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Algorithms Assignment #02

Give asymptotic upper bound for T(n).
Assume $T(n)=n$ for $n \leq 5$.
Select tightest bound and method.

1) $T(n)=T(n-3)+2$

substitute $T(n-3)$

$T(n-3)=T(n-6)+2$

$T(n) = T(n-6)+4$

$T(n-6)=T(n-9)+2$

substitute $T(n-6)$.

$T(n)=T(n-9)+6$

$\vdots$

$T(n)=T(n-3k)+2k$

as we know that

$k=\frac{4}{3}$, $n-3k=5$

$3k=n-5$

$T(n)=T(5)+\frac{2}{3}n$

$T(n)=1+\frac{2}{3}n$

$T(n) = \Theta(n)$

2. $T(n)=T\left(\frac{9n}{10}\right)+1.$

$a=1$, $b=10/9$, $f(n)=1$

$n^{\log_b a}=n^0=1$

This is the second case of master's theoroum so,

$T(n)=O(\log n).$

3. $T(n)=T\left(\frac{\sqrt{n}}{2}\right)+1$

$T(n)=T\left(\frac{n^{1/2}}{2}\right)+1$

$n=2^m$

$T(2^m)=T\left(\frac{2^{m/2}}{2}\right)+1$

let

$T(2^m)=S(m)-T\left(\frac{m/2}{2}\right)+1$

$\cancel{T}$ $S(m)=T(m/4)+1$

$a=1, b=4, f(n)=1$

$m^{\log_4 1}=m^0=1$

This is the second case of master theorum.

$S(m)=O(\log m)$ n

$n=2^m$

$\log n = m$

$S(m) = T(n) = O(\log(\log n))$.

$T(n) = O(\log(\log n))$.

$T(n) = O(\log n)^2$

4. $T(n) = 3T(n-1)$

substitute $T(n-1)$

$T(n-1) = 3T(n-2)$

$T(n) = 3 \cdot 3T(n-2)$

substitute $T(n-2)$

$T(n-2) = 3T(n-3)$

$T(n) = 3 \cdot 3 \cdot 3T(n-3)$

$T(n) = 3^{n-5} T(n-k)$

$n-k=5$

$k = n-5$

$T(n) = 3^{n-5} T(5)$

$T(n) = 3^{n-5}(1)$

$T(n) = O(3^n)$.

5. $T(n) = 2T(n/2) + 2$

$a = 2,\ b = 2,\ f(n) = 1$

$n^{\log_n a} = n^{\log_2 2} = n^1 = n$

let $\varepsilon = 1$ we subtract

$n^{\log_b a - \varepsilon} = f(n)$

$n^{1-1} = 1$

$n^0 = 1$

$1 = 1$

Hence proved this is the first case of Masters Theorum.

$T(n) = O(n)$.

7. $T(n) = 5T(n/3) + n$

$a = 5,\ b = 3,\ f(n) = n$

$n^{\log_b a} = n^{\log_3 5}$

$\log_3 5 = \frac{\log_{10} 5}{\log 3} = 1.46$

$n^{\log_3 5} = n^{1.5}$

so let $\varepsilon = 0.5$ we subtract

$n^{\log_b a - \varepsilon} = f(n)$

$n^{1.5 - 0.5} = n^1$

$n = n$

Hence proved so this is the first case of Masters theorum.

So

$$T(n) = O(n^{3/2})$$

8. $T(n) = 8T(n/4) + n^2$

$a = 8$, $b = 4$, $f(n) = n^2$

$n^{\log_4 8} = n^{1.5}$

so let $\varepsilon = 0.5$ we'll add

$n^{\log_b^{a} + \varepsilon} = n^2$ nearest to $f(n)$

and

$$af(n/b) \leq c f(n)$$

$$8 (n/4)^2 \leq cn^2$$

$$\frac{n^2}{2} \leq cn^2$$

This will always be true so let $c = 1/2$

$$\frac{n^2}{2} \leq n^2/2$$

proved

$$T(n) = O(n^2).$$

9. $T(n) = 27T(n/9) + n\log n$.

$a = 27, b = 9, f(n) = n\log n$

$n^{\log_9 27} = n^{1.5}$

so let $\varepsilon = 0.5$

so $n^{\log_b a - \varepsilon} = n$ near to $f(n)$

This is the first case of Masters Theorum.

$T(n) = O(n^{\log_b a}) = O(n^{3/2})$

$T(n) = O(n^{3/2})$.

10. $T(n) = 4T(n/4) + n/\log n$

$a = 4, b = 4, f(n) = n/\log n$

$n^{\log_b a} = n^{\log_4 4} = n$

We can see that $n^{\log_b a}$ is near to 10

so this is a special case.

Question #06

$$T(n) = (T(\sqrt{n}))^2$$

Replace n by $\sqrt{n}$.

$$T(\sqrt{n}) = T(\sqrt{\sqrt{n}})^2$$

$$T(n) = (T(\sqrt{\sqrt{n}}))^4$$

Again

$$T(n) = T(\sqrt{\sqrt{\sqrt{n}}})^8$$

$$\vdots$$

$$T(n) = (T(n^{\frac{1}{2}^k}))^{2^k}$$

$$n^{\frac{1}{2}^k} = 5$$

Take log $\frac{1}{2}^k \log n = \log 5$

$$k = \log_2 \log n - \log_2 \log 5$$

$$T(5) = 5$$

so

$$T(n) = 5^{2^k} \because k \approx \log\left(\frac{\log n}{\log 5}\right)$$

$$T(n) = 5^{2^{\log\left(\frac{\log n}{\log 5}\right)}}$$

$$T(n) = O(n^{\log_2 5})$$