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Deep Neural Networks for Real-time Trajectory Planning

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Abstract

This project adresses the problem of robotic locomotion under the frameworks of deep learning and optimal control theory. We consider the set of controls, expressed as feedback laws, that allow to transport an unmanned aerial vehicle (UAV) to a given destination. By setting and solving a dynamic optimization problem, we build a synthetic dataset combining current state inputs and applicable optimal control outputs. Accordingly, we cast a supervised learning problem to approximate an optimal feedback law to be used for general locomotion.
Therein, this project is concerned with taking advantage of the versatility of neural networks along with the strong data efficiency provided by the optimal control modeling of the problem.

Keywords: Optimal control · Deep learning · Aerial robotics · Motion planning · Neural networks · Quadrotor control

UAV Model

We study the case of the planar-quadrotor model : a 2-dimensional UAV moving in the $\mathbb{R}^2$ plane.

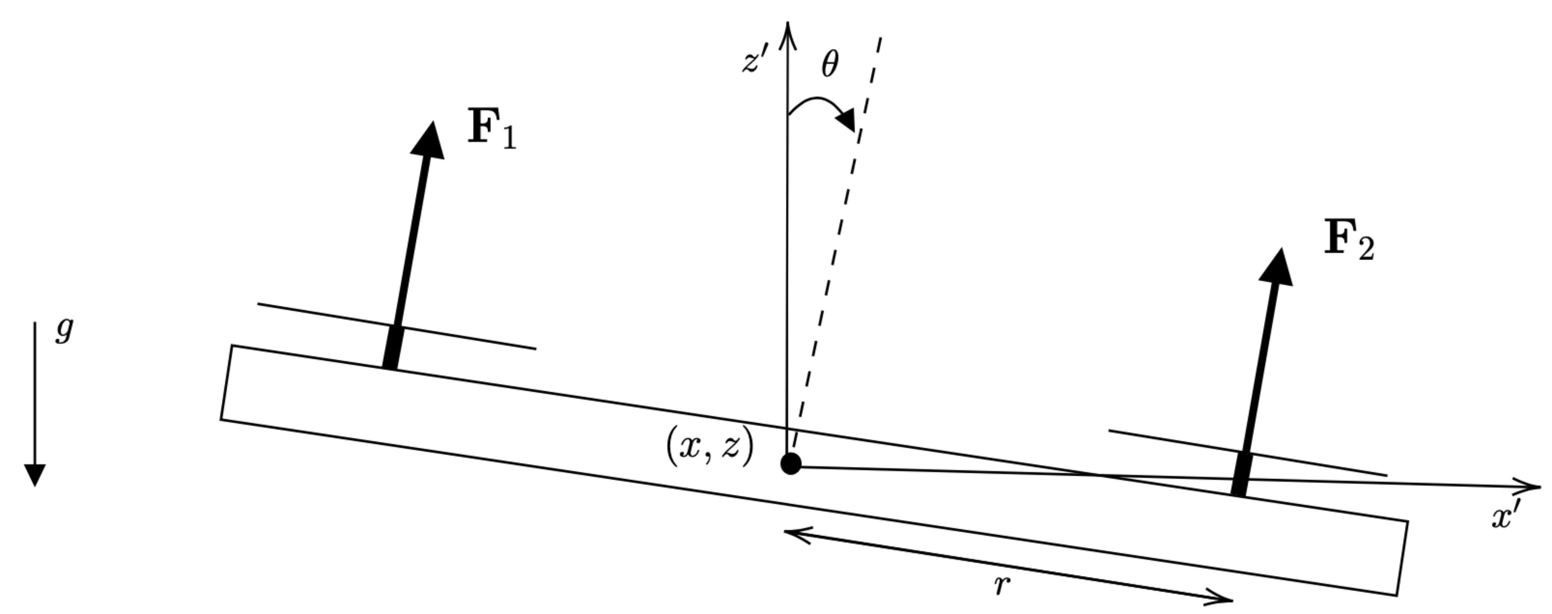

Figure 1: The planar-quadrotor is driven by 2 vertical propellers, which produces 2 controllable upward thrust forces $\mathbf{F_1}$ and $\mathbf{F_2}$. The center of mass of the vehicle is located by (x, z) in the plane. The angle $\theta \in [0, 2\pi)$ measures the aircraft's tilt with respect to the vertical.

The quadrotor UAV is initially characterized by its state $\mathbf{q} = (x, \dot{x}, z, \dot{z}, \theta, \dot{\theta})^T$, where the dot refers to the derivative with respect to time. Starting at $t_0 \in \mathbb{R}$ in state $\mathbf{q}(t = t_0) = \mathbf{q}_0$, we aim reaching $\mathbf{q}_f$ in minimal time t_f. Without loss of generality, we take $t_0 = 0$ as the departing time.

Defining $F_1 := \|\mathbf{F_1}\|$ and $F_2 := \|\mathbf{F_2}\|$, the dynamics governing the UAV's motion are given by :

$$\begin{cases} m\ddot{x} = (F_1 + F_2)\sin\theta \\ m\ddot{z} = (F_1 + F_2)\cos\theta - mg \\ I\ddot{\theta} = r(F_1 - F_2) \end{cases}$$

The parameter I is the given moment of inertia and the parameter r refers to the lever arm. We define $u_T := \frac{F_1+F_2}{m}$ as a control related to thrust. Moreover, as in the literature, we assume that the angular velocity can be directly controlled without dynamical effects and delay, that is, having a corresponding torque controller u_R such that $\dot{\theta} = u_R$. We set our controls as $\mathbf{u} := (u_T, u_R)^T$.

We further characterize the quadrotor UAV by the reduced state vector $\mathbf{q} = (x, \dot{x}, z, \dot{z}, \theta)^T$. The controlled dynamics read $\dot{\mathbf{q}} = f(\mathbf{q}, \mathbf{u})$, where f is given by :

$$f(\mathbf{q}, \mathbf{u}) = \begin{pmatrix} \dot{x} \\ u_T \sin\theta \\ \dot{z} \\ u_T\cos\theta - g \\ u_R \end{pmatrix}$$

Optimal Control Problem and Optimality Conditions

We set our optimal control problem with the objective of minimizing the transportation time t_f. Namely, we set the following *Bolza problem* :

$$\begin{aligned} \min_{\mathbf{u}(\cdot)} \quad & t_f + \int_0^{t_f} \tilde{l}(\mathbf{q}(t), \mathbf{u}(t))dt \\ \text{s.t.} \quad & \mathbf{q}(t = t_f) = \mathbf{q}_f, \\ & \mathbf{q}(t = 0) = \mathbf{q}_0, \\ & \dot{\mathbf{q}} = f(\mathbf{q}, \mathbf{u}), \\ & \mathbf{u} \in U, \\ & t \in [0, t_f] \end{aligned}$$

The running cost $\tilde{l}(\mathbf{q}(t), \mathbf{u}(t))$ accounts for other desired minimization criteria.

The set of admissible controls read : $U := \left\{u \in F(\mathbb{R}, \mathbb{R}^2) \mid \underline{u_T} \le u_T \le \overline{u_T} \text{ and } |u_R| \le \overline{u_R}\right\}$. The constants $\overline{u_T}$ and $\underline{u_T}$ refer to maximum and minimum thrust respectively and $\overline{u_R}$ refers to the maximum torque.

In a first approach, we set $\tilde{l}(\mathbf{q}(t), \mathbf{u}(t)) = 0$ and apply *Pontryagin's Maximum Principle* to obtain necessary optimality conditions. Using a costate vector λ, we build the corresponding Hamiltonian

$$\begin{aligned} H(t, \mathbf{q}, \mathbf{u}, \lambda) &= 1 + \lambda_1\dot{x} + \lambda_2 u_T \sin\theta \\ &+ \lambda_3\dot{z} + \lambda_4(u_T\cos\theta - g) + \lambda_5 u_R \end{aligned}$$

The maximum principle induces a *bang-singular* optimal torque control of the form :

$$u_R^* = \begin{cases} \overline{u_R} \text{ if } \lambda_5 > 0 \\ u_{R,sing}(t) \text{ if } \lambda_5 = 0 \\ -\overline{u_R} \text{ if } \lambda_5 < 0 \end{cases}$$

The rotational control over singular arcs $u_{R,sing}$ is derived using the fifth adjoint equation. Moreover, the maximum principle gives us that the optimal thrust control is *bang-bang*, of the form :

$$u_T^* = \begin{cases} \overline{u_T} \text{ if } \lambda_2\sin\theta + \lambda_4\cos\theta \le 0 \\ \underline{u_R} \text{ if } \lambda_2\sin\theta + \lambda_4\cos\theta > 0 \end{cases}$$

From there, setting 4 constants fully determines the set of controls and the related trajectory. It is still to find a control $\mathbf{u}(\cdot)$ that minimizes the Hamiltonian for all $t \in [0, t_f]$.

Quadrotors

We study said control problem focusing on the case of *quadrotor drones*. Such aircrafts are nowadays widely used in a variety of applications such as infrastructure inspection, search and rescue operations or aerial photography.

Figure 2: The Walkera QR X350 Quadcopter

As they are operated through a simple set of controllers, quadrotors offer a perfect framework to study optimal control problems and are therein broadly used in the related literature.

References

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Future Research

We will firstly be concerned with parametrizing the given optimal control problem in the form of a non-linear system and seek to find an appropriate computerized algebra method to perform the solving. Secondly, we will randomly draw initial and final states and compute the set of controls for transporting the planar-quadrotor along the optimal trajectory. By this mean, we will build an input-output dataset of the form : $\left((\mathbf{q}_f, \mathbf{q}_t); \mathbf{u}_t\right)_i$. Thirdly, we will design a suitable deep neural network and apply a supervised learning process on said data. The neural network will act as a near-optimal feedback law for the UAV. We will assess the obtained law on simulations. We will finally examine the vehicle's behavior and discuss the optimality of the approximated feedback control.