

GSV trick for L6a3

We want to see whether we can use [GSV, eq. (5.5)] to compute unreduced KR homology for the two-component link L6a3. Our results for its n=2 and n=3 KR invariants are as follows:

```
In[1]:= lk = 3; KR2 = Expand[
  (t^6 q^2 + t^6 + t^5 q^2 + t^4 q^6 + t^3 q^6 + t^2 q^10 + q^14 + q^12) / (q^18)];
KR3 = Expand[(t^6 q^6 + 2 t^6 q^4 + 2 t^6 q^2 + t^6 + t^5 q^4 + t^5 q^2 + t^4 q^10 + t^4 q^8 +
  t^3 q^8 + t^3 q^6 + t^2 q^14 + t^2 q^12 + q^18 + q^16 + q^14) / (q^26)];
```

```
In[3]:= U2 = Expand[(q - 1 / q) (q^4 lk) KR2 - t^a (q + 1 / q)^2];
U3 = Expand[Simplify[(q - 1 / q) (q^6 lk) KR3 - t^a ((q^3 - q^(-3)) / (q - 1 / q))^2]]];
```

```
In[5]:= U2 /. {a -> 2 lk}
```

$$\text{Out[5]} = -q^5 + q^9 - q^3 t^2 + q^5 t^2 - \frac{t^3}{q} + q t^3 - \frac{t^4}{q} + q t^4 - \frac{t^5}{q^5} + \frac{t^5}{q^3} - \frac{t^6}{q^7} + \frac{2 t^6}{q^3} + \frac{t^6}{q} - q t^6 - q^3 t^6$$

```
In[6]:= U3 /. {a -> 2 lk}
```

$$\text{Out[6]} = -q^5 + q^{11} - q^3 t^2 + q^7 t^2 - \frac{t^3}{q^3} + q t^3 - \frac{t^4}{q} + q^3 t^4 - \frac{t^5}{q^7} + \frac{t^5}{q^3} - \frac{t^6}{q^9} - \frac{t^6}{q^7} + \frac{t^6}{q^5} + \frac{2 t^6}{q^3} + \frac{2 t^6}{q} - q t^6 - q^3 t^6 - q^5 t^6$$

U3 has more t^6 terms than U2 for all values of lk in {0,1,-1,2,-2,3,-3}, and this continues to be the case if we set a = 6.

All t-monomials above can be easily matched apart from the t^6 -monomials. Now if we have a term $c * q^j * t^r$ in U_N above, according to GSV it should be equal to $\sum_{Q \in \mathbb{Z}} D_{\{Q, j-N*Q, r\}} q^{N*Q + (j - N*Q)} t^r$. Below we will try to solve these equations for all j simultaneously. We use the notation $f[Q,s] = D_{\{Q,s,6\}}$.

```

In[7]:= Solve[
{
  Sum[f[Q, -7 - 2 Q], {Q, -4, 4}] == -1,
  Sum[f[Q, -3 - 2 Q], {Q, -4, 4}] == 2,
  Sum[f[Q, -1 - 2 Q], {Q, -4, 4}] == 1,
  Sum[f[Q, 1 - 2 Q], {Q, -4, 4}] == -1,
  Sum[f[Q, 3 - 2 Q], {Q, -4, 4}] == -1,
  Sum[f[Q, -6 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, -5 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, -4 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, -2 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 0 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 2 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 4 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 5 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 6 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 7 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 8 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 9 - 2 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 10 - 2 Q], {Q, -4, 4}] == 0,

  Sum[f[Q, -9 - 3 Q], {Q, -4, 4}] == -1,
  Sum[f[Q, -7 - 3 Q], {Q, -4, 4}] == -1,
  Sum[f[Q, -5 - 3 Q], {Q, -4, 4}] == 1,
  Sum[f[Q, -3 - 3 Q], {Q, -4, 4}] == 2,
  Sum[f[Q, -1 - 3 Q], {Q, -4, 4}] == 2,
  Sum[f[Q, 1 - 3 Q], {Q, -4, 4}] == -1,
  Sum[f[Q, 3 - 3 Q], {Q, -4, 4}] == -1,
  Sum[f[Q, 5 - 3 Q], {Q, -4, 4}] == -1,
  Sum[f[Q, -11 - 3 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, -10 - 3 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, -8 - 3 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, -6 - 3 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, -4 - 3 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, -2 - 3 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 0 - 3 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 8 - 3 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 6 - 3 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 4 - 3 Q], {Q, -4, 4}] == 0,
  Sum[f[Q, 2 - 3 Q], {Q, -4, 4}] == 0
},
Flatten[Array[f, {4, 47}, {-4, -23}]]]

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Solve::svars : Equations may not give solutions for all "solve" variables. >>

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{{f[-4, 1] -> -f[-3, 16] + f[-2, -4] - f[-2, 13] - f[-2, 14] + f[-1, -7] + f[-1, -6] - f[-1, 10] -
  f[-1, 11] - f[-1, 12] + f[0, -10] + f[0, -9] + f[0, -8] - f[0, 7] - f[0, 8] - f[0, 9] -
  f[0, 10] + f[1, -13] + f[1, -12] + f[1, -11] + f[1, -10] - f[1, 4] - f[1, 5] - f[1, 6] -
  f[1, 7] - f[1, 8] + f[2, -16] + f[2, -15] + f[2, -14] + f[2, -13] + f[2, -12] - f[2, 1] -
  f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -19] + f[3, -18] + f[3, -17] +
  f[3, -16] + f[3, -15] + f[3, -14] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] -
  f[3, 3] - f[3, 4] + f[4, -22] + f[4, -21] + f[4, -20] + f[4, -19] + f[4, -18] + f[4, -17] +
  f[4, -16] - f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
  f[-4, 2] -> 1 - f[-3, 16] + f[-2, -3] - f[-2, 13] - f[-2, 14] + f[-1, -6] + f[-1, -5] -
  f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, -9] + f[0, -8] + f[0, -7] - f[0, 7] - f[0, 8] -
  f[0, 9] - f[0, 10] + f[1, -12] + f[1, -11] + f[1, -10] + f[1, -9] - f[1, 4] - f[1, 5] -

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f[1, 6] - f[1, 7] - f[1, 8] + f[2, -15] + f[2, -14] + f[2, -13] + f[2, -12] + f[2, -11] -
f[2, 1] - f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -18] + f[3, -17] + f[3, -16] +
f[3, -15] + f[3, -14] + f[3, -13] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] -
f[3, 3] - f[3, 4] + f[4, -21] + f[4, -20] + f[4, -19] + f[4, -18] + f[4, -17] + f[4, -16] +
f[4, -15] - f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 3] → -f[-3, 16] + f[-2, -2] - f[-2, 13] - f[-2, 14] + f[-1, -5] + f[-1, -4] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, -8] + f[0, -7] + f[0, -6] - f[0, 7] - f[0, 8] -
f[0, 9] - f[0, 10] + f[1, -11] + f[1, -10] + f[1, -9] + f[1, -8] - f[1, 4] - f[1, 5] -
f[1, 6] - f[1, 7] - f[1, 8] + f[2, -14] + f[2, -13] + f[2, -12] + f[2, -11] + f[2, -10] -
f[2, 1] - f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -17] + f[3, -16] + f[3, -15] +
f[3, -14] + f[3, -13] + f[3, -12] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] -
f[3, 3] - f[3, 4] + f[4, -20] + f[4, -19] + f[4, -18] + f[4, -17] + f[4, -16] + f[4, -15] +
f[4, -14] - f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 4] → -f[-3, 16] + f[-2, -1] - f[-2, 13] - f[-2, 14] + f[-1, -4] + f[-1, -3] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, -7] + f[0, -6] + f[0, -5] - f[0, 7] - f[0, 8] -
f[0, 9] - f[0, 10] + f[1, -10] + f[1, -9] + f[1, -8] + f[1, -7] - f[1, 4] - f[1, 5] -
f[1, 6] - f[1, 7] - f[1, 8] + f[2, -13] + f[2, -12] + f[2, -11] + f[2, -10] + f[2, -9] -
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f[3, -13] + f[3, -12] + f[3, -11] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] -
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f[4, -13] - f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 5] → -1 - f[-3, 16] + f[-2, 0] - f[-2, 13] - f[-2, 14] + f[-1, -3] + f[-1, -2] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, -6] + f[0, -5] + f[0, -4] - f[0, 7] - f[0, 8] -
f[0, 9] - f[0, 10] + f[1, -9] + f[1, -8] + f[1, -7] + f[1, -6] - f[1, 4] - f[1, 5] - f[1, 6] -
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f[3, -12] + f[3, -11] + f[3, -10] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] -
f[3, 3] - f[3, 4] + f[4, -18] + f[4, -17] + f[4, -16] + f[4, -15] + f[4, -14] + f[4, -13] +
f[4, -12] - f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
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f[0, 9] - f[0, 10] + f[1, -8] + f[1, -7] + f[1, -6] + f[1, -5] - f[1, 4] - f[1, 5] - f[1, 6] -
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f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -14] + f[3, -13] + f[3, -12] +
f[3, -11] + f[3, -10] + f[3, -9] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] -
f[3, 3] - f[3, 4] + f[4, -17] + f[4, -16] + f[4, -15] + f[4, -14] + f[4, -13] + f[4, -12] +
f[4, -11] - f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 7] → -2 - f[-3, 16] + f[-2, 2] - f[-2, 13] - f[-2, 14] + f[-1, -1] + f[-1, 0] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, -4] + f[0, -3] + f[0, -2] - f[0, 7] - f[0, 8] -
f[0, 9] - f[0, 10] + f[1, -7] + f[1, -6] + f[1, -5] + f[1, -4] - f[1, 4] - f[1, 5] - f[1, 6] -
f[1, 7] - f[1, 8] + f[2, -10] + f[2, -9] + f[2, -8] + f[2, -7] + f[2, -6] - f[2, 1] - f[2, 2] -
f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -13] + f[3, -12] + f[3, -11] + f[3, -10] +
f[3, -9] + f[3, -8] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] - f[3, 3] - f[3, 4] +
f[4, -16] + f[4, -15] + f[4, -14] + f[4, -13] + f[4, -12] + f[4, -11] + f[4, -10] -
f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
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f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, -3] + f[0, -2] + f[0, -1] - f[0, 7] - f[0, 8] -
f[0, 9] - f[0, 10] + f[1, -6] + f[1, -5] + f[1, -4] + f[1, -3] - f[1, 4] - f[1, 5] - f[1, 6] -
f[1, 7] - f[1, 8] + f[2, -9] + f[2, -8] + f[2, -7] + f[2, -6] + f[2, -5] - f[2, 1] - f[2, 2] -
f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -12] + f[3, -11] + f[3, -10] + f[3, -9] +
f[3, -8] + f[3, -7] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] - f[3, 3] - f[3, 4] +
f[4, -15] + f[4, -14] + f[4, -13] + f[4, -12] + f[4, -11] + f[4, -10] + f[4, -9] -
f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 9] → -1 - f[-3, 16] + f[-2, 4] - f[-2, 13] - f[-2, 14] + f[-1, 1] + f[-1, 2] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, -2] + f[0, -1] + f[0, 0] - f[0, 7] - f[0, 8] -

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f[0, 9] - f[0, 10] + f[1, -5] + f[1, -4] + f[1, -3] + f[1, -2] - f[1, 4] - f[1, 5] -
f[1, 6] - f[1, 7] - f[1, 8] + f[2, -8] + f[2, -7] + f[2, -6] + f[2, -5] + f[2, -4] -
f[2, 1] - f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -11] + f[3, -10] + f[3, -9] +
f[3, -8] + f[3, -7] + f[3, -6] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] -
f[3, 3] - f[3, 4] + f[4, -14] + f[4, -13] + f[4, -12] + f[4, -11] + f[4, -10] + f[4, -9] +
f[4, -8] - f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 10] → -f[-3, 16] + f[-2, 5] - f[-2, 13] - f[-2, 14] + f[-1, 2] + f[-1, 3] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, -1] + f[0, 0] + f[0, 1] - f[0, 7] - f[0, 8] -
f[0, 9] - f[0, 10] + f[1, -4] + f[1, -3] + f[1, -2] + f[1, -1] - f[1, 4] - f[1, 5] -
f[1, 6] - f[1, 7] - f[1, 8] + f[2, -7] + f[2, -6] + f[2, -5] + f[2, -4] + f[2, -3] -
f[2, 1] - f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -9] + f[3, -10] + f[3, -9] + f[3, -8] +
f[3, -7] + f[3, -6] + f[3, -5] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] - f[3, 3] -
f[3, 4] + f[4, -13] + f[4, -12] + f[4, -11] + f[4, -10] + f[4, -9] + f[4, -8] + f[4, -7] -
f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 11] → 2 - f[-3, 16] + f[-2, 6] - f[-2, 13] - f[-2, 14] + f[-1, 3] + f[-1, 4] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, 0] + f[0, 1] + f[0, 2] - f[0, 7] - f[0, 8] -
f[0, 9] - f[0, 10] + f[1, -3] + f[1, -2] + f[1, -1] + f[1, 0] - f[1, 4] - f[1, 5] - f[1, 6] -
f[1, 7] - f[1, 8] + f[2, -6] + f[2, -5] + f[2, -4] + f[2, -3] + f[2, -2] - f[2, 1] -
f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -9] + f[3, -8] + f[3, -7] + f[3, -6] +
f[3, -5] + f[3, -4] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] - f[3, 3] -
f[3, 4] + f[4, -12] + f[4, -11] + f[4, -10] + f[4, -9] + f[4, -8] + f[4, -7] + f[4, -6] -
f[4, -5] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 12] → 3 - f[-3, 16] + f[-2, 7] - f[-2, 13] - f[-2, 14] + f[-1, 4] + f[-1, 5] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, 1] + f[0, 2] + f[0, 3] - f[0, 7] - f[0, 8] -
f[0, 9] - f[0, 10] + f[1, -2] + f[1, -1] + f[1, 0] + f[1, 1] - f[1, 4] - f[1, 5] -
f[1, 6] - f[1, 7] - f[1, 8] + f[2, -5] + f[2, -4] + f[2, -3] + f[2, -2] + f[2, -1] -
f[2, 1] - f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -8] + f[3, -7] +
f[3, -6] + f[3, -5] + f[3, -4] + f[3, -3] - f[3, -2] - f[3, -1] - f[3, 0] - f[3, 1] -
f[3, 2] - f[3, 3] - f[3, 4] + f[4, -11] + f[4, -10] + f[4, -9] + f[4, -8] + f[4, -7] +
f[4, -6] - f[4, -4] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 13] → 2 - f[-3, 16] + f[-2, 8] - f[-2, 13] - f[-2, 14] + f[-1, 5] + f[-1, 6] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, 2] + f[0, 3] + f[0, 4] - f[0, 7] - f[0, 8] -
f[0, 9] - f[0, 10] + f[1, -1] + f[1, 0] + f[1, 1] + f[1, 2] - f[1, 4] - f[1, 5] - f[1, 6] -
f[1, 7] - f[1, 8] + f[2, -4] + f[2, -3] + f[2, -2] + f[2, -1] + f[2, 0] - f[2, 1] - f[2, 2] -
f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -7] + f[3, -6] + f[3, -5] + f[3, -4] + f[3, -3] -
f[3, -1] - f[3, 0] - f[3, 1] - f[3, 2] - f[3, 3] - f[3, 4] + f[4, -10] + f[4, -9] + f[4, -8] +
f[4, -7] + f[4, -6] - f[4, -3] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 14] → 2 - f[-3, 16] + f[-2, 9] - f[-2, 13] - f[-2, 14] + f[-1, 6] + f[-1, 7] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, 3] + f[0, 4] + f[0, 5] - f[0, 7] - f[0, 8] - f[0, 9] -
f[0, 10] + f[1, 0] + f[1, 1] + f[1, 2] + f[1, 3] - f[1, 4] - f[1, 5] - f[1, 6] - f[1, 7] - f[1, 8] +
f[2, -3] + f[2, -2] + f[2, -1] + f[2, 0] - f[2, 1] - f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] +
f[3, -6] + f[3, -5] + f[3, -4] + f[3, -3] - f[3, 0] - f[3, 1] - f[3, 2] - f[3, 3] - f[3, 4] +
f[4, -9] + f[4, -8] + f[4, -7] + f[4, -6] - f[4, -2] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 15] → 1 - f[-3, 16] + f[-2, 10] - f[-2, 13] - f[-2, 14] + f[-1, 7] + f[-1, 8] -
f[-1, 10] - f[-1, 11] - f[-1, 12] + f[0, 4] + f[0, 5] + f[0, 6] - f[0, 7] -
f[0, 8] - f[0, 9] - f[0, 10] + f[1, 1] + f[1, 2] + f[1, 3] - f[1, 4] - f[1, 5] - f[1, 6] -
f[1, 7] - f[1, 8] + f[2, -2] + f[2, -1] + f[2, 0] - f[2, 1] - f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] -
f[2, 6] + f[3, -5] + f[3, -4] + f[3, -3] - f[3, 1] - f[3, 2] - f[3, 3] - f[3, 4] +
f[4, -8] + f[4, -7] + f[4, -6] - f[4, -1] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 16] → 1 - f[-3, 16] + f[-2, 11] - f[-2, 13] - f[-2, 14] + f[-1, 8] + f[-1, 9] - f[-1, 10] -
f[-1, 11] - f[-1, 12] + f[0, 5] + f[0, 6] - f[0, 8] - f[0, 9] - f[0, 10] + f[1, 2] + f[1, 3] -
f[1, 4] - f[1, 5] + f[2, -1] + f[2, 0] - f[2, 1] - f[2, 2] - f[2, 3] - f[2, 4] - f[2, 5] - f[2, 6] + f[3, -4] +
f[3, -3] - f[3, 2] - f[3, 3] - f[3, 4] + f[4, -7] + f[4, -6] - f[4, 0] - f[4, 1] - f[4, 2],
f[-4, 17] → -f[-3, 16] + f[-2, 12] - f[-2, 13] - f[-2, 14] + f[-1, 9] - f[-1, 11] -
f[-1, 12] + f[0, 6] - f[0, 9] - f[0, 10] + f[1, 3] - f[1, 7] - f[1, 8] + f[2, 0] -

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f[-4, 18] → -f[-3, 16] - f[-2, 14] - f[-1, 12] - f[0, 10] - f[1, 8] - f[2, 6] - f[3, 4] - f[4, 2],
f[-4, 20] →
-f[-3, 17] - f[-2, 14] - f[-1, 11] - f[0, 8] - f[1, 5] - f[2, 2] - f[3, -1] - f[4, -4],
f[-3, -2] → f[-3, 16] - f[-2, -5] - f[-2, -4] + f[-2, 13] + f[-2, 14] - f[-1, -8] -
f[-1, -7] - f[-1, -6] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, -11] - f[0, -10] - f[0, -9] -
f[0, -8] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -14] - f[1, -13] - f[1, -12] -
f[1, -11] - f[1, -10] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -17] -
f[2, -16] - f[2, -15] - f[2, -14] - f[2, -13] - f[2, -12] + f[2, 1] + f[2, 2] + f[2, 3] +
f[2, 4] + f[2, 5] + f[2, 6] - f[3, -20] - f[3, -19] - f[3, -18] - f[3, -17] - f[3, -16] -
f[3, -15] - f[3, -14] + f[3, -2] + f[3, -1] + f[3, 0] + f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] -
f[4, -23] - f[4, -22] - f[4, -21] - f[4, -20] - f[4, -19] - f[4, -18] - f[4, -17] -
f[4, -16] + f[4, -5] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, -1] → -1 + f[-3, 16] - f[-2, -4] - f[-2, -3] + f[-2, 13] + f[-2, 14] - f[-1, -7] -
f[-1, -6] - f[-1, -5] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, -10] - f[0, -9] -
f[0, -8] - f[0, -7] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -13] - f[1, -12] -
f[1, -11] - f[1, -10] - f[1, -9] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -16] -
f[2, -15] - f[2, -14] - f[2, -13] - f[2, -12] - f[2, -11] + f[2, 1] + f[2, 2] + f[2, 3] +
f[2, 4] + f[2, 5] + f[2, 6] - f[3, -19] - f[3, -18] - f[3, -17] - f[3, -16] - f[3, -15] -
f[3, -14] - f[3, -13] + f[3, -2] + f[3, -1] + f[3, 0] + f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] -
f[4, -22] - f[4, -21] - f[4, -20] - f[4, -19] - f[4, -18] - f[4, -17] - f[4, -16] -
f[4, -15] + f[4, -5] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 0] → -1 + f[-3, 16] - f[-2, -3] - f[-2, -2] + f[-2, 13] + f[-2, 14] - f[-1, -6] -
f[-1, -5] - f[-1, -4] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, -9] - f[0, -8] - f[0, -7] -
f[0, -6] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -12] - f[1, -11] - f[1, -10] -
f[1, -9] - f[1, -8] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -15] - f[2, -14] -
f[2, -13] - f[2, -12] - f[2, -11] - f[2, -10] + f[2, 1] + f[2, 2] + f[2, 3] + f[2, 4] +
f[2, 5] + f[2, 6] - f[3, -18] - f[3, -17] - f[3, -16] - f[3, -15] - f[3, -14] - f[3, -13] -
f[3, -12] + f[3, -2] + f[3, -1] + f[3, 0] + f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] - f[4, -21] -
f[4, -20] - f[4, -19] - f[4, -18] - f[4, -17] - f[4, -16] - f[4, -15] - f[4, -14] +
f[4, -5] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 1] → f[-3, 16] - f[-2, -2] - f[-2, -1] + f[-2, 13] + f[-2, 14] - f[-1, -5] -
f[-1, -4] - f[-1, -3] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, -8] - f[0, -7] -
f[0, -6] - f[0, -5] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -11] - f[1, -10] -
f[1, -9] - f[1, -8] - f[1, -7] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -14] -
f[2, -13] - f[2, -12] - f[2, -11] - f[2, -10] - f[2, -9] + f[2, 1] + f[2, 2] + f[2, 3] +
f[2, 4] + f[2, 5] + f[2, 6] - f[3, -17] - f[3, -16] - f[3, -15] - f[3, -14] - f[3, -13] -
f[3, -12] - f[3, -11] + f[3, -2] + f[3, -1] + f[3, 0] + f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] -
f[4, -20] - f[4, -19] - f[4, -18] - f[4, -17] - f[4, -16] - f[4, -15] - f[4, -14] -
f[4, -13] + f[4, -5] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 2] → f[-3, 16] - f[-2, -1] - f[-2, 0] + f[-2, 13] + f[-2, 14] - f[-1, -4] -
f[-1, -3] - f[-1, -2] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, -7] - f[0, -6] -
f[0, -5] - f[0, -4] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -10] - f[1, -9] -
f[1, -8] - f[1, -7] - f[1, -6] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -13] -
f[2, -12] - f[2, -11] - f[2, -10] - f[2, -9] - f[2, -8] + f[2, 1] + f[2, 2] + f[2, 3] +
f[2, 4] + f[2, 5] + f[2, 6] - f[3, -16] - f[3, -15] - f[3, -14] - f[3, -13] - f[3, -12] -
f[3, -11] - f[3, -10] + f[3, -2] + f[3, -1] + f[3, 0] + f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] -
f[4, -19] - f[4, -18] - f[4, -17] - f[4, -16] - f[4, -15] - f[4, -14] - f[4, -13] -
f[4, -12] + f[4, -5] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 3] → 3 + f[-3, 16] - f[-2, 0] - f[-2, 1] + f[-2, 13] + f[-2, 14] - f[-1, -3] -
f[-1, -2] - f[-1, -1] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, -6] - f[0, -5] -
f[0, -4] - f[0, -3] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -9] - f[1, -8] -
f[1, -7] - f[1, -6] - f[1, -5] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -12] -
f[2, -11] - f[2, -10] - f[2, -9] - f[2, -8] - f[2, -7] + f[2, 1] + f[2, 2] + f[2, 3] +
f[2, 4] + f[2, 5] + f[2, 6] - f[3, -15] - f[3, -14] - f[3, -13] - f[3, -12] - f[3, -11] -

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f[3, -10] - f[3, -9] + f[3, -2] + f[3, -1] + f[3, 0] + f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] -
f[4, -18] - f[4, -17] - f[4, -16] - f[4, -15] - f[4, -14] - f[4, -13] - f[4, -12] -
f[4, -11] + f[4, -5] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 4] → 3 + f[-3, 16] - f[-2, 1] - f[-2, 2] + f[-2, 13] + f[-2, 14] - f[-1, -2] -
f[-1, -1] - f[-1, 0] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, -5] - f[0, -4] - f[0, -3] -
f[0, -2] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -8] - f[1, -7] - f[1, -6] -
f[1, -5] - f[1, -4] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -11] -
f[2, -10] - f[2, -9] - f[2, -8] - f[2, -7] - f[2, -6] + f[2, 1] + f[2, 2] + f[2, 3] +
f[2, 4] + f[2, 5] + f[2, 6] - f[3, -14] - f[3, -13] - f[3, -12] - f[3, -11] - f[3, -10] -
f[3, -9] - f[3, -8] + f[3, -2] + f[3, -1] + f[3, 0] + f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] -
f[4, -17] - f[4, -16] - f[4, -15] - f[4, -14] - f[4, -13] - f[4, -12] - f[4, -11] -
f[4, -10] + f[4, -5] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 5] → 3 + f[-3, 16] - f[-2, 2] - f[-2, 3] + f[-2, 13] + f[-2, 14] - f[-1, -1] -
f[-1, 0] - f[-1, 1] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, -4] - f[0, -3] - f[0, -2] -
f[0, -1] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -7] - f[1, -6] - f[1, -5] -
f[1, -4] - f[1, -3] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -10] -
f[2, -9] - f[2, -8] - f[2, -7] - f[2, -6] - f[2, -5] + f[2, 1] + f[2, 2] + f[2, 3] +
f[2, 4] + f[2, 5] + f[2, 6] - f[3, -13] - f[3, -12] - f[3, -11] - f[3, -10] - f[3, -9] -
f[3, -8] - f[3, -7] + f[3, -2] + f[3, -1] + f[3, 0] + f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] -
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f[4, -9] + f[4, -5] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 6] → 3 + f[-3, 16] - f[-2, 3] - f[-2, 4] + f[-2, 13] + f[-2, 14] - f[-1, 0] -
f[-1, 1] - f[-1, 2] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, -3] - f[0, -2] - f[0, -1] -
f[0, 0] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -6] - f[1, -5] - f[1, -4] -
f[1, -3] - f[1, -2] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -9] - f[2, -8] -
f[2, -7] - f[2, -6] - f[2, -5] - f[2, -4] + f[2, 1] + f[2, 2] + f[2, 3] + f[2, 4] + f[2, 5] +
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f[4, -14] - f[4, -13] - f[4, -12] - f[4, -11] - f[4, -10] - f[4, -9] - f[4, -8] +
f[4, -5] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 7] → f[-3, 16] - f[-2, 4] - f[-2, 5] + f[-2, 13] + f[-2, 14] - f[-1, 1] - f[-1, 2] -
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f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -5] - f[1, -4] - f[1, -3] - f[1, -2] -
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f[3, -2] + f[3, -1] + f[3, 0] + f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] - f[4, -14] -
f[4, -13] - f[4, -12] - f[4, -11] - f[4, -10] - f[4, -9] - f[4, -8] - f[4, -7] +
f[4, -5] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 8] → f[-3, 16] - f[-2, 5] - f[-2, 6] + f[-2, 13] + f[-2, 14] - f[-1, 2] - f[-1, 3] -
f[-1, 4] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, -1] - f[0, 0] - f[0, 1] - f[0, 2] +
f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -4] - f[1, -3] - f[1, -2] - f[1, -1] -
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f[2, -4] - f[2, -3] - f[2, -2] + f[2, 1] + f[2, 2] + f[2, 3] + f[2, 4] + f[2, 5] +
f[2, 6] - f[3, -10] - f[3, -9] - f[3, -8] - f[3, -7] - f[3, -6] - f[3, -5] - f[3, -4] +
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f[4, -12] - f[4, -11] - f[4, -10] - f[4, -9] - f[4, -8] - f[4, -7] - f[4, -6] +
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f[-3, 9] → -3 + f[-3, 16] - f[-2, 6] - f[-2, 7] + f[-2, 13] + f[-2, 14] - f[-1, 3] - f[-1, 4] -
f[-1, 5] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, 0] - f[0, 1] - f[0, 2] - f[0, 3] + f[0, 7] +
f[0, 8] + f[0, 9] + f[0, 10] - f[1, -3] - f[1, -2] - f[1, -1] - f[1, 0] - f[1, 1] + f[1, 4] +
f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -6] - f[2, -5] - f[2, -4] - f[2, -3] - f[2, -2] -
f[2, -1] + f[2, 1] + f[2, 2] + f[2, 3] + f[2, 4] + f[2, 5] + f[2, 6] - f[3, -9] - f[3, -8] -
f[3, -7] - f[3, -6] - f[3, -5] - f[3, -4] - f[3, -3] + f[3, -2] + f[3, -1] + f[3, 0] + f[3, 1] +
f[3, 2] + f[3, 3] + f[3, 4] - f[4, -12] - f[4, -11] - f[4, -10] - f[4, -9] - f[4, -8] -

```

```

f[4, -7] - f[4, -6] + f[4, -4] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 10] → -3 + f[-3, 16] - f[-2, 7] - f[-2, 8] + f[-2, 13] + f[-2, 14] - f[-1, 4] -
f[-1, 5] - f[-1, 6] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, 1] - f[0, 2] - f[0, 3] -
f[0, 4] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -2] - f[1, -1] - f[1, 0] - f[1, 1] -
f[1, 2] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -5] - f[2, -4] - f[2, -3] -
f[2, -2] - f[2, -1] - f[2, 0] + f[2, 1] + f[2, 2] + f[2, 3] + f[2, 4] + f[2, 5] + f[2, 6] -
f[3, -8] - f[3, -7] - f[3, -6] - f[3, -5] - f[3, -4] - f[3, -3] + f[3, -1] + f[3, 0] +
f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] - f[4, -11] - f[4, -10] - f[4, -9] - f[4, -8] -
f[4, -7] - f[4, -6] + f[4, -3] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 11] → -2 + f[-3, 16] - f[-2, 8] - f[-2, 9] + f[-2, 13] + f[-2, 14] - f[-1, 5] -
f[-1, 6] - f[-1, 7] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, 2] - f[0, 3] - f[0, 4] -
f[0, 5] + f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, -1] - f[1, 0] - f[1, 1] - f[1, 2] -
f[1, 3] + f[1, 4] + f[1, 5] + f[1, 6] + f[1, 7] + f[1, 8] - f[2, -4] - f[2, -3] - f[2, -2] -
f[2, -1] - f[2, 0] + f[2, 2] + f[2, 3] + f[2, 4] + f[2, 5] + f[2, 6] - f[3, -7] - f[3, -6] -
f[3, -5] - f[3, -4] - f[3, -3] + f[3, 0] + f[3, 1] + f[3, 2] + f[3, 3] + f[3, 4] - f[4, -10] -
f[4, -9] - f[4, -8] - f[4, -7] - f[4, -6] + f[4, -2] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 12] → -2 + f[-3, 16] - f[-2, 9] - f[-2, 10] + f[-2, 13] + f[-2, 14] - f[-1, 6] -
f[-1, 7] - f[-1, 8] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, 3] - f[0, 4] - f[0, 5] - f[0, 6] +
f[0, 7] + f[0, 8] + f[0, 9] + f[0, 10] - f[1, 0] - f[1, 1] - f[1, 2] - f[1, 3] + f[1, 5] +
f[1, 6] + f[1, 7] + f[1, 8] - f[2, -3] - f[2, -2] - f[2, -1] - f[2, 0] + f[2, 3] + f[2, 4] +
f[2, 5] + f[2, 6] - f[3, -6] - f[3, -5] - f[3, -4] - f[3, -3] + f[3, 1] + f[3, 2] + f[3, 3] +
f[3, 4] - f[4, -9] - f[4, -8] - f[4, -7] - f[4, -6] + f[4, -1] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 13] → -1 + f[-3, 16] - f[-2, 10] - f[-2, 11] + f[-2, 13] + f[-2, 14] - f[-1, 7] -
f[-1, 8] - f[-1, 9] + f[-1, 10] + f[-1, 11] + f[-1, 12] - f[0, 4] - f[0, 5] - f[0, 6] +
f[0, 8] + f[0, 9] + f[0, 10] - f[1, 1] - f[1, 2] - f[1, 3] + f[1, 6] + f[1, 7] + f[1, 8] -
f[2, -2] - f[2, -1] - f[2, 0] + f[2, 4] + f[2, 5] + f[2, 6] - f[3, -5] - f[3, -4] - f[3, -3] +
f[3, 2] + f[3, 3] + f[3, 4] - f[4, -8] - f[4, -7] - f[4, -6] + f[4, 0] + f[4, 1] + f[4, 2],
f[-3, 14] → -1 + f[-3, 16] - f[-2, 11] - f[-2, 12] + f[-2, 13] + f[-2, 14] - f[-1, 8] -
f[-1, 9] + f[-1, 11] + f[-1, 12] - f[0, 5] - f[0, 6] + f[0, 9] + f[0, 10] -
f[1, 2] - f[1, 3] + f[1, 7] + f[1, 8] - f[2, -1] - f[2, 0] + f[2, 5] + f[2, 6] -
f[3, -4] - f[3, -3] + f[3, 3] + f[3, 4] - f[4, -7] - f[4, -6] + f[4, 1] + f[4, 2],
f[-3, 15] → f[-3, 16] - f[-2, 12] + f[-2, 14] - f[-1, 9] + f[-1, 12] - f[0, 6] +
f[0, 10] - f[1, 3] + f[1, 8] - f[2, 0] + f[2, 6] - f[3, -3] + f[3, 4] - f[4, -6] + f[4, 2]}}

```

Note that the solutions depend on which and how many equations of the form "Sum[f[Q, 10 - 2 Q], {Q, -4, 4}] = 0" we add. For example, if there are no equations of the form LHS == 0 to be solved, then the solution produced by *Mathematica* with the maximal number of f's zero is:

```

In[8]:= {f[-4, 1] → -1, f[-4, 3] → -1, f[-4, 5] → 2, f[-4, 7] → 1, f[-4, 9] → -1, f[-4, 11] → -1,
f[-4, 13] → -1, f[-4, 15] → -1, f[-4, 17] → -1, f[-3, 2] → -3, f[-3, 6] → 3, f[-3, 8] → 3};

```

However, if we solve all the equations in Solve[...] as written above, the *Mathematica* gives this as the solution with the maximal number of f's zero:

```

In[9]:= {f[-4, 1] → f[-4, 2] → 1, f[-4, 5] → -1, f[-4, 6] → -3, f[-4, 7] → -2,
f[-4, 8] → -3, f[-4, 9] → -1, f[-4, 11] → 2, f[-4, 12] → 3, f[-4, 13] → 2,
f[-4, 14] → 2, f[-4, 15] → 1, f[-4, 16] → 1, f[-3, -1] → -1, f[-3, 0] → -1,
f[-3, 3] → 3, f[-3, 4] → 3, f[-3, 5] → 3, f[-3, 6] → 3, f[-3, 9] → -3,
f[-3, 10] → -3, f[-3, 11] → -2, f[-3, 12] → -2, f[-3, 13] → -1, f[-3, 14] → -1};

```

Strangely, in both cases only f's with Q=-4 or Q=-3 appear, and "large" values like 3 appear.

Anyhow, let's now try to proceed as usual, i.e. write down what U should be for arbitrary n=N. For t^r-monomials with r<5 we do this by hand as before, and for r=6 we use the solution in the "paragraph" above, i.e.:

```

In[10]:= For[Q = -4, Q ≤ 4, Q++, For[s = -23, s ≤ 23, s++, f[Q, s] = 0]];
f[-4, 2] = 1;
f[-4, 5] = -1;
f[-4, 6] = -3;
f[-4, 7] = -2;
f[-4, 8] = -3;
f[-4, 9] = -1;
f[-4, 11] = 2;
f[-4, 12] = 3;
f[-4, 13] = 2;
f[-4, 14] = 2;
f[-4, 15] = 1;
f[-4, 16] = 1;
f[-3, -1] = -1;
f[-3, 0] = -1;
f[-3, 3] = 3;
f[-3, 4] = 3;
f[-3, 5] = 3;
f[-3, 6] = 3;
f[-3, 9] = -3;
f[-3, 10] = -3;
f[-3, 11] = -2;
f[-3, 12] = -2;
f[-3, 13] = -1;
f[-3, 14] = -1;

```

```

In[35]:= UU[n_] := Expand[- q^(0 n + 5) + q^(2 n + 5) - q^(0 n + 3) t^2 + q^(2 n + 1) t^2 -
q^(-2 n + 3) t^3 + q^(0 n + 1) t^3 - q^(0 n - 1) t^4 + q^(2 n - 3) t^4 - q^(-2 n - 1) t^5 +
q^(0 n - 3) t^5 + t^6 Sum[f[Q, s] q^(n Q + s), {Q, -4, 4}, {s, -23, 23}]]

```

This function indeed reproduces the experimental results above (note however that I could only find the right t^6 -term by including sufficiently many LHS == 0 terms in Solve[...] -- it might still be that there are non-vanishing $D_{\{Q,s,r\}}$ missing):

```

In[38]:= {UU[2] - U2 /. {a → 2 lk}, UU[3] - U3 /. {a → 2 lk}}

```

```

Out[38]:= {0, 0}

```

Now we use [GSV, eq. (5.5)] with $a=\alpha=6$ to define a candidate for the KR invariants for arbitrary n :

```

In[39]:= KR[n_] := Expand[FullSimplify[Expand[Simplify[(q^(-2 n lk)
(t^a Simplify[(q^n - q^(-n)) / (q - 1 / q))^2] + UU[n] / (q - 1 / q)) /. {a → 6}]]]]

```

Check that we have not made a mistake so far:

```

In[61]:= {KR[2] - KR2, KR[3] - KR3}

```

```

Out[61]:= {0, 0}

```

That looks very good. Now I went on to write down the general candidate formula for KR_n , but then I noticed that we actually have an $N=4$ result here against which $KR[4]$ must be checked. OK, so our result is:

```

In[62]:= KR4 =
Expand[(t^6 q^(10) + 2 t^6 q^8 + 3 t^6 q^6 + 3 t^6 q^4 + 2 t^6 q^2 + t^6 + t^5 q^6 + t^5 q^4 +
t^5 q^2 + t^4 q^(14) + t^4 q^(12) + t^4 q^(10) + t^3 q^(10) + t^3 q^8 + t^3 q^6 +
t^2 q^(18) + t^2 q^(16) + t^2 q^(14) + q^(22) + q^(20) + q^(18) + q^(16)) / (q^(34))];

```

Now the moment of truth:

```

In[63]:= KR[4] - KR4

```

```

Out[63]:= - t^6 / q^37 + t^6 / q^36 + t^6 / q^34 + 3 t^6 / q^33 - t^6 / q^32 + 3 t^6 / q^31 - 3 t^6 / q^30 - 4 t^6 / q^28 - 3 t^6 / q^27 - t^6 / q^26 - 2 t^6 / q^25 + 2 t^6 / q^24 - t^6 / q^23 + 3 t^6 / q^22 + 2 t^6 / q^20 + t^6 / q^18

```


ARRRGGGHH!!!

So this means that if the GSV trick is applicable here, I would have to add a lot more $LHS == 0$ equations to solve above. I don't want to do that, there is too much to be done by hand the way I went about it.