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Model Predictive Control for Real-Time Irrigation Scheduling

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Abstract: Irrigation underpins agricultural productivity. The purpose of irrigation is to match water supply to crop water demand. The effectiveness of irrigation depends on the quality of the timing and duration of watering events, also called irrigation scheduling. Most farmers use heuristic rules to determine irrigation events. This often leads to over-watering which results in lower crop yields and wasted water. By acquiring good estimates of a plant's water demand and local weather, it is possible to use optimization theory to compute an irrigation schedule that matches supply and demand thereby improving crop yields. Previous work has focused on scheduling irrigation over long time frames such as seasonal water allocations. Real-time irrigation scheduling, e.g. hourly or daily, has received little attention. Farmers rely on heuristic approaches implemented using simple spreadsheet tools to help them in this task. This approach cannot deal effectively with operational constraints and thereby results in poor performance. In this paper we develop a Model Predictive Control framework for real-time irrigation scheduling. The proposed formulation can take into account common operational constraints, including limitations on water availability as well as practical limits on the maximum or minimum amount of water that should be applied. We use measured climate data coupled with a simulation model to evaluate the proposed algorithm.

Keywords: agriculture, control system design, irrigation, modeling, optimization, water

1. INTRODUCTION

Irrigation scheduling is the process by which a farmer determines the timing and duration of crop watering events. A successful irrigation scheduling regime is one in which the farmer is able to closely match water supply with crop water demand, Brouwer et al. (1989). This paper addresses real-time irrigation scheduling where we are specifically concerned with daily or hourly crop watering events.

To compute an irrigation schedule it is necessary to measure crop water demand. Crop water demand can be inferred from direct measurements on the plant. Examples include stem or leaf water potential, or leaf vigor. More commonly however, crop water demand is inferred from indirect measurements. Examples include the soil water content in the plant's root zone, and more recently, reflectance in the near-infrared bands, Jones (2004). The former is the most common measurement for inferring crop water demand.

Irrigation scheduling is also subject to a many constraints and uncertainties. There are often constraints on the total volume of water available for irrigation purposes, as well as flow and volume constraints imposed by the delivery infrastructure. There is significant uncertainty associated with climate inputs to the water balance model, precipitation and evapotranspiration. Irrigation schedules can be affected by other factors. For example, crop water event timings are subject to the availability of labor and power for operating pumping equipment. Farmers must often adjust irrigation scheduling around these limitations, and it is quite common for irrigation to occur at set times during the day.

The previous paragraph outlines some of the challenges in irrigation scheduling. A widely studied and practiced solution to this problem is the so-called *water balance* approach, Steduto et al. (2009). This methodology uses a water flow balance, see Fig. 1. The figure depicts a soil column that contains a plant's roots, the arrows depict the water flows into and out of the soil column. Inflows are irrigation and precipitation. Outflows are evapotranspiration, runoff and deep percolation below the plant's root zone. The plant's interaction with the soil column is through the transpiration component in evapotranspiration which also includes extraction of water from the soil via the root zone. Runoff and deep percolation should generally be minimized. We do not consider them further in this work since they usually occur when too much irrigation

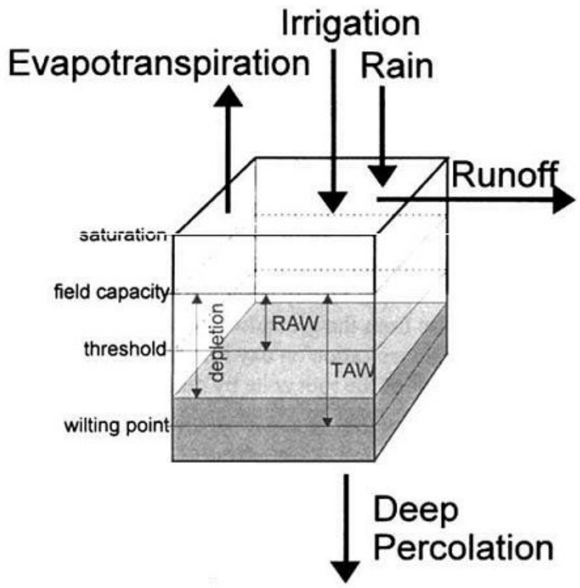

Fig. 1. Basic water balance model (from Steduto et al. (2009))

water is applied. The model developed herein assumes that the soil water content does not exceed field capacity hence it does not capture dynamics associated with deep percolation and runoff. In some case, deep percolation is used to manage salinity by effectively using irrigation to flush salts from the soil. This aspect will be part of future work.

Some important soil water content values are illustrated in the figure. The measurement unit, in this case, is volumetric water content (Vol %) or millimeters of water per 100 mm of soil, which is usually expressed simply as (mm) or equivalenty as (%). The first water content value is *saturation*. At this level there is no air in the soil and there is significant surface runoff. The second is *field capacity*. At this water content level, free drainage has ceased, that is there is no drainage from shallow to deeper soil layers. This is the ideal soil water content for plant growth. The third is *wilting point*. At this point, the water content is such that the plant roots cannot extract water from the soil. The level *threshold* denotes a minimum water content in the soil for unrestricted plant growth. These levels lead to the two important total water content values highlighted in the figure. The first is *TAW* or total available water. It is the difference between field capacity and wilting point and is the maximum water available to the plant for extraction. The second total water content value is *RAW* or readily availble water. It is the difference between field capacity and threshold. It is the water that is available to the plant that it can easily extract from the soil. Because of this, the threshold level is often called the *refill* point. The farmer uses irrigation to maintain the water content in the soil column above this level.

Using the model in Fig. 1, a common measure of crop water demand is the difference between field capacity and actual soil moisture content. This is called the *soil moisture deficit*. The objective of irrigation scheduling is to ensure that the soil moisture content does not exceed field capacity and does not drop below the refill point. This is the most common approach to irrigation scheduling used by farmers, and can be implemented using computer spreadsheet programs.

The accuracy of the water balance approach to scheduling irrigation events is affected by uncertain measurements, namely evapotranspiration and precipitation, constraints such as limited water, and tunable variables such as the field capacity and refill point. Evapotranspiration and precipitation, for example, are usually published daily by meteorological services however they are often affected by significant spatial and temporal uncertainty and errors. For example, measured evapotranspiration may only be available at a limited number of weather stations, and is then interpolated to a coarse grid where each cell may span one or more square kilometers. This resolution is usually not sufficiently accurate for paddock scale irrigation decisions. Variability in soil type can also lead to significant errors. The soil column model and the associated water content levels will not be constant over a large area with different soil types, soil layer depths and above ground topgoraphy.

There has been substantial effort at developing more systematic frameworks employing estimation, optimization and control theory to address these issues. Since the effect of irrigation on crop yield is generally cumulative, and irrigation schedules are discrete events, most previous efforts have focused on the application of Dynamic Programming methods. Some example of such previous work includes Bras and Cordova (1981), Dudley et al. (1971), Dudley et al. (1972), Dudley (1972), Yaron and Dinar (1982), Rao et al. (1990), Sunantara and Ramirez (1997), Harris and Mapp (1980), Protopapas and Georgakakos (1990), and Rao et al. (1992). These studies have considered a range of issues in irrigation scheduling including limited water supply for both single- and multi-crop scenarios, as well as inter- and intra-seasonal water allocation, uncertainly in climate inputs, as well as crop selection. There are also studies on the use of machine learning methods to solve this problem, Brown et al. (2010) and Ticlavilca et al. (2011). More recently Park et al. (2009) have also applied receding horizon control to soil moisture management.

The studies cited above have been mainly concerned with optimization of water allocations in the paddock or field. But over the last decade there has been significant interest in improving the operation of the water supply systems for irrigated agriculture, namely open canal networks. One of the challenges in this problem has been the complexity in developing plant models that can be applied to controller design. This has been tackled using system identification concepts in numerous studies including Mareels et al. (2003), Mareels et al. (2005), Ooi and Weyer (2003), Ooi and Weyer (2007), Weyer (2002), Weyer (2003), and Weyer (2006). These newer, data-based models, have resulted in the development of successful decentralized feedback controllers for large-scale open canal networks. This success has motivated others to explore the application of novel models and feedback control on the paddock. One such attempt includes Ooi et al. (2008) which demonstrated thata relatively simple first-order model based on water balance concepts can be used to reliably predict soil water deficit in a plant's root zone. With these new concepts,

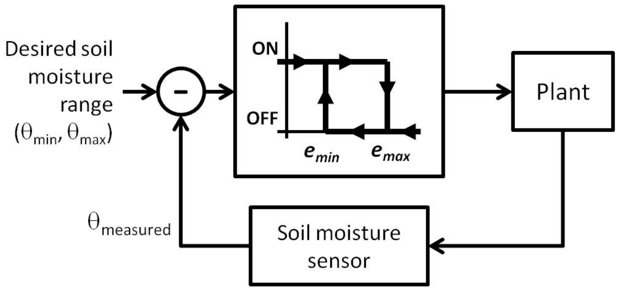

Fig. 2. Schematic overview of the irrigation scheduling problem as a feedback control problem.

irrigation scheduling can be caste in the familiar feedback control framework in Fig. 2. In this case, irrigation is the input, the soil moisture (or deficit) is the system output. Our work is motivated by these developments.

This paper applies Model Predictive Control (MPC) to the irrigation scheduling problem. MPC is widely used in the process control industry to solve large scale multivariable problems with multiple constraints. The objective of MPC is to calculate plant inputs (controls) to minimize future values of a performance criterion that will possibly be subject to constraints on both manipulated inputs and outputs. Future values of the performance criterion are computed according to a model of the plant, called the internal model, Wang (2009). MPC appears to be well suited to the problem at hand. The problem has input constraints in the form of maximum daily irrigation volumes as well as maximum seasonal volumes. This paper only deals with the former. The problem also has output constraints denoted by the *field capacity* and the *refill point*. A significant advantage of MPC is that the controller can be executed in real-time. This is not possible with solutions based on Dynamic Programming.

In this paper, we develop and evaluate an MPC controller to schedule paddock-scale irrigation events subject to water availabilityconstraints. A summary of the remainder of the paper is as follows. Section 2 outlines the problem formulation, including the system dynamics, a state space model, and a formulation of the constrained optimization. Section 3 outlines some results using measured evapotranspiration and precipitation data. Finally Section 4 includes some conclusions and extensions to this work currently underway.

2. PROBLEM FORMULATION

The starting point in the design of our controller is the development of the system model. Following Ooi et al. (2008) we note that changes in the root-zone soil moisture can be modeled using the expression

$$\begin{aligned}\frac{d\theta}{dt} &\propto E_c(t) - I(t) - P(t) \\ &= c_1 E_c(t) - c_2 I(t) - c_3 P(t),\end{aligned} \tag{1}$$

where θ is the soil moisture deficit in the plant's root zone, E is the evapotranspiration,I is the applied irrigation, and P is the precipitation. It is assumed that E is known, P is known, I is the control input, and θ is the objective variable. In Ooi et al. (2008) the coefficients c_i are estimated using system identification principles. Our objective in this paper however is the controller formulation for a model of the form in (1). To simplify the exposition, we assume that all coefficients are unity $c_i = 1,\ i = 1, 2, 3$. This selection of coefficient values does not affect the controller formulation. On the other hand, model mismatch is an important issue affecting controller performance, which we do not cover in this paper and is the subject of future work. In a future paper we will further extend this model to other more general time-series models. Note that all quantities are measured in metric millimeters (mm).

Using a Euler approximation we can write

$$\frac{d\theta}{dt} \approx \frac{\theta(k+1) - \theta(k)}{T_s}, \tag{2}$$

where T_s is the sample interval. Using (2) in (1), the discrete time water balance dynamics are given by

$$\begin{aligned}\theta(k+1) = \theta(k) + E_c(k) - I(k) - P(k), \\ k = 0, 1, ..., N-1\end{aligned} \tag{3}$$

where NT_s denotes the period over which irrigation is performed. We assume $\theta(0) = 0$, that is, the irrigation period is started with soil moisture at field capacity.

2.1 State space model

Armed with the discrete time model, the next step towards developing the MPC controller is to express (3) in state space form. The following procedure is based on that outlined in Wang (2009). First, the following state variable is defined

$$x_m(k) \triangleq \theta(k). \tag{4}$$

We can using this to write the following single input single output state space model

$$\begin{aligned}x_m(k+1) &= x_m(k) + u(k) + D_m d(k), \\ y(k) &= x_m(k),\end{aligned} \tag{5}$$

where

$$u(k) = -I(k),\ d(k) = [E_c(k)\ P(k)]^T,\ D_m = [1\ -1]. \tag{6}$$

Integral action is introduced by defining the following difference

$$\Delta x_m(k) \triangleq x_m(k) - x_m(k-1). \tag{7}$$

We now introduce a new state vector

$$\mathbf{x}(k) = [\,\Delta x_m(k)\ y(k)\,]^T. \tag{8}$$

Using these equations we can write the following augmented state space model

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\Delta u(k) + \mathbf{B}_d\mathbf{e}(k), \\ y(k) &= \mathbf{C}\mathbf{x}(k),\end{aligned} \tag{9}$$

where

$$\begin{aligned}\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\ \mathbf{B}_d = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix},\ \mathbf{C} = [\,0\ 1\,],\end{aligned} \tag{10}$$

and

$$\Delta u(k) = u(k) - u(k-1), \tag{11}$$

and

$$\mathbf{e}(k) = d(k) - d(k-1). \tag{12}$$

2.2 MPC controller

Having developed the state space equation, we can now implement the controller. The idea behind MPC can be summarized as follows Goodwin et al. (2005):

- At time i and for the current state $x(i)$, solve an optimal control problem over a fixed future interval, e.g. $[i; i+N_c-1]$, taking into account the current and future constraints.
- Apply only the first step in the resulting optimal control sequence.
- Measure the state reached at time $i+1$.
- Repeat the fixed horizon optimization at time $i+1$ over the future interval $[i+1; i+N_c]$, starting from the (now) current state $x(i+1)$.

The first step in this task is to calculate predicted outputs as a function of future control signals. The prediction is performed over an optimization window N_p. The predicted outputs are given by

$$\mathbf{Y}(k') = [y(k'+1|k')\, y(k'+1|k') \cdots y(k'+N_p|k')]^T, \tag{13}$$

and the future control signals are given by

$$\Delta\mathbf{U}(k') = [\Delta u(k')\, \Delta u(k'+1) \cdots \Delta u(k'+N_c-1)]^T, \tag{14}$$

where k' is the current time-step, N_c is the control horizon, and $N_c \leq N_p$. Using these vectors and the augmented state space model we can write

$$\mathbf{Y}(k') = \mathbf{F}\mathbf{x}(k') + \Phi\Delta\mathbf{U}(k') + \Psi\mathbf{E}(k'), \tag{15}$$

where

$$\mathbf{E}(k') = [e(k')\, e(k'+1) \cdots e(k'+N_c-1)]^T, \tag{16}$$

and

$$\mathbf{F} = \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{N_p} \end{bmatrix}, \tag{17}$$

and

$$\Phi = \begin{bmatrix} \mathbf{CB} & 0 & \cdots & 0 \\ \mathbf{CAB} & \mathbf{CB} & \cdots & 0 \\ \mathbf{CA}^2\mathbf{B} & \mathbf{CAB} & \cdots & 0 \\ \vdots & & & \\ \mathbf{CA}^{N_p-1}\mathbf{B} & \mathbf{CA}^{N_p-2}\mathbf{B} & \cdots & \mathbf{CA}^{N_p-N_c}\mathbf{B} \end{bmatrix}, \tag{18}$$

and

$$\Psi = \begin{bmatrix} \mathbf{CB}_d & 0 & \cdots & 0 \\ \mathbf{CAB}_d & \mathbf{CB}_d & \cdots & 0 \\ \mathbf{CA}^2\mathbf{B}_d & \mathbf{CAB}_d & \cdots & 0 \\ \vdots & & & \\ \mathbf{CA}^{N_p-1}\mathbf{B}_d & \mathbf{CA}^{N_p-2}\mathbf{B}_d & \cdots & \mathbf{CA}^{N_p-N_c}\mathbf{B}_d \end{bmatrix}. \tag{19}$$

We are now in a position to formally state the optimization problem. Let θ^* denote the target soil moisture deficit which is assumed to be constant for all k and define the following $(N_p \times 1)$ set-point vector

$$\mathbf{R}_s^T = [1\,1 \cdots 1]\,\theta^*. \tag{20}$$

The optimization cost function that reflects the control objectives is

$$J = (\mathbf{R}_s - \mathbf{Y}_{k'})^T (\mathbf{R}_s - \mathbf{Y}_{k'}) + \Delta\mathbf{U}_{k'}^T \bar{\mathbf{R}} \Delta\mathbf{U}_{k'}, \tag{21}$$

where $\bar{\mathbf{R}} = r_w \mathbf{I}_{N_c \times N_c}$ and $r_w \geq 0$ is a tuning parameter. This tuning parameter can be used to tradeoff output tracking response time. Decreasing r_w leads to a more sluggish controller through more conservative irrigation effort. Increasing r_w leads to increased output errors but faster tracking.

Note in this equation that the cost function is in terms of $\Delta\mathbf{U}$ and not absolute irrigation values. Recall that this is as a consequence of including integral control to eliminate steady state errors, see (7). The MPC formulation for the irrigation scheduling problem using the simplified model in (1) is now complete and can be stated as follows

$$\begin{aligned} &\min_{\Delta\mathbf{U}} \; J \\ &\text{subject to } u(k) < I_{\max} \\ &\qquad \theta_{\min} \leq \theta \leq \theta_{\max} \end{aligned} \tag{22}$$

where $I_{\max}$ is the maximum daily irrigation. The Hildreth quadratic programming procedure outlined in Wang (2009) is used to solve (22).

3. SIMULATION EXAMPLE

The examples presented in this section show the benefits of MPC as a method for real-time scheduling of irrigation events. Before we demonstrate the MPC simulation example, Fig. 4 illustrates the outcome of a common heuristic irrigation scheduling regime. In this example, the sample rate in (2) is $T_s = 1$ day. The soil moisture set point, field capacity, and refill point are, $\theta^* = 350$ mm, $\theta_{\max} = 355$ mm, and $\theta_{\min} = 345$ mm, respectively. The ET and precipitation data are plotted in Fig. 3. These are derived from a weather station located at the University of Melbourne's agricultural research station in Dookie, Victoria, Australia. They have selected as a typical set of inputs that would be presented to a farmer considering irrigation scheduling. It is assumed the farmer irrigates at most every two days. Irrigation is applied in three bands depending on the measured deficit, the algorithm is as follows:

$$\begin{array}{rl} \theta(k) - \theta^* > 5 & I(k) = 0 \\ 5 \geq \theta(k) - \theta^* \geq -5 & I(k) = 5 \\ \theta(k) - \theta^* < -5 & I(k) = 15 \end{array}, \tag{23}$$

$$k = 0, 2, 4, ...$$

The irrigation values $I(k) = 5$ and $I(k) = 15$, the threshold values 5 and -5, and the minimum irrigation interval of two days, have been selected somewhat arbitrarily. In practice the farmer may use further intuition to fine tune these values. Nonetheless, the outcome in Fig. 4 highlights some of the key disadvantages of this approach. These include over-watering and under-watering, and poor set point tracking. Such heuristic methods are very common

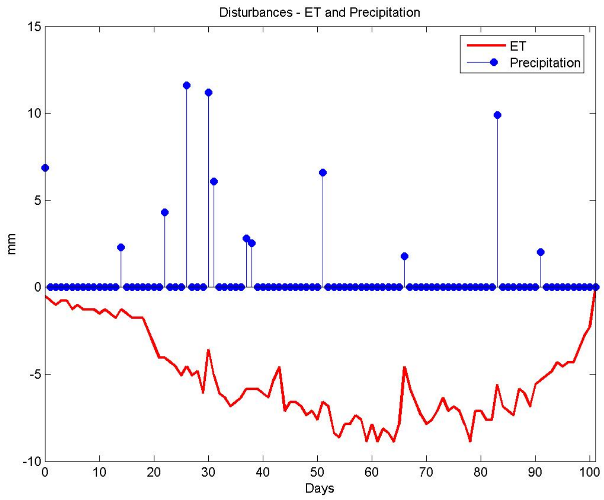

Fig. 3. Measured ET and precipitation.

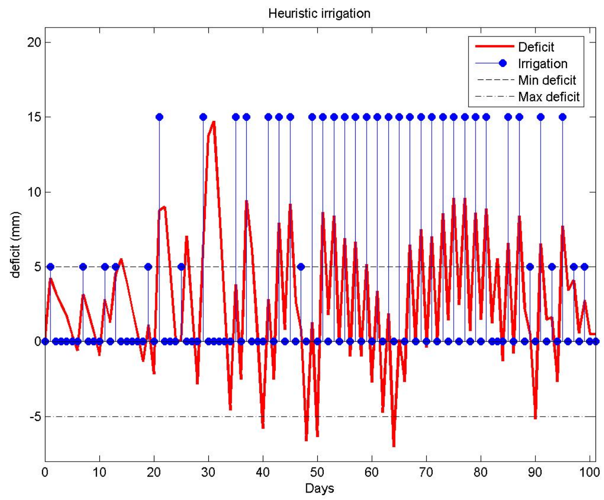

Fig. 4. Output of heuristic controller.

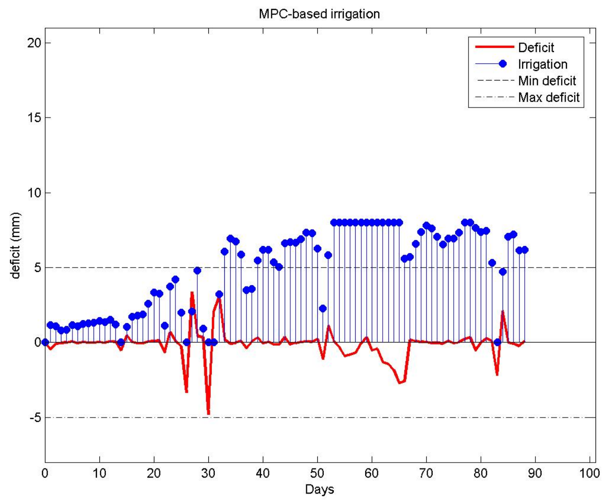

Fig. 5. Output of MPC controller.

in practice. The MPC algorithm presented herein provides a more systematic methodology.

The MPC controller's control and prediction horizon are $N_c = 7$ and $N_p = 14$, respectively. The maximum daily irrigation is $I_{\max} = 8$mm. The commanded irrigation input and the resultant soil moisture for the MPC controller are plotted in Fig. 5. The main points to observe are improved set-point tracking and the deficit constraints are satisfied.

4. CONCLUSION

We have developed an MPC controller for scheduling irrigation events in the paddock. The system dynamics are based on the well known water balance model that is used by many heuristic scheduling approaches. A state space formulation of the water balance model is developed. The MPC controller is designed for soil moisture deficit set-point tracking and also incorporates input and output constraints. Measured ET and precipitation data is used stimulate a simulation model and the MPC controller. The controller successfully tracks the commanded set-point as well as meet the constraints.

The work presented herein is a preliminary investigation into the application of feedback control and optimization for irrigation scheduling. There are a number of extensions that are part of ongoing studies. Firstly, the simplistic model in (1) must be generalized based on system identification experiments. Secondly, the model assumes that ET and precipitation are directly available. This is not always the case. Often ET must be estimated from other climate variables and then scaled for the specific crop under consideration. This requires an observer coupled with the controller. Future work will also investigate the use of other climate variables as a substitute for ET, such as temperature, humidity, solar radiation and wind speed. Another extension will investigate the inclusion of a constraint to capture the total volume of water available.

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