Irrigation Control Based on Model Predictive Control (MPC): Formulation of Theory and Validation Using Weather Forecast Data and AQUACROP Model

Article *in* Environmental Modelling & Software · April 2016

DOI: 10.1016/j.envsoft.2015.12.012

4 authors, including:


Dilini Delgoda
University of Melbourne


Hector M. Malano
University of Melbourne


Malka N. Halgamuge
RMIT University

Environmental Modelling & Software 78 (2016) 40–53

Contents lists available at ScienceDirect

Environmental Modelling & Software

journal homepage: www.elsevier.com/locate/envsoft

Irrigation control based on model predictive control (MPC): Formulation of theory and validation using weather forecast data and AQUACROP model

Dilini Delgoda [a, *], Hector Malano [a], Syed K. Saleem [b], Malka N. Halgamuge [b]

[a] Department of Infrastructure Engineering, The University of Melbourne, Parkville, Victoria, Australia
[b] Department of Electrical and Electronics Engineering, The University of Melbourne, Parkville, Victoria, Australia

ARTICLE INFO

Article history:
Received 11 September 2015
Received in revised form
8 December 2015
Accepted 11 December 2015
Available online 31 December 2015

Keywords:
Model predictive control (MPC)
Automation
Weather forecasts
Irrigation
Uncertainty
Simple model

ABSTRACT

This research proposes A THEORETICAL FRAMEWORK based on model predictive control (MPC) for irrigation control to minimize both root zone soil moisture deficit (RZSMD) and irrigation amount under a limited water supply. We (i) investigate means to incorporate direct measurements to MPC (ii) introduce two Robust MPC techniques – Certainty Equivalence control (CE) and Disturbance Affine Feedback Control (DA) – to mitigate the uncertainty of weather forecasts, and (iii) provide conditions to obtain two important theoretical aspects of MPC – feasibility and stability – in the context of irrigation control. Our results show that system identification enables automation while incorporating direct measurements. Both DA and CE minimize RZSMD and irrigation amount under uncertain weather forecasts and always maintain soil moisture above wilting point subject to water availability. The theoretical results are compared against the model AQUACROP, weather data and forecasts from Shepparton, Australia. We also discuss the performance of Robust MPC under different water availability, soil, crop conditions. In general, MPC shows to be a promising tool for irrigation control.

1. Introduction

From the perspective of a typical farmer, an ideal irrigation control system is one that looks ahead at the water availability and weather forecasts, and adjusts the present irrigation amount to reduce the irrigation demand. As root zone soil moisture level is strongly coupled with irrigation demand, this aim can be interpreted as maintaining a reference soil moisture level through the use of irrigation.

The term root zone soil moisture deficit (RZSMD) is defined as the difference between a given reference level and current root zone soil moisture level. As such, the ideal irrigation control method would be one that maintains the RZSMD close to zero, while minimizing irrigation amount subject to water availability and weather forecasts. Automation is required to achieve this goal as manual control is inadequate.

Automated irrigation control has been given considerable attention during the past decade. State-of-the-art technologies have been developed and tested (Allam (2002); Hibbs et al., (1992); Hornbuckle et al., (2009)). Some irrigation control methods depend on complex physical models, which closely resemble the actual physical system, based on principles of crop phenology, soil physics and hydrology (Steduto et al., (2009); Raes et al., (2009); Jones et al., (2003); Rossi et al., (2004)). Another important avenue is data assimilation. Data associated with proxy variables such as sap flow, stomatal conductance and trunk diameter are used to infer the soil water requirements (Lu et al., (2004); Kanemasu et al., (1969); Goldhamer and Fereres, (2003)). Irrigation decisions are based on these inferences or estimates which are almost always precise. Nevertheless, the underlying irrigation control logic is limited to only few categories. One method is to replenish the soil moisture when RZSMD or water demand exceeds a certain level. They are called rule-based or 'ON–OFF' category. Some methods follow a predefined irrigation schedule and belong to open-loop control methods. Former is reactive to current soil moisture conditions (closed-loop) however in both cases, the control method cannot utilize future weather information. In contrast, Giusti and Marsili-Libelli (2015) use weather forecasts and fuzzy rules for irrigation control, based on approximate fuzzy models of the complex physical model. Kia et al. (2009); Bahat et al. (2000); Zhang et al.

* Corresponding author.
E-mail address: d.delgoda@pgrad.unimelb.edu.au (D. Delgoda).

http://dx.doi.org/10.1016/j.envsoft.2015.12.012
1364-8152/

List of symbols and abbreviations

Abbreviation

MPC	model predictive control
RMPC	robust model predictive control
CE	Certainty Equivalence Control
DA	Disturbance Affine Feedback Control
RZSMD	root zone soil moisture deficit[1]
FC	field capacity [1]
RP	refill point [1]
WP	wilting point [1]
ISS	input to state stability

Symbol

D	current RZSMD
D^+	RZSMD at next time step
E	current crop evapotranspiration
P/P^e	current rainfall/effective rainfall
I/I^e	current irrigation amount/effective irrigation amount
x	system state
u	control input
w	disturbance
N	control horizon
H	water holding capacity
S,Q,R	weights on MPC objectives
$\langle c\rangle_{max}$	upper bound on variable $\langle c\rangle$
$\langle c\rangle_{min}$	lower bound on variable $\langle c\rangle$
$\langle\tilde{c}\rangle$	estimate of variable $\langle c\rangle$
$\mathbb{R}$	real number set
X_f	target set

(1996) use fuzzy rules on simple evapotranspiration models. However, considering the objectives and constraints mentioned previously, defining fuzzy rules is a difficult task which demands perfect knowledge on the system and most of the time the decision making becomes ad-hoc. In all the above cases, no attention is given to optimizing the irrigation amount and in most occasions, the methods assume an unlimited supply of water to the field.

On the contrary, model predictive control (MPC) based irrigation control systems are proactive in that they aim to achieve a desired soil moisture level by adjusting the present irrigation amount. Examples include Park et al. (2009); McCarthy et al. (2014); Romero Vicente et al. (2011); Romero et al. (2008) where receding horizon control based on complex physical models are used to optimize irrigation. All except McCarthy et al. (2014) used nonlinear optimization which utilized trial and error method. All methods need a high level of calibration due to use of the complex physical model. If not properly analyzed, constrained nonlinear optimization can be infeasible or suboptimal, making the optimization process redundant and irrigation control unreliable. This could be further complicated by uncertainty in rainfall which none of the methods have considered and actual weather data are used instead of forecasts to test the method.

Some works use dynamic programming to optimize inter-seasonal and intra-seasonal water allocations subject to seasonal water limitations (Bras and Cordova (1981); Dudley et al. (1971); Yaron and Dinar (1982); Rao et al. (1992); Protopapas and Georgakakos (1990); Sunantara and Ramirez (1997)). Nonetheless, getting no feedback on crop soil conditions during the calculation stages, could lead to large propagation errors. Among these, the work in Bras and Cordova (1981); Protopapas and Georgakakos (1990); Sunantara and Ramirez (1997) and references therein consider rainfall forecasts and their uncertainty. However, due to the incorporation of a closed form expression for the rainfall forecast, the optimization becomes nonlinear and no guarantee can be given on the reliability of irrigation control. Uncertainty only in crop evapotranspiration is considered in Aboitiz and Labadie (1986).

In Saleem et al. (2013), we propose to use MPC, based on a system model which is a simplified water balance model, that captures main dynamics of the actual physical system. The irrigation control problem is solved by minimizing both the irrigation amount and RZSMD. It also considers maximum allowed irrigation amount and maximum and minimum soil moisture deficits. In the current paper, we extend our previous work presented in Saleem et al. (2013) and propose a theoretical framework based on MPC for irrigation control.

The MPC approach and that in Giusti and Marsili-Libelli (2015) are equivalent in that both use simplified system models adequately representative of the actual physical system instead of complex physical models and both control methodologies are based on these system models. Using these approximate models reduce the calibration requirement significantly. However, authors of Giusti and Marsili-Libelli (2015) do not focus on the control action and their approach does not optimize RZSMD. This can be attributed to the ad-hoc manner of defining the rules. Further, the method assumes an unlimited amount of water supply to the field. In other words, the control method in Giusti and Marsili-Libelli (2015) minimizes the total irrigation amount subject to a given soil moisture threshold. The MPC method described herein (1) minimizes RZSMD and daily irrigation amount (and reduces total amount subsequently) (2) subject to daily irrigation water availability and RZSMD thresholds.

Saleem et al. (2013) introduced a few assumptions when developing the MPC approach which are removed in this paper. It was assumed in Saleem et al. (2013), that the effective values of all variables in water balance model are known, when in reality they are not. In this study, we propose to use system identification so that direct measurements can be incorporated into MPC to accommodate online calculations.

Second, Saleem et al. (2013) used actual rainfall data as weather forecasts which removes the uncertainty in weather forecasts. We now relax this assumption to match the real field application by designing MPC to accommodate uncertainty in weather forecasts. In this regard, we consider and compare two MPC formulations that are well studied in the area of MPC under uncertainty in disturbance also known as 'robust MPC' (RMPC): Certainty Equivalence Control (CE) and Disturbance Affine Feedback Control (DA).

Third, it was assumed in Saleem et al. (2013) that the MPC is feasible and (possibly) stable at all times. In this paper, we first explain how these aspects are important in irrigation control then discuss how they can be guaranteed.

A case study is selected to verify the theory developed in this research. The weather data required are obtained from The Bureau of Meteorology (BoM), Australia for Shepparton, Victoria, Australia. The model AQUACROP (FAO (2011); Steduto et al. (2009); Raes et al. (2009)) by United Nations Food and Agriculture Organization was used to simulate the actual physical system.

Section 2 describes the formulation of irrigation control using MPC.

[1] *volume/volume%* converted to *mm*.

Section 3 introduces system identification to the model and explains the formulation of RMPC based irrigation control under uncertainty in weather forecasts. This is followed by the analysis of feasibility and stability and related modifications. In Section 4, the demonstration of the proposed theoretical framework under different levels of water availability, crop and soil conditions are discussed through simulations. Section 5 contains the concluding remarks.

2. Structure of MPC

This section summarizes the concepts presented in Saleem et al. (2013) for applying MPC in irrigation control.

2.1. System model

Using a simplified system model to represent the actual system is common in MPC. We use a simplified water balance model as the system model for irrigation control. Given D as RZSMD at the current time step, RZSMD at the next time step is obtained by,

$$D^{+} = D + E^{*} - P^{e} - I^{e} \tag{1}$$

where E^* is the crop evapotranspiration, I^e is the effective irrigation amount and P^e is the effective rainfall. Runoff and deep percolation are neglected assuming negligible amounts of gravitational flows during the control phase. This simple model is a first order linear time series in which all the time steps are in days, although this could be an integer multiple of days.

Based on Equation (1), a state space model is formulated such that the state at the next time step is given by,

$$x^{+} = Ax + Bu + \omega \tag{2}$$

where $x=D$, input $u=I^e$, disturbance $\omega=E^*-P^e$, $A=1$ and $B=-1$.

2.2. Control objective

The objective of the proposed irrigation control method is to maintain RZSMD close to zero (i.e. minimize RZSMD) throughout the time period known as the control horizon, while minimizing the amount of irrigation water use. This leads to the following (multi) objective optimization function that is the standard equation in MPC (Wang (2009); Goulart et al. (2006)):

$$J = \underset{u=u_0:u_N}{\text{minimize}} \left[\sum_{i=0}^{N-1} \frac{1}{2} \left(x_i^2 Q + u_i^2 R \right) + \frac{1}{2} x_N^2 S \right] \tag{3}$$

where S,Q and R are positive numbers, i is the time step number and N is the control horizon. This optimization is repeated for the time steps $1 \cdots N_o$ where N_o is the operational period (e.g. the length of the growing season). The value $\frac{1}{2}x_N^2 S$ is called a terminal cost. This is well known in MPC literature as a term that guarantees stability in an undisturbed control system. When the system is disturbed, we use the notion 'Input to State Stability' (ISS). Stability gives an indication of how close the system state is to the target state. In a stable undisturbed system, the state would converge asymptotically, whereas under ISS the state always remains bounded (Mayne et al., (2005)).

2.3. System constraints

The root zone soil moisture level is considered to be optimal if it is maintained at or near to a point called field capacity (FC). If it goes below the so called permanent wilting point (WP), plants would wilt and the damage to growth and yield is irreversible. If it exceeds the FC, gravitational flows will occur causing water waste and plant damage. Thus, there will be two safety levels above and below the zero RZSMD as x_{max} and x_{min}. As zero RZSMD we select a point RP which is close to but below FC. Note that the volumetric soil moisture contents must be converted into irrigation depth, based on the plant root depth.

The upper limit on irrigation amount u_{max} depends on the water supply to the field through delivery. This can be either the actual volume, depth or even a rate defined on 'volume or depth per day' basis. The minimum value of irrigation amount u_{min} is always zero. These provide the system constraints,

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} x \le \begin{bmatrix} x_{max} \\ -x_{min} \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} u \le \begin{bmatrix} u_{max} \\ -u_{min} \end{bmatrix}$$

where x_{max} = [reference soil moisture level (RP) − lowest allowable soil moisture level (e.g. WP)]; x_{min}=[reference soil moisture level − highest allowable soil moisture level (e.g. FC)]; u_{max}=[maximum irrigation depth/amount per day]; u_{min}=0. This is equivalent to

$$\mathbf{C}x + \mathbf{D}u \le b \tag{4}$$

where $\mathbf{C} = [1 \quad -1 \quad 0 \quad 0]^T$, $\mathbf{D} = [0 \quad 0 \quad 1 \quad -1]^T$ and $b = [x_{max} \quad (-x_{min}) \quad u_{max} \quad (-u_{min})]^T$. Imposing time variable state constraints create more challenges in MPC. Thus we set constant values as state constraints (e.g. $x_{max}=-x_{min}$=50mm) more conservatively in a narrower range such that they will accommodate possible variations of WP and FC over time.

Note that the dimensions of the above variables are specific to the irrigation control problem and therefore are scalars unless otherwise mentioned. Vectors are denoted with bold letters belonging to real valued set (e.g. vector $A \in \mathbb{R}^d$) where d denotes the dimension.

Given the state space model in Equation (2), the goal is to solve the optimization problem in Equation (3) subject to the system constraints given by Equation (4). The steps to solve the optimization problem are detailed in Supplementary Section A. The first element of the resultant input $\boldsymbol{U}$ (defined in Supplementary Section A) is taken as the irrigation amount that should be applied on day i=0. This is the receding horizon control principle which would help to overcome the effects of small nonlinearities in the actual physical system. The next day is reset as day i=0 and the optimization process is repeated.

Other goals related to RZSMD can also be optimized by calculating the target RZSMD value exogenously. MPC can be applied to precise water application methods such as drip or sprinkler systems and can also be adapted to a surface irrigation system if the gross irrigation amount (considering the additional water amount to account for losses) can be inferred from the target net amount. It is also applicable together with modern technology such as sensor and actuator based automatic systems. Next section explains our contributions in this paper to improve MPC in irrigation control as it seems to be a promising tool in future.

3. Materials and methods

In this section, we discuss removing the assumptions explained in Section 1 (Introduction): (i) perfect knowledge of water balance variables, (ii) perfect knowledge of future weather and (iii) guaranteed feasibility of optimization and stability of the system.

3.1. Incorporating direct measurements to MPC

When using the system model in Equation (1), it is necessary to know the effective values of rainfall, irrigation and crop evapotranspiration. An efficient way of calculating these effective values is important for the purpose of automatic irrigation control.

In Delgoda et al. (2015), we propose a linear first order time series model based on system identification from raw data in place of Equation (1) as,

$$D^{+} = c_1 D + c_2 E + c_3 P + c_4 I \qquad (5)$$

where $c_i{:}i \in [1,4]$ are coefficients, E is the calculated crop evapotranspiration using methods as in Allen et al. (1998), P and I are measured rainfall and irrigation respectively. Coefficients c_1, c_2, c_3 and c_4 represent contribution of RZSMD from the previous day, correction factor on calculated crop evapotranspiration, effectiveness of rainfall and efficiency of irrigation respectively. They must be estimated from raw data through system identification.

In essence, Equation (5) is equivalent to Equation (1) except that raw values can be directly input into Equation (5). As Equation (5) is a linear time series, 'the linear least square method' is used to identify the coefficients $c_i{:}i \in [1,4]$. If there is measurement noise in the soil moisture readings, the 'prediction error method' should be applied to identify the above coefficients. An additional step needs to be followed if there are large gravitational flows occurring due to saturation. Details on these methods are given in Ljung (1999) and Delgoda et al. (2015) and summarized in Supplementary Section B. These coefficients are assumed to be constant over a short period of time such as the growing season.

Performance of the model is verified using AQUACROP and field data based on the tests explained in Delgoda et al. (2015) and listed in Section 3.4. Once the system parameters $c_i{:}i \in [1,4]$ are identified through one of the above system identification methods, we can build a state space model as in Equation (2). Now $x=D$, $u=I$, $w=c_2E+c_3P$, $A=c_1$ and $B=c_4$.

3.2. Addressing uncertainty in weather forecasts

Disturbance to irrigation system is driven by rainfall and crop evapotranspiration. To apply MPC on a given day, a weather forecast for N days in advance is required. Reference evapotranspiration has a seasonal nature thus its trend is more predictable. Fig. 1(a) shows the deviation of reference evapotranspiration from a basic seasonal forecast obtained using the method outlined in Supplementary Section C. There are more accurate methods to estimate reference evapotranspiration (Perera et al. (2014); Aboitiz and Labadie (1986); Or and Hanks (1992)). Thus, the uncertainty in crop evapotranspiration (product of crop coefficient and reference evapotranspiration) can be reduced considerably. Rainfall forecasts are more uncertain than crop evapotranspiration. Fig. 1(b) shows a forecast from the ACCESS-G model of ACCESS Numerical Weather Prediction (NWP) system of the BoM, Australia. Despite that ACCESS-G being a sophisticated model, forecasts deviate from the true values at a considerable number of times. In Saleem et al. (2013) we used actual weather data as forecasts, thus, there was no uncertainty in them. However, this is not the real situation and irrigation control based on MPC should be able to accommodate the uncertainties in the weather forecasts.

As mentioned above, predicting crop evapotranspiration is straightforward and gives numerical values. The proposed method uses numerical rainfall forecasts due to a few reasons: (i) As explained in Section 1, incorporating rainfall forecasts to the model using probability mass functions (as in Sunantara and Ramirez (1997)) can do more harm than good due to its mathematical limitations; (ii)

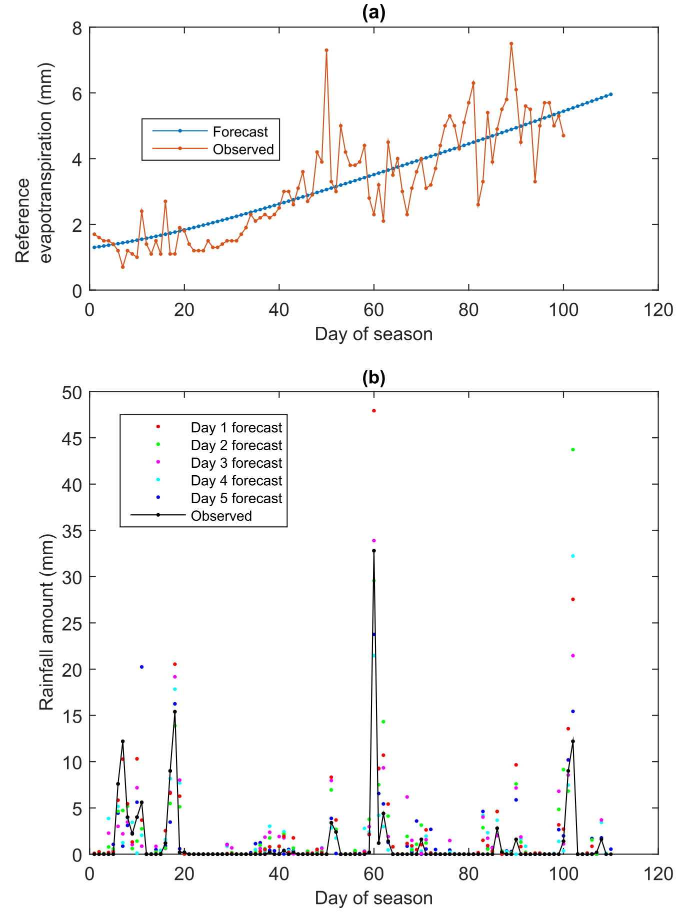

Fig. 1. Uncertainty in weather forecasts compared to the actual situation (Shepparton, Australia: Year 2011) (a) A basic seasonal forecast on reference evapotranspiration. (b) A deterministic rainfall forecast from the ACCESS-G model of ACCESS Numerical Weather Prediction system of The Bureau of Meteorology Australia. Day n forecast is the rainfall forecast for a given day, n days in advance.

More recent methods of rainfall forecasts are more accurate than the values based on these expressions and are readily available (BoM (2010, 2012)); (iii) Directly incorporating a single numerical value instead of a mathematical expression for rainfall amount in Equation (2) preserves the linear structure of the model making the optimization problem in MPC easy to analyze using well established linear MPC principles; (iv) If the MPC method is formulated such that it accepts numerical rainfall forecasts, numerical output from any rainfall forecast model can be fed to it making the MPC method independent of the rainfall forecast method. Considering these advantages, it is appropriate to use numerical weather forecasts for rainfall and evapotranspiration in MPC.

An MPC system which meets the performance specifications under uncertainty in disturbance is called Robust MPC (RMPC). A number of RMPCs are described in the literature (Bemporad and Morari (1999); Kothare et al. (1996); Langson et al. (2004); Mayne et al. (2005)), however, the focus of this paper is on two methods: Certainty Equivalence Control and Disturbance Affine Feedback Control.

3.2.1. Certainty Equivalence Control (CE)

In this type of control, it is assumed that the future disturbances to the system are known with certainty. The optimization problem is solved using the estimated disturbance that is assumed to be perfect. The form of MPC used in Saleem et al. (2013) is CE, however, using actual data in place of estimates. In this paper, we make deterministic estimates of the future disturbance based on weather forecasts, an approach that warrants further investigation.

The formulations described in Section 2 are applied here directly. However, deterministic forecasts of evapotranspiration $\widetilde{\mathbf{E}}_a$ and rainfall $\widetilde{\mathbf{P}}$ calculated according to Supplementary Section C are used to estimate the disturbance $\mathbf{w}\in\mathbb{R}^N$ such that,

$$\mathbf{w} = c_2\widetilde{\mathbf{E}}_a + c_3\widetilde{\mathbf{P}}. \tag{6}$$

Past values of $\mathbf{w}$ are updated using actual weather data.

3.2.2. Disturbance Affine Feedback Control (DA)

In this approach, the current and future values of $\mathbf{w}$ are unknown and may change unpredictably from one time period to the next. However, these values are assumed to be contained in a convex and compact (closed and bounded) set $\mathscr{W}$ that also contains the origin. The control input is calculated as an affine function of past disturbances as,

$$u_i = \sum_{j=0}^{i-1} M_{i,j}w_j + v_i \tag{7}$$

where $\mathbf{M}\in\mathbb{R}^{N\times N}$ and $\mathbf{v}\in\mathbb{R}^N$ are 'disturbance to input gain matrix' (lower triangular) and an offset vector, respectively.

DA allows solving the optimization problem using convex programming. The definition of the DA formulation requires that the **input set contains the origin in its interior**. However, as the irrigation amount cannot be negative, the requirement cannot be fulfilled in our system. Despite this, the DA method has been successfully applied in two situations (Kearney and Cantoni (2012); Oldewurtel et al. (2010)) with non-negative inputs. We use a similar approach in this DA application and the implications are addressed in Section 3.3. Given the nature of our system, the bounds of the disturbance set $\mathscr{W}$ can be expressed as a polytope such that,

$$\mathscr{W} = \{\mathbf{w}|\mathbf{S}\mathbf{w} \le \mathbf{h}\} \tag{8}$$

where $\mathbf{S} = [1 \quad -1]^T$, $\mathbf{h} = [\mathbf{w}_{max} \quad (-\mathbf{w}_{min})]^T$, $\mathbf{w}_{max}{=}c_2\mathbf{E}_{max}{+}c_3\mathbf{P}_{min}$, $\mathbf{w}_{min}{=}c_2\mathbf{E}_{min}{+}c_3\mathbf{P}_{max}$, the postscripts *min* and *max* denote extremes of the corresponding variable and $\mathbf{E}_{max}$,$\mathbf{P}_{max}$,$\mathbf{E}_{min}$,$\mathbf{P}_{min}\in\mathbb{R}^N$.

It must be pointed out that $\mathbf{w}_{max}$ is always positive and $\mathbf{w}_{min}$ is always negative or zero. The lower bounds of disturbance are smoothed out (constraint tightening) in order to maintain the disturbance set convex and compact. This means that $\mathbf{w}_{max}$ and $\mathbf{w}_{min}$ are the maximum and minimum of N number of days ahead, respectively. It is obvious that this step would make the control more conservative. Next, we find the admissible pair ($\mathbf{M}^*(x)$,$\mathbf{v}^*(x)$) that optimizes Equation (3) subject to Equation (4) and calculate the optimal input u using Equation (7). However, as this is a receding horizon control problem, we calculate u for i=0 only by making u=v_0. These steps from Goulart et al. (2006) are summarized as follows:

$$u = v_0^*$$

where

$$(\mathbf{M}^*(x), \mathbf{v}^*(x)) = \underset{(M,v)\in\Pi(x,w)}{argmin} \quad J \tag{9}$$

and

$$\Pi(x,w) = \left\{ (\mathbf{M},\mathbf{v}) \left| \begin{array}{c} (2),(4),(7) \text{ hold} \\ x = x_0 \quad x_N\in X_f \\ \text{for all } w\in W \end{array} \right. \right\}. \tag{10}$$

Goulart et al. (2006) explain how to solve for optimal ($\mathbf{M}^*(x)$,$\mathbf{v}^*(x)$) and the steps are summarized in Supplementary Section D. The target set X_f, which is the neighborhood set of the target state is given by,

$$X_f = \{x|\mathbf{Y}x \le \mathbf{z}\}$$

where $\mathbf{Y} = [1 \quad -1]^T$, $\mathbf{z} = [\alpha_{max} \quad (-\alpha_{min})]^T$ and α_{max} and α_{min} are the upper and lower bounds of the target set. It is recommended in Goulart et al. (2006) that the target set be chosen to satisfy certain given conditions (refer to A_1 in Goulart et al. (2006)). However, because the input range does not contain origin in its interior, we cannot define a target set systematically for our system. Thus, we select an arbitrary region in the vicinity of zero as the target set.

The following section describes two important concepts discussed in MPC literature: feasibility and stability (in the form of ISS) which were not addressed in Saleem et al. (2013).

3.3. Feasibility and stability in irrigation systems context

When the optimization (3) is feasible, its solution is optimal and the irrigation input values obtained are optimal. Otherwise, the attempts for optimization and automation of irrigation control would be redundant. Therefore, optimization feasibility is essential in automated optimized irrigation control. The state of the Systems with ISS enter X_f towards the end of the time horizon, indicating the closeness of the current RZSMD to the target RZSMD over this time. Thus, ISS is desirable in irrigation control.

In DA, there are already established system properties guaranteeing ISS and feasibility. However, such ready-made theory cannot be applied here, due to the non-negative input. CE does not guarantee these defined properties except for a given set of initial conditions (Chuang et al. (2014)). To overcome this limitation, we propose to remove state constraints in both MPC systems and make X_f in DA equal to $\mathbb{R}$. The explanation to this action is given in Supplementary Section E. Is there a new risk in exceeding the original state constraints at the extremes of disturbance? We answer this question by looking at the (original) state constraints.

After the modification, **original** soil moisture constraints can be violated in the following few instances: WP can be violated 1. due to crop evapotranspiration or 2. by withholding irrigation; FC can be violated 3. due to rainfall or 4. by over-irrigation. Instances 2 and 4 caused by infeasibility are now completely overcome by removing the state constraints. RMPC laws cannot be used to deal with instance 1 where the problem is the under-supply of irrigation water. If the additional water quantity from a rainfall event exceeds the space left in the water holding capacity, the violation of x_{min} due to instance 3 occurs inevitably. After the recession of gravitational flows, the system reverts to normal. This situation is unavoidable unlike permanent saturation caused by over-irrigation on a continuous basis. Therefore, removing x_{min} completely or setting it to a very low value would make no difference at all to the system state. As a result of removing x_{min}, careful monitoring of x_{max} becomes needless, as the risk of violating x_{max} due to instance 2 to accommodate rainfall is completely eliminated.

If D_0 is the RZSMD with respect to FC at the beginning of the control horizon, considering the inflows and outflows to and from the soil profile, the RZSMD (without irrigation) over the horizon is given by, D_0+$(E-P)_{1:N}$ where 1:N denotes the cumulative value of the corresponding variable (crop evapotranspiration and rainfall in this case) over days 1 to N. The maximum water availability to the field is N times u_{max}. Under water limitations, the maximum irrigation amount would be applied resulting in the minimum possible RZSMD,

$$d_{min} = D_0 + (E - P)_{1:N} - Nu_{max}.$$

At the WP, RZSMD with respect to FC is equal to TAW and this is the limit on maximum RZSMD that the plant can tolerate. Thus, in order to avoid plant wilting,

$$d_{min} \leq H$$

where H is TAW.

Even if the lower bound (FC) is violated due to high rainfall, the system rebounds to ISS through gravitational flows (Note that the original ISS definition in RMPC is slightly abused here because the lower state constraint can be momentarily violated.) This condition is not typically guaranteed in a process control system.

On the above basis, it can be stated that, RZSMD is always bounded between WP and FC subject to water availability. Thus, the ISS of the system would be naturally guaranteed provided that,

$$D_0 - H + (E - P)_{1:N} \leq Nu_{max}. \tag{11}$$

Considering the uncertainty of rainfall forecasts (neglecting the uncertainty of evapotranspiration forecasts) farmers can determine the worst case scenarios $D_0 - H + (E - P_{min})_{1:N} \leq Nu_{max}$ for DA and $D_0 - H + (E - \widetilde{P})_{1:N} \leq Nu_{max}$ for CE.

In summary, the (original) state constraints can be violated only when Equation (11) is violated or by a very large rainfall, both of which are beyond control. Hence, the proposed method will always minimize RZSMD and irrigation amount and maintain ISS (hence the soil moisture level above WP) subject to water availability.

The concept behind the RMPC implementation in irrigation control is shown in Fig. 2. Model training based on Equation (5) is given in Fig. 2(a). Validation of Equation (5) is demonstrated with AQUACROP data and field data in Delgoda et al. (2015). The present paper demonstrates RMPC using AQUACROP data (Fig. 2(b)). Future work will extend the RMPC implementation to using field data.

3.4. Performance evaluation

A five step generic procedure is proposed in Bennett et al. (2013) for performance evaluation of environmental models. Evaluation of the simple linear model proposed in Delgoda et al. (2015) fits in with these concepts in the following context.

Reassessment of the model's aim, scale and scope: The only aim of this model is to be used in irrigation control. The model uses a daily scale and the critical event considered is the occurrence of gravitational flows. The model does not consider capillary forces hence it is not expected to be successful under capillary flow conditions.

Characterization of the data for calibration and testing: Data used for modeling and validation are split with the model application in irrigation control in mind. Accordingly, a dataset from one season is used for model training whereas the dataset from the next season is used for validation.

Visual and other analysis to detect under- or non-modeled behavior and to gain an overview of overall performance: The model performance is visually analyzed by plotting the measured and modeled data in time scale, including the behavior under gravitational flows. They are also plotted against each other (not shown in Delgoda et al. (2015) due to space limit) where the slope is equal to one and the point dispersion is low. The visualization results indicate that the model is able to capture main trends of the actual system dynamics.

Selection of basic performance criteria: The model performance is measured using a combination of methods: (i) coupling real and modeled values: root mean square error (RMSE) and Whiteness Test of residuals; (ii) preserving data pattern: coefficient of determination; and (iii) indirect metrics based on parameter values: estimated variance of parameter values (graphically indicated).

In addition, the Independence Test of residuals (Ljung (1999)) is carried out to confirm that the model performance does not depend on specific input values in the dataset, by verifying that the model residuals are independent of the input values. This means that the model performance will be similar with a fresh dataset different to the original dataset. The model is based on a hypothesis that input output relationship is linear under the training conditions. Independence of the model performance from specific input values confirms this assumed linear structure as well.

The model is not tested for over-fitting as the number of parameters in the model is limited and the parameters have physical meanings. Persistency of excitation also cannot be assured as frequencies of two of the inputs to the model, rainfall and evapotranspiration, cannot be controlled. However, unique parameter values could be obtained by constraining the parameter values to ranges [0,1] and [−1,0] considering their physical properties.

Consideration of more advance methods to handle problems observed during previous steps: The model refinement is not carried out as the main purpose was to obtain a model simple rather than precise.

Similarly, the proposed control method performance also needs to be tested.

Reassessment of the model/method's aim, scale and scope: The objectives of the control method are to minimize RZSMD and irrigation amount under uncertainty of weather forecasts. The soil moisture level should be always above WP subject to irrigation water availability.

Characterization of the data for calibration and testing: In this case, model training is based on the previous season data and control is based on the data from the next season.

Visual and other analysis to detect under- or non-modeled behavior and to gain an overview of overall performance: The minimization of RZSMD and irrigation amount, withholding of irrigation prior to rainfall events and performance under irrigation water limitations and extreme events (that force the soil moisture level to violate the soil moisture constraints) are visually demonstrated through numerical simulations in Section 4.

Selection of basic performance criteria: In order to quantify the RMPC performance RMSE of RZSMD and total irrigation amount are calculated. In addition, three indicators are proposed. The *water requirement ratio:*

$$R_1 = \frac{\text{total water deficit}}{(\text{total irrigation amount} + \text{total rainfall amount})}$$

demonstrates the level of optimization of both RZSMD and irrigation amount, in other words, the level of match between the rainfall and irrigation total to the collective crop evapotranspiration and previous RZSMD. The ideal value is 1 whereas values greater than 1 could occur under water deficiency. Values less than 1 indicate the inefficiency of the control method.

Minimization of irrigation amount is demonstrated by calculating total irrigation water usage. However, to obtain an absolute performance indicator, define *water wastage ratio*:

$$R_2 = \frac{\text{total gravitational flows}}{(\text{total irrigation amount} + \text{total rainfall amount})}$$

that has to be as small as possible unless leaching is required by the crop. In this paper, it is not expected to have leaching in the soil. R_2 is a measurement of water waste in the form of gravitational flows, mainly due to over-irrigation.

A third indicator *irrigation water usage ratio*:

$$R_3 = \frac{\text{total irrigation amount}}{\text{total available irrigation amount by field supply}}$$

would usually be less than 1. It needs to be close to 1 displaying the high level of utilization of the available irrigation amount, in the event of R_1 being greater than 1 (water deficiency situation).

Note that R_1 and R_2 are somewhat similar to 'beneficial consumptive use fraction' and 'beneficial reusable fraction' respectively in Pereira et al. (2012). However, considering water quality is beyond the scope of this work. Thus, the above definitions are slightly different to those used in Pereira et al. (2012), by their neglect of contaminated water amounts. The point to make is the similarity in conceptualization of the two works, in measuring the performance level of irrigation control methods. Indicator R_3 is a new concept to measure the utilization level of delivery water at the fields. This also implies the level of match between the field application and field delivery amounts. Other definitions related to irrigation efficiency are given in Jensen (1980) as well. However, they are not used as performance measures of the proposed method, because they do not account for rainfall as a possible water source apart from irrigation.

Consideration of more advance methods to handle problems observed during previous steps: This step will be the focus of possible future work.

4. Numerical experiments and discussion

This section compares the theoretical results obtained in Section 3 with AQUACROP data, actual weather data and forecasts, and investigates the RMPC performance under different crop, soil and water availability conditions. Six test cases are designed to demonstrate the specific aspects of the proposed method:

- Case 1: (i) The use of system identification to incorporate direct measurements to RMPC system (corresponds to Section 3.1) (ii) The tolerance of the RMPC methodology to the uncertainty in weather forecasts (corresponds to Section 3.2).
- Case 2: Importance of feasibility and ISS in RMPC in the context of irrigation control systems (corresponds to Section 3.3).
- Case 3: ISS under extreme rainfall events (corresponds to Section 3.3).
- Case 4: Comparison of RMPC performance with two control cases.
- Case 5: Performance of RMPC under soil type variations.
- Case 6: Performance of RMPC under crop type variations.

The basic concept of irrigation control in this paper is to control the actual physical system, using an RMPC rule based on the system model in Equation (5). Here, the actual physical system is represented by AQUACROP, a model that has been validated against field data by several authors (Alizadeh et al. (2010); Andarzian et al. (2011); Heng et al. (2009); Salemi et al. (2011); Garcia-Vila and Fereres (2012)). Table 1 shows the comparison between the system model and AQUACROP model. The simulations for the selected loam soil and a tomato variety from the AQUACROP library were carried out using Matlab, 2014*a* (Mat (2014)) and AQUACROP 4.1 (FAO (2011)). The weather data and forecasts are obtained for Shepparton, Victoria, Australia.

The training phase uses weather data from the year 2010 and a simple rule-based irrigation schedule set up in AQUACROP. The AQUACROP model generates irrigation events and the resultant soil moisture data time series. System identification is then applied on the data based on Equation (5) and the resulting coefficients $c_i \in [1{:}4]$ are used to build the state space model in Equation (2).

The rainfall forecast data for the year 2011 used in the control phase were obtained from the ACCESS Numerical Weather Prediction (NWP) system of the BoM, which consists of several ACCESS models with varying temporal and spatial resolutions (BoM (2010, 2012)). The ACCESS-G model has the largest temporal resolution (+240 h). For our simulations, we selected N=5 days. Accordingly, we selected the first 5 day forecast (+120 h) from the ACCESS-G model as our data source as the next available resolution is $+72h$ which is less than 5 days. Model outputs are numerical, therefore we sorted them into different ranges as explained in Supplementary Section C, i.e. $\mathbf{P}_{min}$ and $\mathbf{P}_{max} \in \mathbb{R}^N$ for DA and original numerical values $\widetilde{\mathbf{P}} \in \mathbb{R}^N$ for CE. It should be noted that the forecasts issued to the general public on the BoM website can be slightly different to these ACCESS-G forecasts as the outputs from the ACCESS models are combined before rainfall forecasts are released to the public.

Mean reference evapotranspiration was calculated for the recorded data of the period (2009–2013) at Shepparton and crop coefficients for a tomato variety with growing period over summer were selected from (Allen et al. (1998)). The estimate of $\widetilde{\mathbf{E}}_a$ was obtained using Equation (S1) and was used for CE. In the case of DA, the value of E_m (for which we selected the absolute maximum value of $\widetilde{\mathbf{E}}$ through visual inspection) was added to $\widetilde{\mathbf{E}}_a$ to obtain $\mathbf{E}_{max}$ and $\mathbf{E}_{min}$=0.

The value of S was set to be greater than Q and R according to the guidelines in Goulart et al. (2006) and the latter two are equal to unity. Soil moisture level $RP = FC-10$ is selected as the reference. Table 2 summarizes the parameter values used in the simulations described in this paper. The optimization problem was setup using YALMIP toolbox in MATLAB (Lofberg (2004)) with these parameters and solved using *CPLEX*12.6 (Cpl, (2009)).

4.1. Case 1 – system identification and RMPC performance under weather forecast uncertainty

We set up the irrigation control using the two RMPC methods: CE and DA. In both cases, the input constraint u_{max}=10 mm and state constraints are removed. As there is sufficient water, i.e. $D_0 - \mathrm{H} + (E-P)_{1:N} \le Nu_{max}$, the optimization is feasible (see Fig. 3(e) and (f) for CE and DA respectively). The value of R_1 is 0.85 and 0.82 for CE and DA respectively. This indicates that both methods provide a high level of optimality in RZSMD and irrigation amount. R_2 is 0.10 and 0.10 for the two cases showing that the wastage is minimal. R_3 is 0.24 and 0.25 implying that there is a high potential to save the irrigation water. Building on-site storage and recycling the irrigation water, or integrating the field delivery system to the RMPC system would make the water savings possible and increase R_3. As shown by Fig. 3(a) and (b), the soil moisture level is always above WP and at or below FC providing performance guarantees to the method in terms of fulfilling its goals and constraints when there is a sufficiency in available water amount. The impact of irrigation is shown by a high level of crop growth (see Fig. 3(c) and (d).

DA is more conservative than CE in triggering irrigation before rainfall events. This is because, unlike CE, DA sees the disturbance from rainfall as an event lasting over N days due to the smoothing of the forecasts. CE may sometimes be slightly less reliable if the rainfall forecasts are incorrect due to its consideration of only one

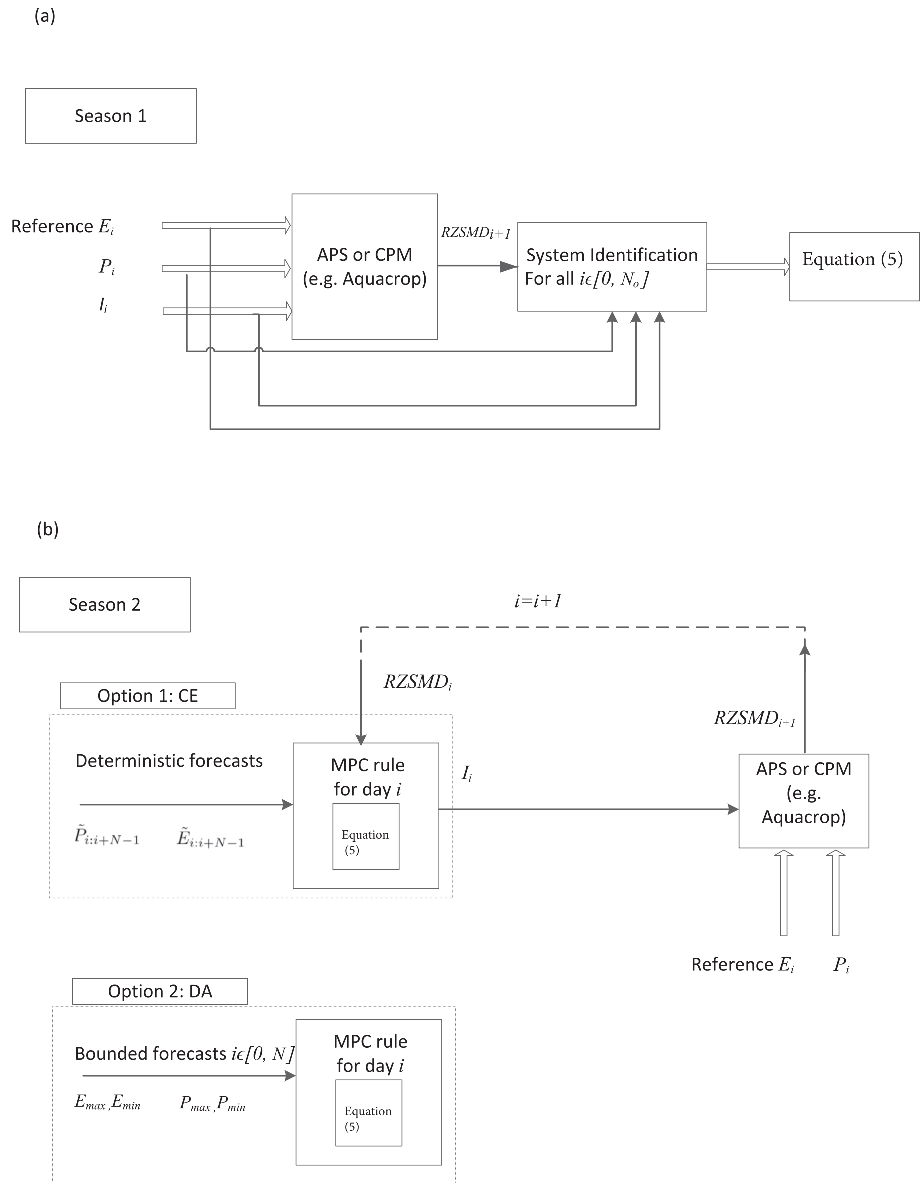

Fig. 2. Structure of the irrigation control methodology (a) Training phase: weather and soil moisture data are used to calibrate the system model using the method in Delgoda et al. (2015). (b) Control phase: weather forecasts and soil moisture data are fed to the RMPC controller to calculate the optimal irrigation amount. The resultant soil moisture deficit value is fed in the next time step, as the new system state. CE and DA methodologies are interchangeable according to preference. APS = actual physical system; CPM = complex physical model (e.g. AQUACROP model).

definite value as there is no provision in CE to maintain the optimization if the actual value significantly differs from the forecast. This is further explained in Supplementary Section E.

In contrast, there are instances where the rainfall forecasts are not only different to the actual rainfall by value, but are also classified into the wrong rainfall range among those shown in Supplementary Fig. S1 (see Fig. 1). In such a case, the main assumption in DA that 'the actual rainfall will take any value inside the forecast range' is violated. This was not an issue in the case study because the ACCESS-G misidentifies only between narrowly different ranges (e.g. 0–1 mm with 1–5 mm). However, if the forecasts are significantly less reliable, selecting a safe higher value

Table 1
A short comparison between actual physical system and control system model.

	Actual physical system	System model
Represented by	AQUACROP model	Equation (5)
Effective irrigation amount estimation	By considering soil physical properties and irrigation application method	System identification
Effective rainfall estimation	USDA-SCS method (Steduto et al. (2009); Raes et al. (2009))	System identification
Consider deep percolation/runoff?	Yes	Yes (training phase) No (control phase)

Table 2
Main parameter values used in simulations.

	System identification	Control
Simulation period	1 Aug 2010–18 Nov 2010	1 Aug 2011–18 Nov 2011
Irrigation application method	Sprinkler	Sprinkler
Irrigation control logic	Maximum depletion in readily available water is 10% (rule-based)	RMPC (CE or DA)
Percentage of soil surface to be wetted by irrigation	100	100
Crop type	Tomato	Tomato
Soil type	Loam (base case)	Loam (base case)
Reference *E*, rainfall, temperature data	Shepparton (2010)	Shepparton (2011)
P forecasts	–	NWP forecast from The Bureau of Meteorology, Australia
E forecasts	–	$\mathbf{K}_c$(from Allen et al. (1998)) $^*\mathbf{T}_a$ (from Equation (S3) and historical data)

(e.g. upper value of the upper rainfall range) for $\mathbf{P}_{max}$ would be more appropriate. This will make the irrigation control more conservative. Hence, it can be asserted that DA is more appropriate for humid areas with frequent rain fall and/or when weather forecasts are less reliable. Conversely, CE is better suited for arid and semi-arid regions and/or accurate weather forecasts. It is therefore up to the practitioner to consider this trade-off for a given set of forecasts from the meteorological agency.

4.2. Case 2 – importance of feasibility and ISS

The input constraint u_{max} is set to a very small value (2*mm*) to represent water limitations. The state constraints were initially set to (x_{max}=$-x_{min}$=50). Fig. 4 shows the results for the case of DA, and the results are similar when using CE. Due to tight state constraints and input constraints, the optimization becomes infeasible (see Fig. 4(c)). Even if there is some limited amount of irrigation water

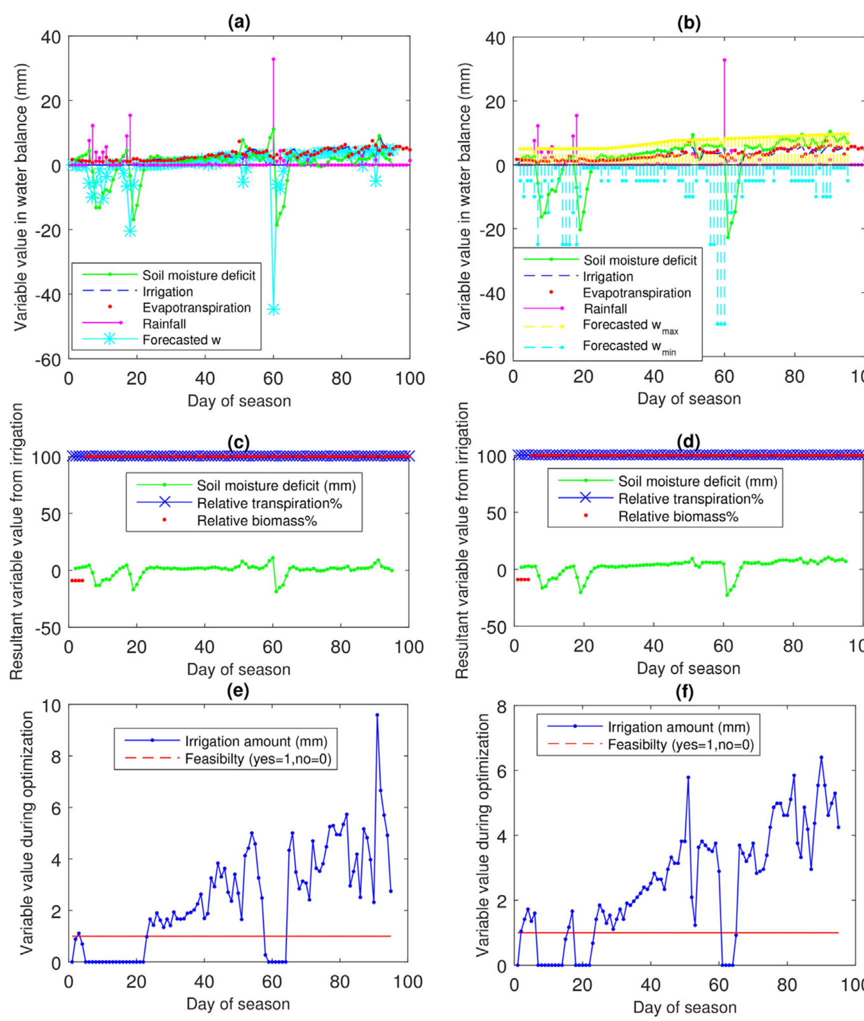

Fig. 3. Irrigation control based on RMPC: ROW 1: RZSMD close to zero under a sufficient amount of available irrigation (a) CE, (b) DA. ROW 2: Resultant relative transpiration and relative biomass (c) CE, (d) DA. ROW 3: Irrigation amount optimized (e) CE, (f) DA. CE = Certainty Equivalence control; DA = Disturbance Affine feedback Control. (Note that $\mathbf{w}_{min}$ and $\mathbf{w}_{max}$ shown at time k is common for the period k:$k+N-1$).

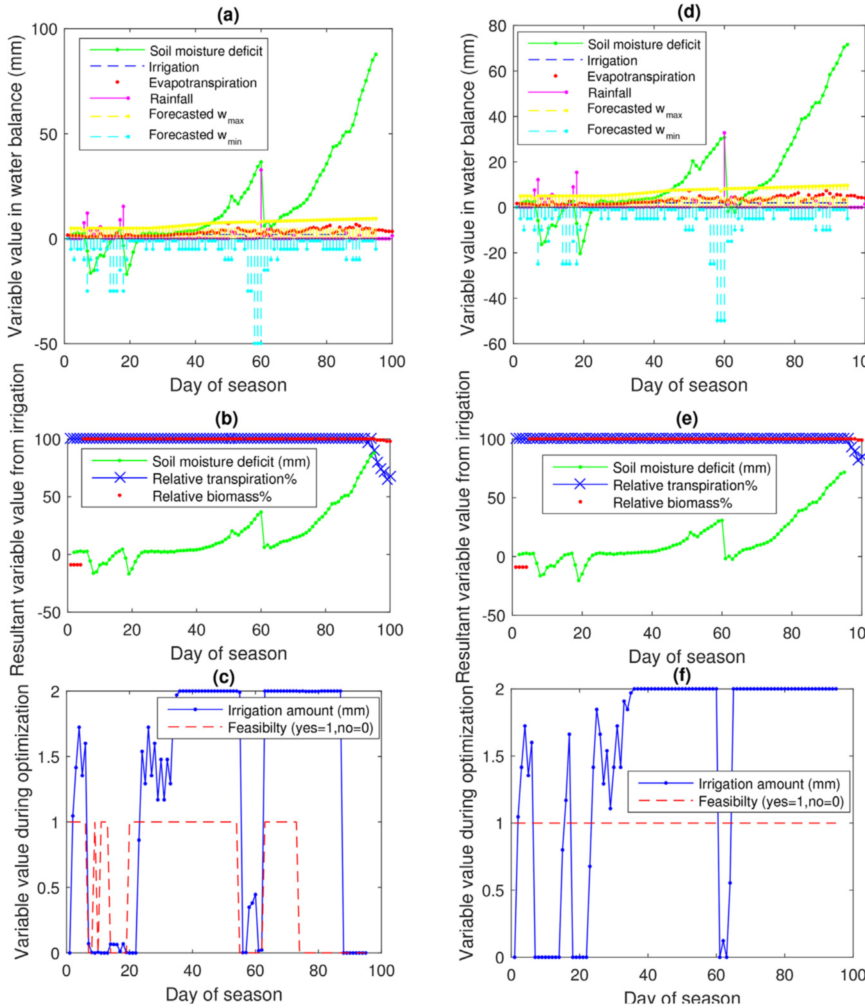

Fig. 4. Feasibility and ISS of RMPC: COLUMN 1: With state constraints (a) Soil moisture deficit increasing continuously under a limited amount of available irrigation. No ISS possible (b) Resultant decline of relative transpiration and relative biomass (c) Irrigation amount being zeroed due to infeasibility of optimization COLUMN 2: Without state constraints (d) Soil moisture deficit decreased compared to (a). No ISS, still it may return to ISS (e) Resultant increase of relative transpiration and relative biomass compared to (b). (f) Irrigation amount maximized due to feasibility of optimization. (Note that $\mathbf{w}_{min}$ and $\mathbf{w}_{max}$ shown at time k is common for the period k:$k+N-1$).

available, the system is not able to trigger irrigation for about 20 days. The performance ratios (R_1= 1.20; R_2= 0.15; R_3= 0.62) also confirm this observation. Despite the presence of a water deficiency, R_3 is small indicating that the maximum available water amount is not applied, which shows the importance of feasibility in attaining optimization. As $D_0 - \mathrm{H} + (E-P)_{1:N} \geq Nu_{max}$, ISS does not exist. If the deviation of the state from the target state is exceedingly large, the system will not return to ISS without manual intervention.

Next, the state constraints are removed as explained in Section 3.3, and the RMPC triggers maximum irrigation. The optimization is kept feasible as shown in Fig. 4(f). The crop suffers due to lack of water supply to the field, however, system has applied all the water available. As $D_0 - \mathrm{H} + (E-P)_{1:N} \geq Nu_{max}$, ISS is still not possible, nevertheless, a rainfall event of sufficiently large magnitude can bring back the system close to the target and back to ISS. The performance ratios (R_1= 1.11; R_2= 0.13; R_3= 0.75) are significantly better than in the previous case. Value of R_3 is not equal to 1 due to the fact that there was a small water requirement during the early crop stages and maximum irrigation was not needed at that stage. This observation also highlights the benefit of having on-site storage facilities to recycle unused water.

4.3. Case 3 – feasibility and stability under extreme cases

In order to check if the system state would reach an extreme end at a rainfall event of large magnitude, we consider a modification of Case 1 where there is sufficient irrigation supply and the state constraints are removed. Further, we created a large rainfall event (282 *mm* and 100 *mm*) artificially on two consecutive days. Such extreme rainfall events create large gravitational flows. Despite this, the system recovers soon after the rainfall and x_{max} is not violated before the rainfall event. As irrigation is sufficiently withheld before the rainfall, there is no water logging caused due to over-irrigation (Fig. 5).

As a point of interest it can be seen that the values of Q and R can change the weights on the two objectives of minimizing RZSMD and minimizing irrigation amount respectively. Variation of the performance indicators (R_1=0.47,0.48,0.50; R_2=0.08, 0.06, 0.06 R_3=0.23, 0.21, 0.19) for three Q/R ratios (1/1, 1/5, 1/10) confirms this observation (see Fig. 5). Apparently small values in R_1 are due to the excess of soil moisture from the large rainfall event. When this amount is deducted from the denominator, (R_1= 0.68, 0.70, 0.73) tally with the nominal scenario shown in Case 1.

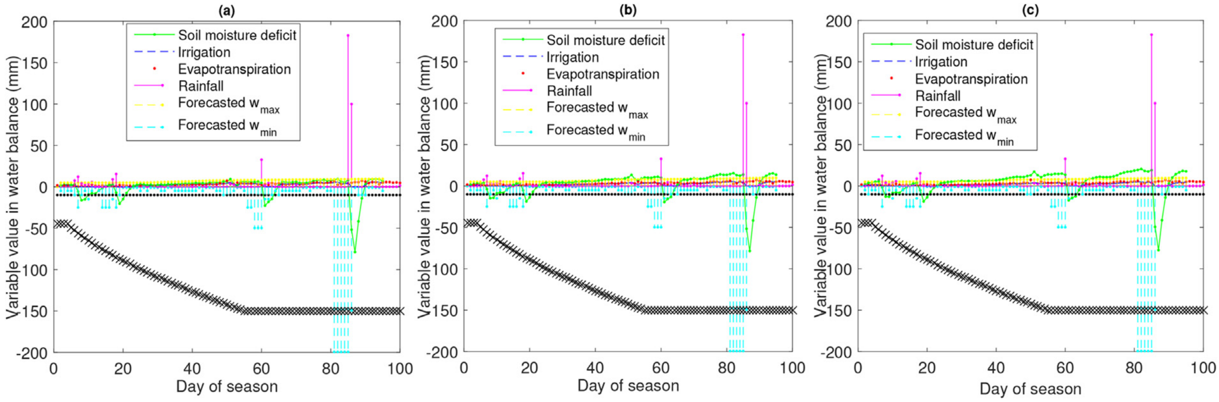

Fig. 5. System recovering after an extreme rainfall event. Line marked with 'x' is the soil moisture deficit corresponding to saturation point. (a) Q/R=1/1 (b)Q/R=1/5 (c) Q/R=1/10. Changing Q/R ratio varies the weights on minimizing RZSMD and irrigation amount. (Note that $\mathbf{w}_{min}$ and $\mathbf{w}_{max}$ shown at time k is common for the period k:$k+N-1$).

4.4. Case 4 – comparison of RMPC performance with two control cases

Two control cases are selected: a set schedule where the irrigation amounts are gradually increasing from 1 *mm* to I_s *mm* to cater the increasing crop water needs over the season with $I_s \in$ [2,5,9]; an improved rule-based irrigation control method combined with water balance that irrigates when the soil moisture level drops below a given threshold ($FC-10$), to replenish the soil moisture by an irrigation amount equal to the removed water amount.

The situation where u_{max}=10 (corresponds to Case 1) is considered first. Under this maximum water availability, two set schedules are carried out where I_s is equal to 5 and 9 *mm*. Then the rule-based method subject to a maximum 10 *mm* water availability is applied. The results are shown in Supplementary Fig. S2 where first two rows demonstrate the performance of the two set schedules and the last row of the rule-based method. The performance of DA and CE corresponding to same conditions is shown in Fig. 3. The total water use for the season, RMSE of RZSMD and final relative biomass (actual biomass with respect to the potential biomass under no water stress) are summarized in Table 3. As the target soil moisture level is $FC-10$, none of the systems undergo water stress conditions hence the crop yield is very high. However, the water use is clearly less under the two RMPC methods. In addition, all the other control method performances are inferior to RMPC in terms of the three performance indicators. RZSMD also is minimized as shown by the low RMSE and is second to only rule-based method by a small value, however, rule-based method consumes a relatively large amount of water.

Then, the case under water limitations i.e. u_{max}=2 (corresponding to Case 2) is taken into consideration. One set schedule is carried out with I_s=2 and rule-based control is performed with 2 *mm* being the maximum water availability. The results are shown in Supplementary Fig. S3 where the three rows demonstrate the performance of the set schedule, the rule-based method and CE respectively. The performance of DA in this case is shown in Fig. 4 (COLUMN 2). The results are summarized in Table 3. All methods undergo a minor level of water stress as indicated by the relative biomass values however as explained above this is not a good indicator to compare the performances as the target level is $FC-10$. Limited by water availability all the methods use a less amount of water. Rule-based method has a similar performance to RMPC methods in terms of minimizing RZSMD and irrigation amount. However, it can be observed that R_2 is high indicating that the method does not consider rainfall forecasts and cause gravitational flows by over-irrigation, thus, in a season with high rainfall amounts, the performance is bound to deteriorate.

According to the results in both cases, RMPC in general stands out among other methods by minimizing irrigation water use and RZSMD, matching the combined irrigation and rainfall amounts with water demand (indicated by R_1), minimizing irrigation water loss as gravitational flows (indicated by R_2) and maximizing the usage of available water in the case of water shortages (indicated by R_3) simultaneously. The latter aspect where, maximizing water utilization under water shortages is available in rule-based control as well, however, cannot be guaranteed to be always 100% until water storages are built to reuse the water throughout the season or the delivery to the field is synchronized with irrigation control, which is not the focus of this paper. In general, water use is

Table 3
Comparison of the proposed method with other control methods.

Maximum available daily irrigation amount = 10 *mm*						
Control method	**Total water use (mm)**	**RMSE of RZSMD (mm)**	**Final relative biomass**	R_1	R_2	R_3
Set schedule I_s = 9 *mm*	410	9.98	100	0.61	0.37	0.43
Set schedule I_s = 5 *mm*	275	8.70	100	0.82	0.22	0.29
Rule-based (threshold: FC-10)	243	6.43	100	0.80	0.14	0.26
RMPC (CE)	221.78	5.23	100	0.85	0.10	0.24
RMPC (DA)	209.23	7.24	100	0.82	0.10	0.25
Maximum available daily irrigation amount = 2 *mm*						
Control method	**Total water use (mm)**	**RMSE of RZSMD**	**Final relative biomass**	R_1	R_2	R_3
Set schedule I_s = 2 *mm*	140	46.10	96	1.21	0.19	0.15
Rule-based (threshold: FC-10)	144	24.65	97	1.10	0.19	1
RMPC (CE)	143.27	24.59	97	1.11	0.13	0.75
RMPC (DA)	133.47	24.35	97	1.15	0.14	0.71

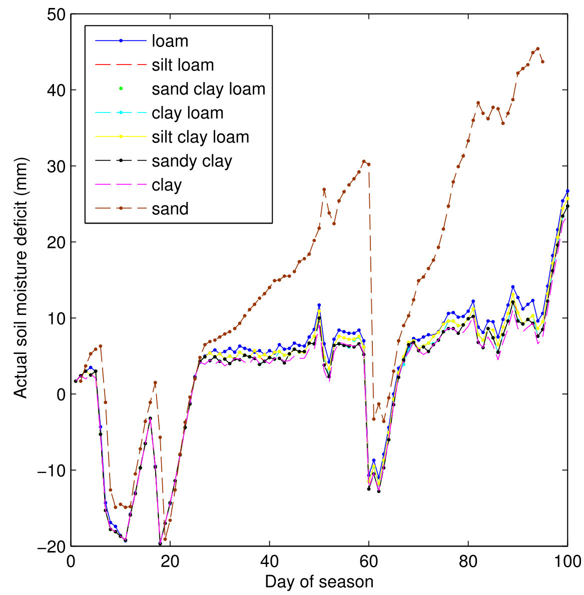

Fig. 6. Robust model predictive control (RMPC) with model mismatches due to soil type differences: Model calibration is done for loam soil however system responds in a similar manner for other soil types except sandy soil.

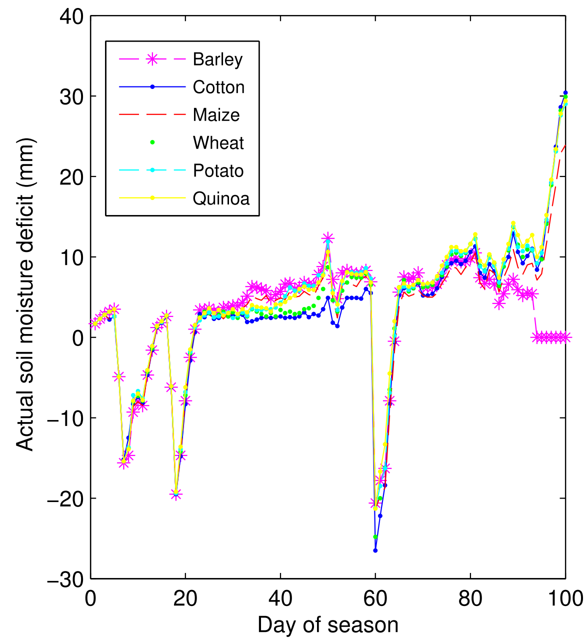

Fig. 7. Robust model predictive control (RMPC), with models calibrated for different crop types: RMPC performs differently with contrasting crop types.

relatively less and RZSMD is relatively high in DA, confirming its conservativeness than CE.

4.5. Case 5 — RMPC performance with soil type variations

The RMPC model calibrated for loam soil was applied to several other soil types covering a wide range of soil parameters (Table S1). It can be observed in Fig. 6 that the resulting curves show a very similar trace for all soil types except sandy soil. They follow a similar pattern to the soil-moisture characteristic curves for these soil types. This similarity facilitates the modeling and tuning of the system model and RMPC approach for a field with different soil types without considering their differences. However, the observation may not be valid for spatial variability of RZSMD itself, because different initial conditions may cause variable RZSMD over the field. The RMPC performance was not satisfactory for sandy soil because of the low water holding capacity and the resulting drainage flows. This implies that separate calibration has to be carried out for sandy soil.

4.6. Case 6 — RMPC performance with crop type variations

An observation of the system identified coefficients shows that they are less sensitive to soil type than to crop type. Fig. 7 shows the RMPC results using a single loam soil type under different crop types. It can be observed that the trends of soil moisture deficit are different from each other. When the system is modeled for one crop type, it is not possible to implement RMPC on any other crop. However this should not matter for the irrigators as changing the irrigation practice according to the crop type than soil types is a common practice.

In summary, Case 1 shows that the proposed irrigation control method based on one of the two RMPC techniques is easily implemented with raw measurements and is robust to ACCESS-G rainfall forecasts and very approximate crop evapotranspiration forecasts. It also shows a slight difference between the two approaches in terms of responsiveness of the irrigation control. Case 2 emphasizes the importance of feasibility and ISS in an irrigation control system based on RMPC. Case 3 further affirms ISS of RMPC in response to an extreme rainfall event. It also gives a guideline to tune the control through weights. In Case 4, the two RMPC approaches are compared with two irrigation control methods: a set schedule and an improved rule-based method combined with water balance. The performance of the two RMPC methods is superior to the other two methods in terms of achieving its set goals. Case 5 and Case 6 demonstrated that RMPC performance is sensitive to crop type than soil type. On overall, the simulations confirm the claims made during the development of the methodology. ACCESS-G rainfall forecasts may be less accurate than ACCESS forecasts and crop evapotranspiration in this study is calculated using a simple method given in Supplementary Section C, whereas there are more sophisticated methods available (e.g. Perera et al. (2014); Aboitiz and Labadie (1986)). Yet, the proposed methodology was able to provide optimized irrigation control using these forecasts.

Ensemble forecasts: Due to the fact that the rainfall amounts change at the same frequency as the system (both at 1/day), it is not possible to apply the method in Delgoda et al. (2013), where ensemble of forecasts are used to get a probabilistic solution to RMPC during the control input calculation. Therefore, the correct approach to be taken with ensemble forecasts is to use either the ensemble average or the boundary values of the ensemble forecast as the weather forecast **w** in CE and DA respectively. This would mean considering the variability of the forecast instead of the control input.

Performance with noisy soil measurements: In this research, we have perfect soil moisture data (zero measurement noise) from AQUACROP model. However, if field measurements are used, a Kalman filter needs to be used along with RMPC (Wang (2009)) to eliminate the effect of measurement noise.

5. Conclusion

MPC can be applied in irrigation control to manage the complex crop-soil-water relationships in the actual physical system through a control law based on a simple system model. This paper presents the theoretical framework required to implement MPC in a field situation. The model AQUACROP representing actual physical system is used to demonstrate the theoretical results through simulations using weather data and ACCESS NWP forecasts from the BoM, Australia for Shepparton, Victoria, Australia.

First, the paper shows that it is possible to incorporate direct measurements of variables (rainfall, irrigation, calculated crop evapotranspiration) to MPC through system identification. This enables automation of the control process by letting direct measurements be input to the decision support system. Second, we show that it is possible to mitigate uncertainty in weather forecasts through two RMPC techniques: CE and DA. DA is slightly more conservative than CE and is therefore more suitable for humid areas and/or less reliable forecast data whereas CE would perform better in arid areas and/or when weather forecasts are more reliable. Third, we show that RMPC will always minimize RZSMD and irrigation amount, and the soil moisture will always stay above WP subject to water availability $[D_0 - H + (E-P)_{1:N} \leq Nu_{max}]$. Performance of the proposed method is quantified using a number of metrics and compared with two other irrigation control methods. Both CE and DA based approaches have a superior performance to other methods. Optimizing irrigation amount along with RZSMD prevents water logging due to excessive irrigation. The main differences in the RMPC performance were due to crop types more than soil types, a finding that can be used to ease the effort to model RZSMD for different soil types. Incorporation of numerical forecasts makes the method independent of the weather forecast method. To make the most out of the proposed method, the field water delivery system must be integrated to the irrigation control or on-site storage should be provided.

All our results show that RMPC is, in general, a promising tool which can be practically implemented for irrigation control under uncertainty of weather forecasts. The system model is already verified with AQUACROP and field data. In future, AQUACROP model data would be replaced by field data during the control phase.

Acknowledgments

The authors wish to express their gratitude to The Bureau of Meteorology, Australia for providing NWP forecasts derived from ACCESS-G for research purposes and Dr. Alan Seed, Dr. Shaun Cooper of The Bureau of Meteorology, Australia; Prof. Andrew Western, PhD Candidate Kushan C. Perera of The University of Melbourne for their support in obtaining the data. Authors would also like to thank Amir Reza Neshastehriz of The University of Melbourne for his clarifications on robust model predictive control.

Appendix A. Supplementary data

Supplementary data related to this article can be found at http://dx.doi.org/10.1016/j.envsoft.2015.12.012.

References

Aboitiz, M., Labadie, J.W., 1986. Stochastic soil moisture estimation and forecasting for irrigated fields. Water Resour. Res. 22, 180–190.
Alizadeh, H., Nazari, B., Parsinezhad, M., Etedali, H.R., Janbaz, H., 2010. Evaluation of aquacrop model on wheat deficit irrigation in karaj area. Iran. J. Irrig. Drain. 4, 273–283.
Allam, M., 2002. Survey on Irrigation Modernization Beni Ebeid Area (Technical Report).
Allen, R.G., Pereira, L.S., Raes, D., Smith, M., 1998. Crop Evapotranspiration (Guidelines for Computing Crop Water Requirements). Technical Report. FAO – Food and Agriculture Organization of the United Nations.
Andarzian, B., Bannayan, M., Steduto, P., Mazraeh, H., Barati, M., Barati, M., Rahnama, A., 2011. Validation and testing of the aquacrop model under full and deficit irrigated wheat production in Iran. Agric. Water Manag. 100, 1–8.
APS1 Upgrade of the ACCESS-G Numerical Weather Prediction System, 2012. NMOC Operations Bulletin No. 93. Technical Report. The Bureau of Meteorology, Australia.
Bahat, M., Inbar, G., Yaniv, O., Schneider, M., 2000. A fuzzy irrigation controller system. Eng. Appl. Artif. Intell. 13, 137–145.
Bemporad, A., Morari, M., 1999. Robust model predictive control: a survey. Lect. Notes Control Inf. Sci. 245, 207–226.
Bennett, N.D., Croke, B.F., Guariso, G., Guillaume, J.H., Hamilton, S.H., Jakeman, A.J., Marsili-Libelli, S., Newham, L.T., Norton, J.P., Perrin, C., et al., 2013. Characterising performance of environmental models. Environ. Model. Softw. 40, 1–20.
Bras, R.L., Cordova, J.R., 1981. Intraseasonal water allocation in deficit irrigation. Water Resour. Res. 17, 866–874.
Chuang, F., Danielson, C., Borrelli, F., 2014. Optimality of certainty equivalence in expected value problems for uncertain linear systems. In: Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on. IEEE, pp. 2822–2827.
Delgoda, D.K., Saleem, S.K., Halgamuge, M.N., Malano, H., 2013. Multiple model predictive flood control in regulated river systems with uncertain inflows. Water Resour. Manag. 27 (3), 765–790.
Delgoda, D.K., Saleem, S.K., Malano, H., Halgamuge, M.N., 2015. Root zone soil moisture prediction models based on system identification: formulation of the theory and validation using field and aquacrop data. Agric. Water Manag. http://dx.doi.org/10.1016/j.agwat.2015.08.011.
Dudley, N.J., Howell, D.T., Musgrave, W.F., 1971. Optimal intraseasonal irrigation water allocation. Water Resour. Res. 8, 586–594.
FAO, 2011. Aquacrop Version 4.0. http://www.fao.org/nr/water/aquacrop.html.
Garcia-Vila, M., Fereres, E., 2012. Combining the simulation crop model aquacrop with an economic model for the optimization of irrigation management at farm level. Eur. J. Agron. 36, 21–31.
Giusti, E., Marsili-Libelli, S., 2015. A fuzzy decision support system for irrigation and water conservation in agriculture. Environ. Model. Softw. 63, 73–86.
Goldhamer, D.A., Fereres, E., 2003. Irrigation scheduling of almond trees with trunk diameter sensors. Irrig. Sci. 23, 11–19.
Goulart, P.J., Kerrigan, E.C., Maciejowski, J.M., 2006. Optimization over state feedback policies for robust control with constraints. Automatica 42, 523–533.
Heng, L.K., Hsiao, T., Evett, S., Howell, T., Steduto, P., 2009. Validating the fao aquacrop model for irrigated and water deficient field maize. Agronomy 101, 488–498.
Hibbs, R.A., James, L.G., Cavalieri, R.P., 1992. A furrow irrigation automation system utilizing adaptive control. Am. Soc. Agric. Eng. 35, 1063–1067.
Hornbuckle, J.W., Car, N.J., Christen, E.W., Stein, T.-M., Williamson, B., 2009. Irrigation Water Management by Satellite and SMS: a Utilisation Framework. Technical Report. Cooperative Research Centre for Irrigation Futures.
Ibm cplex optimization studio, 2009. URL: http://www.ibm.com/developerworks/downloads/ws/ilogcplex/.
Jensen, M. (Ed.), 1980. Design and Operation of Farm Irigation Systems. The American Society of Agricultural Engineers.
Jones, J., Hoogenboom, G., Porter, C., Boote, K., Batchelor, W., Hunt, L., Wilkens, P., Singh, U., Gijsman, A., Ritchie, J., 2003. The DSSAT cropping system model. Eur. J. Agron. 18, 235–265.
Kanemasu, E.T., Thurtell, G., Tanner, C., 1969. Design calibration and field use of a stomatal diffusion porometer. Plant Physiol. 44, 881–885.
Kearney, M., Cantoni, M., 2012. Mpc-based reference management for automated irrigation channels. In: Australian Control Conference.
Kia, P.J., Far, A.T., Omid, M., Alimardani, R., Naderloo, L., et al., 2009. Intelligent control based fuzzy logic for automation of greenhouse irrigation system and evaluation in relation to conventional systems. World Appl. Sci. J. 6, 16–23.
Kothare, M.V., Balakrishnan, V., Morari, M., 1996. Robust constrained model predictive control using linear matrix inequalities. Automatica 32, 1361–1379.
Langson, W., Chryssochoos, I., Rakovic, S., Mayne, D., 2004. Robust model predictive control using tubes. Automatica 40, 125–133.
Ljung, L., 1999. System Identification: Theory for the User, second ed. Prentice Hall.
Lofberg, J., 2004. Yalmip: a toolbox for modeling and optimization in matlab. In: IEEE International Symposium on Computer Aided Control Systems Design.
Lu, P., Urban, L., Zhao, P., 2004. Granier's thermal dissipation probe (tdp) method for measuring sap flow in trees: theory and practice. Acta Bot. Sin. Engl. Ed. 46, 631–646.
Matlab 2014b, 2014. URL: http://au.mathworks.com/support/sysreq/sv-r2014a/.
Mayne, D., Seron, M., Rakovic, S., 2005. Robust model predictive control of constrained linear systems with bounded disturbances. Automatica 41, 219–224.
McCarthy, A., Hancock, N., Raine, S., 2014. Simulation of irrigation control strategies for cotton using model predictive control within the VARIwise simulation framework. Comput. Electron. Agric. 101, 135–147.
Oldewurtel, F., Parisio, A., Jones, C.N., Morari, M., Gyalistras, D., Gwerder, M., Stauch, V., Lehmann, B., Wirth, K., 2010. Energy efficient building climate control using stochastic model predictive control and weather predictions. In: American Control Conference (ACC).
Operational Implementation of the ACCESS Numerical Weather Prediction Systems, 2010. NMOC Operations Bulletin No. 83. Technical Report. The Bureau of Meteorology, Australia.
Or, D., Hanks, J., 1992. Spatial and temporal soil water estimation considering soil variability and evapotranspiration uncertainty. Water Resour. Res. 28, 803–814.
Park, Y., Shamma, J.S., Harmon, T.C., 2009. A Receding Horizon Control Algorithm for Adaptive Management of Soil Moisture and Chemical Levels during Irrigation. Technical Report. Center for Embedded Network Sensing, University of California.

Pereira, L.S., Cordery, I., Iacovides, I., 2012. Improved indicators of water use performance and productivity for sustainable water conservation and saving. Agric. Water Manag. 108, 39–51.
Perera, K.C., Western, A.W., Nawarathna, B., George, B., 2014. Forecasting daily reference evapotranspiration for australia using numerical weather prediction outputs. Agric. For. Meteorol. 194, 50–63.
Protopapas, A.L., Georgakakos, A.P., 1990. An optimal control method for real-time irrigation scheduling. Water Resour. Res. 26, 647–669.
Raes, D., Steduto, P., Hsiao, T.C., Fereres, E., 2009. Aquacrop the fao crop model to simulate yield response to water: Ii. Main algorithms and software description. Agron. J. 101, 438–447.
Rao, N., Sarma, P., Chander, S., 1992. Real-time adaptive irrigation scheduling under a limited water supply. Agric. Water Manag. 20, 267–279.
Romero, R., Muriel, J., Garcia, I., 2008. Automatic irrigation system in almonds and walnuts trees based on sap flow measurements. In: VII International Workshop on Sap Flow, 846, pp. 135–142.
Romero Vicente, R., et al., 2011. Hydraulic Modelling and Control of the Soil-plant-atmosphere Continuum in Woody Crops.
Rossi, F., Nardino, M., Mannini, P., Genovesi, R., 2004. Irrinet emilia romagna: online decision support on irrigation. Online agrometeological applications with decision support on the farm level. Cost Action 718.
Saleem, S.K., Delgoda, D.K., Ooi, S.K., Dassanayake, K.B., Liu, L., Halgamuge, M.N., Malano, H., 2013. Model predictive control for real-time irrigation scheduling. In: The 4th IFAC Conference on Modelling and Control in Agriculture. Horticulture and Post Harvest Industry.
Salemi, H., Soom, M.A.M., Lee, T.S., Mousavi, S.F., Ganji, A., Yusoff, M.K., 2011. Application of aquacrop model in deficit irrigation management of winter wheat in arid region. Afr. J. Agric. Res. 6, 2204–2215.
Steduto, P., Hsiaob, T.C., Raes, D., Fereres, E., 2009. AquaCrop: the FAO crop model to simulate yield response to water: I. Concepts and underlying principles. United Nations Food and Agriculture Organization.
Sunantara, J.D., Ramirez, J.A., 1997. Optimal stochastic multicrop seasonal and intraseasonal irrigation control. J. Water Resour. Plan. Manag. 39–48.
Wang, L., 2009. Model Predictive Control System Design and Implementation Using Matlab, first ed. Springer-Verlag London Limited.
Yaron, D., Dinar, A., 1982. Optimal allocation of farm irrigation water during peak seasons. Am. J. Agric. Econ. 64, 681–689.
Zhang, Q., Wu, C.-H.J., Tilt, K., 1996. Application of fuzzy logic in an irrigation control system. In: Industrial Technology, 1996. (ICIT'96), Proceedings of the IEEE International Conference on. IEEE, pp. 593–597.