

N-Dimensional Lists (ndlist)

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<https://github.com/ambaker1/ndlist>

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Abstract

The “ndlist” package provides tools for vector, matrix, and tensor manipulation and processing, where vectors are represented by Tcl lists, matrices by nested Tcl lists, and higher dimension lists by additional levels of nesting.

1-Dimensional Lists (Vectors)

Lists are foundational to Tcl, so in addition to providing utilities for ND lists, this package also provides utilities for working with 1D lists, or vectors.

Range Generator

The command *range* simply generates a range of integer values. This can be used in conjunction with the Tcl *foreach* loop to simplify writing “for” loops. There are two ways of calling this command, as shown below.

```
range $n
range $start $stop <$step>
```

\$n	Number of indices, starting at 0 (e.g. 3 returns 0 1 2).
\$start	Starting value.
\$stop	Stop value.
\$step	Step size. Default 1 or -1, depending on direction of start to stop.

Example 1: Integer range generation

Code:

```
puts [range 3]
puts [range 0 2]
puts [range 10 3 -2]
```

Output:

```
0 1 2
0 1 2
10 8 6 4
```

Example 2: Simpler for-loop

Code:

```
foreach i [range 10] {
    puts -nonewline $i
}
```

Output:

```
0 1 2 3 4 5 6 7 8 9
```

Logical Indexing

The command *find* returns the indices of non-zero elements of a boolean list, or indices of elements that satisfy a given criterion. Can be used in conjunction with *nget* to perform logical indexing.

```
find $list <$op $scalar>
```

\$list	List of values to compare.
\$op	Comparison operator. Default “!=”.
\$scalar	Comparison value. Default 0.

Example 3: Filtering a list

Code:

```
set x {0.5 2.3 4.0 2.5 1.6 2.0 1.4 5.6}
puts [nget $x [find $x > 2]]
```

Output:

```
2.3 4.0 2.5 5.6
```

Linear Interpolation

The command *linterp* performs linear 1D interpolation. Converts input to “double”.

```
linterp $x $xList $yList
```

\$x	Value to query in \$xList
\$xList	List of x points, strictly increasing
\$yList	List of y points, same length as \$xList

Example 4: Linear interpolation

Code:

```
puts [linterp 2 {1 2 3} {4 5 6}]
puts [linterp 8.2 {0 10 20} {2 -4 5}]
```

Output:

```
5.0
-2.92
```

Vector Generation

The command *linspace* can be used to generate a vector of specified length and equal spacing between two specified values. Converts input to “double”

```
linspace $n $start $stop
```

<code>\$n</code>	Number of points
<code>\$start</code>	Starting value
<code>\$stop</code>	End value

Example 5: Linearly spaced vector generation

Code:

```
puts [linspace 5 0 1]
```

Output:

```
0.0 0.25 0.5 0.75 1.0
```

The command *linspace* generates intermediate values given an increment size and a sequence of targets. Converts input to “double”.

```
linspace $step $x1 $x2 ...
```

<code>\$step</code>	Maximum step size
<code>\$x1 \$x2 ...</code>	Targets to hit.

Example 6: Intermediate value vector generation

Code:

```
puts [linspace 0.25 0 1 0]
```

Output:

```
0.0 0.25 0.5 0.75 1.0 0.75 0.5 0.25 0.0
```

Functional Mapping

The command *lapply* simply applies a command over each element of a list, and returns the result. Basic math operators can be mapped over a list with the command *lop*.

```
lapply $command $list $arg ...
```

```
lop $list $op $arg...
```

<code>\$list</code>	List to map over.
<code>\$command</code>	Command prefix to map with.
<code>\$op</code>	Math operator (see <code>::tcl::mathop</code> documentation).
<code>\$arg ...</code>	Additional arguments to append to command after each list element.

Example 7: Applying a math function to a list

Code:

```
# Add Tcl math functions to the current namespace path
namespace path [concat [namespace path] ::tcl::mathfunc]
puts [lapply abs {-5 1 2 -2}]
```

Output:

```
5 1 2 2
```

Mapping Over Two Lists

The commands *lapply* and *lop* only map over one list. The commands *lapply2* and *lop2* allow you to map, element-wise, over two lists. List lengths must be equal.

```
lapply2 $command $list1 $list2 $arg ...
```

```
lop2 $list1 $op $list2 $arg...
```

<code>\$list1 \$list2</code>	Lists to map over, element-wise.
<code>\$command</code>	Command prefix to map with.
<code>\$op</code>	Math operator (see <code>::tcl::mathop</code> documentation).
<code>\$arg ...</code>	Additional arguments to append to command after list elements.

Example 8: Mapping over two lists

Code:

```
lapply puts [lapply2 {format "%s %s"} {hello goodbye} {world moon}]
```

Output:

```
hello world
goodbye moon
```

Example 9: Adding two lists together

Code:

```
puts [lop2 {1 2 3} + {2 3 2}]
```

Output:

```
3 5 5
```

List Math

The Tcl command *lmap* allows you to loop over an arbitrary number of lists in parallel, evaluating a script at each iteration, and collecting the results of each loop iteration into a new list. The command *lexpr* is an extension of this concept, just calling *lmap* and passing the input through the Tcl *expr* command.

```
lexpr $varList $list <$varList $list ...> $expr
```

\$varList ...	List(s) of variables to iterate with.
\$list ...	List(s) to iterate over.
\$expr	Tcl expression to evaluate at every loop iteration.

Example 10: Filtering a list

Code:

```
set numbers [range 10]
set odds [lexpr x $numbers {$x % 2 ? $x : [continue]}]; # only odd numbers
puts $odds
```

Output:

```
1 3 5 7 9
```

Example 11: Adding three lists together

Code:

```
set x {1 2 3}
set y {2 9 2}
set z {5 -2 0}
puts [lexpr xi $x yi $y zi $z {$xi + $yi + $zi}]
```

Output:

```
8 9 5
```

List Statistics

The commands *max*, *min*, *sum*, *product*, *mean*, *median*, *variance*, and *stdev* compute the maximum, minimum, sum, product, mean, median, variance, and standard deviation of values in a list. For more advanced statistics, check out the Tellib `math::statistics` package.

```
max $list
```

```
min $list
```

```
sum $list
```

```
product $list
```

```
mean $list
```

```
median $list
```

```
variance $list <$pop>
```

```
stdev $list <$pop>
```

`$list` List (at least length 1) to compute statistic of.

`$pop` Compute population statistic instead of sample statistic. Default false.

Example 12: List Statistics

Code:

```
set list {-5 3 4 0}
foreach stat {max min sum product mean median variance stdev} {
    puts [list $stat [$stat $list]]
}
```

Output:

```
max 4
min -5
sum 2
product 0
mean 0.5
median 1.5
variance 16.333333333333332
stdev 4.041451884327381
```


Vector Algebra

The dot product of two equal length vectors can be computed with *dot*. The cross product of two vectors of length 3 can be computed with *cross*.

```
dot $a $b
```

```
cross $a $b
```

`$a` First vector.
`$b` Second vector.

Example 13: Dot and cross product

Code:

```
set x {1 2 3}  
set y {-2 -4 6}  
puts [dot $x $y]  
puts [cross $x $y]
```

Output:

```
8  
24 -12 0
```

The norm, or magnitude, of a vector can be computed with *norm*.

```
norm $a <$p>
```

`$a` Vector to compute norm of.
`$p` Norm type. 1 is sum of absolute values, 2 is euclidean distance, and Inf is absolute maximum value. Default 2.

Example 14: Normalizing a vector

Code:

```
set x {3 4}  
set x [lcp $x / [norm $x]]  
puts $x
```

Output:

```
0.6 0.8
```

For more advanced vector algebra routines, check out the Tcllib `math::linearalgebra` package.

2-Dimensional Lists (Matrices)

A matrix is a two-dimensional list, or a list of row vectors. This is consistent with the format used in the Tcllib `math::linearalgebra` package. See the example below for how matrices are interpreted.

$$A = \begin{bmatrix} 2 & 5 & 1 & 3 \\ 4 & 1 & 7 & 9 \\ 6 & 8 & 3 & 2 \\ 7 & 8 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 9 \\ 3 \\ 0 \\ -3 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 7 & -5 & -2 \end{bmatrix}$$

Example 15: Matrices and vectors

Code:

```
set A {{2 5 1 3} {4 1 7 9} {6 8 3 2} {7 8 1 4}}
set B {9 3 0 -3}
set C {{3 7 -5 -2}}
```

Identity Matrix

The command `eye` generates an identity matrix of a specified size.

```
eye $n
```

`$n` Size of identity matrix

Example 16: Generating an identity matrix

Code:

```
puts [eye 3]
```

Output:

```
{1 0 0} {0 1 0} {0 0 1}
```

Matrix Transpose

The command *transpose* simply swaps the rows and columns of a matrix.

```
transpose $A
```

\$A Matrix to transpose, nxm.

Returns an mxn matrix.

Example 17: Transposing a matrix

Code:

```
puts [transpose {{1 2} {3 4}}]
```

Output:

```
{1 3} {2 4}
```

Matrix Multiplication

The command *matmul* performs matrix multiplication for two matrices. Inner dimensions must match.

```
matmul $A $B
```

\$A Left matrix, nxq.

\$B Right matrix, qxm.

Returns an nxm matrix (or the corresponding dimensions from additional matrices)

Example 18: Multiplying a matrix

Code:

```
puts [matmul {{2 5 1 3} {4 1 7 9} {6 8 3 2} {7 8 1 4}} {9 3 0 -3}]
```

Output:

```
24.0 12.0 72.0 75.0
```

Iteration Tools

The commands *zip* zips two lists into a list of tuples, and *zip3* zip three lists into a list of triples. Lists must be the same length.

```
zip $a $b
```

```
zip3 $a $b $c
```

`$a $b $c`

Lists to zip together.

Example 19: Zipping lists

Code:

```
puts [zip {A B C} {1 2 3}]  
puts [zip3 {Do Re Mi} {A B C} {1 2 3}]
```

Output:

```
{A 1} {B 2} {C 3}  
{Do A 1} {Re B 2} {Mi C 3}
```

The command *cartprod* computes the Cartesian product of an arbitrary number of vectors, returning a matrix where the columns correspond to the input vectors and the rows correspond to all the combinations of the vector elements.

```
cartprod $list1 $list2 ...
```

`$list1 $list2 ...`

Lists, or vectors, to take Cartesian product of.

Example 20: Cartesian product

Code:

```
puts [cartprod {A B C} {1 2 3}]
```

Output:

```
{A 1} {A 2} {A 3} {B 1} {B 2} {B 3} {C 1} {C 2} {C 3}
```

N-Dimensional Lists (Tensors)

A ND list is defined as a list of equal length (N-1)D lists, which are defined as equal length (N-2)D lists, and so on until (N-N)D lists, which are scalars of arbitrary size. This definition is flexible, and allows for different interpretations of the same data. For example, the list “1 2 3” can be interpreted as a scalar with value “1 2 3”, a vector with values “1”, “2”, and “3”, or a matrix with row vectors “1”, “2”, and “3”.

The command *ndlist* validates that the input is a valid ndlist. If the input value is “ragged”, as in it has inconsistent dimensions, it will throw an error. In general, if a value is a valid for N dimensions, it will also be valid for dimensions 0 to N-1. All other ND list commands assume a valid ND list.

```
ndlist $nd $value
```

<code>\$nd</code>	Dimensionality of ND list (e.g. 2D, 2d, or 2 for a matrix).
<code>\$value</code>	List to interpret as an ndlist

Shape and Size

The commands *nshape* and *nsiz*e return the shape and size of an ND list, respectively. The shape is a list of the dimensions, and the size is the product of the shape.

```
nshape $nd $ndlist <$axis>
```

```
nsiz
```

<code>\$nd</code>	Dimensionality of ND list (e.g. 2D, 2d, or 2 for a matrix).
<code>\$ndlist</code>	ND list to get dimensions of.
<code>\$axis</code>	Axis to get dimension along. Blank for all.

Example 21: Getting shape and size of an ND list

Code:

```
set A [ndlist 2D {{1 2 3} {4 5 6}}]
puts [nshape 2D $A]
puts [nsiz
```

Output:

```
2 3
6
```

Initialization

The command *nfull* initializes a valid ND list of any size filled with a single value.

```
nfull $value $n ...
```

\$value Value to repeat
\$n ... Shape (list of dimensions) of ND list.

Example 22: Generate ND list filled with one value

Code:

```
puts [nfull foo 3 2]; # 3x2 matrix filled with "foo"  
puts [nfull 0 2 2 2]; # 2x2x2 tensor filled with zeros
```

Output:

```
{foo foo} {foo foo} {foo foo}  
{0 0} {0 0} {0 0} {0 0}
```

The command *nrand* initializes a valid ND list of any size filled with random values between 0 and 1.

```
nrand $n ...
```

\$n ... Shape (list of dimensions) of ND list.

Example 23: Generate random matrix

Code:

```
expr {srand(0)}; # resets the random number seed (for the example)  
puts [nrand 1 2]; # 1x2 matrix filled with random numbers
```

Output:

```
{0.013469574513598146 0.3831388500440581}
```

Repeating and Expanding

The command *nrepeat* repeats portions of an ND list a specified number of times.

```
nrepeat $ndlist $n ...
```

\$value	Value to repeat
\$n ...	Repetitions at each level.

Example 24: Repeat elements of a matrix

Code:

```
puts [nrepeat {{1 2} {3 4}} 1 2]
```

Output:

```
{1 2 1 2} {3 4 3 4}
```

The command *nexpand* repeats portions of an ND list to expand to new dimensions. New dimensions must be divisible by old dimensions. For example, 1x1, 2x1, 4x1, 1x3, 2x3 and 4x3 are compatible with 4x3.

```
nexpand $ndlist $n ...
```

\$ndlist	ND list to expand.
\$n ...	New dimensions of ND list.

Example 25: Expand an ND list to new dimensions

Code:

```
puts [nexpand {1 2 3} 3 2]  
puts [nexpand {{1 2}} 2 4]
```

Output:

```
{1 1} {2 2} {3 3}  
{1 2 1 2} {1 2 1 2}
```

Flattening and Reshaping

The command *nreshape* reshapes a vector into a compatible shape. Vector length must equal ND list size.

```
nreshape $vector $n ...
```

\$vector Vector (1D list) to reshape.

\$n ... Shape (list of dimensions) of ND list.

Example 26: Reshape a vector to a matrix

Code:

```
puts [nreshape {1 2 3 4 5 6} 2 3]
```

Output:

```
{1 2 3} {4 5 6}
```

The inverse is *nflatten*, which flattens an ND list to a vector, which can be then used with *nreshape*.

```
nflatten $nd $ndlist
```

\$nd Dimensionality of ND list (e.g. 2D, 2d, or 2 for a matrix).

\$ndlist ND list to flatten.

Example 27: Reshape a matrix to a 3D tensor

Code:

```
set x [nflatten 2D {{1 2 3 4} {5 6 7 8}}]
puts [nreshape $x 2 2 2]
```

Output:

```
{{1 2} {3 4}} {{5 6} {7 8}}
```


Index Notation

This package provides generalized n-dimensional list access/modification commands, using an index notation parsed by the command `::ndlist::ParseIndex`, which returns the index type and an index list for the type.

```
::ndlist::ParseIndex $n $input
```

<code>\$input</code>	Index input. Options are shown below:
<code>* or :</code>	All indices
<code>\$start:\$stop</code>	Range of indices (e.g. 0:4 or 1:end-2).
<code>\$start:\$step:\$stop</code>	Stepped range of indices (e.g. 0:2:-2 or 2:3:end).
<code>\$iList</code>	List of indices (e.g. {0 end-1 5} or 3).
<code>\$i.</code>	Single index with a dot, “flattens” the ndlist (e.g. 0. or end-3.).
<code>\$n</code>	Number of elements in list.

Additionally, indices get passed through the `::ndlist::Index2Integer` command, which converts the inputs “end”, “end-integer”, “integer±integer” and negative wrap-around indexing (where -1 is equivalent to “end”) into normal integer indices. Note that this command will return an error if the index is out of range.

```
::ndlist::Index2Integer $n $index
```

<code>\$index</code>	Single index.
<code>\$n</code>	Number of elements in list.

Example 28: Index Notation

Code:

```
set n 10
puts [::ndlist::ParseIndex $n * $n]
puts [::ndlist::ParseIndex $n 1:8]
puts [::ndlist::ParseIndex $n 0:2:6]
puts [::ndlist::ParseIndex $n {0 5 end-1}]
puts [::ndlist::ParseIndex $n end.]
```

Output:

```
A {}
R {1 8}
L {0 2 4 6}
L {0 5 8}
S 9
```

Access

Portions of an ND list can be accessed with the command *nget*, using the index parser *::ndlist::ParseIndex* for each dimension being indexed. Note that unlike the Tcl *lindex* and *lrange* commands, *nget* will return an error if the indices are out of range.

```
nget $ndlist $i ...
```

\$ndlist ND list value.

\$i ... Index inputs, parsed with *::ndlist::ParseIndex*. The number of index arguments determines the interpreted dimensions.

Example 29: ND list access

Code:

```
set A {{1 2 3} {4 5 6} {7 8 9}}
puts [nget $A 0 *]; # get row matrix
puts [nget $A 0. *]; # flatten row matrix to a vector
puts [nget $A 0:1 0:1]; # get matrix subset
puts [nget $A end:0 end:0]; # can have reverse ranges
puts [nget $A {0 0 0} 1.]; # can repeat indices
```

Output:

```
{1 2 3}
1 2 3
{1 2} {4 5}
{9 8 7} {6 5 4} {3 2 1}
2 2 2
```

Modification

A ND list can be modified by reference with *nset*, and by value with *nreplace*, using the index parser *::ndlist::ParseIndex* for each dimension being indexed. Note that unlike the Tcl *lset* and *lreplace* commands, the commands *nset* and *nreplace* will return an error if the indices are out of range. If all the index inputs are “*” except for one, and the replacement list is blank, it will delete values along that axis by calling *nremove*. Otherwise, the replacement ND list must be expandable to the target index dimensions.

```
nset $varName $i ... $sublist
```

```
nreplace $ndlist $i ... $sublist
```

\$varName	Name of ND list to modify.
\$ndlist	ND list to modify.
\$i ...	Index inputs, parsed with <i>::ndlist::ParseIndex</i> . The number of index inputs determines the interpreted dimensions.
\$sublist	Replacement list, or blank to delete values.

Example 30: Swapping rows in a matrix

Code:

```
# ND List Value Modification
set a {{1 2} {3 4} {5 6}}
nset a {1 0} * [nget $a {0 1} *]; # Swap rows and columns (modify by reference)
puts $a
```

Output:

```
{3 4} {1 2} {5 6}
```

Removal

The command *nremove* removes portions of an ND list at a specified axis.

```
nremove $nd $ndlist $i <$axis>
```

<code>\$nd</code>	Dimensionality of ND list (e.g. 2D, 2d, or 2 for a matrix).
<code>\$ndlist</code>	ND list to modify.
<code>\$i</code>	Index input, parsed with <code>::ndlist::ParseIndex</code> .
<code>\$axis</code>	Axis to remove at. Default 0.

Example 31: Deleting a column from a matrix

Code:

```
set a {{1 2 3} {4 5 6} {7 8 9}}
puts [nremove $a 2 1]; # Delete column 2
```

Output:

```
{1 2} {4 5} {7 8}
```

Example 32: Removing list elements that satisfy criteria

Code:

```
set x [range 10]
puts [nremove $x [find $x > 4]]
```

Output:

```
0 1 2 3 4
```

Insertion and Concatenation

The command *ninsert* allows you to insert an ND list into another ND list at a specified index and axis, as long as the ND lists agree in dimension at all other axes. If “end” or “end-integer” is used for the index, it will insert after the index. Otherwise, it will insert before the index.

```
ninsert $nd $ndlist1 $index $ndlist2 <$axis>
```

\$nd	Dimensionality of ND list (e.g. 2D, 2d, or 2 for a matrix).
\$ndlist1 \$ndlist2	ND lists to combine.
\$index	Index to insert at.
\$axis	Axis to insert at (default 0).

The command *nstack* is shorthand for inserting at “end”, and concatenates two ND lists.

```
nstack $nd $ndlist1 $ndlist2 <$axis>
```

\$nd	Dimensionality of ND list (e.g. 2D, 2d, or 2 for a matrix).
\$ndlist1 \$ndlist2	ND lists to concatenate.
\$axis	Axis to concatenate at (default 0).

Example 33: Inserting a column into a matrix

Code:

```
set matrix {{1 2} {3 4} {5 6}}
set column {A B C}
puts [ninsert 2D $matrix 1 $column 1]
```

Output:

```
{1 A 2} {3 B 4} {5 C 6}
```

Example 34: Concatenate tensors

Code:

```
set x [nreshape {1 2 3 4 5 6 7 8 9} 3 3 1]
set y [nreshape {A B C D E F G H I} 3 3 1]
puts [nstack 3D $x $y 2]
```

Output:

```
{{1 A} {2 B} {3 C}} {{4 D} {5 E} {6 F}} {{7 G} {8 H} {9 I}}
```

Changing Order of Axes

The command *nswapaxes* is a general purpose transposing function that swaps the axes of an ND list. For simple matrix transposing, the command *transpose* can be used instead.

```
nswapaxes $ndlist $axis1 $axis2
```

\$ndlist ND list to manipulate.

\$axis1 \$axis2 Axes to swap.

The command *nmoveaxis* moves a specified source axis to a target position. For example, moving axis 0 to position 2 would change “i,j,k” to “j,k,i”.

```
nmoveaxis $ndlist $source $target
```

\$ndlist ND list to manipulate.

\$source Source axis.

\$target Target position.

The command *npermute* is more general purpose, and defines a new order for the axes of an ND list. For example, the axis list “1 0 2” would change “i,j,k” to “j,i,k”.

```
npermute $ndlist $axis ...
```

\$ndlist ND list to manipulate.

\$axis ... List of axes defining new order.

Example 35: Changing tensor axes

Code:

```
set x {{{1 2} {3 4}} {{5 6} {7 8}}}  
set y [nswapaxes $x 0 2]  
set z [nmoveaxis $x 0 2]  
puts [lindex $x 0 0 1]  
puts [lindex $y 1 0 0]  
puts [lindex $z 0 1 0]
```

Output:

```
2  
2  
2
```

ND Functional Mapping

The command *napply* simply applies a command over each element of an ND list, and returns the result. Basic math operators can be mapped over an ND list with the command *nop*, which is a special case of *napply*, using the `::tcl::mathop` namespace.

```
napply $nd $command $ndlist $arg ...
```

```
nop $nd $ndlist $op $arg...
```

<code>\$nd</code>	Dimensionality of ND list (e.g. 2D, 2d, or 2 for a matrix).
<code>\$ndlist</code>	ND list to map over.
<code>\$command</code>	Command prefix to map with.
<code>\$op</code>	Math operator (see <code>::tcl::mathop</code> documentation).
<code>\$arg ...</code>	Additional arguments to append to command after ND list element.

Example 36: Chained functional mapping over a matrix

Code:

```
napply 2D puts [napply 2D {format %.2f} [napply 2D expr {{1 2} {3 4}} + 1]]
```

Output:

```
2.00
3.00
4.00
5.00
```

Example 37: Element-wise operations

Code:

```
puts [nop 1D {1 2 3} + 1]
puts [nop 2D {{1 2 3} {4 5 6}} > 2]
```

Output:

```
2 3 4
{0 0 1} {1 1 1}
```

Mapping Over Two ND Lists

The commands *napply* and *nop* only map over one ND list. The commands *napply2* and *nop2* allow you to map, element-wise, over two ND lists. If the input lists have different shapes, they will be expanded to their maximum dimensions with *nexpand* (if compatible).

```
napply2 $nd $command $ndlist1 $ndlist2 $arg ...
```

```
nop2 $nd $ndlist1 $op $ndlist2 $arg...
```

<code>\$nd</code>	Dimensionality of ND list (e.g. 2D for a matrix).
<code>\$ndlist1 \$ndlist2</code>	ND lists to map over, element-wise.
<code>\$command</code>	Command prefix to map with.
<code>\$op</code>	Math operator (see <code>::tcl::mathop</code> documentation).
<code>\$arg ...</code>	Additional arguments to append to command after ND list elements.

Example 38: Format columns of a matrix

Code:

```
set data {{1 2 3} {4 5 6} {7 8 9}}
set formats {%1f %2f %3f}
puts [napply2 2D format $formats $data]
```

Output:

```
{1.0 2.00 3.000} {4.0 5.00 6.000} {7.0 8.00 9.000}
```

Example 39: Adding matrices together

Code:

```
set A {{1 2} {3 4}}
set B {{4 9} {3 1}}
puts [nop2 2D $A + $B]
```

Output:

```
{{5 11} {6 5}}
```


Reducing an ND List

The command *nreduce* combines *nmoveaxis* and *napply* to reduce an axis of an ND list with a function that reduces a vector to a scalar, like *max* or *sum*.

```
nreduce $nd $command $ndlist <$axis>
```

\$nd	Dimensionality of ND list (e.g. 2D, 2d, or 2 for a matrix).
\$command	Command prefix to map with.
\$ndlist	ND list to map over.
\$axis	Axis to reduce. Default 0.

Example 40: Matrix row and column statistics

Code:

```
set x {{1 2} {3 4} {5 6} {7 8}}
puts [nreduce 2D max $x]; # max of each column
puts [nreduce 2D max $x 1]; # max of each row
puts [nreduce 2D sum $x]; # sum of each column
puts [nreduce 2D sum $x 1]; # sum of each row
```

Output:

```
7 8
2 4 6 8
16 20
3 7 11 15
```

Generalized N-Dimensional Mapping

The command *nmap* is a general purpose mapping function for N-dimensional lists in Tcl, and the command *nexpr* a special case for math expressions. If multiple ND lists are provided for iteration, they must be expandable to their maximum dimensions. The actual implementation flattens all the ND lists and calls the Tcl *lmap* command, and then reshapes the result to the target dimensions. So, if “continue” or “break” are used, it will return an error; it cannot be used for filtering.

```
nmap $nd $varName $ndlist <$varName $ndlist ...> $body
```

```
nexpr $nd $varName $ndlist <$varName $ndlist ...> $expr
```

\$nd	Dimensionality of ND list (e.g. 2D, 2d, or 2 for a matrix).
\$varName	Variable name to iterate with.
\$ndlist	ND list to iterate over.
\$body	Tcl script to evaluate at every loop iteration.
\$expr	Tcl expression to evaluate at every loop iteration.

Example 41: Expand and map over matrices

Code:

```
set phrases [nmap 2D greeting {{hello goodbye}} subject {world moon} {  
    list $greeting $subject  
}]  
napply 2D puts $phrases
```

Output:

```
hello world  
goodbye world  
hello moon  
goodbye moon
```

Example 42: Adding two matrices together, element-wise

Code:

```
set x {{1 2} {3 4}}  
set y {{4 1} {3 9}}  
set z [nexpr 2D xi $x yi $y {$xi + $yi}]  
puts $z
```

Output:

```
{5 3} {6 13}
```

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