The Genetic Algorithm

- Directed search algorithms based on the mechanics of biological evolution
- Developed by John Holland, University of Michigan (1970's)
  - To understand the adaptive processes of natural systems
  - To design artificial systems software that retains the robustness of natural systems

Genetic Algorithms

- Provide efficient, effective techniques for optimization and machine learning applications
- Widely-used today in business, scientific and engineering circles

Genetic Algorithms (GA)

- Function Optimization
- AI (Games,Pattern recognition ...)
- OR after a while
- Basic idea:
  - intelligent exploration of the search space based on random search
  - analogies from biology

GA - Analogies with biology

- Representation of complex objects by a vector of simple components
- Chromosomes
- Selective breeding
- Darwinistic evolution

- Classical GA: Binary encoding

Components of a GA

A problem to solve, and ...

- Encoding technique (*gene, chromosome*)
- Initialization procedure *(creation)*
- Evaluation function *(environment)*
- Selection of parents *(reproduction)*
- Genetic operators *(mutation, recombination)*
- Parameter settings *(practice and art)*

Classical GA: Binary Chromosomes

Chromosome, component vector, vector, string, solution, individual $\mathbf{x}=(x_1, \ldots , x_7)$

1	2	3	4	5	6	7
1	0	1	0	0	1	0

Gene, Component, Variable, x_3

$x_3 \in \{0,1\}$

Locus, position

Allele, value

Alleles, domain

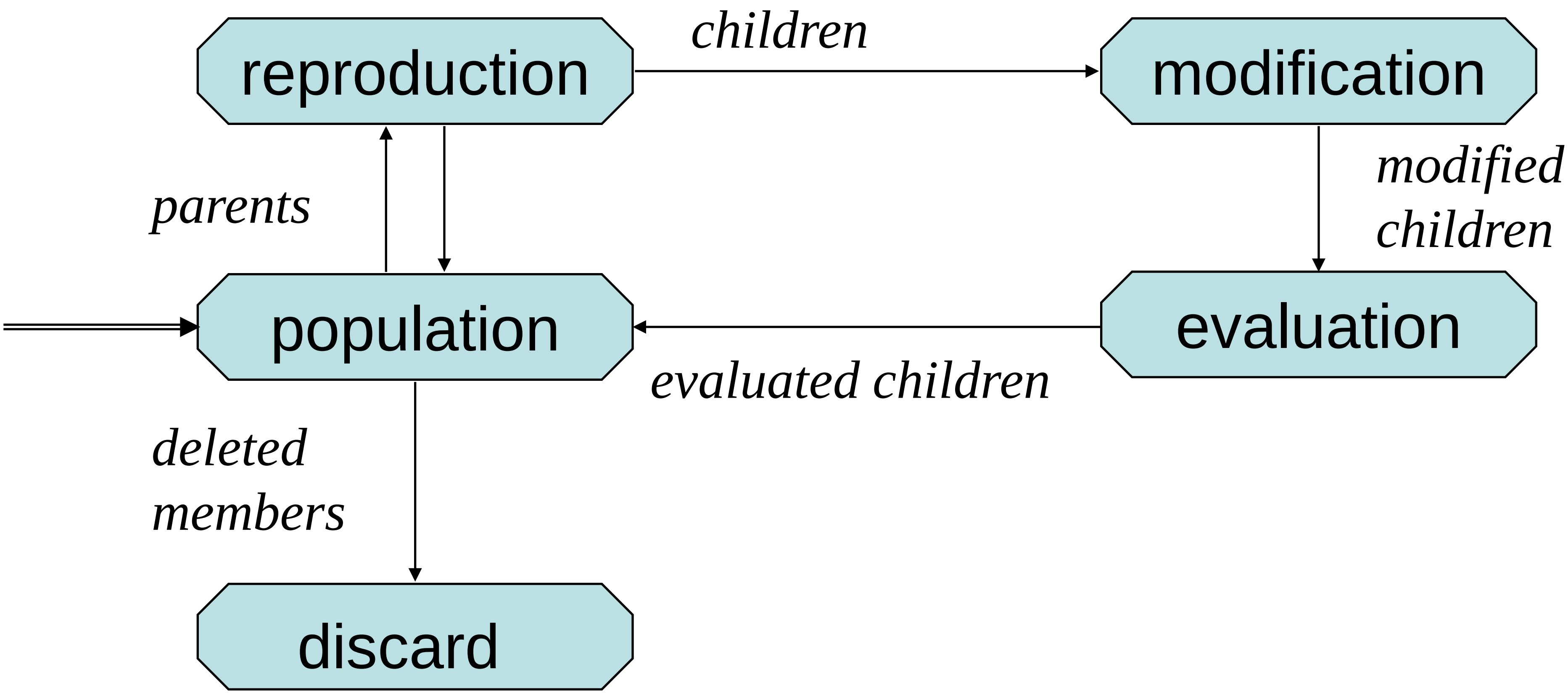
Genotype, Phenotype, Population

- Genotype
  - chromosome
  - Coding of chromosomes
  - coded string, set of coded strings
- Phenotype
  - The physical expression
  - Properties of a set of solutions
- Population – a set of solutions

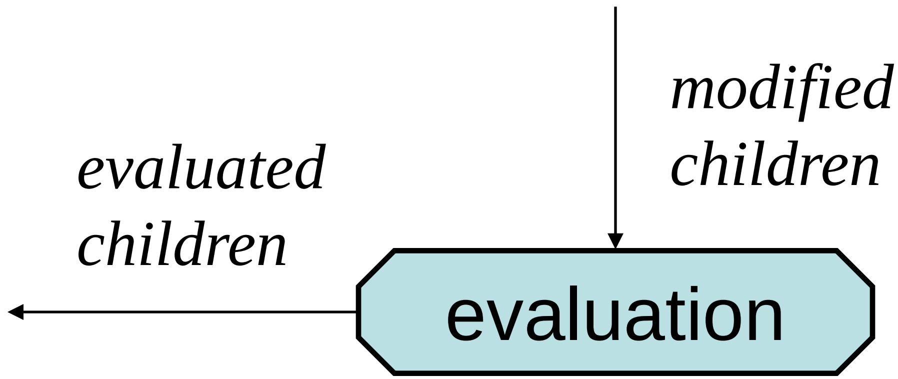
Genetic Algorithm

```
Choose an initial population of chromosomes
while stopping criterion not met do
    while sufficient offspring has not been created do
        if condition for crossover is satisfied then
            Select parent chromosomes
            Choose crossover parameters
            Perform crossover
        end if
        if condition for mutation is satisfied then
            Choose mutation points
            Perform mutation
        end if
        Evaluate fitness of offspring
    end while
end while
```

The GA Cycle of Reproduction



Evaluation



- The evaluator decodes a chromosome and assigns it a fitness measure
- The evaluator is the only link between a classical GA and the problem it is solving

Evaluation of Individuals

- Adaptability – ”fitness”
- Relates to the objective function value for a DOP
- Fitness is maximized
- Used in selection (”*Survival of the fittest*”)
- Often *normalized*

$$f : S \to [0,1]$$

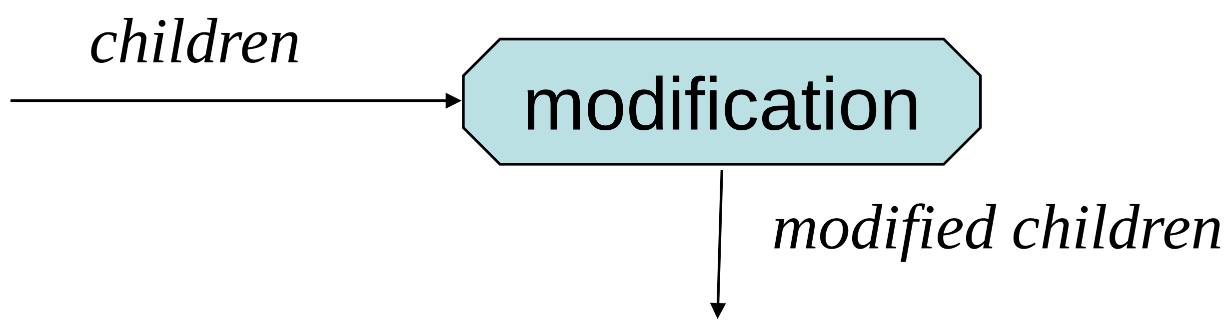
Genetic Operators

- Manipulates chromosomes/solutions
- Mutation: Unary operator
  - Inversions
- Crossover: Binary operator

GA - Evolution

- N generations of populations
- For every step in the evolution
  - Selection of individuals for genetic operations
  - Creation of new individuals (reproduction)
  - Mutation
  - Selection of individuals to survive
- Fixed population size M

Chromosome Modification



- Modifications are stochastically triggered
- Operator types are:
  - Mutation
  - Crossover (recombination)

GA - Mutation

1	2	3	4	5	6	7
1	0	1	0	0	1	0

$$\downarrow$$

1	2	3	4	5	6	7
1	0	1	1	0	1	0

Mutation: Local Modification

Before:	(1 0 1 1 0 1 1 0)
After:	(1 0 1 0 0 1 1 0)

Before:	(1.38 -69.4 326.44 0.1)
After:	(1.38 -67.5 326.44 0.1)

- Causes movement in the search space (local or global)
- Restores lost information to the population

Crossover: Recombination

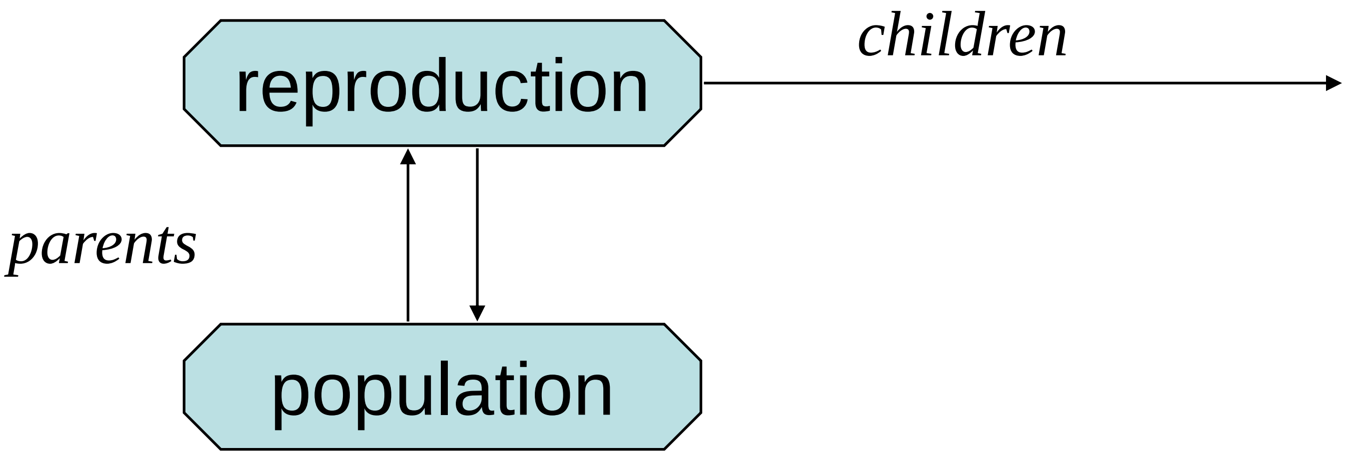
P1 (0 1 1 | 0 1 0 0 0) → (0 1 1 | 1 1 0 1 0) C1

P2 (1 1 0 | 1 1 0 1 0) → (1 1 0 | 0 1 0 0 0) C2

Crossover is a critical feature of genetic algorithms:

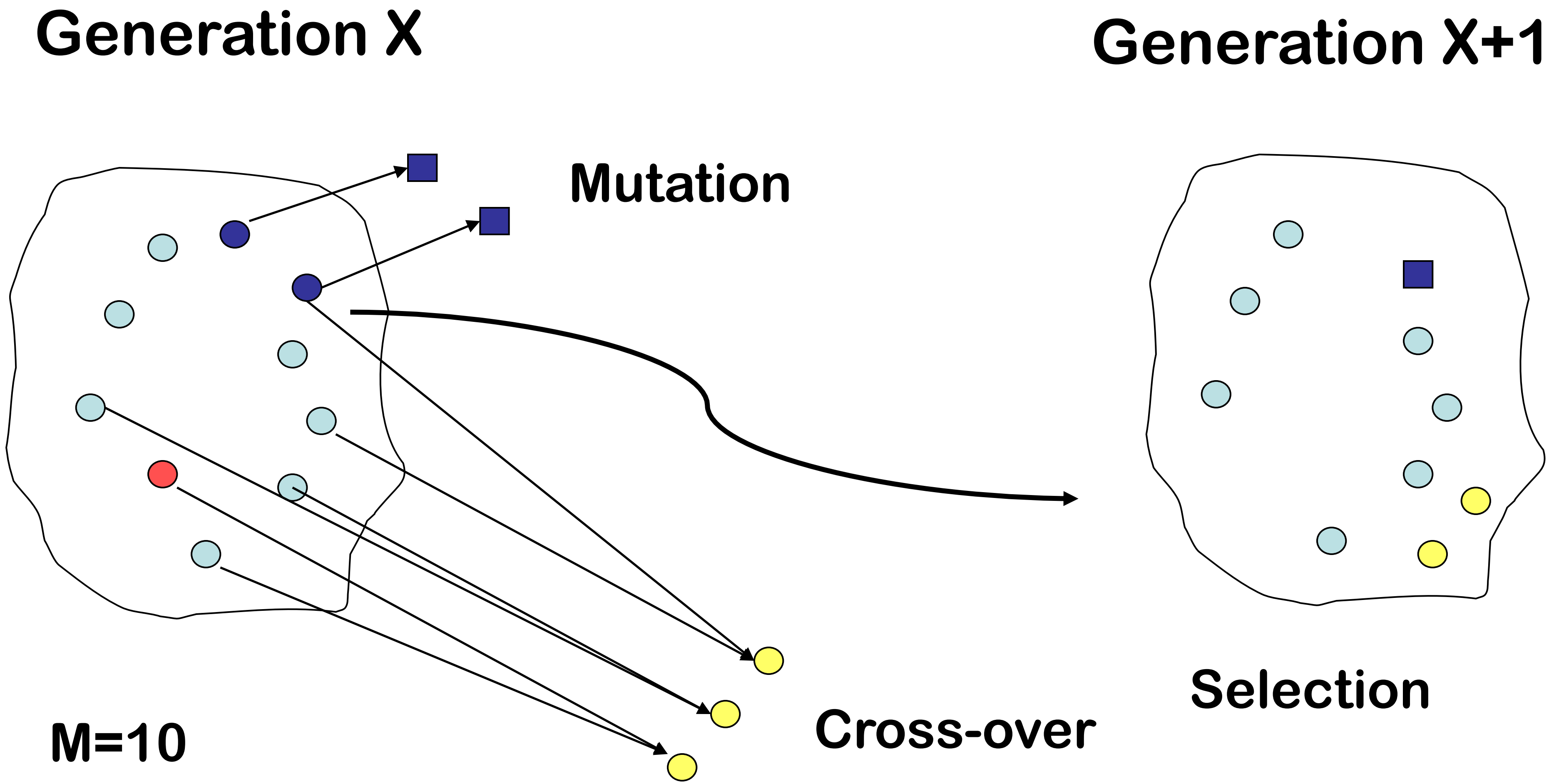
- It greatly accelerates search early in evolution of a population
- It leads to effective combination of schemata (subsolutions on different chromosomes)

Reproduction

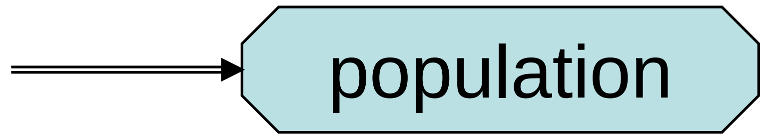


Parents are selected at random with selection chances biased in relation to chromosome evaluations

GA - Evolution



Population



Chromosomes could be:

- Bit strings (0101 ... 1100)
- Real numbers (43.2 -33.1 ... 0.0 89.2)
- Permutations of element (E11 E3 E7 ... E1 E15)
- Lists of rules (R1 R2 R3 ... R22 R23)
- Program elements (genetic programming)
- ... any data structure ...

Classical GA: Binary chromosomes

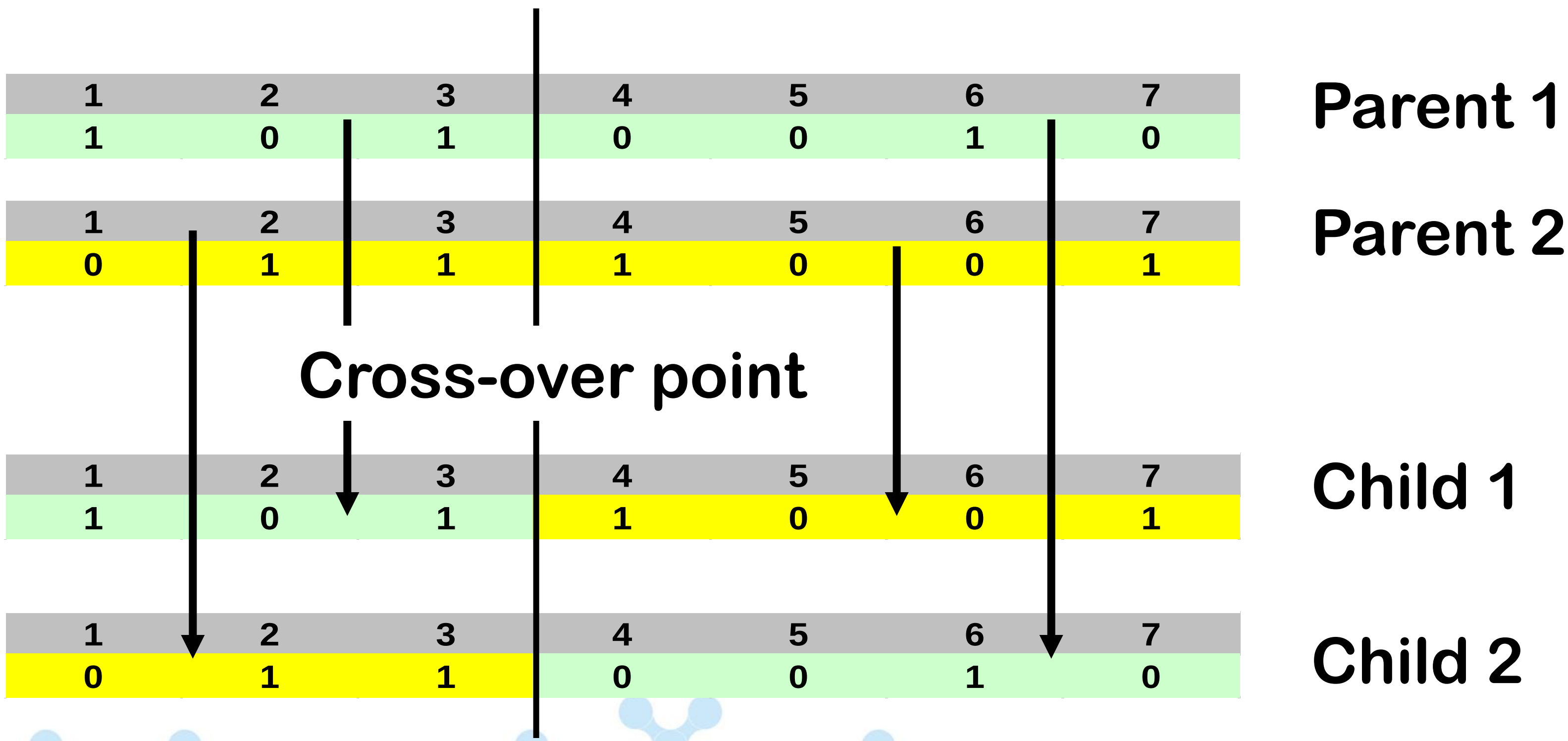
1	2	3	4	5	6	7
1	0	1	0	0	1	0

1	2	3	4	5	6	7
0	1	1	1	0	0	1

- Functional optimization
  - Chromosome corresponds to a binary encoding of a real number - min/max of an arbitrary function
- COP, TSP as an example
  - Binary encoding of a solution
  - Often better with a more direct representation (e.g. sequence representation)

GA - Classical Crossover (1-point)

- One parent is selected based on *fitness*
- The other parent is selected randomly
- Random choice of cross-over point



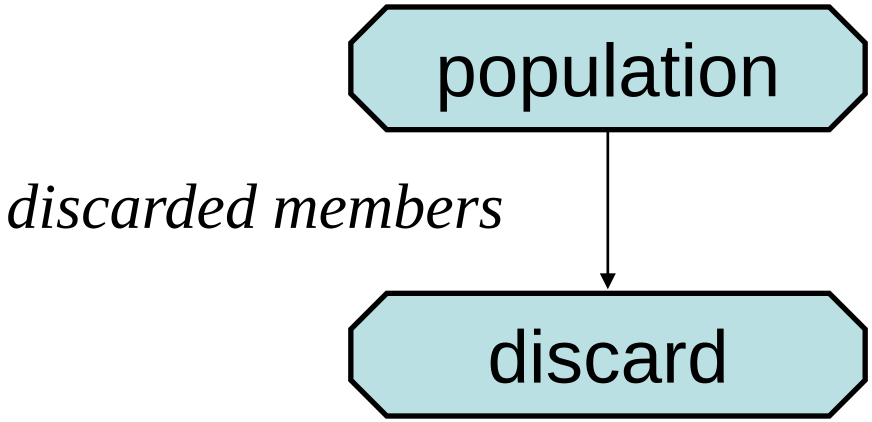
GA – Classical Crossover

- Arbitrary (or worst) individual in the population is changed wi\
- th one of the two offspring (e.g. the best)
- Reproduce as long as you want
- Can be regarded as a sequence of almost equal populations
- Alternatively:
  - One parent selected according to fitness
  - Crossover until (at least) M offspring are created
  - The new population consists of the offspring
- Lots of other possibilities ...
- Basic GA with classical crossover and mutation often works well

GA – Standard Reproduction Plan

- Fixed population size
- Standard cross-over
  - One parent selected according to fitness
  - The other selected randomly
  - Random cross-over point
  - A random individual is exchanged with one of the offspring
- Mutation
  - A certain probability that an individual mutate
  - Random choice of which gene to mutate
  - Standard: mutation of offspring

Deletion



- *Generational* GA:
  entire populations replaced each iteration
- *Steady-state* GA:
  a few members replaced each generation

Methods of Selection

- *Roulette-wheel selection*.
- *Elitist selection*.
- *Fitness-proportionate selection*.
- *Scaling selection*.
- *Rank selection*.
- …

Roulette wheel selection

- Conceptually, this can be represented as a game of roulette - each individual gets a slice of the wheel, but more fit ones get larger slices than less fit ones.

Roulette wheel selection

No.	String	Fitness	% Of Total
1	01101	169	14.4
2	11000	576	49.2
3	01000	64	5.5
4	10011	361	30.9
Total		1170	100.0

Other selection methods

- *Elitist selection*:

  Chose only the most fit members of each generation.
- *Cutoff selection*:

  Select only those that are above a certain cutoff for the target function.

Methods of Reproduction

- There are primary methods:
  - *Crossover*
  - *Mutation*

Methods of Reproduction: Crossover

– Two parents produce two offspring

– Two options:

1. The chromosomes of the two parents are copied to the next generation

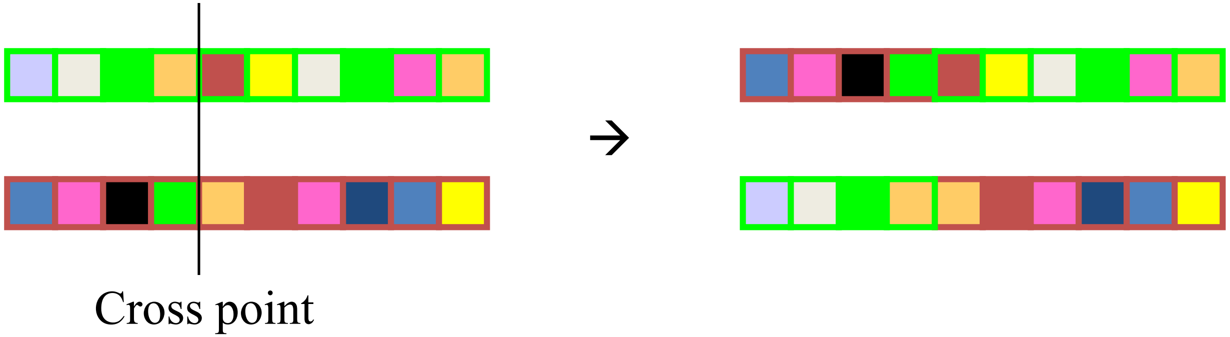
2. The two parents are randomly recombined (crossed-over) to form new offsprings

Several possible crossover strategies

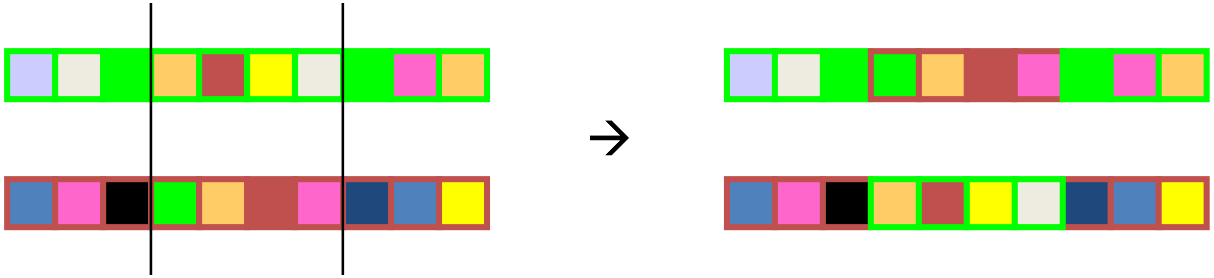
- Randomly select a single point for a crossover
- Multi point crossover
- Uniform crossover

Crossover

- Single point crossover

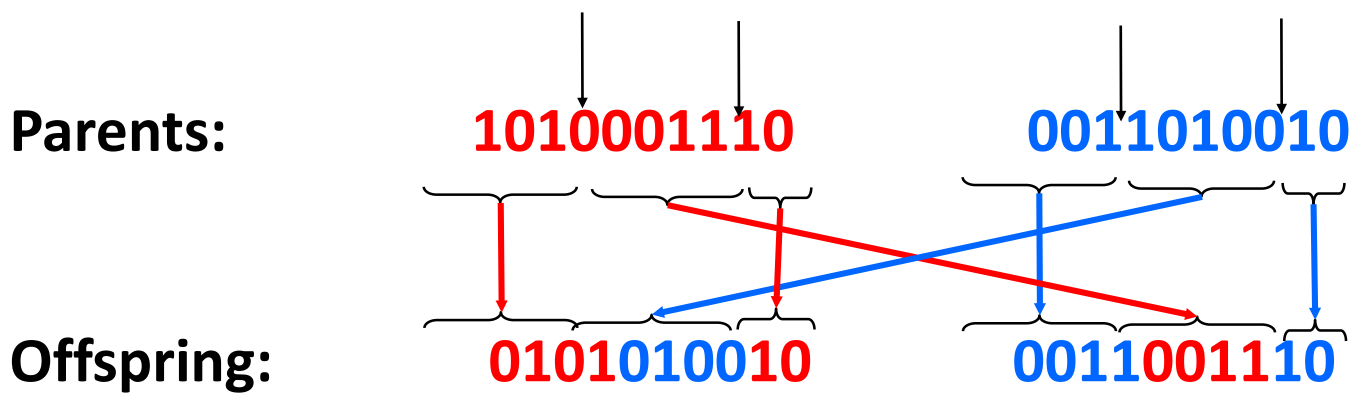


- Two point crossover (Multi point crossover)



Two-point crossover

- Avoids cases where genes at the beginning and end of a chromosome are always split



Uniform crossover

- A random subset is chosen
- The subset is taken from parent 1 and the other bits from parent 2.

Subset: BAABBAABBB (Randomly generated)

Parents: 1010001110 0011010010

Offspring: 0011001010 1010010110

A Simple Example

The Traveling Salesman Problem:

Find a tour of a given set of cities so that

- each city is visited only once
- the total distance traveled is minimized

Representation

Representation is an ordered list of city numbers known as an *order-based* GA.

1) London	3) Dunedin	5) Beijing	7) Tokyo
2) Venice	4) Singapore	6) Phoenix	8) Victoria

City List 1 (3 5 7 2 1 6 4 8)

City List 2 (2 5 7 6 8 1 3 4)

Crossover

Crossover combines inversion and recombination:

			*			*		
Parent1	(3	5	7	2	1	6	4	8)
Parent2	(2	5	7	6	8	1	3	4)
Child	(5	8	7	2	1	6	3	4)

This operator is called order-based crossover.

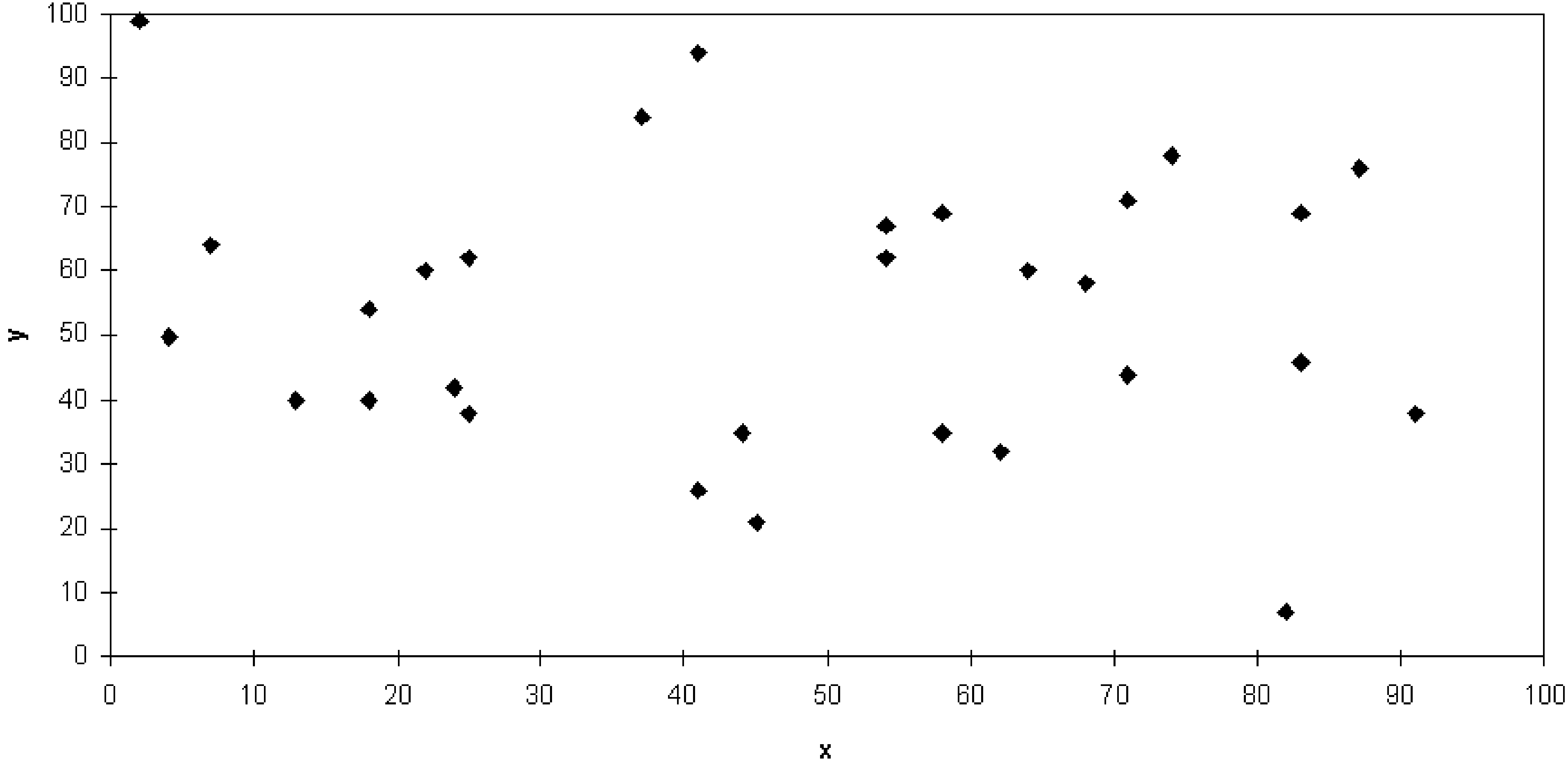
Mutation

Mutation involves reordering of the list:

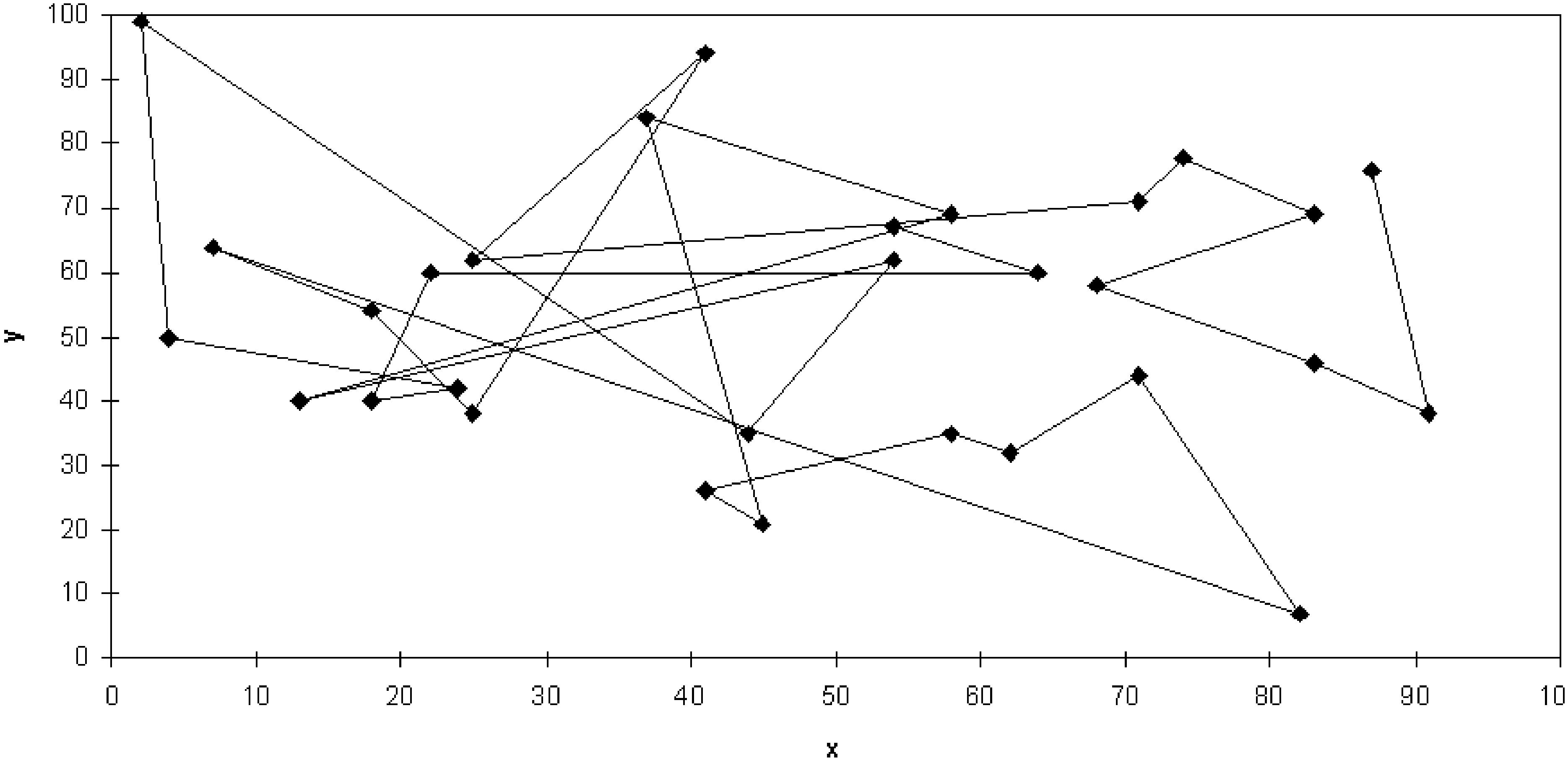
Before: (5 8 7 2 1 6 3 4)

After: (5 8 6 2 1 7 3 4)

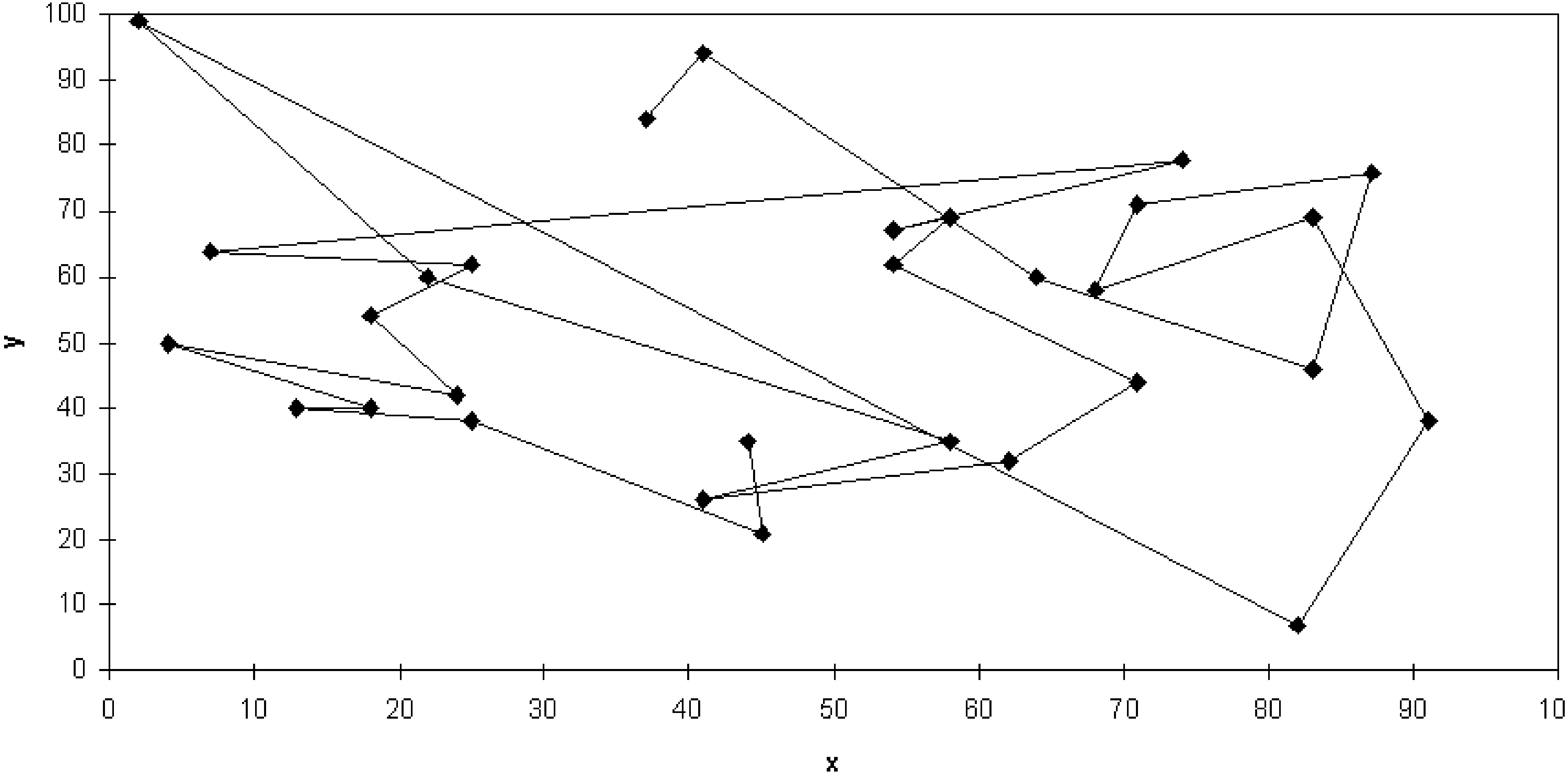
TSP Example: 30 Cities



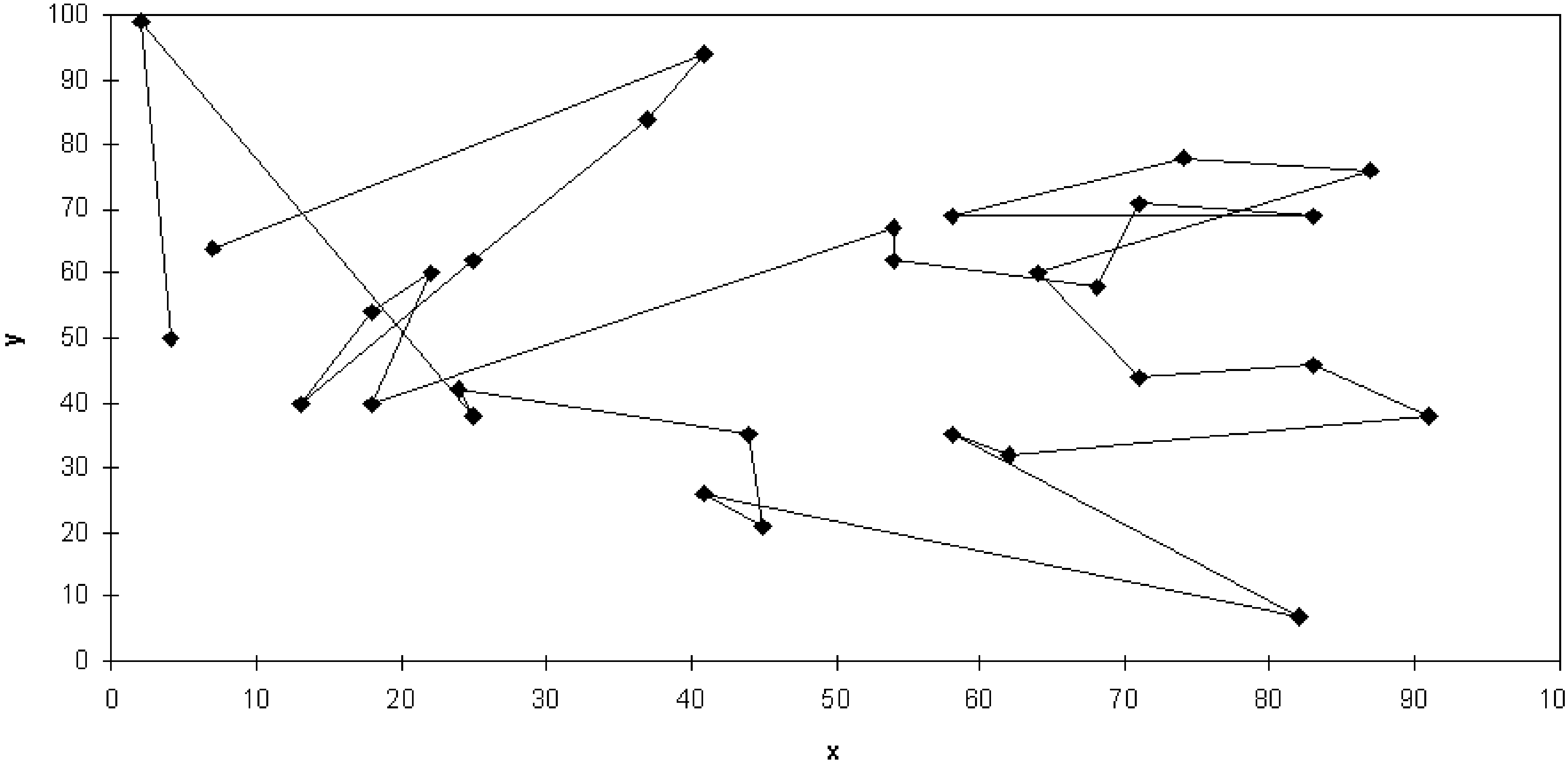
Solution $_i$ (Distance = 941)



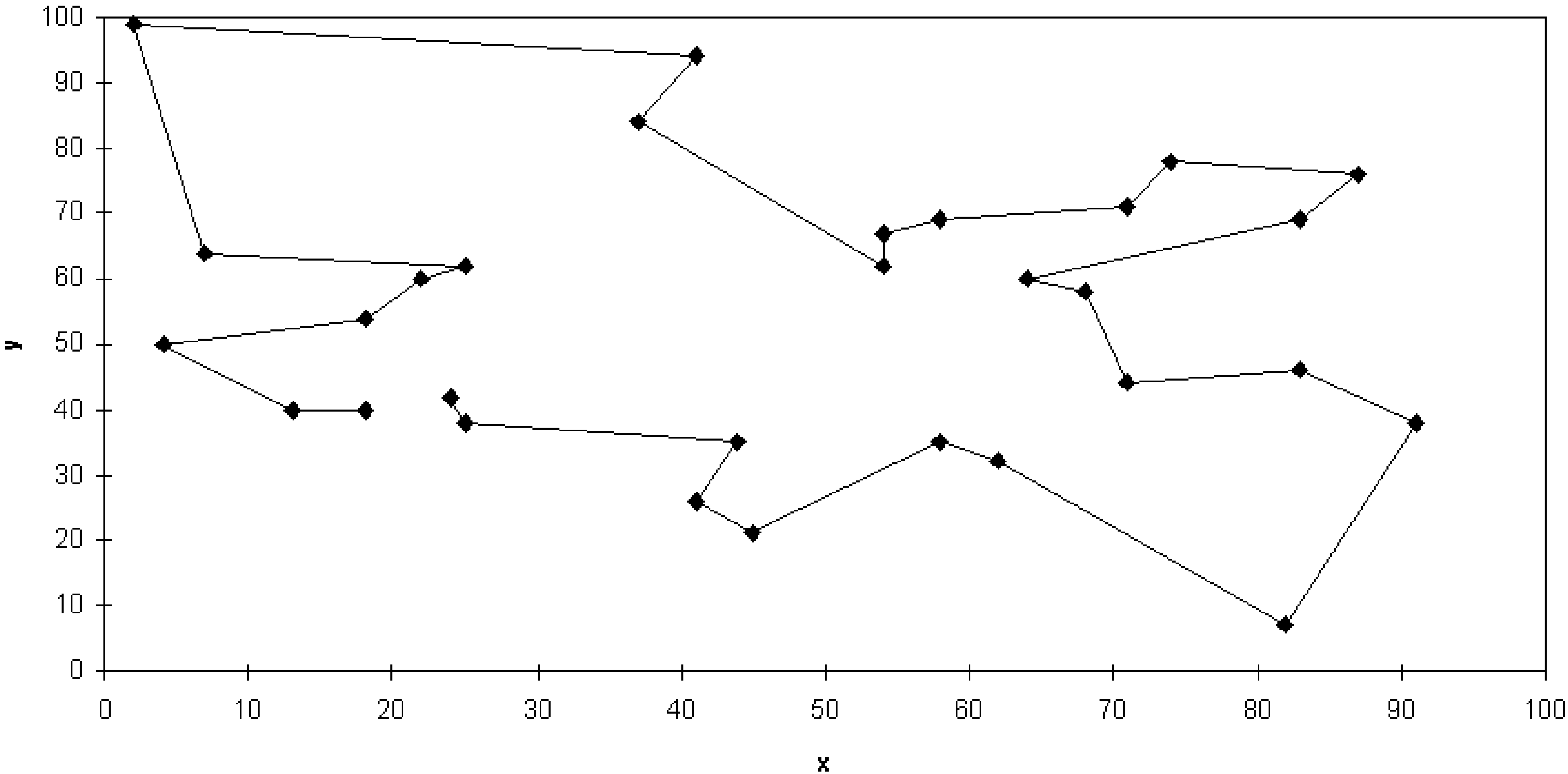
Solution $_j$ (Distance = 800)



Solution $_k$ (Distance = 652)



Best Solution (Distance = 420)



Overview of Performance