Checking KR's morphisms chi0 and chi1 [KR, pages 48-51]

Define the function g,u1,u2,a1,a2,a3:

In[1]:=
```
g[x_, y_] := x^(n + 1) +
    (n + 1) Sum[j^(-1) (-1)^j Binomial[n - j, j - 1] y^j x^(n + 1 - 2 j), {j, 1, (n + 1) / 2}];
u1[x1_, x2_, x3_, x4_] := (g[x1 + x2, x1 x2] - g[x3 + x4, x1 x2]) / (x1 + x2 - x3 - x4);
u2[x1_, x2_, x3_, x4_] := (g[x3 + x4, x1 x2] - g[x3 + x4, x3 x4]) / (x1 x2 - x3 x4);
aa1 = (mu - 1) u2[x1, x2, x3, x4] + (u1[x1, x2, x3, x4] + x1 u2[x1, x2, x3, x4] - pi23) / (x1 - x4);
aa2 = lambda u2[x1, x2, x3, x4] + (u1[x1, x2, x3, x4] + x1 u2[x1, x2, x3, x4] - pi23) / (x4 - x1);
aa3 = lambda (x3 + x4 - x1 - x2) + x1 - x3;
```

Define the double-identity defect P, the wide-edge defect Q, and the maps chi0, chi1:

In[7]:=
```
P = ( 0                                     0          x1 - x4
      0                                     0          (x2^(n + 1) - x3^(n + 1)) / (x2 - x3)   -(x1^(n +
      (x1^(n + 1) - x4^(n + 1)) / (x1 - x4)  x2 - x3    0
      (x2^(n + 1) - x3^(n + 1)) / (x2 - x3)  x4 - x1    0 )

Q = ( 0                    0                  x1 + x2 - x3 - x4      x1 x2 - x3 x4
      0                    0                  u2[x1, x2, x3, x4]    -u1[x1, x2, x3, x4]
      u1[x1, x2, x3, x4]   x1 x2 - x3 x4      0                      0
      u2[x1, x2, x3, x4]   x3 + x4 - x1 - x2  0                      0 );

c0 = ( x4 - x2 + mu (x1 + x2 - x3 - x4)  0   0                    0
       aa1                               1   0                    0
       0                                 0   x4 + mu (x1 - x4)    mu (x2 - x3) - x2
       0                                 0   -1                   1 );

c1 = ( 1    0    0   0
       aa2  aa3  0   0
       0    0    1   x3 + lambda (x2 - x3)
       0    0    1   x1 + lambda (x4 - x1) );
```

Mathematica cannot check that chi0 is a morphism for any n and any integer mu:

In[11]:=
```
FullSimplify[
  Assuming[n ∈ Integers && n > 1 && mu ∈ Integers, Simplify[MatrixForm[Q . c0 - c0 . P]]]]
```

Out[11]//MatrixForm=

$$\left(\begin{array}{l} 0 \\ 0 \\ \frac{((-1+mu)\ x2-mu\ x3)\ \left(-x2^{1+n}+x3^{1+n}\right)}{x2-x3}+\frac{pi23\ (-x1\ x2+x3\ x4)}{x1-x4}+\frac{(mu\ (x1-x4)+x4)\ \left(-x1^{1+n}+x4^{1+n}\right)}{x1-x4}+\frac{(mu\ (x1-x4)+x4)\ Gamma[-n]\ \left((x1+x2)^{1+n}\ Hype\ldots\right.}{\ldots} \\ \frac{x1^{1+n}+\frac{x1\ \left(-x2^{1+n}+x3^{1+n}\right)}{x2-x3}+pi23\ (x1+x2-x3-x4)+x4\ \left(\frac{x2^{1+n}-x3^{1+n}}{x2-x3}-x4^{n}\right)-(x1+x2)^{1+n}\ Hypergeometric2F1\left[\frac{1}{2}\ \ldots\right.}{x1-x4} \end{array}\right.$$

However, chi0 is a morphism for _any_ value of mu for n<20:

In[12]:=
```
EverythingOK = 1;
For[n = 2, n < 20, n++, If[Simplify[MatrixForm[Q . c0 - c0 . P]] == 0 IdentityMatrix[4],
   1, EverythingOK++]]; EverythingOK
```

Out[12]= 1

Mathematica cannot check that chi1 is a morphism for any n and any integer lambda:

In[13]:=
```
FullSimplify[
 Assuming[n ∈ Integers && n > 1 && lambda ∈ Integers, Simplify[MatrixForm[P . c1 - c1 . Q]]]]
```

$$\begin{pmatrix} 0 \\ 0 \\ \frac{x1^{1+n}+pi23\,x2-pi23\,x3-x4^{1+n}-(x1+x2)^{1+n}\,Hypergeometric2F1\left[\frac{1}{2}(-1-n),-\frac{n}{2},-n,\frac{4\,x1\,x2}{(x1+x2)^2}\right]+(x3+x4)^{1+n}\,Hypergeometric2F1\left[\frac{1}{2}(-1-n),-\frac{n}{2},-n,\frac{4\,x3\,x}{(x3+x4}\right.}{x1-x4} \\ \frac{x2^{1+n}-x3^{1+n}+pi23\,(-x2+x3)}{x2-x3} \end{pmatrix}$$

However, chi1 is a morphism for _any_ value of lambda for n<20:

In[14]:=
```
EverythingOK = 1;
For[n = 2, n < 20, n++, If[Simplify[MatrixForm[P . c1 - c1 . Q]] == 0 IdentityMatrix[4],
  1, EverythingOK++]]; EverythingOK
```

Out[14]= 1