$\gamma_{\tau m}=(\gamma_m^2-\gamma_w^2)^{1/2},\ \gamma_m=\gamma_0 n_m$$

$$n_m=[\varepsilon_{rm}+\sigma_m/(j\omega\varepsilon_0)]^{1/2},\ \gamma_0=j\omega(\varepsilon_0\mu_0)^{1/2}$$

$$d'=[(y-h)^2+x^2]^{1/2};\ d''=[(y+h)^2+x^2]^{1/2}$$

$$u_m=(k^2+\gamma_{\tau m}^2)^{1/2}$$

$$n=n_2/n_1$$

The path of integration is along the real axis. The potential A_{xm} can be taken to be equal to zero. The expression for A_{ym} is immaterial for this study and is not shown. These potentials satisfy the Helmholz equation in both media, have the right singularities at the line-source, and satisfy the boundary conditions for the E- and H-field at the interface of the media. The square-root function appearing is the principal branch rendering the real part positive. The quantities γ_m and $\gamma_{\tau m}$ are the propagation constants of the medium itself and in the transverse (x,y)-plane, respectively. K_0 is the modified Bessel-function of the second kind of order zero, [3].

Actually these equations were derived for the case h>0 but by reciprocity of field- and source-points they will apply also for the case h=0 with $\Lambda_1=0$.

The generalized unit length self series impedance Z and shunt admittance Y of a single wire, as defined by the "telegrapher's equations"

$$ZI=-\partial V/\partial z,\ YV=-\partial I/\partial z \tag{2}$$

are from above, with index 1 for Λ, P and Q implicitely understood, found to be given by

$$Z=j\omega\mu_0(\Lambda+P)/(2\pi),\ Y=j\omega\varepsilon_0 n_1^2(\Lambda+Q)^{-1}2\pi \tag{3}$$

noting that $\partial V/\partial z=-j\omega A_z$ for a lossless wire ($E_z=0$ on the wire) and that $\partial I/\partial z=-\gamma_w I$. For a wire with losses, a proper term representing the internal series impedance of the wire has to be added to Z. Solving (2) for I and V it follows that γ_w must satisfy the modal equation

$$\gamma_w=(ZY)^{1/2}=\gamma_1[(\Lambda+P)/(\Lambda+Q)]^{1/2} \tag{4}$$

where it is observed that Λ, P and Q are functions of γ_w. The characteristic impedance Z_w of the wire defined by $Z_w=V/I$ is moreover given by

$$Z_w=(Z/Y)^{1/2}=Z_1[(\Lambda+P)(\Lambda+Q)]^{1/2}/(2\pi) \tag{5}$$

$$Z_1=Z_0/n_1,\ Z_0=(\mu_0/\varepsilon_0)^{1/2}$$

where the Λ, P and Q are given for the modal γ_w solving (4). Here Z_1 is the characteristic impedance of Medium 1.

Two solutions of the mode equation consistent with field decay at large transverse distances from the line are known from the literature, see e.g. Olsen et. al. [4]. For a wire in the air, one of the solutions ("structure-mode") represents the classic quite lossy mode of propagation with a significant part of its fields situated in the earth, while the other one ("surface mode") has its fields more spread out along the earth's surface. Only the classic mode will be considered here.

The mode equation can be solved numerically, e.g. by iterations. Approximate evaluation of γ_w can also be performed by a perturbation method, meaning that γ_w is set equal to some initially assumed value in the right-hand side of (4). If the resulting γ_w is close to the initial one, then it is likely to be close to the correct value. As initial value, $\gamma_w=\gamma_1$ is used for the case with h>0 and $\gamma_w=[(\gamma_1^2+\gamma_2^2)/2]^{1/2}$ for h=0. The effectiveness of these choices will be shown in the sequel. This will here be referred to as the quasi-TEM (Transverse Electric and Magnetic fields) solution, which is known to be sufficiently accurate for most transmission line transients computations.

Historically, it seems that Carson [5] and Pollaczek [6] were the first to express P, and Wise [7] Q for the case with h>0 by the quasi-TEM approximation. Kikuchi found P and Q as functions of γ_w. Consolidation of the theory was made at the University of Boulder, Colorado, see e.g. Kuester et. al. [8], where also the Russian research is surveyed. The early theories for the case with h=0 are comprised in Sunde [9] and Coleman [10].

IMAGE REPRESENTATIONS

The rigorous theory of above is extremely complicated from a conceptual point of view and also from a numerical one as computer surge calculations by use of Fourier transforms would require time consuming integrations for the entire range of frequencies. For this purpose sound approximate methods are wanted. The concept of image representation practicable in the static theory to handle planar discontinuities in the medium serves as a model. There seems to be a close connection between a useful image theory for the problem of interest and exactly integrable approximations to the integrands of P and Q, which are working over the whole integration range. The approximation scheme to follow has the pedagogical advantage of comprising the cases with h>0 and h=0 in one single unified model.

Starting with the case h>0, inserting $\gamma_w=\gamma_1$ into (1) yields for P and Q expressions of the following common structure when $y\geq 0$

$$2\int_0^\infty [bk+(k^2+\beta^2)^{1/2}]^{-1}\exp[-k(y+h)]\cos(kx)dk \tag{6}$$

$$\beta=\gamma_1(n^2-1)^{1/2}$$

with b=1 for P (the Carson integral) and $b=n^2$ for Q. This integral cannot be evaluated analytically. In what will here be referred to as the Quasi-TEM method, the integral is evaluated numerically.In order to find a good approximation to it, which will be called the Image method, the following key identity is observed

$$\int_0^\infty k^{-1}[1-\exp(-kc)]\exp[-k(y+h)]\cos(kx)dk=\ln(d^*/d'') \tag{7}$$

$$d^*=[(y+h+c)^2+x^2]^{1/2}$$

where c is any complex number with a real part such that the integral converges. The proof of this is quite easy by using the exponential representation of the cosine factor, writing

$$k^{-1}[1-\exp(-kc)]=\int_0^c \exp(-kw)dw \tag{8}$$

and reversing the order of integration in the resulting double-integral. The crucial approximation now, is

$$[bk+(k^2+\beta^2)^{1/2}]^{-1}\simeq k^{-1}\{1-\exp[-k(1+b)/\beta]\}/(1+b) \tag{9}$$

which combined with (6) and (7) gives the desired approximations for P and Q. It is easily verified that the two sides of (9) agree asymptotically for small as well as large values on k/β, which so far makes the approximation promising.

We are then in a position to write

$$\Lambda=\ln(d''/d') \tag{10}$$

$$P\simeq\ln(d_P/d''),\ d_P=[(y+h+2/\beta)^2+x^2]^{1/2}$$

$$Q\simeq[2/(n^2+1)]\ln(d_Q/d''),\ d_Q=\pm\{[y+h+(n^2+1)/\beta]^2+x^2\}^{1/2}$$

where (7) was used for evaluation of Λ. This is well known as the "complex depth" approximation as far as P is concerned. For continuity of Q as a function of frequency, the sign of d_Q has to be chosen so that the imaginary part becomes negative when the wire is in the air and positive when it is in the ground. In passing it is noted that $\Lambda+P\simeq\ln(d_P/d')$ and that a corresponding simplification does not exist for $\Lambda+Q$.

For the self-coupling case we may use x=0 and y=h+a whereby (10) simplifies into

$$\Lambda\simeq\ln(2h/a) \tag{11}$$

$$P\simeq\ln[1+1/(\beta h)]$$

$$Q\simeq[2(n^2+1)]\ln\{\pm[1+(n^2+1)/(2\beta h)]\}$$

for a thin wire, i.e. h/a >>1.

The P-approximation of (10) is known since long, and was in this form probably first found by Kostenko [11], and has been repeatedly rediscovered. However, still earlier Sunde [9] suggested the P of (11) without proof for the self coupling case. Reference is also made to Wait and Spies [12] and Deri et.al. [13].The Q-approximation may be new. The resemblance with the electrostatic case is suggestive, see e.g. Wait [14], Chapter 1.7.

It remains to study the case h=0. Inserting $\gamma_w=\gamma_1[(n^2+1)/2]^{1/2}$ into (1) gives the following structure for the P- and Q-integrals when $y\geq 0$.

$$2\int_0^\infty [b(k^2-\beta^2/2)^{1/2}+(k^2+\beta^2/2)^{1/2}]^{-1}\exp(-ky)\cos(kx)dk \tag{12}$$

with b and β as of above. The following approximation corresponding to (9) is in the same vein

$$[b(k^2-\beta^2/2)^{1/2}+(k^2+\beta^2/2)^{1/2}]^{-1} \tag{13}$$

$$\simeq k^{-1}\{1-\exp[-k\sqrt{2}(1+b)(1-jb)^{-1}/\beta]\}/(1+b)$$

and has the right asymptotic behaviour. Thus P and Q will have the same structure as in the case h>0, but now with

$$\Lambda=0 \tag{14}$$

$$d_P\simeq\{[y+\sqrt{2}(1+j)/\beta]^2+x^2\}^{1/2}$$

$$d_Q\simeq\{[(y+j\sqrt{2}(n^2+1)(n^2+j)^{-1}/\beta]^2+x^2\}^{1/2}$$

Inserting x=0 and y=a into (14), we get for the self-coupling case

$$\Lambda=0 \tag{15}$$

$$P\simeq\ln[2\sqrt{j}/(\beta a)]$$

$$Q\simeq[2/(n^2+1)]\ln[j\sqrt{2}(n^2+1)(n^2+j)^{-1}/(\beta a)]$$

where the unit term appearing in the arguments of the logarithmic functions of P and Q have been neglected, since $|\beta a|<<1$ in practical cases. Further simplification of Q could be achieved by noting that typically $(n^2+1)/(n^2+j)\simeq 1$ for field point in air.

The P- and Q-approximations of (14) are believed to be new. The P and Q of (15) have, however, close resemblances to those of Sunde [9] Chapter 5.4, which were derived quite differently.

The presented image approximation scheme may be viewed as a generalization of the method Kostenko used to estimate P for the overhead line case. Actually, for small k/β-values the approximation (9) is a very good one in the case b=1, because series-expansion of both sides proves the first two terms to be equal and the third one to be almost equal.

VALIDATION

The objective is to demonstrate the precision of the image method for approximating γ_w and Z_w. Instead of working directly with these quantities it will be more convenient to have them normalized. For fixing suitable normalizations we are led by the image method expressions for γ_w and Z_w which contain factors that may be tentatively scaled off from both the exact and the approximate quantities, leaving as remainders quantities believed to be close to unity in the complex plane. These normalizations have an interest in their own right by representing more or less crude first approximations to γ_w and Z_w.

For the case h>0 we thus write

$$\gamma_w=\alpha\gamma_{ref},\ \gamma_{ref}=\gamma_1 \tag{16}$$

$$Z_w=\psi Z_{ref},\ Z_{ref}=Z_1\ln(2h/a)/(2\pi)$$

where α and ψ are called the relative propagation constant and characteristic impedance, respectively, which will be compared using the approximate and exact methods. γ_{ref} and Z_{ref} are serving as reference values. When the wire is located in the air, α has a physical interpretation, viz Re(α) is the inverted value of the phase velocity of the wave relative to the velocity of light, and -Im(α) is the attenuation in Nepers per wave-length divided by 2π. Z_{ref} is identified as the characteristic impedance of a wire above a perfectly conducting plane.

For the case h=0, as suggested by the image method, we use for reference values

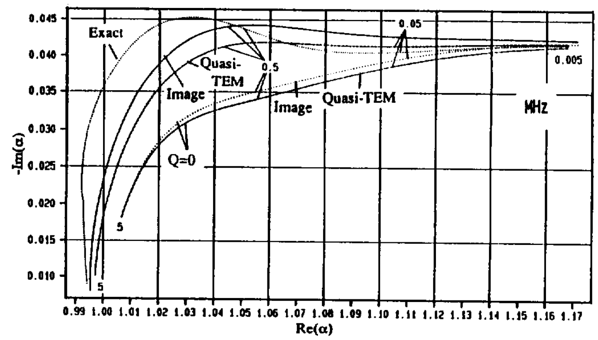
$$\gamma_{ref}=\gamma_1[(1+n^2)/2]^{1/2} \tag{17}$$

$$Z_{ref}=Z_1[2/(1+n^2)]^{1/2}\ln[j\sqrt{2}/(\beta a)]/(2\pi)$$

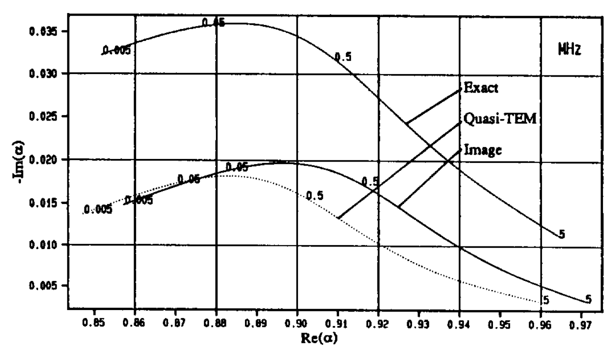
The close resemblance of half Z_{ref} to the expression of Sunde [9], Chapter 8.6 for the impedance of a doubly infinite counterpoise is noted.

Computer calculations for comparison of the three methods presented above for calculation of α and ψ, namely the Exact, the Quasi-TEM and the Image method, were performed on a VAX 4000-200. In Exact, iterations were used to assess α. Diagram 1 shows the trajectories of the imaginary (note negative sign) and real part of α as functions of frequency for the range 5 kHz-5 MHz and Diagram 2 the same for ψ. In a, b and c of the diagrams, the wire is placed 10 m above ground, 1 m under ground, and at the ground surface, respectively. The wire radius used is 1 cm. For the ground, relative permittivity 10 and resistivity 1000 Ωm are used.

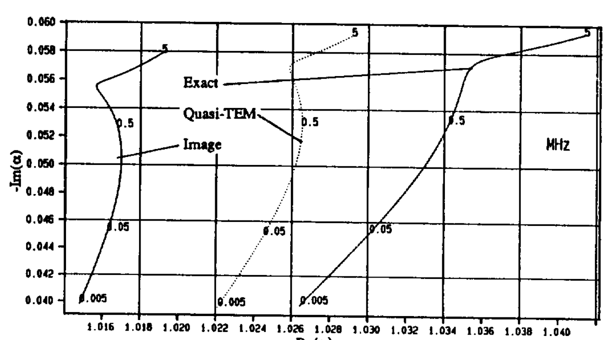
For the overhead wire of Diagram 1a, it appears that all three methods agree very well for frequencies at least up to 1 MHz. Above 1 MHz, Image is surprisingly seen to be closer to Exact than is Quasi-TEM. For the buried wire of Diagram 1b, both approximate methods give too small values to Im(α). For the lower frequencies,



a: Wire 10 m above ground



b: Wire 1 m under ground

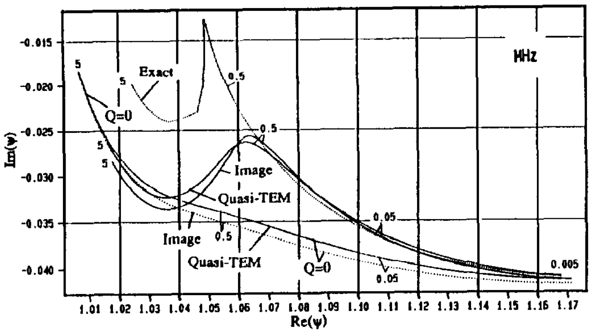


c: Wire at ground surface

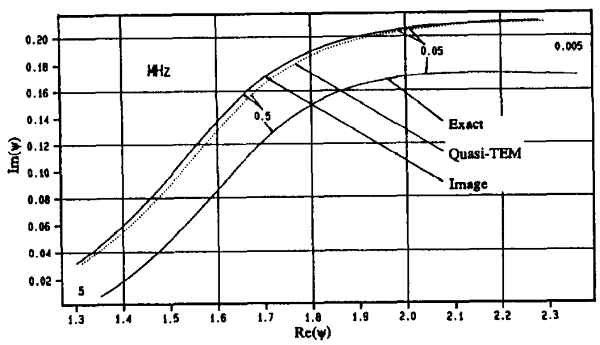
Diagram 1 Relative propagation constant for a wire near the surface of ground as function of frequency (MHz). Comparison of three methods.

however, this will not mean any significant loss in accuracy because the angle of the propagation constant of ground will then be near 45°. An exact range of applicability of the approximations is hard to define. In regard to the narrow scale of Re(α) in Diagram 1c, both approximations must be judged to work satisfactorily for the surface wire. The diagrams confirm the soundness of the reference propagation constants.

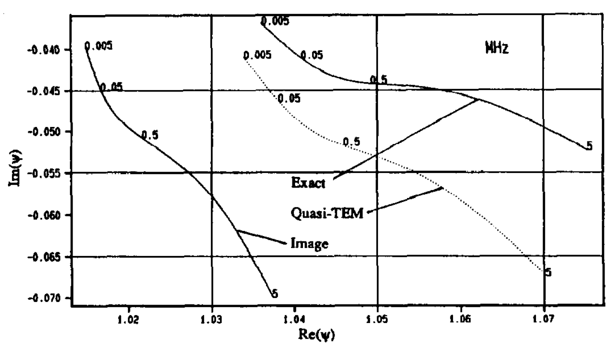
Concerning the characteristic impedance Diagram 2 shows that very little accuracy is lost when using the approximate methods, which have very similar trajectories. Similar to Diagram 1 it is found that the buried wire case has a seemingly worse accuracy. For this case it appears also that the reference characteristic impedance used works markedly worse compared to the other two cases.



a: Wire 10 m above ground



b: Wire 1 m under ground



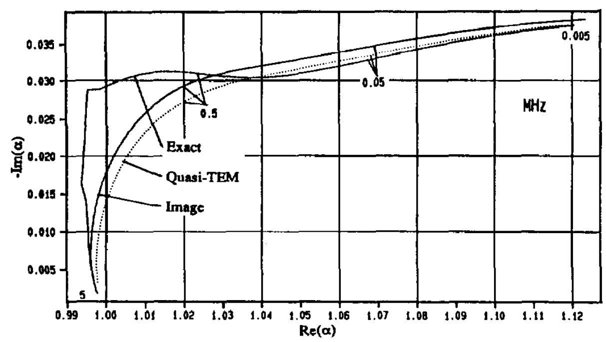
c: Wire at ground surface

Diagram 2 Relative characteristic impedance for a wire near the surface of ground as function of frequency (MHz). Comparison of three methods.

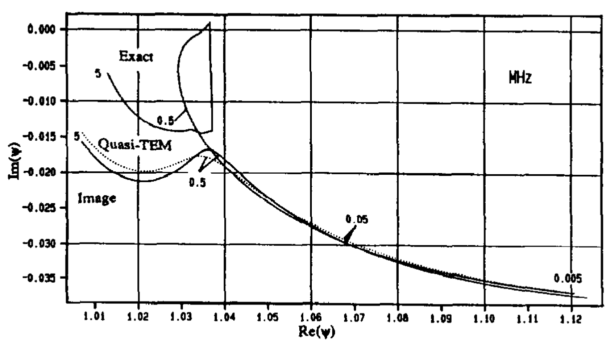
Also shown in Diag. 1a and 2a are the trajectories resulting when Q is set equal to zero for the approximate methods. It is concluded that inclusion of shunt correction becomes important for frequencies above about 0.05 MHz. For higher frequencies, neglecting Q will modify the attenuation characteristic markedly. Moreover, phase velocities above that of light cannot be attained.

One peculiarity deserves mentioning, namely the strange behaviour of ψ between 0.5 and 5 MHz in Diagram 2a. This is most probably due to the fact that the mode equation has two solution trajectories, as was mentioned earlier, and there is a jump from one trajectory to the other at about 2 MHz. Checking Diagram 1a, no such anomaly is spotted for α. However, by increasing the height by some 5 to 10 m, the existence of a corresponding discontinuity for the propagation constant shows up as well. Diagrams 3a and b show α and ψ for h increased to 20 m. This phenomenon will not be further studied in this paper.

One more experience from the calculations was that there seemed to be a limit frequency above which the iterations in Exact broke down. This phenomenon can probably be explained by the rigorous modal theory. The frequency limit was about 10 MHz for the surface and buried wires and 80 MHz for the overhead wire.



a: Relative propagation constant



b: Relative characteristic impedance

Diagram 3 The effect of height increase. Wire 20 m above ground.

CONCLUSION

Image type approximations to the correction terms of series impedance and shunt admittance from a non-perfectly conducting ground are derived for self and mutual couplings in a system of parallel wires above, at, and under ground. Departing from a single approximation scheme for the integrals involved, image representations for series and shunt couplings are found for the time-harmonic state. For overhead wires, the series impedance correction term turns out to coincide with that of the now classical image approximation with a complex depth for the image source. The shunt admittance correction term found is also of image type but is slightly more complex and has a strong resemblance to image results in the electrostatic theory. Both coupling approximations are extended to cover wires buried in the ground as well as at the ground surface.

Using the image representation, simple closed-form expressions for the axial propagation constant and characteristic impedance of wires above, at and under ground have been established. The accuracy of these expressions are tested in the frequency range 5 kHz to 5 MHz by comparison to the exact results for all three cases of wire location. The exact propagation constant is given implicitly as the solution of a certain mode equation, and iterations are used to find its pertinent root. Also participating in the comparison is the so called quasi-TEM (Transverse Electric Magnetic fields) solution, which is the first iteration result using the propagation constant of the medium in which the wire is located as the initial value. For the case with surface location, the root-mean-square value of the propagation constants of the media is used as initial value.

For the overhead wire case, the accuracy was found to be excellent for both approximation methods up to frequencies about 0.5 MHz, and fairly good above. Inclusion of shunt correction becomes important for frequencies above about 0.05 MHz. In the buried and surface wire cases, the image method works with good accuracy in the frequency range studied. In none of the three cases of wire location was the quasi-TEM approximation found notably superior to the numerically much more tractable image approximation.

GLOSSARY OF SYMBOLS

a	Radius of wire
A_z	Axial component of magnetic vector potential
d	Distance
d'	Distance defined in Figure 1
d''	D:o
h	Distance of wire from ground
I	Wire current
n	Relative refractive index, $n=n_2/n_1$
P	Series impedance correction term
Q	Shunt admittance correction term
n_m	Refractive index of Medium m,m=1;2
V	Wire voltage
Y	Shunt admittance
Z	Series impedance
Z_m	Characteristic impedance of Medium m,m=1;2
Z_w	Characteristic impedance of wire
α	Relative propagation constant of wire
β	$=\gamma_1(n^2-1)^{1/2}$
γ_m	Propagation constant of Medium m,m=1;2
γ_w	Propagation constant of wire
ε_0	Permittivity of free space
ψ	Relative characteristic impedance of wire
μ_0	Permeability of free space
Λ	Basic term of immittance
σ	Conductivity of ground
ω	Angular frequency

REFERENCES

[1] Olsen, R.G., Pankaskie, T.A., (1983), "On the Exact, Carson and Image Theories for Wires At or Above the Earth's Interface", IEEE Trans. PAS, PAS-102, 4, 769-778

[2] Kikuchi, H, (1956), "Wave Propagation Along an Infinite Wire Above Ground at High Frequencies", Electrotech. J., Japan, 2, 73-78

[3] Abramowitz, M., and Stegun, I.A., (1964), "Handbook of Mathematical Functions", Washington, DC: U.S. Government Printing Office

[4] Olsen, R.G., Chang, D.C., (1974), "New Modal Representation of Electromagnetic Waves Supported by Horizontal Wire Above Dissipative Earth", Electronics Letters, 10, 92-94

[5] Carson, J.R., (1926), "Wave Propagation in Overhead Wires With Ground Return", Bell Syst. Tech. J., 5, 539-554

[6] Pollaczek, F., (1926), "Über das Feld einer unendlichen langen wechselstromdurchflossenen Einfachleitung", E.N.T., 3, 339-360

[7] Wise, W.H., (1948), "Potential Coefficients for Ground Return Circuits", Bell Syst. Tech. J., 27, 365-371

[8] Kuester, E.F., Chang, D.C., Plate, S.W., (1981), "Electromagnetic Wave Propagation Along Horizontal Wire Systems In or Near a Layered Earth", Electromagnetics, 1, 243-265

[9] Sunde, E.D., (1949), "Earth Conduction Effects in Transmission Systems", New York: D van Nostrand

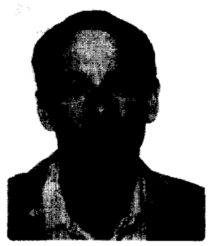
[10] Coleman, B.L., (1950), "Propagation of Electromagnetic Disturbances Along a Thin Wire in a Horizontally Stratified Medium", Phil. Mag. Ser. 7, 41, 276-288

[11] Kostenko, M.V., (1955), "Mutual Impedance of Earth-Return Overhead Lines taking into account the Skin Effect", (In Russian), Elektritchestvo, 10, 29-34

[12] Wait, J.R., and Spies, K.P., (1969), "On the Image Representation of the Quasi-Static Fields of a Line Current Source Above the Ground", Canad. Jour. Phys., 47, 2731-2733

[13] Deri, A., Tevan, G., Semlyen, A., Castanheira, A., (1981), "The Complex Ground Return Plane. A Simplified Model for Homogenous and Multi-Layer Earth Return", IEEE Trans. PAS, PAS-100, 8, 3686-3693

[14] Wait, J.R., (1985), "Electromagnetic Wave Theory", New York: Harper and Row



Per Pettersson (M'90) was born in Kristinehamn, Sweden on February 20, 1942. He received the M.S. degree in Electrical Engineering from the Chalmers Institute of Technology, Gothenburg, Sweden in 1967, and 1973 he received the Technical Licentiate degree (equiv. Ph.D.) in Mathematical Statistics from the same institute.

Mr. Pettersson is presently with Vattenfall Utveckling AB (derivative company of former Swedish State Power Board) as a Senior Research Engineer. He is a member of the Swedish National Committee of IEC/TC81: Lightning Protection, and a member of Cigré TF33.11-0.1: Lightning.