Course XYZ Problem Bank

Course XYZ Fall 2020

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Contents

1 Rotations

Problem 1. Rodriguez' Formula

Answer the following questions:

(a) Using Rodriguez' rotation formula, write down an expression for $e^{\hat{\omega}}$ where ω is a given vector in $\mathbb{R}^3$.

(b) Write down an expression for a rotation matrix implementing a rotation by θ radians counterclockwise about an axis in the direction of $\omega \in \mathbb{R}^3$.

Solution:

1. Let $\theta = ||\omega||$ and $K = \hat{\omega}/\theta$. Then

$$e^{\hat{\omega}} = I + (\sin\theta)K + (1 - \cos\theta)K^2 \tag{1}$$

2. Recall that this rotation matrix is exactly the exponential $e^{\hat{u}\theta}$ where u is a unit vector in the direction of ω. So let $u = \omega/||\omega||$. Then the required matrix is

$$\mathbf{R} = I + (\sin\theta)\hat{u} + (1 - \cos\theta)\hat{u}^2 \tag{2}$$