Electrical properties of a filament lamp

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Abstract

The purpose of this experiment was to study the electrical properties of a tungsten filament, specifically by analysing the dissipation of power versus temperature. The relevance of this work is clear in the use of light bulbs in everyday life. First, the measurements were fitted to the expected equation for radiation and conduction. Then, the measurements at low and high temperatures were analysed separately, letting the exponents of the fitting functions free, choosing as fitting function the one for conduction at low temperatures and the one for radiation at high temperatures. It was found that the measurements follow the expected laws of heat transfer but with some discrepancy. The measurements fit well to the theoretical curve. However, at high temperatures, the exponent of temperature for radiation is lower than the expected 4 and at low temperatures, the exponent of temperature for conduction is higher than the expected 1.

1 Introduction

Light bulbs light as a result of the microscopic behaviour of metals, including the ones with a filament of tungsten. In metallic solids, electrons are the main actors of electrical conduction. At low temperatures, the mean free path, the distance travelled by electrons between two collisions, is limited by the collisions between the electrons and impurities of the material. At high temperatures, by the collisions between the electrons and phonons, wave packets originated by vibrations of the atoms due to heat. Phonons also conduct heat, but the significance is lower than that of electrons. The latter type of collision results in a shorter mean free path, so lower conductivity. Then, light starts being radiated and radiation starts being an important source of power dissipation[1].

Therefore, the physics of this experiment seems to rely on heat transfer. There are three main types of transfer mechanisms: conduction, convection and radiation. Convection occurs when a fluid is in contact with a hotter object. Since the filament of tungsten is inside a bulb, in vacuum, there is no convection. Conduction is the process by which heat is transferred as a change in temperature. The power dissipated by conduction is linearly proportional to the temperature difference, $P = k(T - T_0)$, where k is the thermal resistance of the material and, in this case, T_0 is the ambient temperature. The third way is radiation, that consists of exchange of heat via electromagnetic waves. This heat is called thermal radiation and no medium is required for its transfer. Because the filament radiates energy to the environment while it absorbs energy from the environment, the object's net dissipated power through radiation is $P = \epsilon S\sigma(T^4 - T_0^4)$, where S is the surface area, ϵ is the filament emissivity and σ is the Stefan–Boltzmann constant. The emissivity is a property of the material, which for blackbodies in thermal equilibrium equals to 1[2]. Thus, the total dissipated power in the bulb for a given temperature is:

$$P = \epsilon S\sigma(T^4 - T_0^4) + k(T - T_0), \qquad (1)$$

To get the relationship P-T from the measurements, it is necessary to know the temperature. For that purpose, it is useful to know that, in nonohmic materials like tungsten, the temperature is related to the resistivity. Tungsten is an isotropic material, which means that its electrical properties are the same in all directions. For these materials, the resistivity is defined as

$$\rho = \frac{E}{J} = \frac{V/L}{I/S} = R\frac{S}{L}, \qquad (2)$$

where E is the electric field, J the current density in the filament, V is the voltage drop, L is the length of the filament and I is the intensity[2].

The relationship between resistivity and resistance can be considered linear if $\frac{S}{L}$ is constant for all temperatures. This is the case when the thermal expansion for length and cross area is negligible. In this experiment, $L \simeq 0.01$ m, the maximum change in temperature $\Delta T_{max} \simeq 2000$ K and the coefficient of linear expansion of tungsten $\alpha_L = 4.5 \cdot 10^{-6}$ K^{-1}[3], so the relative maximum linear expansion $\frac{\Delta L}{L} = \alpha_L \Delta T \simeq 0.01 = 1\%$ of the length of the filament. Since the area thermal expansion $\alpha_S = 2\alpha_L$, $\frac{\Delta S}{S} << 0.01$. Both changes in the range of temperatures of interest are negligible. Hence, $\frac{S}{L}$ can be considered constant during this experiment.

For tungsten, resistivity shows a quadratic relationship with temperature as seen in Fig. 1:

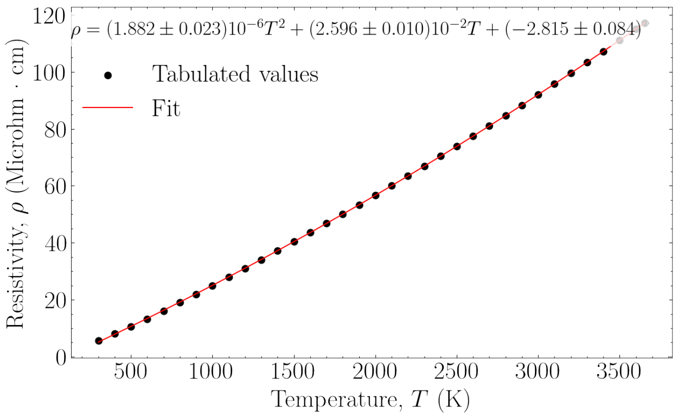

Figure 1: Resistivity of tungsten versus temperature. The tabulated values of resistivity as a function of temperature used in the experiment are from Jones, H. A.[4].

Knowing this and having measurements of intensity and voltage, it is possible then to know the power $P = IV$ and temperature and draw conclusions from the results[5], comparing them with known equations, which is the main objective of this work.

2 Methods

The experimental set up consisted of an electric circuit with a direct circuit (DC) voltage source, an incandescent bulb of tungsten filament, a voltmeter parallel to the bulb and an ammeter in series, as shown in Fig. 2. Like that, even if the wire has a non-zero resistance, the voltage dropped in the wire, not in the filament, is not relevant, since the voltmeter measures only the voltage dropped in the filament. The measurements were taken manually. Also, the bulb, OSRAM model, had a maximum rated voltage value of 12 V, so a maximum of 10.10 V was applied to it.

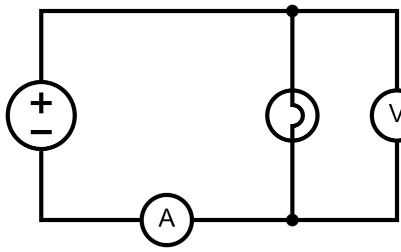

Figure 2: The circuit. $\pm$ stands for the voltage source, A is the ammeter and V is the voltmeter, which is parallel to the bulb.

First, the source voltage was varied from 0.03 to 0.10 V in steps of 0.01 V to get the resistance of the tungsten filament at ambient temperature, (296.2 ± 0.1) K. Since the temperature changes significantly with the voltage supplied and the voltage-current relationship cannot be assumed linear, the optimal approach is to get the resistance at an extrapolated value of $V = 0$. This way, the voltage versus current relationship was fit to a second order polynomial and after getting the current for which $V = 0$, the derivative of this polynomial was calculated at this point. Like this, the effect of any offset in the voltage-current measurements was also diminished by taking the derivative[6]. Then, the proportionality constant $\alpha = \frac{S}{L} = \frac{\rho_0}{R_0}$ was calculated, where R_0 is the value for resistance and ρ_0 is the fitted value of resistivity of tungsten at ambient temperature, using the best fit to a second order polynomial of the tabulated resistivities as a function of temperature showed in Fig. 1. α was later used to get the resistivity from the measured resistances. Next, the source voltage was varied from 0.10 to 10.10 V in steps of 0.20 V to get the resistance at each step with Ohm's law.

The time step between measurements was about 30 seconds, considered appropriate by one of the sources[6]. The error in not taking the measurements at the exact steady state was reduced by taking the measurements twice, half of them increasing the voltage, hence the temperature, the other half decreasing it back to 0.03 V and then taking the average. Lastly, the values of the temperature of the filament in the second range of voltages were obtained with the inverse of the best fit equation of resistivity versus temperature. All the fits were done with the Python function *curve_fit* from the *SciPy* library. The uncertainties were calculated by error propagation and the one for temperature was calculated with the Python module *uncertainties*, because of the complexity of the calculation.

3 Results

Fig. 3 shows the measured current-voltage at steps of 0.20 Volts for the voltage source. All the points are the mean values of the measurements taken at increasing and decreasing voltage. The uncertainty due to the precision of the measuring tools is negligible at the scale of the measurements, especially after taking the mean of two measurements for the same voltage. However, the error of not taking the measurements at the steady state in both ways (increasing and decreasing voltage) lowers the precision gain with the mean.

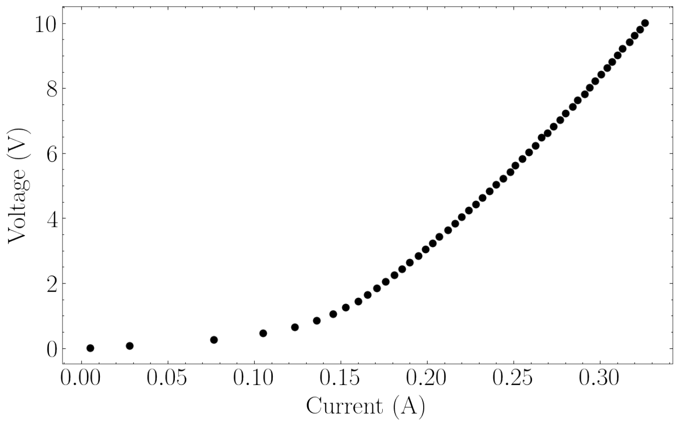

Figure 3: Measurements of current and voltage drop through the filament.

The varying slope of the current-voltage measurements, the resistance, is shown in Fig. 4 as a function of current.

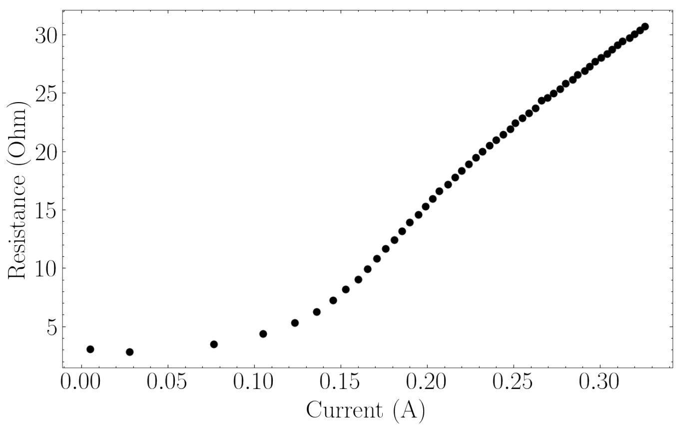

Figure 4: Resistance of the filament as a function of current.

As a consequence of error propagation, the values of power at high temperatures have larger uncertainties than at lower temperatures. That way, the power versus temperature curve fits into a range of coefficients for the linear and fourth order term of Eq. 1, especially the fourth order one, as can be seen in Fig. 5. The error in power is negligible compared to the error in temperature.

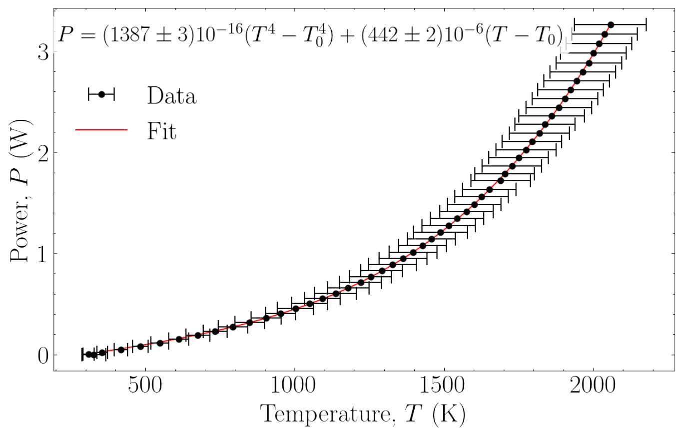

Figure 5: Power dissipated through the bulb versus the temperature of the filament.

The best fit equation using Eq. 1 is

$$P = (1387 \pm 3)10^{-16}(T^4 - T_0^4) + (442 \pm 2)10^{-6}(T - T_0). \tag{3}$$

The measurements are of the order of 1000 K. Taking this value and using Eq. 3, the radiation term is $\simeq 1767 \cdot 10^{-16} \cdot (1000^4 - 296^4) \simeq 0.14$ W and the conduction term $\simeq 484 \cdot 10^{-6} \cdot (1000 - 296) \simeq 0.31$ W. The order of magnitude of both terms is the same.

To analyse how well the measurements fit to Stefan-Boltzmann's Law, an effective approach is to focus on the high temperatures regime. Choosing the data from around $T = 1450$ K up, the exponent of the fitting function $P = A(T^n + T_0^n) \simeq AT^n$ is $n = 3.1$. The constant term is neglected so that it is possible to do a linear fitting taking logarithms. The approximation is justified, since the proportion $\frac{T_0^3}{T^3} = \frac{296.2^3}{1450^3} = 0.009$ is low.

In the low temperatures regime, up to around $T = 734$ K, the measurements should follow the fitting function $P = A(T - T_0)^n$, which gives an exponent of $n = 1.9$.

4 Discussion

The current-voltage relationship in Fig. 3 is clearly non-linear, as a consequence of tungsten being a non-ohmic material. There is less data at low voltage than at high voltage, but this differences is compensated in the power calculations in such a way that the final result, shown in Fig. 5, has well distributed values for temperature and power.

As can be seen in Fig. 4, tungsten behaves approximately as an ohmic material at low current values, being the resistance approximately constant. At higher current values, higher temperature, the resistance increases steadily, having an inflexion point at around 0.180 A. The fact that at high temperatures the resistance has less slope seems to have a relationship with the smaller relative importance of conduction compared to radiation. This inflection point corresponds approximately to the start of the high temperatures regime.

The data fits well to the expected Eq. 1, which suggests that this fitting function is considering the main sources of power dissipation in the filament. Both phenomena, conduction and radiation, seem to have the same importance, considering their orders of magnitude using Eq. 3, and none of them can be disregarded in the analysis.

With respect to radiation, the exponent at high temperatures is lower than the expected $n = 4$ from the Stefan-Boltzmann law. This suggests that the filaments dissipates less power than expected for each temperature. With respect to conduction, the exponent at low temperatures is higher than the expected $n = 1$. At low temperatures, the filament seems to dissipate more power than expected. As suggested by one of the sources, this might be due to some convection happening in the bulb, since the same exponent for convection is expected to be $n > 1$[6]. The fact that conductivity and emissivity depend on temperature might also have an impact on the results, since the fitting coefficients have been taken as constant, but in fact they depend on temperature[1], [6].

The ranges for the higher temperatures in Fig. 5 include other possible coefficients of the best fit curve inside them. However, the ranges of error seem to overestimate the uncertainty in temperature, given how close the measurements lie to the best fit line, and this limits the validity of the range of coefficients.

The fact that the temperature regimes do not completely follow the expected fitting functions suggests that other material-related phenomena should be taken into account in order to fully understand the electrical properties of tungsten. The experiment could be improved, for example, using more precise measuring tools so that the error in resistivity, which was of the order of $\mu\Omega << \mathrm{m}\Omega = \frac{\mathrm{mV}}{\mathrm{mA}}$ (the precision of the measuring tools) and cm, was smaller and so the ranges of error in temperature, which would make the best fit equations more accurate. The fitting of the measurements could also be improved by using more advanced numerical methods and less approximations of the fitting functions and ranges to a more realistic non-linear fitting function of the form $P = A(T^a - T_0^a) + B(T - T_0)^b$[6].

5 Conclusions

The analysis of the electrical properties of tungsten via a voltage source, an ammeter and a voltmeter has given the expected results in the correlation of dissipated power and laws of radiation and conduction[2], [5]. These are the main ways the bulb dissipates power and are equally important. Some discrepancy has been found in the exponents of high and low temperatures regimes compared to the expected equations. At high temperatures, the exponent of temperature for radiation is lower than the expected 4 and at low temperatures, the exponent of temperature for conduction is higher than the expected 1. This can be reasoned by the properties of the material and the experimental set up. However, the analysis could be improved by a more thorough interpretation of the behaviour of the material, beyond the scope of this work, and more advanced numerical methods than the ones used in this experiment.

References

[1] Alan J. Walton. *Three phases of matter.* 2nd ed. Oxford science publications. Oxford: Clarendon Press, 1983, pp. 353–90.

[2] David Halliday. *Fundamentals of physics.* eng. 7th ed., extended. Hoboken, N.J.: Wiley, 2005, pp. 493–6, 688–90, 482.

[3] *ASM Ready Reference: Thermal Properties of Metals.* `http://www.owlnet.rice.edu/~msci301/ThermalExpansion.pdf`. ASM International. Chap. 2.

[4] H. A. Jones. "A temperature scale for tungsten". In: *Physical Review* 28.202 (1926).

[5] I. R. Edmonds. "Stephan-Boltzmann Law in the Laboratory". In: *American Journal of Physics* 36 (1968). `https://doi.org/10.1119/1.1975165`, pp. 845–6.

[6] Marcello Carlà. "Stefan–Boltzmann law for the tungsten filament of a light bulb: Revisiting the experiment". In: *American Journal of Physics* 81 (2013). `https://doi.org/10.1119/1.4802873`, pp. 512–7.