Rise of the Planet of the Coevaluations

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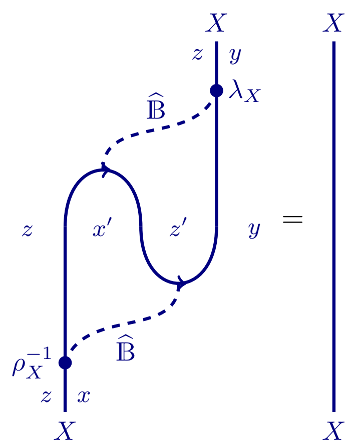
Abstract

They take over.

1. Zorro moves

In this section we will show that the bicategory $\mathcal{LG}$ of Landau-Ginzburg models has dualities [TODO: come up with better nomenclature]. Let us fix two arbitrary potentials $W \in A := k[x_1, \dots, x_n] \equiv k[x]$ and $V \in B := k[z_1, \dots, z_m] \equiv k[z]$. Then we want to prove that for any matrix factorisation $X \in \mathrm{hmf}(B \otimes_k A, V - W)$ and its dual $X^* = X^\vee[n] \in \mathrm{hmf}(A \otimes_k B, W - V)$ the Zorro moves (**??**) are satisfied.

Let us consider the identity (**??**) in more detail:



(1.1)

Here we indicated our choices for variable names in the various domains, as well as the fact that we use the completed bar complex $\widehat{\mathbb{B}}$ as a model for the unit endomorphism of the 0-cell W. Writing

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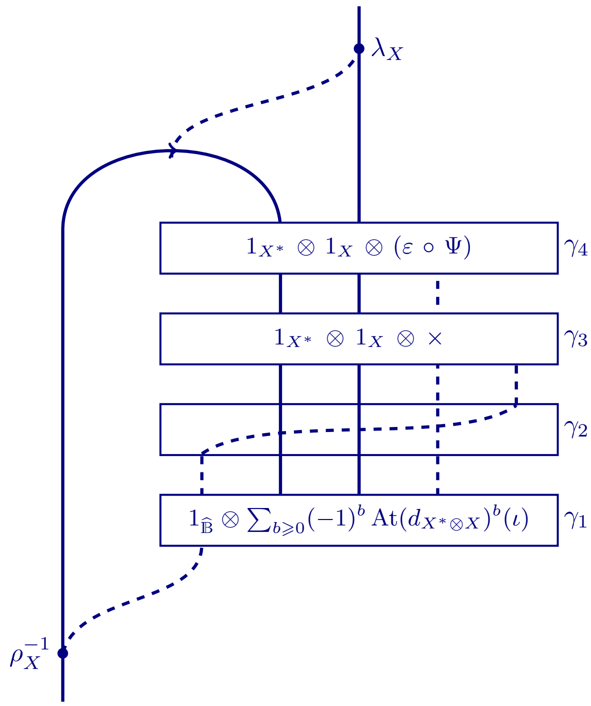
out the coevaluation (**??**) the left-hand side of (1.1) becomes

$Z :=$



(1.2)

where $\iota = \sum_j (-1)^{|e_j|} e_j^* \otimes e_j$ for an A-basis $\{e_j\}$ of X in γ_1, and the twist map γ_2 produces Koszul signs coming from commuting non-commutative forms from the left to the very right.

To prove that (1.2) is indeed homotopic to the identity we first concentrate on the map

$$\Gamma := \gamma_3 \circ \gamma_2 \circ \gamma_1 \circ \rho_X^{-1} : X \longrightarrow X \otimes_A X^* \otimes_B X \otimes_{A^e} \widehat{\mathbb{B}}\,.$$

If we understand the bar complex as $\Omega_A(A \otimes_k B)$ then as explained in section **??** only d_{X^*} contributes to the Atiyah class in γ_1. As a result Γ is basically the shuffle product of two telescopic series of the Atiyah classes for X and X^*. This can be expressed in terms of the Atiyah class of the tensor product $X^* \otimes X$:

LEMMA 1.1. $\Gamma(e_q) = \sum_{n\geqslant 0} \sum_j (-1)^{|e_j|+n} (\operatorname{At}(d_{X\otimes X^*}))^n (e_q \otimes e_j^* \otimes e_j)$.

Proof. We compute $\Gamma(e_q)$, using $\operatorname{At}(d_X)(e_q) = (-1)^{|e_q|+1} e_k \otimes d(d_X)_{kq}$ and paying attention to Koszul signs:

$$
\begin{aligned}
e_q \xmapsto{\rho_X^{-1}} & \sum_{a\geqslant 0}(-1)^a \operatorname{At}(d_X)^a(e_q) \\
& = \sum_{a\geqslant 0}(-1)^{a+(|e_q|+1)+\ldots+(|e_q|+a)} e_{k_a}\otimes d(d_X)_{k_a k_{a-1}}\ldots d(d_X)_{k_1 q} \\
\xmapsto{\gamma_1} & \Big(\sum_{a\geqslant 0}(-1)^{a+a|e_q|+\binom{a+1}{2}} e_{k_a}\otimes d(d_X)_{k_a k_{a-1}}\ldots d(d_X)_{k_1 q}\Big) \\
& \otimes\Big(\sum_{b\geqslant 0}\sum_j(-1)^{|e_j|+b}(-1)^{(|e_j|+1)+\ldots+(|e_j|+b)+b|e_j|} e^*_{l_b}\otimes e_j\otimes d(d_{X^*})_{l_b l_{b-1}}\ldots d(d_{X^*})_{l_1 j}\Big) \\
\xmapsto{\gamma_3\circ\gamma_2} & \sum_{n\geqslant 0}\sum_j(-1)^{|e_j|+n}\sum_{a=0}^{n}(-1)^{a|e_q|+\binom{a+1}{2}+\binom{n-a+1}{2}}\sum_{\sigma\in\mathrm{Sh}(n-a,a)}(-1)^{|\sigma|} e_{k_a}\otimes e^*_{l_{n-a}}\otimes e_j \\
& \otimes\sigma_\bullet\big(d(d_{X^*})_{l_{n-a}l_{n-a-1}}\ldots d(d_{X^*})_{l_1 j} d(d_X)_{k_a k_{a-1}}\ldots d(d_X)_{k_1 q}\big) \\
& = \sum_{n\geqslant 0}\sum_j(-1)^{|e_j|+n}\sum_{a=0}^{n}(-1)^{a|e_q|+\binom{a+1}{2}+\binom{n-a+1}{2}+a(n-a)}\sum_{\sigma\in\mathrm{Sh}(a,n-a)}(-1)^{|\sigma|} \qquad (1.3)\\
& \qquad\cdot e_{k_a}\otimes e^*_{l_{n-a}}\otimes e_j\otimes\sigma_\bullet\big(d(d_X)_{k_a k_{a-1}}\ldots d(d_X)_{k_1 q} d(d_{X^*})_{l_{n-a}l_{n-a-1}}\ldots d(d_{X^*})_{l_1 j}\big) \\
& = \sum_{n\geqslant 0}\sum_j(-1)^{|e_j|+n}\big(\operatorname{At}(d_X)+\operatorname{At}(d_{X^*})\big)^n(e_q\otimes e_j^*\otimes e_j) \qquad (1.4)\\
& = \sum_{n\geqslant 0}\sum_j(-1)^{|e_j|+n}\operatorname{At}(d_{X\otimes X^\vee})^n(e_q\otimes e_j^*\otimes e_j)\,.
\end{aligned}
$$

To understand the penultimate step we note that the sign $(-1)^{a|e_q|+\binom{a+1}{2}+\binom{n-a+1}{2}+a(n-a)}$ with $\sigma=\mathrm{id}$ in (1.3) is precisely that of the contribution to (1.4) where $\operatorname{At}(d_{X^*})$ first acts $n-a$ times on $e_q\otimes e_j^*\otimes e_j$, followed by $\operatorname{At}(d_X)^a$. The sign $(-1)^{|\sigma|}$ appears in (1.4) if some $\operatorname{At}(d_X)$ acts before some of the $\operatorname{At}(d_{X^*})$. $\square$

Since the Zorro map Z is $k[z,x]$-linear it is fixed by its action on basis elements e_q, so we find

$$
Z=\sum_j(-1)^{|e_j|+n+\binom{n+1}{2}}\operatorname{Res}_{k[x']}\left[\frac{\operatorname{str}(\underline{\lambda}'(\varepsilon\circ\Psi)\operatorname{At}(d_{X\otimes X^\vee})^n(-\otimes e_j^*\otimes e_j))\underline{dx'}}{\partial_{x'_1}W\ldots\partial_{x'_n}W}\right] \qquad (1.5)
$$

where $\underline{dx'}=dx'_1\ldots dx'_n$ and $\underline{\lambda}'=\lambda'_1\ldots\lambda'_n$ with $\lambda'_i=\partial_{x'_i}d_X(z,x')$. To arrive at the above expression for Z we also used that by naturality we have $(\widetilde{\operatorname{ev}}_X\otimes 1_X)\circ\gamma_4=(1_{\widehat{\mathbb{B}}}\otimes 1_X\otimes(\varepsilon\Psi))\circ(\widetilde{\operatorname{ev}}_X\otimes 1_X\otimes 1_{\widehat{\mathbb{B}}})$.

The Zorro map (1.5) will generally be the identity on X up to homotopy, but determining this homotopy directly is not easy. However, we can and will make use of the fact that the Kapustin-Li trace (**??**) is non-degenerate, so Z equals 1_X up to homotopy if

$$
\langle Z\psi'\rangle_X=\langle\psi'\rangle_X \qquad (1.6)
$$

for all closed endomorphisms ψ' of X. If we write $\psi:=\psi'\underline{\lambda}\underline{\mu}$ with $\lambda_i=\partial_{x_i}d_X(z,x)$ and $\mu_i=\partial_{z_i}d_X(z,x)$ then (1.6) holds if

$$
\operatorname{str}(Z\psi)=\operatorname{str}(\psi)\qquad\operatorname{mod}\ \partial_{y_i}W,\partial_{z_i}V\,.
$$

We will denote equality modulo the derivatives $f_i(y):=\partial_{y_i}W$ and $g_i(z):=\partial_{z_i}V$ by "$\equiv$", and we write $\langle\!\langle -\rangle\!\rangle$ for the map $\widehat{\mathbb{B}}\longrightarrow k[z,y]$ which acts as

$$
\beta\longmapsto(-1)^{\binom{n+1}{2}}\operatorname{Res}_{k[x']}\left[\frac{(\varepsilon\circ\Psi)(\beta)\underline{dx'}}{\partial_{x'_1}W\ldots\partial_{x'_n}W}\right].
$$

Then we can compute

$$\begin{aligned}
\operatorname{str}(Z\psi) &= \sum_i (-1)^{|e_i|} e_i^* \big(Z(\psi(e_i)) \big) \\
&= \sum_{i,j} (-1)^{|e_i|+|e_j|+n} e_i^* \Big(\big\langle\!\big\langle \operatorname{str}(\underline{\lambda}' \operatorname{At}(d_{X\otimes X^*})^n (\psi(e_i) \otimes e_j^* \otimes e_j)) \big\rangle\!\big\rangle \Big) \\
&= \sum_{i,j} (-1)^{|e_i|+|e_j|+n+n|e_j|} e_i^* \Big(\big\langle\!\big\langle \operatorname{str}(\underline{\lambda}' \operatorname{At}(d_{X\otimes X^*})^n (\psi(e_i) \otimes e_j^*)) \big\rangle\!\big\rangle e_j \Big) \\
&= \big\langle\!\big\langle \operatorname{str} \big(\underline{\lambda}' \operatorname{At}(d_{X\otimes X^*})^n \big(\sum_i \psi(e_i) \otimes e_j^* \big) \big) \big\rangle\!\big\rangle \\
&= \big\langle\!\big\langle \operatorname{str} \big(\underline{\lambda}' \operatorname{At}(d_{X\otimes X^*})^n (\psi) \big) \big\rangle\!\big\rangle \, . \qquad (1.7)
\end{aligned}$$

To see that this is indeed equal to $\operatorname{str}(\psi)$ the idea is to convert the action of Atiyah classes on ψ to an action on $\underline{\lambda}'$ instead. More precisely, we will repeatedly use the super Jacobi identity

$$\big[\varphi, \operatorname{At}(d_{X\otimes X^*})\big] = (-1)^{|\varphi|} \big[d_{X\otimes X^*}, [\varphi, \nabla]\big] - \big[\nabla, [d_{X\otimes X^*}, \varphi]\big] \qquad (1.8)$$

for homogeneous $\varphi \in \operatorname{End} X$, together with the fact that the Atiyah class precomposed with the supertrace is zero.

In the first step we apply (1.8) with $\varphi = \underline{\lambda}'$ to see that (1.7) equals

$$\Big\langle\!\!\Big\langle \operatorname{str} \Big(\Big\{ (-1)^n \big[d_{X\otimes X^*}, [\underline{\lambda}', \nabla] \big] - \big[\nabla, [d_{X\otimes X^*}, \underline{\lambda}'] \big] \Big\} \big(\operatorname{At}(d_{X\otimes X^*})^{n-1}(\psi) \big) \Big) \Big\rangle\!\!\Big\rangle \, . \qquad (1.9)$$

Since we defined $\psi = \psi' \underline{\lambda} \underline{\mu}$ with $[d_{X\otimes X^*}, \psi'] = 0$ we have $[d_{X\otimes X^*}, \psi] = \sum_i (\zeta_i f_i(x) + \xi_i g_i(z))$ for some maps ζ_i and ξ_i. Thus by cyclicity of str the first term in (1.9) is

$$\begin{aligned}
&\Big\langle\!\!\Big\langle \operatorname{str} \Big([\underline{\lambda}', \nabla] \big([d_{X\otimes X^*}, \operatorname{At}(d_{X\otimes X^*})^{n-1}(\psi)] \big) \Big) \Big\rangle\!\!\Big\rangle \\
= &\Big\langle\!\!\Big\langle \operatorname{str} \Big([\underline{\lambda}', \nabla] \Big(\sum_i \operatorname{At}(d_{X\otimes X^*})^{n-1} \big(\zeta_i f_i(x) + \xi_i g_i(z) \big) \Big) \Big) \Big\rangle\!\!\Big\rangle \equiv 0 \, . \qquad (1.10)
\end{aligned}$$

Here the last step follows immediately for the terms involving $g_i(z) = \partial_{z_i} V(z)$ as they are modded out by. The argument for the terms with $f_i(x) = \partial_{x_i} W(x)$ is slightly more subtle as they depend on the x-variables and not the y-variables featuring in the residue expression for the Kapustin-Li trace. However, the $f_i(x)$ in (1.10) act as zero-forms multiplying n-forms (in the image under $(\varepsilon \circ \Psi)$ in $\langle\!\langle - \rangle\!\rangle$) from the right. Hence by the following lemma this is the same as scalar multiplication with $f_i(y)$, which does not survive the quotient.

Lemma 1.2. *Let $\omega \in \Omega(k[x_1, \ldots, x_n])$ be and n-form. Then $\Psi(\omega f) = \Psi(\omega) f(y)$ for any $f \in k[x]$.*

Proof. We may assume that ω is of the form $da_1 \dots da_n$ for some $a_i \in k[x]$. Then

$$\begin{aligned}
\Psi(\omega f) &= \Psi\Big(\sum_{i=1}^{n}(-1)^{n-i} da_1 \dots da_{i-1} d(a_i a_{i+1}) da_{i+2} \dots da_n df + (-1)^n a_1 da_2 \dots da_n df\Big) \\
&= \sum_{i=1}^{n}(-1)^{n-i} \partial_{[1]} a_1 \dots \partial_{[i-1]} a_{i-1} \partial_{[i]}(a_i a_{i+1}) \partial_{[i+1]} a_{i+2} \dots \partial_{[n-1]} a_n \partial_{[n]} f \\
&\qquad + (-1)^n a_1 \partial_{[1]} a_2 \dots \partial_{[n-1]} a_n \partial_{[n]} f \\
&= \sum_{i=1}^{n}(-1)^{n-i} \partial_{[1]} a_1 \dots \partial_{[i-1]} a_{i-1} \Big(\partial_{[i]} a_i{}^{t_1 \dots t_i} a_{i+1} + {}^{t_1 \dots t_{i-1}} a_i \partial_{[i]} a_{i+1}\Big) \\
&\qquad \cdot \partial_{[i+1]} a_{i+2} \dots \partial_{[n-1]} a_n \partial_{[n]} f + (-1)^n a_1 \partial_{[1]} a_2 \dots \partial_{[n-1]} a_n \partial_{[n]} f \\
&= \partial_{[1]} a_1 \dots \partial_{[n]} a_n f(y) \\
&= \Psi(\omega) f(y) \,,
\end{aligned}$$

where we used lemma **??** in the third step. $\square$

Thus we find that only the second term in (1.9) remains. It is equal to

$$\begin{aligned}
&- \Big\langle\!\!\Big\langle \operatorname{str}\Big(\nabla\Big(\big[d_{X\otimes X^*}, \underline{\lambda}'\big]\big(\operatorname{At}(d_{X\otimes X^*})^{n-1}(\psi)\big)\Big)\Big)\Big\rangle\!\!\Big\rangle \\
&= - \sum_{j=1}^{n}(-1)^{j+1} \Big\langle\!\!\Big\langle \nabla\Big\{ f_j \operatorname{str}\big(\underline{\lambda}'_j \operatorname{At}(d_{X\otimes X^*})^{n-1}(\psi)\big)\Big\}\Big\rangle\!\!\Big\rangle
\end{aligned} \tag{1.11}$$

where $\underline{\lambda}'_j := \lambda'_1 \dots \lambda'_{j-1} \widehat{\lambda'_j} \lambda'_{j+1} \dots \lambda'_n$. Now we are in a similar situation as we were in (1.7), only this time the relevant term $\operatorname{str}(\underline{\lambda}'_j \operatorname{At}(d_{X\otimes X^*})^{n-1}(\psi))$ has one Atiyah class less. Again we use $\operatorname{str} \circ \operatorname{At} = 0$, apply the super Jacobi identity (now with $\varphi = \underline{\lambda}'_j$) and see that the first term on its right-hand side gives no contribution for the same reason as before, leading to

$$\operatorname{str}(Z\psi) \equiv \sum_{j_2<j_1} \sum_{\sigma\in S_2} (-1)^{j_1+j_2+|\sigma|} \Big\langle\!\!\Big\langle \nabla\Big\{ f_{j_{\sigma(1)}} \nabla\Big\{ f_{j_{\sigma(2)}} \operatorname{str}\big(\underline{\lambda}'_{j_{\sigma(1)} j_{\sigma(1)}} \operatorname{At}(d_{X\otimes X^*})^{n-2}(\psi)\big)\Big\}\Big\}\Big\rangle\!\!\Big\rangle \,.$$

Continuing in this fasion we find that

$$\begin{aligned}
\operatorname{str}(Z\psi) &\equiv \sum_{j_n<\dots<j_1} \sum_{\sigma\in S_n} (-1)^{j_1+\dots+j_n+|\sigma|} \Big\langle\!\!\Big\langle \nabla\Big\{ f_{j_{\sigma(1)}} \nabla\Big\{ f_{j_{\sigma(2)}} \nabla\Big\{ \dots \nabla\Big\{ f_{j_{\sigma(n)}} \operatorname{str}\big(\psi\big)\Big\} \dots \Big\}\Big\}\Big\}\Big\rangle\!\!\Big\rangle \\
&\equiv (-1)^{\binom{n+1}{2}} \sum_{\sigma\in S_n} (-1)^{|\sigma|} \Big\langle\!\!\Big\langle df_{\sigma(1)} df_{\sigma(2)} \dots df_{\sigma(n)} \operatorname{str}(\psi)\Big\rangle\!\!\Big\rangle \\
&= \operatorname{Res}_{k[x']}\left[\frac{(\varepsilon\circ\Psi)\operatorname{str}(\delta\psi)}{f_1(x')\dots f_n(x')}\right] \\
&= \operatorname{str}(\psi)\Big|_{x'\longmapsto y}
\end{aligned}$$

where

$$\delta = \det\begin{pmatrix} df_1 & df_2 & \cdots & df_n \\ \vdots & \vdots & & \vdots \\ df_1 & df_2 & \cdots & df_n \end{pmatrix}.$$

This concludes the proof that the Zorro map (1.2) is homotopic to the identity.

The three other Zorro moves are proven analogously. What remains to be done is to show that the same is true if we replace the completed bar complex $\widehat{\mathbb{B}}$ by the unit object Δ_W of the monoidal category $\mathrm{HMF}(A^{\mathrm{e}}, \widetilde{W})$.

[TODO: better write this bit only once the section on $\widehat{\mathbb{B}}$ as a MF is in place]

Theorem 1.3. *TODO: LG has dualities. . .*

2. Defect action on bulk fields

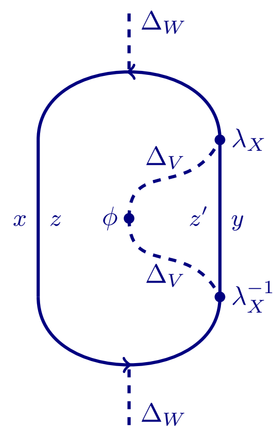
In any bicategory with duals there are natural maps between the endomorphism spaces of unit 1-cells. Roughly, these maps are constructed by capturing a 2-morphism of a unit 1-cell inside a loop labelled by an arbitrary 1-cell (and its dual). Below we present the details for the case of the bicategory $\mathcal{LG}$. We will also give the interpretation in terms of defect actions on bulk fields in Landau-Ginzburg models.

Let $X \in \operatorname{hmf}(k[z,x], V-W)$ as before. We define a map

$$\mathcal{D}_X : \operatorname{End}(\Delta_V) \longrightarrow \operatorname{End}(\Delta_W)$$

in terms of the morphisms encoding the monoidal and duality structures as follows. For $\phi \in \operatorname{End}(\Delta_V)$ we set $\mathcal{D}_X(\phi) = \operatorname{ev}_X \circ (1_{X^*} \otimes (\lambda_X \circ (\phi \otimes 1_X) \circ \lambda_X^{-1})) \circ \widetilde{\operatorname{coev}}_X$. Diagrammatically this definition reads

$$\mathcal{D}_X(\phi) = \qquad (2.1)$$



where again we indicated our choice of variable names in the four domains.

Remark 2.1. $\operatorname{End}(\Delta_W) = k[x]/(\partial_{x_i} W)$ is the Hochschild cohomology of $\operatorname{hmf}(k[x], W)$ [Dyc]. This space also precisely describes bulk fields of Landau-Ginzburg models with potential W. Furthermore, matrix factorisations of $V - W$ describe defect conditions between different Landau-Ginzburg models. Hence the map (2.1) has the natural interpretation of defect operators on bulk fields: a bulk field ϕ in the theory with potential V is mapped to the bulk field $\mathcal{D}_X(\phi)$ in the theory with potential W by wrapping around its insertion on the worldsheet a defect line labelled by X, and then collapsing this loop onto the insertion point. This limiting process is non-singular as the bicategory $\mathcal{LG}$ describes the purely topological sector of Landau-Ginzburg models.

Using the "folding trick" (which relates defects to boundary conditions in a product theory) one can argue for an explicit expression for $\mathcal{D}_X(\phi)$. This was done in [CR] for the case $V = W$. Here we use the duality structure to directly prove it for the general case:

Proposition 2.2. *For any $X \in \operatorname{hmf}(k[z,x], V-W)$ and $\phi \in \operatorname{End}(\Delta_V)$ we have*

$$\mathcal{D}_X(\phi) = (-1)^n \operatorname{Res}_{k[z]} \left[\frac{\phi(z) \operatorname{str}\left(\partial_{x_1} d_{X^*} \dots \partial_{x_n} d_{X^*} \partial_{z_1} d_{X^*} \dots \partial_{z_m} d_{X^*}\right)}{\partial_{z_1} V \dots \partial_{z_m} V} \right].$$

Proof. Since $\operatorname{End}(\Delta_W) = k[x]/(\partial_{x_i} W)$ and $\operatorname{End}(\Delta_V) = k[z]/(\partial_{z_i} V)$ we are free to set $x = y$ and $z = z'$ at appropriate places, cf. (2.1). Furthermore, λ_X will project out all non-zero degree contributions coming from the action of λ_X^{-1}, so $\lambda_X \circ (\phi \otimes 1_X) \circ \lambda_X^{-1}$ is simply multiplication by the polynomial $\phi(z)$.

In the lower part of (2.1) we have

$$\begin{aligned}
\widetilde{\operatorname{coev}}(1) &= \sum_j (-1)^{|e_j|} (\varepsilon\Psi) \left((-\operatorname{At}(d_{X^*}))^n (e_j^* \otimes e_j) \right) \\
&= \sum_j (-1)^{|e_j|+n} (-1)^{(|e_j|+n)+\ldots+(|e_j|+n)+n|e_j|} e_{l_n}^* \otimes e_j \otimes (\varepsilon\Psi) \left(d(d_{X^*})_{l_n l_{n-1}} \ldots d(d_{X^*})_{l_1 j} \right) \\
&= \sum_j (-1)^{|e_j|+n+\binom{n+1}{2}} \left(\partial_{[1]} d_{X^*} \ldots \partial_{[n]} d_{X^*}(e_j^*) \right) \otimes e_j \\
&= (-1)^{n+\binom{n+1}{2}} \Big(\left(\partial_{x_1} d_{X^*} \ldots \partial_{x_n} d_{X^*} \right) \otimes 1_X \Big) \circ \Big(\sum_j (-1)^{|e_j|} e_j^* \otimes e_j \Big)
\end{aligned} \tag{2.2}$$

which we identify with $(-1)^{n+\binom{n+1}{2}} \partial_{x_1} d_{X^*} \ldots \partial_{x_n} d_{X^*}$ in $\operatorname{End}(X^*)$. Note that in the last step leading to (2.2) we set $\partial_{[i]} d_{X^*}(x,z) = \partial_{x_i} d_{X^*}(x,z)$ since $x = y$ in $\operatorname{End}(\Delta_W)$.

Next we apply the upper part of (2.1) to (2.2) to get

$$\mathcal{D}_X(\phi) = (-1)^n \operatorname{Res}_{k[z]} \left[\frac{\phi(z) \operatorname{str}\left(\partial_{x_1} d_{X^*} \ldots \partial_{x_n} d_{X^*} \partial_{z_1} d_{X^*} \ldots \partial_{z_m} d_{X^*} \right)}{\partial_{z_1} V \ldots \partial_{z_m} V} \right] + \mathcal{O}(\theta) \, .$$

Here we collectively denote the contributions from ev_X of non-zero degree in the Koszul complex Δ_W by $\mathcal{O}(\theta)$. Since we know that $\mathcal{D}_X(\phi)$ is a morphism in $\operatorname{End}_{\operatorname{hmf}(k[x,y],\widetilde{W})}(\Delta_W) = k[x]/(\partial_{x_i})$ it follows that $\mathcal{O}(\theta)$ must be null-homotopic, thus concluding the proof. □

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