Name: Amber Khurshid
Section: BAI-5A
Roll no: 22P-9295

Assignment #01

1. Theoretical Questions.

1.1

(a) $f(n) = 3n^3 + 5n^2 + 7$

$3n^3$ here is higher order term and grows faster than other terms for bigger values of n. We will ignore 3 and time complexity is $O(n^3)$.

b) $g(n) = 2^{\sqrt{n}}$

$n > \sqrt{n}$ which means $\sqrt{n}$ grows slower than n so as we dont have any other term and $2^{\sqrt{n}}$ is also exponential. The time complexity is $O(\sqrt{2^n})$.

c) $h(n) = n\log^2(n)$

As we know $\log n < n$ but if we take product $n\log^2(n)$ then the function grows faster. So the time complexity is $O(n\log^2 n)$.

d) $k(n) = n!$

The factorial function grows fast compared to polynomials, log or exponential functions, for large values of n, $n!$ dominates other functions so time complexity is $O(n!)$.

1.2

When we say $f(n)$ is $O(g(n))$. It means that $f(n)$ grows as fast as $g(n)$ for sufficiently large values of n.

In simple words, it shows that $g(n)$ is upper bound on the growth rate of $f(n)$.

Example:

When $f(n)$ is $O(g(n))$.

$f(n) = n^2$ $\quad\quad$ $g(n) = n^3$

then $f(n)$ is $O(g(n))$ since n^2 grows as fast as $g(n)$.

When $f(n)$ is not $O(g(n))$.

then $f(n)$ is not $O(g(n))$ because logarithmic growth of $\log n$ is slower than n which is linear growth.

1.3.

a) $P(n) = 5n^2 + 3n + 1$

It is polynomial because it has variable n and coefficients (5, 3 & 1). It also has non negative exponents (2, 1, 0). So it fulfills polynomial definition so yes it is quadratic polynomial.

(b) $Q(n) = 4^n$

This is exponential function because the variable n appears as an exponent while in a polynomial the variable is base & is raised to non-negative integer powers.

(c) $R(n) = \log n \cdot n$

It is logarithmic function.

d) $S(n) = \sqrt{n} \cdot 2^n$

This function is neither a polynomial nor exponential.
As it has $\sqrt{n}$ multiplied by exponential term 2^n. Function involving roots and exponents both together are not polynomial or exponents.

2 - Practical Coding Problems

2.1 loop 1.

$i=0$	$i=1$	$i=2$	$i=n-1$
inner loop	inner loop	inner loop	inner loop
n	$(n-1)$	$(n-2)$	1

So $n+(n-1)+(n-1)+\cdots+1.$

total iterations $= \dfrac{n(n+1)}{2}$

Time complexity is $O(n^2)$.

2.2 loop 2

$i=1$	$i=2$	$i=n-1$
middle loop	middle loop	middle loop
$n-1$	$n-2$	1

total no of iterations of middle loop

$= (n-1)+(n-2)+\cdots+1 = \dfrac{n(n-1)}{2}$

$T(n)$ for middle loop $= O(n^2)$.

inner loop :

$J=1$	$J=2$	$J=n-1$
inner loop	inner loop	inner loop
n	n	n

Total no of iterations of inner loop $= n$
$T(n)$ for inner loop $= O(n)$.

Iteration of nested loop is
$O(n^2) \times O(n) = O(n^3)$
overall $T(n) = O(n^3)$.

2.3 Loop 3.

middle loop runs n times for each iteration of the outer loop.
Inner loop:

$i=0$	$i=1$	$i=2$	$i=n-1$
inner loop	inner loop	inner loop	inner loop
1	2	3	n.

T.n of iterations of inner most statement is the product of all three loops
so $\sum_{i=0}^{n-1} (i+1) \cdot n = n \sum_{i=0}^{n-1} (i+1)$.

$= n\left(\frac{n(n+1)}{2}\right) = \frac{n^3+n^2}{2}$

$T(n) = O(n^3)$.

CamScanner

2-4 Loop 4.

inner loop:

$i=0$	$i=1$	$i=i$
inner loop	inner loop	inner loop
n	$(n-2)$	$n-4$

it all stop after $i = n/2$.

total iterations $= n + (n-2) + (n-4) + \dots$

sum of arithmetic series is

$$S = \frac{\text{no of terms}}{2} (\text{first term} + \text{last})$$

$$S = \frac{n/2}{2} \cdot (n+0) = \frac{n^2}{4}$$

$T(n) = O(n^2)$.

2-5 Loop 5.

1- Outer loop runs from it on -1 $O(n)$ times.

2- for each iteration of i, middle loop runs 1 to $i-1$ means it runs $O(i)$ times.

3- for each iteration of J, inner loop runs from 1 to $J-1$, means it runs $O(J)$ times.

so total $= \sum_{i=1}^{n-1} \sum_{J=1}^{i=1} \sum_{k=1}^{J=1} 1$

$= \sum_{i=1}^{n-1} \frac{i(i-1)}{2} = O(n3)$

time complexity of this code is $O(n3)$.

3. Advanced Analysis

3.1 Prove or disprove.

$T(n) = \frac{5}{2} 2^n + 3n^2$ is $O(2^n)$.

we know $f(n) \leq cg(n)$ where c is positive for all $n \geq n_0$.

$\frac{5}{2} 2^n + 3n^2 \leq c\,g(n)$

$\frac{5}{2} 2^n + 3n^2 \leq c \cdot 2^n$

$\frac{5}{2} \cdot \frac{\cancel{2^n}}{\cancel{2^n}} + \frac{3n^2}{2^n} \leq c$

$\frac{5}{2} + \frac{3n^2}{2^n} \leq c.$

Now as n grows large the terms $\frac{3n^2}{2^n}$ approach to zero. It means we have to add something is $5/2$ let say we add 1 then $5/2 + 1 = 7/2$

so take $c = 7/2$ $g(n) = 2^n$ $\forall n \geq 8$

so yes it proves that $T(n)$ has ~~to~~ $O(2^n)$.