```
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
'https://raw.githubusercontent.com/amaydixit11/Academics/refs/heads/
main/DSL251/data.csv'
data = pd.read csv(file)
data.head()
{"summary":"{\n \"name\": \"data\",\n \"rows\": 43,\n \"fields\":
[\n {\n \"column\": \"Location Name\",\n \"properties\":
       \"dtype\": \"string\",\n \"num unique values\": 35,\
{\n
         \"samples\": [\n
                                  \"Ongole \",\n
\"guntur\",\n \"Varanasi\"\n
                                            ],\n
\"semantic_type\": \"\",\n \"description\": \"\"\n
n },\n {\n \"column\": \"Time to Reach (hr)\",\n \"properties\": {\n \"dtype\": \"number\",\n \"std\": 7.836545410100203,\n \"min\": 0.5,\n \"max\": 48.0,\n
\"Distance (km)\",\n \"properties\": {\n \"dtype\":
\"number\",\n \"std\": 354,\n \"min\": 9,\n \"max\": 1782,\n \"num_unique_values\": 38,\n \"samples\": [\n 900,\n 1159,\n
      \"semantic_type\": \"\",\n \"description\": \"\"\n
],\n
0,\n \"min\": 0,\n \"max\": 1,\n
\"num_unique_values\": 2,\n \"samples\": [\n
                                                             0, n
1\n     ],\n \"semantic_type\": \"\",\n
\"column\": \"Train+Road\",\n \"properties\": {\n \"dtype\": \"number\",\n \"std\": 0,\n \"min\": 0,\n \"max\": 1,\n \"num_unique_values\": 2,\n \"samples\": [\n 1,\n 0\n ],\n \"semantic type\":
[\n 1,\n 0\n ],\n \"\description\": \"\"\n }\n
                                                  }\n ]\
n}","type":"dataframe","variable_name":"data"}
data.describe()
```

```
{"summary":"{\n \"name\": \"data\",\n \"rows\": 8,\n \"fields\": [\
n {\n \"column\": \"Time to Reach (hr)\",\n
\"properties\": {\n \"dtype\": \"number\",\n \"std\": 16.43339031746364,\n \"min\": 0.5,\n \"max\": 48.0,\n
\"num_unique_values\": 8,\n \"samples\": [\n 17.28418604651163,\n 18.0,\n 43.0\
                                                         43.0\n
                                                                         ],\n
\"semantic_type\": \"\",\n \"description\": \"\"\n
                                                                           }\
      \"properties\": {\n \"dtype\": \"number\",\n \"std\": 581.0372438911829,\n \"min\": 9.0,\n \"max\": 1782.0,\n
                                          \"samples\": [\n
\"num unique values\": 8,\n
865.5581395348837,\n
                                    861.0,\n
                                                          43.0\n
                                                                           ],\n
\"semantic_type\": \"\",\n \"description\": \"\"\n
                                                                         }\
      \ \,\n \"column\": \"Train Only\",\n
\"properties\": {\n \"dtype\": \"number\",\n \"std\": 15.058543923692291,\n \"min\": 0.0,\n \"max\": 43.0,\n
\"num_unique_values\": 5,\n \"samples\": [\n 0.46511627906976744,\n 1.0,\n 0.5046845884077511\n ],\n \"semantic_type\": \"\",\n \"description\": \"\"\n
        \ \,\n {\n \"column\": \"Road Only\",\n
}\n
\"properties\": {\n \"dtype\": \"number\",\n \"std\": 15.136616595487634,\n \"min\": 0.0,\n \"max\": 43.0,\n
\"num_unique_values\": 5,\n \"samples\": [\n 0.09302325581395349,\n 1.0,\n 0.293902598732179\n ],\n \"semantic_type\": \"\",\n \"description\": \"\"\n
}\n     },\n     {\n         \"column\": \"Train+Road\",\n
\"properties\": {\n         \"dtype\": \"number\",\n         \"std\":
15.058543923692291,\n         \"min\": 0.0,\n         \"max\": 43.0,\n
\"num_unique_values\": 5,\n \"samples\": [\n 0.46511627906976744,\n 1.0,\n 0.50
                                                          0.5046845884077511\n
                                                   \"description\": \"\"\n
],\n \"semantic type\": \"\",\n
}\n }\n ]\n}","type":"dataframe"}
plt.figure(figsize=(10, 6))
plt.scatter(data["Time to Reach (hr)"], data["Distance (km)"],
c=colors, s=100, alpha=0.8, edgecolor="k")
for i, location in enumerate(data["Location Name"]):
     plt.text(data["Time to Reach (hr)"][i] + 0.2, data["Distance
(km)"][i], '.', fontsize=9)
plt.scatter([], [], color="blue", label="Train Only", s=100,
edgecolor="k")
plt.scatter([], [], color="green", label="Road Only", s=100,
edgecolor="k")
plt.scatter([], [], color="red", label="Train + Road", s=100,
edgecolor="k")
plt.title("Time vs Distance with Travel Modes", fontsize=16)
```

```
plt.xlabel("Time to Reach (hr)", fontsize=12)
plt.ylabel("Distance (km)", fontsize=12)
plt.legend(title="Travel Mode", loc="upper right")
plt.grid(alpha=0.5)

plt.tight_layout()
plt.show()
```

## Time vs Distance with Travel Modes Travel Mode Train Only Road Only Train + Road Travel Mode Train Only Road Only Train + Road

Time to Reach (hr)

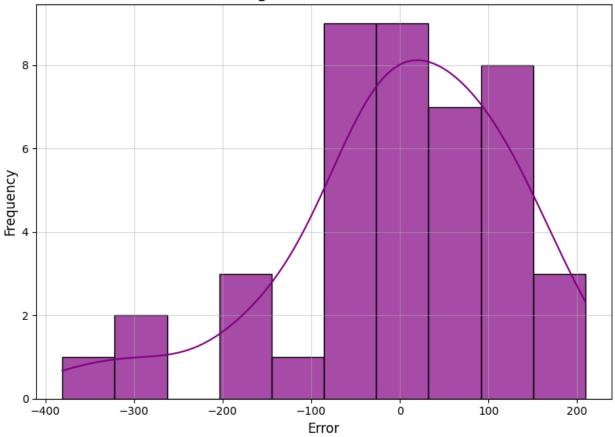
40

```
A = np.array([np.ones(data['Time to Reach (hr)'].shape), data['Time to
Reach (hr)']])
array([[ 1.
                                             1.
           1.
                   1.
                            1.
                                     1.
                                             1.
                                                      1.
                                                               1.
                                                                       1.
                                                                                1.
           1.
                   1.
                            1.
                                     1.
                                             1.
                                                      1.
                                                               1.
                                                                       1.
                                                                                1.
                            1.
                                     1.
                                             1.
                                                      1.
                                                              1.
                                                                       1.
           1.
                   1.
                                                                                1.
           1.
                            1.
                                     1.
                                             1.
                   1.
                                                      1.
                                                              1.
         [18.
                  18.
                           13.5
                                   13.5
                                            18.
                                                     14.
                                                             18.
                                                                      21.
                                     0.5
          15.
                  13.
                            1.25,
                                            14.
                                                     35.
                                                             19.
                                                                      18.
                                                                              12.
          14.
                  22.
                           16.
                                   21.
                                            13.
                                                      5.47,
                                                             10.
                                                                     20.
                                                                              12.
          48.
                  11.
                           18.
                                   12.
                                            19.
                                                    19.
                                                             18.
                                                                      28.
                                                                              23.
          19.
                        , 13.
                                                  , 20.
                  20.
                                   18.
                                          , 14.
                                                           , 22.
y = np.array(data['Distance (km)'])
У
```

```
array([1020, 870, 676, 680, 977, 580, 977, 1157, 1264,
669,
         44,
               9, 756, 1743, 970, 1044, 727, 683, 1200, 750,
1232.
        700, 309, 600, 949, 600, 1782, 700, 720, 620, 965,
982,
        624, 1354, 1200, 1006, 900, 646, 584, 700, 1159, 1230])
w hat = np.linalg.inv(A.dot(A.T)).dot(A).dot(y)
w hat
array([135.62292013, 42.23139102])
# verify that the error vector is perpendicular to any vector in
column space of A
e = (y - A.T.dot(w hat))
е
array([ 124.2120415 , -25.7879585 ,
                                     -29.74669891,
                                                    -25.74669891,
        81.2120415 , -146.86239442,
                                      81.2120415 .
                                                    134.51786844.
         30.36091333,
                       91.90621456,
                                     -15.6310034 , -144.41215891,
       -147.73861564,
                       29.13760558,
                                     129.27839415,
                                                    31.98065048,
                       84,60038762.
                                     -43.86239442.
        148.2120415 ,
                                                    135.28647741.
        -61.32517646,
                      209.51786844,
                                      15.3689966 ,
                                                    -57.62862901,
        42.06316966,
                      -31.25074054,
                                     -42.39961238, -380.72968912,
        99.83177864, -175.7879585,
                                     -22.39961238,
                                                     26.98065048.
        43.98065048, -271.7879585,
                                      35.89813129,
                                                     93.05508639,
        67.98065048,
                     -80.25074054,
                                     -38.6310034 , -311.7879585 ,
        -26.86239442, 178.74925946,
                                     165.28647741])
data['error'] = e
data.describe()
{"summary":"{\n \"name\": \"data\",\n \"rows\": 8,\n \"fields\": [\
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\"properties\": {\n \"dtype\": \"number\",\n \"6.43339031746364,\n \"min\": 0.5,\n \"
                                                          \"std\":
                                               \mbox{"max}: 48.0,\n
                                  \"samples\": [\n
\"num unique values\": 8,\n
17.28418604651163,\n
                             18.0, n
                                                            ],\n
                                              43.0\n
\"semantic type\": \"\",\n
                            \"description\": \"\"\n
                                                             }\
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\"properties\": {\n
                          \"dtype\": \"number\",\n
                                                          \"std\":
                      \"min\": 9.0,\n
581.0372438911829,\n
                                                 \"max\": 1782.0,\n
\"num unique_values\": 8,\n
                                  \"samples\": [\n
865.5581395348837,\n
                             861.0,\n
                                               43.0\n
                                                             ],\n
\"semantic_type\": \"\",\n
                                \"description\": \"\"\n
                                                              }\
           {\n \"column\": \"Train Only\",\n
     },\n
                          \"dtype\": \"number\",\n
\"properties\": {\n
                                                          \"std\":
15.058543923692291,\n
                            \"min\": 0.0,\n
                                                   \mbox{"max}": 43.0,\n
```

```
\"num unique values\": 5,\n \"samples\": [\n
                               1.0,\n
0.46511627906976744,\n
                                                 0.5046845884077511\n
            \"semantic_type\": \"\",\n \"description\": \"\"\n
],\n
               {\n \"column\": \"Road Only\",\n
}\n },\n
                           \"dtype\": \"number\",\n
\"properties\": {\n
                                                          \"std\":
                            \"min\": 0.0,\n
15.136616595487634,\n
                                              \mbox{"max}": 43.0,\n
                                   \"samples\": [\n
\"num unique values\": 5,\n
0.09302325581395349,\n
                                1.0, n
                                                 0.293902598732179\n
            \"semantic_type\": \"\",\n
],\n
                                              \"description\": \"\"\n
}\n
       },\n {\n \"column\": \"Train+Road\",\n
\"properties\": {\n \"dtype\": \"number\",\n \"std\": 15.058543923692291,\n \"min\": 0.0,\n \"max\": 43.0,\n
\"num_unique_values\": 5,\n \"samples\": [\n 0.46511627906976744,\n 1.0,\n 0.56
      1627906976744,\n 1.0,\n 0.5046845884077511\n \"semantic_type\": \"\",\n \"description\": \"\"\n
],\n
       },\n {\n \"column\": \"error\",\n
}\n
                                                       \"properties\":
           \"dtype\": \"number\",\n \"std\":
{\n
176.00203165146118,\n\\"min\\": -380.72968912285705,\n
\"max\": 209.5178684352809,\n \"num unique values\": 8,\n
\label{eq:samples} $$\scalebox{": [n 3.2784111342977643e-13, n 26.98065047662442, n 43.0 n ], n }
\"semantic type\": \"\",\n
                                 \"description\": \"\"\n
                                                                }\
     # error is perpendicular to A
A.dot(e)
array([1.42108547e-11, 3.01952241e-10])
plt.figure(figsize=(8, 6))
sns.histplot(data["error"], kde=True, bins=10, color="purple",
alpha=0.7, edgecolor="black")
plt.title("Histogram of Error Vector", fontsize=16)
plt.xlabel("Error", fontsize=12)
plt.ylabel("Frequency", fontsize=12)
plt.grid(alpha=0.5)
plt.tight_layout()
plt.show()
```

## Histogram of Error Vector



```
road only = data[data['Road Only'] == 1]
road only A = np.array([np.ones(road only['Time to Reach
(hr)'].shape), road only['Time to Reach (hr)']])
road only y = np.array(road only['Distance (km)'])
road only w hat =
np.linalg.inv(road only A.dot(road only A.T)).dot(road only A).dot(roa
d only y)
road only w hat
array([-11.06834871, 58.80124774])
road only e = (road only y - road only A.T.dot(road only w hat))
road only e
array([ -9.33227516, -53.34787194, -1.57447644, 64.25462354])
road_only['road_only_error'] = road_only_e
<ipython-input-136-4733fb8f6d1c>:1: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row indexer,col indexer] = value instead
```

```
See the caveats in the documentation:
https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#
returning-a-view-versus-a-copy
   road only['road only error'] = road only e
road only.describe()
{"summary":"{\n \"name\": \"road_only\",\n \"rows\": 8,\n \"fields\": [\n {\n \"column\": \"Time to Reach (hr)\",\n \"properties\": {\n \"dtype\": \"number\",\n \"std\":
\"properties\": {\n \"dtype\": \"number\",\n \"std\": 4.115198107636982,\n \"min\": 0.5,\n \"max\": 13.0,\n
\"num_unique_values\": 8,\n \"samples\": [\n 7.4925,\n 8.235,\n 4.0\n ],\n \"semantic_type\": \"\",\n \"description\": \"\"\n }\n }\n {\n \"column\":
\"Distance (km)\",\n \"properties\": {\n \"dtype\": \"number\",\n \"std\": 273.3802196657122,\n \"min\"
                                                                             \"min\":
4.0,\n \"max\": 700.0,\n \"num_unique_values\": 7,\n \"samples\": [\n 4.0,\n 429.5,\n 504.5\n
        \"semantic_type\": \"\",\n
                                                            \"description\": \"\"\n
}\n    },\n    {\n     \"column\": \"Train Only\",\n
\"properties\": {\n         \"dtype\": \"number\",\n         \"std\":
1.4142135623730951,\n         \"min\": 0.0,\n         \"max\": 4.0,\n
\"num_unique_values\": 2,\n \"samples\": [\n
                                                                                 0.0, n
0.0,\n \"max\": 4.0,\n \"num_unique_values\": 3,\n
\"samples\": [\n 4.0,\n
                                                       1.0\n
\"semantic_type\": \"\",\n \"description\": \"\"\n }\
n },\n {\n \"column\": \"Train+Road\",\n \"properties\": {\n \"dtype\": \"number\",\n \"std\":
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4.0\n ],\n \"semantic_type\": \"\",\n
\"std\": 85.80887311510278,\n\\"min\": -147.73861564094767,\n\\"max\": 105.38477035018127,\n\\"num_unique_values\": 8,\n
53.34787194296655,\n\\"max\": 64.25462354245019,\n
\"num_unique_values\": 8,\n \"samples\": [\n 1.7763568394002505e-13,\n -5.453375799742176\n \"semantic_type\": \"\",\n \"description\": \"\"\n
                                                                                   ],\n
                                                                                   }\
      }\n ]\n}","type":"dataframe"}
```