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ECE4960
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ECE 4960 Spring 2018: Computational and Software Engineering
Reading 2: Differentiation in Local Analysis
Deposit a pdf file of the two tables below to your Git directory before 11:59pm of 2/11

Document your programming environment: Language; development platform; operating system
Language: C++
Development Platform: XCode (GCC 9.0.0)
Operating System: MacOS 10.13.1 (High Sierra)

Prob. 1. (Quadratic function to observe the tradeoffs between the truncation error and round-off error): For $f(x) = x^2$, we know the exact $f'(x=1) = 2$.
1.1   Use Eq. (1) below to estimate $f'(x=1)$ varying the value of h from 0.1 to 10^{-18} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.
1.2   Repeat your calculation with $f(x) = x^2 + 10^8$. Add your results to the same table.
1.3   Repeat the above two procedures by using Eq. (2). Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + O(h) \qquad (1)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2) \quad (2)$$

Error values recorded here are absolute value error from the expected result 2.

h	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2 + 10^8$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2 + 10^8$
10^{-1}	0.1	0.0999999	4.44E-16	4.47E-08
10^{-2}	0.01	0.00999975	1.78E-15	4.17E-07
10^{-3}	0.001	0.00100243	1.65E-13	4.05E-06
10^{-4}	0.0001	3.39E-05	7.75E-13	4.07E-05
10^{-5}	1.00E-05	0.000264168	2.00E-12	0.000264168
10^{-6}	1.00E-06	0.0032444	2.00E-12	0.0032444
10^{-7}	1.01E-07	0.062849	5.75E-11	0.062849
10^{-8}	1.22E-08	0.509884	6.60E-09	0.509884
10^{-9}	1.65E-07	2	5.45E-08	2
10^{-10}	1.65E-07	2	1.65E-07	2
10^{-11}	1.65E-07	2	1.65E-07	2
10^{-12}	0.000177801	2	6.68E-05	2
10^{-13}	0.00159856	2	0.000488333	2
10^{-14}	0.00159856	2	0.00159856	2
10^{-15}	0.220446	2	0.109424	2
10^{-16}	2	2	0.889777	2
10^{-17}	2	2	2	2
10^{-18}	2	2	2	2

Prob. 2. (Cubic function to observe the Richardson error estimation): For $f(x) = x^3$, we know the exact value of $f'(x=1) = 3$.
2.1   Use Eqs. (3) – (5) below to estimate $f'(x=1)$ varying the value of h from 2^{-4} to 2^{-40} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.
2.2   Estimate η from Eqs. (6) and (7) for each choice of h. Add your results to the same table.

$$f'(x) = \frac{f(x+h) - f(x)}{h} + E(h); \quad E(h) = O(h) = \frac{1}{2}hf''(x) + O(h^2) \qquad (3)$$

$$f'(x) = \frac{f(x+2h) - f(x)}{2h} + E(2h); \quad E(2h) = O(h) = \frac{1}{2}2hf''(x) + O(h^2) \quad (4)$$

$$f'(x) = \frac{-1}{2h}f(x+2h) - \frac{3}{2h}f(x) + \frac{2}{h}f(x+h) + O(h^2) \quad (5)$$

$$R(h) \equiv \frac{E(2h)}{E(h)} \cong \eta \quad (6)$$

$$R(h) \cong \frac{\hat{A}(4h) - \hat{A}(2h)}{\hat{A}(2h) - \hat{A}(h)} \cong \eta \quad (7)$$

Error values recorded here are absolute value error from the expected result 3.

h	Error in $f'(x=1)$ by Eq. (3)	Error in $f'(x=1)$ by Eq. (4)	Error in $f'(x=1)$ by Eq. (5)	η by Eq. (6)	η by Eq. (7)
2^{-4}	0.191406	0.390625	0.0078125	2.04082	2.11765
2^{-5}	0.0947266	0.191406	0.00195312	2.02062	2.06061
2^{-6}	0.0471191	0.0947266	0.000488281	2.01036	2.03077
2^{-7}	0.0234985	0.0471191	0.00012207	2.00519	2.0155
2^{-8}	0.011734	0.0234985	3.05E-05	2.0026	2.00778
2^{-9}	0.00586319	0.011734	7.63E-06	2.0013	2.0039
2^{-10}	0.00293064	0.00586319	1.91E-06	2.00065	2.00195
2^{-11}	0.00146508	0.00293064	4.77E-07	2.00033	2.00098
2^{-12}	0.000732481	0.00146508	1.19E-07	2.00016	2.00049
2^{-13}	0.000366226	0.000732481	2.98E-08	2.00008	2.00024
2^{-14}	0.000183109	0.000366226	7.45E-09	2.00004	2.00012
2^{-15}	9.16E-05	0.000183109	1.86E-09	2.00002	2.00006
2^{-16}	4.58E-05	9.16E-05	4.66E-10	2.00001	2.00003
2^{-17}	2.29E-05	4.58E-05	1.16E-10	2.00001	2.00002
2^{-18}	1.14E-05	2.29E-05	0	2.00001	2.00001
2^{-19}	5.72E-06	1.14E-05	0	2	2.00001
2^{-20}	2.86E-06	5.72E-06	0	2	2
2^{-21}	1.43E-06	2.86E-06	0	2	2
2^{-22}	7.15E-07	1.43E-06	0	2	2
2^{-23}	3.58E-07	7.15E-07	0	2	2
2^{-24}	1.79E-07	3.58E-07	0	2	2
2^{-25}	8.94E-08	1.79E-07	0	2	2
2^{-26}	4.47E-08	8.94E-08	0	2	2
2^{-27}	2.98E-08	4.47E-08	0	1.5	3
2^{-28}	0	2.98E-08	0	inf	0.5
2^{-29}	0	0	0	nan	inf
2^{-30}	0	0	0	nan	nan
2^{-31}	0	0	0	nan	nan
2^{-32}	0	0	0	nan	nan
2^{-33}	0	0	0	nan	nan
2^{-34}	0	0	0	nan	nan
2^{-35}	0	0	0	nan	nan
2^{-36}	0	0	0	nan	nan
2^{-37}	0	0	0	nan	nan
2^{-38}	0	0	0	nan	nan
2^{-39}	0	0	0	nan	nan
2^{-40}	0	0	0	nan	nan