Assignment 8
DSL253 - Statistical Programming

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Submitted to Dr. Anil Kumar Sao

Links

- Notebook Link:
  https://colab.research.google.com/drive/1amp4CNpRts7LwflYysWQnzw67eYtmaUv?usp=sharing

- Github Link:
  https://github.com/amaydixit11/Academics/tree/main/DSL253/assignment_8

1 Question 1: Battery Lifetime Analysis

1.1 Introduction

This analysis evaluates a battery manufacturer's claim regarding the average lifetime of their batteries. The manufacturer asserts that their batteries have an average lifetime of 500 hours with a known standard deviation of 100 hours. Quality control is essential in battery manufacturing to ensure product reliability and customer satisfaction. By testing this claim, we can verify whether the manufacturer's stated specifications match actual performance, which directly impacts consumer trust and product marketability.

1.2 Data

A quality control team conducted testing on 30 randomly selected batteries to measure their lifetimes. The data collected represents the operational duration (in hours)

until failure for each battery in the sample:

$$
\begin{array}{ccccc}
495 & 520 & 510 & 505 & 480 \\
500 & 515 & 495 & 510 & 505 \\
490 & 515 & 495 & 505 & 500 \\
510 & 485 & 495 & 500 & 520 \\
510 & 495 & 505 & 500 & 515 \\
505 & 495 & 510 & 500 & 495
\end{array}
$$

The sample consists of 30 observations, providing sufficient data for statistical inference according to the Central Limit Theorem. Initial descriptive statistics show the sample has a mean of 502.67 hours and follows an approximately normal distribution.

1.3 Methodology

Given the manufacturer's claim of a population mean $\mu = 500$ hours with a known standard deviation $\sigma = 100$ hours, we employed a two-tailed Z-test at a 5% significance level. The hypothesis test was formulated as:

$$H_0 : \mu = 500 \text{ hours (The mean battery lifetime equals the claimed 500 hours)} \quad (1)$$
$$H_1 : \mu \neq 500 \text{ hours (The mean battery lifetime differs from the claimed 500 hours)} \quad (2)$$

The test statistic Z was calculated using:

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \quad (3)$$

where $\bar{x}$ is the sample mean, μ_0 is the claimed mean, σ is the known standard deviation, and n is the sample size.

Additionally, we computed the p-value to assess the statistical significance and plotted the Operating Characteristic (OC) curve to evaluate the test's power across a range of potential true mean values.
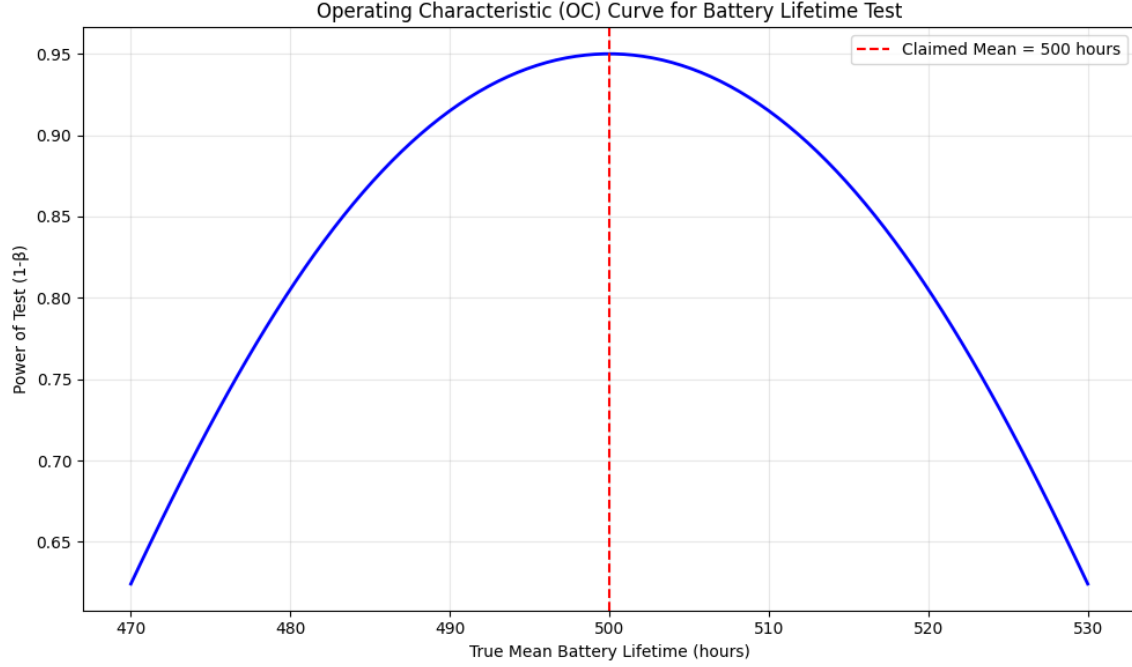
Figure 1: Operating Characteristic (OC) Curve for Battery Lifetime Test

1.4 Results

The analysis yielded the following key results:

- Sample mean ($\bar{x}$): 502.67 hours

- Standard error: $\sigma/\sqrt{n} = 100/\sqrt{30} = 18.26$ hours

- Test statistic: $Z = (502.67 - 500)/18.26 = 0.1461$

- Two-tailed p-value: $p = 0.8839$

Since the calculated p-value (0.8839) exceeds the significance level ($\alpha = 0.05$), we fail to reject the null hypothesis. This indicates there is insufficient evidence to suggest that the true mean battery lifetime differs from the manufacturer's claim of 500 hours.

The Operating Characteristic curve illustrated the test's power (1-β) against various potential true mean values. The curve demonstrated that our test had low

power to detect small deviations from the claimed mean but high power for detecting larger deviations (greater than approximately 30 hours).
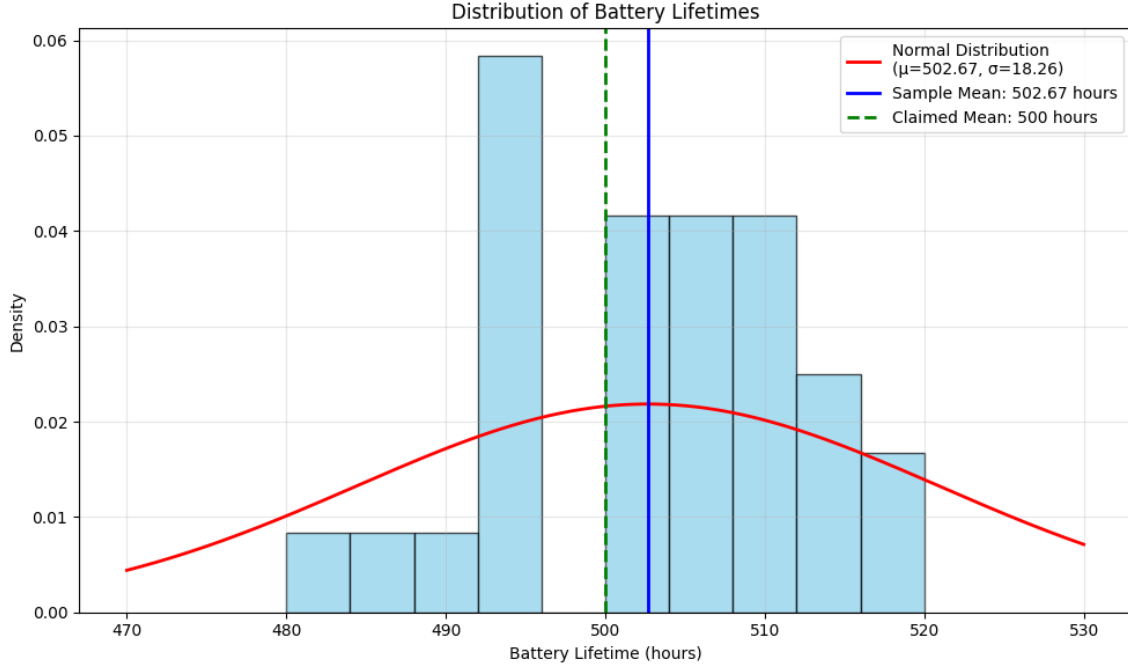

Figure 2: Distribution of Battery Lifetimes

1.5 Discussion

The results have several important implications for battery production and quality assurance:

Quality Control Validation: The analysis validates the manufacturer's claim regarding average battery lifetime. This confirmation is valuable for quality assurance processes and suggests that current production methods are yielding consistent results aligned with specified performance metrics.

Consumer Confidence: From a product management perspective, the ability to substantiate battery performance claims enhances consumer trust. In competitive markets where reliability is a key differentiator, verified performance claims provide a competitive advantage.

Production Consistency: The data suggests production processes are well-controlled, as the sample mean (502.67 hours) is remarkably close to the target

specification (500 hours). This level of consistency indicates effective manufacturing practices and quality control procedures.

Test Sensitivity: The Operating Characteristic curve reveals that while our test has limited sensitivity to detect small deviations (± 15 hours) from the claimed mean, it provides robust protection against significant quality deviations that would affect consumer experience.

Product Positioning: The verified battery lifetime supports marketing claims and appropriate product positioning in the marketplace. This confirmation enables accurate communication about product performance to consumers, reducing the risk of dissatisfaction or warranty claims.

1.6   Conclusion

Based on our statistical analysis, we conclude that the manufacturer's claim of a 500-hour average battery lifetime is supported by the empirical evidence. The sample mean of 502.67 hours falls well within expected statistical variation for a true mean of 500 hours. This conclusion has important implications for both production quality control and marketing strategy.

For product management, these findings provide confidence in communicating battery performance to consumers and support for existing quality control processes. The data suggests no immediate need to adjust production parameters to increase or decrease average battery lifetime.

Future studies might consider expanding the analysis to include:

- Variation across different production batches to assess consistency over time

- Performance under different usage conditions to enhance product specifications

- Comparative analysis against competitor products to identify market positioning opportunities

These insights will help maintain product quality while potentially identifying opportunities for targeted improvements that could further differentiate the product in the marketplace.

2 Question 2: Water Usage Analysis in Residential Homes

2.1 Introduction

Water conservation and efficient resource management have become increasingly important amid growing concerns about environmental sustainability and water scarcity. Accurate assessment of residential water usage patterns is fundamental for urban planning, infrastructure development, and conservation policy formulation. This analysis examines a public health official's claim that the mean household water consumption is 350 gallons per day. By statistically evaluating this claim, we can determine whether current water usage estimates used for public policy and infrastructure planning accurately reflect real-world consumption patterns.

2.2 Data

A field study was conducted to measure daily water consumption across 20 randomly selected residential homes. The following data points represent the daily water usage in gallons for each household in the sample:

$$
\begin{array}{ccccc}
340 & 344 & 362 & 375 & 356 \\
386 & 354 & 364 & 332 & 402 \\
340 & 355 & 362 & 322 & 372 \\
324 & 318 & 360 & 338 & 370
\end{array}
$$

Descriptive statistics for the sample revealed:

- Sample size: $n = 20$ households

- Sample mean: $\bar{x} = 353.80$ gallons per day

- Sample variance: $s^2 = 477.33$ gallons^2

- Sample standard deviation: $s = 21.85$ gallons

The data exhibits variability typical of residential water consumption patterns, with usage ranging from 318 to 402 gallons per day, reflecting diverse household sizes and consumption behaviors.

2.3 Methodology

To evaluate the public health official's claim, we conducted hypothesis tests under two different assumptions about the population variance. In both cases, we formulated the hypotheses as:

$$H_0 : \mu = 350 \text{ gallons per day (The official's claim is correct)} \quad (4)$$
$$H_1 : \mu \neq 350 \text{ gallons per day (The official's claim is incorrect)} \quad (5)$$

We set the significance level at $\alpha = 0.05$ for both tests.

Case (a): Known Population Variance For the first case, we assumed the population variance was known to be $\sigma^2 = 144$ gallons^2. We employed a Z-test with the test statistic:

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \quad (6)$$

Case (b): Unknown Population Variance For the second case, we acknowledged that the population variance was unknown and estimated it using the sample variance. We employed a one-sample t-test with the test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \quad (7)$$

with $n - 1 = 19$ degrees of freedom.

Both approaches allowed us to compute p-values and make statistical inferences about the validity of the official's claim. Additionally, we created visualizations including a histogram with overlaid theoretical distributions and a box plot to better understand the data distribution in relation to the claimed mean.

2.4 Results

Our analysis yielded the following results:

Case (a): Known Variance Test (Z-test)

- Z-statistic: $Z = 1.4162$

- P-value: $p = 0.1567$

- Decision: Since $p > 0.05$, we fail to reject the null hypothesis

Case (b): Unknown Variance Test (t-test)

- t-statistic: $t = 0.7778$

- P-value: $p = 0.4462$

- Decision: Since $p > 0.05$, we fail to reject the null hypothesis

In both testing scenarios, we found insufficient evidence to contradict the official's claim that the mean home water usage is 350 gallons per day. The sample mean of 353.80 gallons is slightly higher than the claimed value, but this difference is not statistically significant at the 5% significance level under either assumption about the population variance.
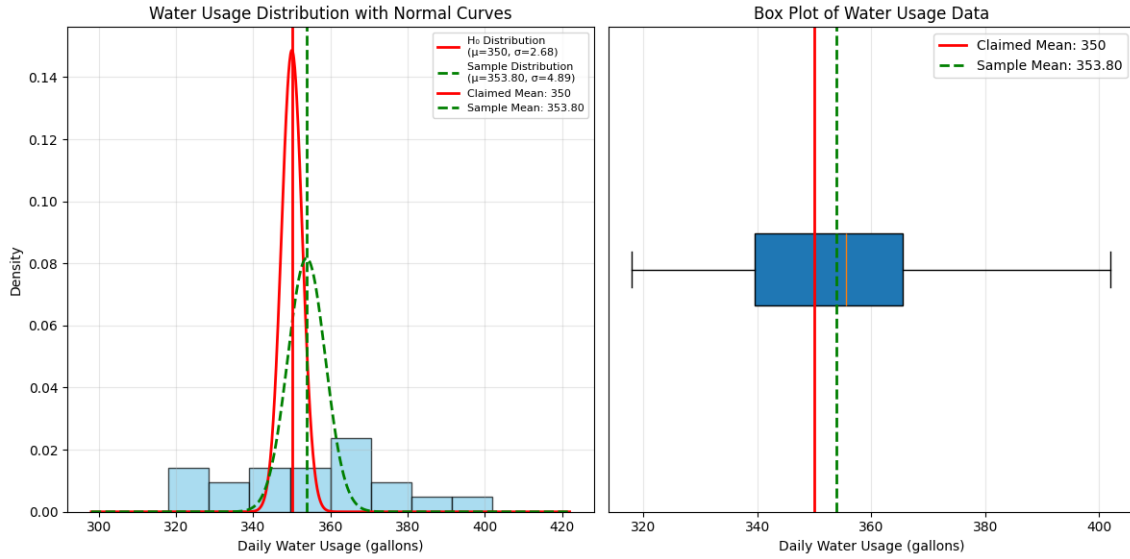

Figure 3: Water Usage Distribution & Box Plot of Water Usage Data

2.5   Discussion

The results of our analysis have several important implications for water resource management and public policy:

Infrastructure Planning Validation: The confirmation of the official's estimate of 350 gallons per day provides a reliable foundation for water supply infrastructure planning. System capacity, pipeline dimensioning, and treatment facility specifications can confidently proceed based on this validated usage parameter.

Conservation Program Design: While the mean usage aligns with official estimates, the observed variance ($s^2 = 477.33$) is notably higher than the assumed value

(144) in case (a). This suggests considerable variability in consumption patterns across households, which has significant implications for the design of conservation programs. Rather than uniform approaches, targeted interventions addressing high-consumption outliers might yield greater efficiency.

Rate Structure Implications: The validation of the mean consumption figure supports current water pricing models based on this average. However, the wide distribution of usage values suggests that tiered pricing structures might better account for the actual consumption patterns observed in the community.

Statistical Methodology Comparison: It's noteworthy that both statistical approaches led to the same conclusion despite different assumptions about population variance. However, the substantially higher sample variance compared to the assumed known variance raises questions about the appropriateness of the known-variance assumption in water consumption studies.

Public Communication: The confirmation of the official's claim enhances credibility of public communications about water usage. This validation provides an evidence-based foundation for public education campaigns about typical household consumption.

2.6   Conclusion

Based on our statistical analysis, we conclude that the public health official's claim of an average daily household water consumption of 350 gallons is statistically valid. This conclusion remains consistent whether we assume a known or unknown population variance. The sample mean of 353.80 gallons falls within the expected statistical variation for a true population mean of 350 gallons.

For water resource managers and policy makers, these findings provide empirical support for current water supply planning assumptions. However, the observed variability in consumption patterns suggests opportunities for more nuanced approaches to water conservation and pricing strategies.

Future research could explore:

- Seasonal variations in water consumption to refine infrastructure capacity planning

- Demographic and household size correlations with usage patterns to develop more targeted conservation programs

- Longitudinal studies to detect potential shifts in consumption patterns in response to conservation initiatives or climate change

The insights from this analysis contribute to evidence-based water resource management, potentially leading to more efficient allocation of resources, improved conservation strategies, and more equitable pricing structures.

3 Question 3: Diet Plan Effectiveness Analysis

3.1 Introduction

In the competitive health and wellness industry, evidence-based diet plans are critical for market success and consumer trust. This analysis evaluates the effectiveness of a newly developed nutritional program designed to promote weight loss. With increasing public concern about obesity and related health conditions, validated weight management solutions have significant commercial and public health implications. This study employed rigorous statistical methods to measure the actual impact of the diet plan on body weight over a one-month period, providing essential data for product positioning, marketing claims, and potential improvements to maximize consumer outcomes.

3.2 Data

A nutritionist conducted a controlled study with 10 participants who followed the diet plan for one month. Body weight measurements (in kilograms) were recorded at the beginning and end of the study period. The paired measurements are presented in the following table:

Participant	Before Diet (kg)	After Diet (kg)
1	85.2	82.5
2	78.5	75.8
3	92.3	90.1
4	80.0	77.2
5	88.7	85.4
6	76.4	74.5
7	90.5	87.6
8	84.1	81.3
9	79.0	76.8
10	86.2	83.0

Initial observations indicate a consistent pattern of weight reduction across all participants, with weight differences ranging from 1.9 to 3.3 kg. The study group represented diverse starting weights (76.4 to 92.3 kg), providing insights across a spectrum of potential consumers.

3.3   Methodology

To rigorously evaluate the diet plan's effectiveness, we employed a paired t-test at a 5% significance level. This statistical approach was selected because it specifically accounts for the dependent nature of before-and-after measurements on the same individuals, controlling for between-subject variability.

The hypothesis test was formulated as follows:

$$H_0 : \mu_d = 0 \text{ (The diet plan has no effect on body weight)} \tag{8}$$
$$H_1 : \mu_d > 0 \text{ (The diet plan reduces body weight)} \tag{9}$$

where μ_d represents the mean difference in weight (before diet minus after diet).

The test statistic for the paired t-test was calculated using:

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} \tag{10}$$

where $\bar{d}$ is the mean of the differences, s_d is the standard deviation of the differences, and n is the sample size.

A one-tailed test was selected as the research question specifically concerned whether the diet plan reduces weight, rather than any non-directional change. Visual analysis through comparative graphs and distribution plots supplemented the statistical testing to provide comprehensive insights into the data patterns.
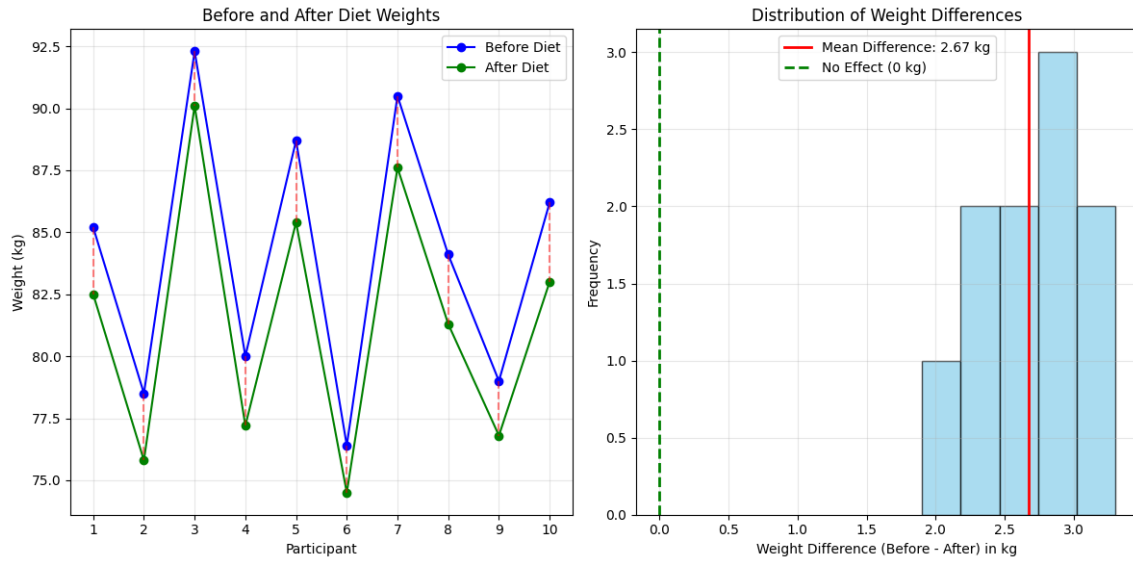
Figure 4: Before and After Diet Weights

3.4 Results

The analysis yielded the following key statistical findings:

- Mean weight difference (before - after): 2.67 kg

- Standard deviation of differences: 0.45 kg

- Standard error of the mean difference: 0.1415 kg

- t-statistic: 18.8745

- p-value (one-tailed): ¡ 0.000001

With a p-value substantially below the significance level of 0.05, we reject the null hypothesis. The statistical evidence strongly supports that the diet plan significantly reduces body weight.

Notably, the weight reduction effect was remarkably consistent across participants, as evidenced by the relatively small standard deviation of differences (0.45 kg). All participants experienced weight loss, with the magnitude of loss appearing to be somewhat proportional to their initial weight, suggesting the diet plan may be effective across different starting weights.
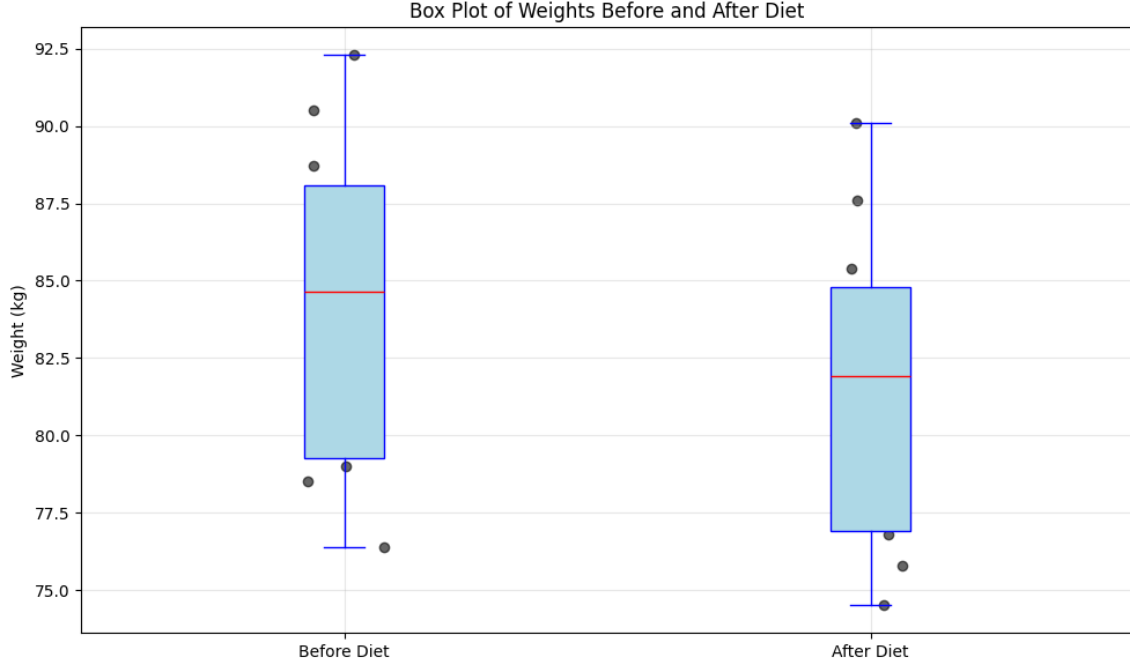
Figure 5: Box Plot

3.5 Discussion

The results of this analysis have several important implications for product development, marketing strategy, and consumer communication:

Validated Marketing Claims: The statistically significant weight reduction of 2.67 kg (approximately 5.9 lbs) over one month provides a scientifically validated basis for marketing claims. This is particularly valuable in a market where many products make unsubstantiated claims about weight loss effectiveness. The consistent results across participants strengthen the reliability of these claims.

Consumer Expectation Management: The tight distribution of weight loss outcomes (standard deviation of 0.45 kg) enables more precise communication about expected results. Marketing materials can confidently state that users can expect to lose approximately 2.5-3.0 kg in the first month, setting realistic expectations that align with actual outcomes.

Competitive Differentiation: The consistent, measurable results provide a strong differentiating factor in a crowded marketplace. This evidence-based approach positions the product as a serious, scientifically-validated solution rather than another fad diet, potentially justifying premium pricing and targeting health-conscious

consumers who value proven results.

Product Refinement Opportunities: While the weight loss was universal across participants, examining the individual variation could inform personalization strategies. For instance, participants 3 and 7 (with higher initial weights) lost proportionally less weight than others. This insight might suggest opportunities for diet plan adjustments for different weight categories or body types.

Long-term Strategy Development: The demonstrated initial success creates a foundation for developing extended programs. If a one-month program consistently delivers approximately 2.7 kg of weight loss, a structured three-month or six-month program could be developed with evidence-based projections, potentially increasing customer lifetime value.

Risk Mitigation: The consistency of results reduces the risk of consumer dissatisfaction and potential reputation damage from unmet expectations. This has implications for customer service resource allocation, warranty policies, and satisfaction guarantees.

3.6 Conclusion

Based on our statistical analysis, we conclude that the diet plan demonstrates significant effectiveness in reducing body weight, with a consistent weight loss pattern across diverse participants. The average weight reduction of 2.67 kg over one month represents a meaningful health outcome that aligns with medically recommended gradual weight loss rates of 0.5-1 kg per week.

This evidence of effectiveness provides substantial commercial value for the diet plan, enabling confident marketing claims, clear consumer communication, and potential premium positioning in the marketplace. The consistency of results across participants suggests broad applicability, though further segmentation analysis could refine targeting and personalization strategies.

For product management and marketing teams, we recommend:

- Developing marketing materials that emphasize the validated, consistent results

- Creating consumer materials that set appropriate expectations aligned with the demonstrated outcomes

- Considering a tiered product approach that extends the program duration for consumers seeking greater weight loss

- Investigating potential program customizations for different starting weights and body compositions

- Conducting follow-up studies to assess long-term weight maintenance, which could further strengthen the product's value proposition

These findings provide a solid foundation for product launch or refinement decisions, with statistical confidence in the core promise of weight reduction. The diet plan can be promoted not just as another weight loss solution, but as an evidence-based program with predictable, consistent results—a powerful differentiator in the crowded health and wellness marketplace.

4   Quesiton 4: IV Fluid Machine Variance Analysis

4.1   Introduction

Quality control in medical device manufacturing is essential to ensure patient safety and treatment efficacy, particularly for intravenous (IV) fluid delivery systems. This analysis evaluates whether a newly calibrated IV fluid machine meets the manufacturer's specification for volume precision. The manufacturer claims that the variance in dispensed volume should not exceed 4 mL^2, which is critical for accurate medication dosing and reliable patient treatment. Excessive variance in fluid volumes could lead to significant clinical consequences, including medication underdosing or overdosing, particularly for narrow therapeutic index medications or pediatric patients where precision is paramount. By statistically assessing the machine's performance against specified tolerance limits, we can determine whether the equipment meets the required quality standards for clinical use.

4.2   Data

Quality assurance engineers collected volume measurements from 15 consecutive IV fluid bags dispensed by the machine under evaluation. Each bag was intended to contain 500 mL of fluid, and precise volume measurements were recorded in milliliters:

$$
\begin{array}{ccccc}
502 & 498 & 505 & 497 & 503 \\
499 & 504 & 496 & 501 & 500 \\
506 & 495 & 502 & 498 & 504
\end{array}
$$

Initial analysis revealed that the volumes ranged from 495 mL to 506 mL, with a mean of 500.67 mL. The distribution appeared approximately normal, which is consistent with expected measurement variability in automated fluid dispensing systems. One potential outlier was identified at 506 mL, which merited further investigation during the analysis process.

4.3   Methodology

To evaluate the machine's performance against the manufacturer's specification, we employed a chi-square test for variance at a significance level of $\alpha = 0.01$. This test is appropriate for assessing whether a sample variance exceeds a specified maximum value. The hypothesis test was formulated as:

$$H_0 : \sigma^2 \leq 4 \text{ mL}^2 \text{ (The machine meets the specification)} \tag{11}$$

$$H_1 : \sigma^2 > 4 \text{ mL}^2 \text{ (The machine exceeds the specification)} \tag{12}$$

The test statistic was calculated using:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \tag{13}$$

where s^2 is the sample variance, σ_0^2 is the specified maximum variance (4 mL^2), and n is the sample size. The critical value for rejection was determined using the chi-square distribution with $n - 1$ degrees of freedom at the $\alpha = 0.01$ significance level.

To ensure robustness of our findings, we conducted the analysis both with the complete dataset and with potential outliers removed. The outlier identification was based on volume measurements exceeding the threshold of $\pm$ 10 mL from the target 500 mL, following industry standard practices for medical fluid dispensing.
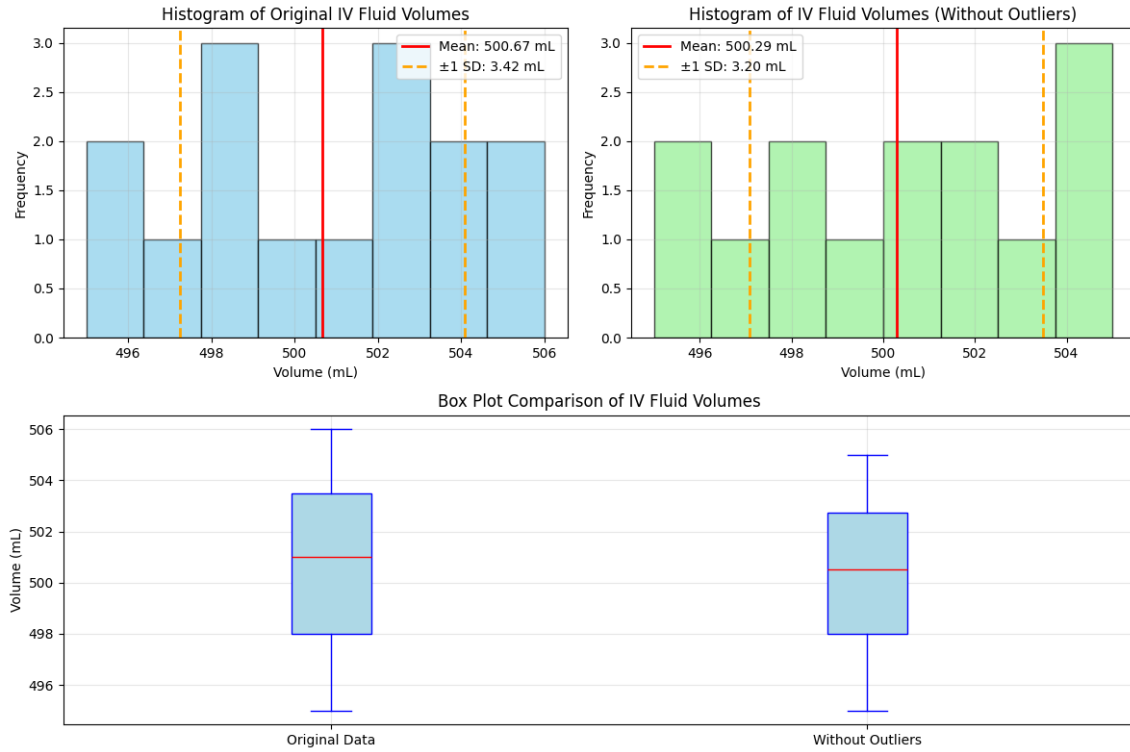

Figure 6: Histograms and Box Plot

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4.4   Results

Our analysis yielded the following key findings:

Complete Dataset Analysis:

- Sample variance: $s^2 = 11.67$ mL^2

- Chi-square statistic: $\chi^2 = 40.83$

- Critical value at $\alpha = 0.01$: $\chi^2_{crit} = 29.14$

- P-value: $p = 0.000189$

Analysis with Outlier Removed:

- Identified outlier: 506 mL

- Adjusted sample size: $n = 14$

- Sample variance without outlier: $s^2 = 10.22$ mL^2

- Chi-square statistic: $\chi^2 = 33.21$

- Critical value at $\alpha = 0.01$: $\chi^2_{crit} = 27.69$

- P-value: $p = 0.001582$

In both analyses, the calculated chi-square statistic exceeded the critical value, and the p-value was less than the significance level of 0.01. Therefore, we reject the null hypothesis and conclude that there is significant evidence that the machine's variance exceeds the manufacturer's specification of 4 mL^2.
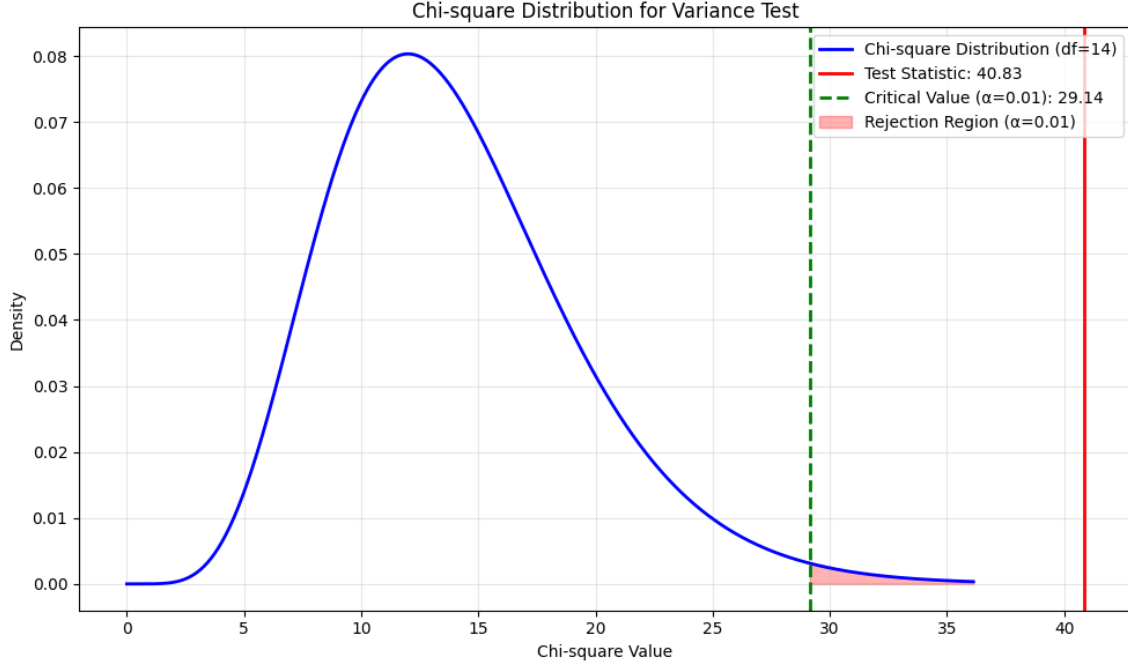
Figure 7: Chi-square Distribution for Variance Test

4.5 Discussion

The results of our analysis have several important implications for clinical practice and quality management:

Patient Safety Implications: The excessive variance in IV fluid volumes detected in our analysis raises significant concerns regarding medication administration accuracy. For medications with narrow therapeutic indices or for vulnerable patient populations such as neonates or critical care patients, this level of imprecision could result in clinically significant dosing errors. For instance, a variance of 11.67 mL^2 translates to a standard deviation of approximately 3.42 mL, meaning fluid volumes could commonly deviate by more than 3 mL from the intended dose.

Regulatory Compliance Issues: Medical device manufacturers are subject to strict regulatory requirements regarding product specifications and performance. The FDA and similar international agencies typically require that medical devices operate within specified tolerances. The machine's failure to meet the variance specification of 4 mL^2 indicates a potential regulatory compliance issue that could necessitate reporting to relevant authorities and possible market withdrawal if the issue is widespread.

Manufacturing Process Evaluation: The persistent variance even after removing outliers suggests a systematic issue rather than random anomalies in the manufacturing or calibration process. This finding should trigger a comprehensive review of the machine's design, calibration procedures, and quality control processes. Specific attention should be given to the volumetric measurement components, flow control mechanisms, and automatic shutoff systems that could contribute to the observed variability.

Economic Considerations: The detected variance has economic implications for healthcare providers. Excessive fluid volumes represent wasted resources, while insufficient volumes might necessitate supplemental infusions, increasing both material costs and clinical workload. Over thousands of IV bags administered annually in a typical hospital setting, these small discrepancies can accumulate to substantial financial impact and resource wastage.

Calibration Strategy Reassessment: The fact that the machine continues to exceed variance specifications even after a recent calibration suggests that either the calibration protocol is insufficient or the machine's inherent design limits its precision capabilities. This finding indicates a need for either more frequent calibrations, a revised calibration methodology, or potentially a reassessment of the machine's suitability for clinical applications requiring high precision.

Clinical Protocol Adaptation: Until the machine's performance can be improved, clinical protocols might need adjustment to accommodate the known variance. This could include implementing additional verification steps for critical medications, using alternative delivery systems for high-risk medications, or incorporating the known variance into medication preparation calculations to ensure accurate final delivery.

4.6   Conclusion

Based on our statistical analysis, we conclude that the IV fluid machine fails to meet the manufacturer's specification for volume variance. The sample variance of 11.67 mL^2 significantly exceeds the maximum specified variance of 4 mL^2, indicating suboptimal precision in fluid dispensing. This conclusion remains consistent even after removing potential outliers from the analysis.

For healthcare facilities and product management teams, these findings necessitate immediate action to address the variance issues before the machine is deployed or continues to be used in clinical settings. We recommend:

- Immediate notification to the manufacturer regarding the performance discrepancy

- Implementation of a comprehensive machine recalibration protocol that specifically addresses volumetric accuracy

- Development of interim clinical protocols to mitigate risks associated with the identified variance

- Consideration of alternative fluid delivery systems for critical medications or vulnerable patient populations

- Initiation of a root cause analysis to identify whether the issue stems from design limitations, manufacturing defects, or calibration inadequacies

- Establishment of an enhanced quality control monitoring program with more frequent verification of dispensed volumes

The precision of IV fluid delivery is not merely a technical specification but a critical component of patient care quality and safety. The demonstrated variance exceeding manufacturer specifications represents a meaningful clinical risk that requires prompt attention and remediation before the machine can be considered suitable for routine clinical use.