## Notebook

### March 15, 2021

Question 1.a. Set the sample size at 1,000 and generate an error term,  $u_i$ , by randomly selecting from a normal distribution with mean 0, and standard deviation 5. Draw an explanatory variable,  $X_{1i}$ , from a standard normal distribution,  $\mathcal{N}(0,1)$ , and then define a second explanatory variable,  $X_{2i}$ , to be equal to  $e^{X_{1i}}$  for all i. Finally, set the dependent variable to be linearly related to the two regressors plus an additive error term:  $y_i = 2 + 4X_{1i} - 6X_{2i} + u_i$ . Note that, by construction, the error term of this multivariate linear regression is homoskedastic.

*Hint*: You may want to refer to how you did this in Problem Set 2. Also, the function np.exp() takes a list/array of numbers and applies the exponential function to each element. This is basically the opposite function of np.log().

Question 1.b. Regress y on  $X_1$  with homoskedasticity-only standard errors (statsmodels does this by default, just don't specify a cov\_type like we usually do to get robust errors). Do the same analysis for y and  $X_2$ . Compare the results with the true data generating process. Explain why differences arise between the population slopes and the estimated slopes, if there are any.

This question is for your code, the next is for your explanation.

```
[31]: X1_const = sm.add_constant(X1)
model_1b_X1 = sm.OLS(y, X1_const)
results_1b_X1 = model_1b_X1.fit()
results_1b_X1.summary()
```

[31]: <class 'statsmodels.iolib.summary.Summary'>

#### OLS Regression Results

Dep. Variable:	у	R-squared:	0.314
Model:	OLS	Adj. R-squared:	0.313
Method:	Least Squares	F-statistic:	456.7
Date:	Mon, 15 Mar 2021	Prob (F-statistic):	9.94e-84
Time:	16:14:39	Log-Likelihood:	-3520.4
No. Observations:	1000	AIC:	7045.
Df Residuals:	998	BIC:	7055.

Df Model: 1
Covariance Type: nonrobust

	coef	std err	t	P> t	[0.025	0.975]
const x1	-7.7323 -5.6348	0.259 0.264	-29.868 -21.370	0.000	-8.240 -6.152	-7.224 -5.117
Omnibus: Prob(Omnib Skew: Kurtosis:	======== us):	0	.000 Jarq .547 Prob	======================================	:	1.967 2058.779 0.00 1.02
=======	========					========

#### Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

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### Question 1.c. Explain.

According to the equation given yi = 2 + 4X1i - 6X2i + ui, the true parameter for 0 should be 2 and the coefficient for X1 and X2 should be 4 and -6 respectively. However, the regression model we conducted missed both of these coefficients. Being more specific, the confidence interval generated for X1 is (-6.361, -5.209), which undoubtedly exclude the true parameter 4. Similarly, the the confidence interval generated for X2 is (-6.361, -5.209), which also does not include the true parameter -6. The reason for this accuracy is the violation of the first assumption of OSL, which is the conditional mean zero errors assumption. Being more specific, the error term for X1 is -6X2i + ui. However, the expectation for this error term given X1i does not equal to 0. Similarly, the error term for X2 is 4X1i + ui, whose expectation also does not evaluate to 0 given X2i

**Question 1.d.** Next, regress y on both  $X_1$  and  $X_2$ . Compare the estimation results with those you did in part (b/c), especially the model with only the regressor  $X_1$ . Examine differences across the three regressions in terms of the coefficient estimates, their standard errors, the  $R^2$ , and the adjusted  $R^2$ .

This question is for your code, the next is for your explanation.

[33]: <class 'statsmodels.iolib.summary.Summary'>

OLS Regression Results

Dep. Variable: y R-squared: 0.729

Model:	OLS	Adj. R-squared:	0.728
Method:	Least Squares	F-statistic:	1340.
Date:	Mon, 15 Mar 2021	Prob (F-statistic):	2.59e-283
Time:	16:14:41	Log-Likelihood:	-3056.2
No. Observations:	1000	AIC:	6118.
Df Residuals:	997	BIC:	6133.
Df Model:	2		

Covariance Type: nonrobust

========	========	========	======	-====	==========	-========	=======
	coef	std err		t	P> t	[0.025	0.975]
const	1.9954	0.298	6.	707	0.000	1.412	2.579
x1	3.8166	0.293	13.	012	0.000	3.241	4.392
x2	-6.0597	0.155	-39.	065	0.000	-6.364	-5.755
		========	======				
Omnibus:		1	.057	Durb	in-Watson:		2.050
Prob(Omnib	us):	C	.590	Jarqı	ıe-Bera (JB):		1.088
Skew:		-0	.016	Prob	(JB):		0.580
Kurtosis:		2	2.841	Cond	. No.		6.60

### Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

## Question 1.e. Explain.

Compared to the OLS conducted in 1b, this OLS is much more accurate on capturing the coefficient 0, X1, and X2. The true parameter for these three terms are 2, 4, and -6. From the chart above, we can see that all three numbers are included in the confidence interval. This increment in accuration is also reflected in the increased R and the adjusted R. In this OLS, Both the R and the adjusted R2 for this OLS increases to 0.859. Compared to the R2s conducted above, this increased closer-to-1 R2 and adjusted R2indicates a comparatively stronger relationship between the independent variables and the dependent variable.

Question 1.f. Generate a third regressor:  $X_{3i} = 1 + X_{1i} - X_{2i} + v_i$  where  $v_i$  is drawn from a normal distribution with mean 0 and standard deviation 0.5. Estimate the model  $y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + w_i$ . Compare the result with part (d/e). Do changes in OLS estimates, standard errors, the  $R^2$ , and the adjusted  $R_2$  make sense to you? Explain why or why not.

Hint: Think about the concept of "imperfect multicollinearity".

This question is for your code, the next is for your explanation.

```
[34]: v = np.random.normal(0, 0.5, 1000)
X3 = 1 + X1 - X2 + v
X_const_f = sm.add_constant(np.stack([X1, X2, X3], axis=1))
model_lf = sm.OLS(y, X_const_f)
```

```
results_1f = model_1f.fit()
results_1f.summary()
```

[34]: <class 'statsmodels.iolib.summary.Summary'>

## OLS Regression Results

===========			=========
Dep. Variable:	у	R-squared:	0.729
Model:	OLS	Adj. R-squared:	0.728
Method:	Least Squares	F-statistic:	892.7
Date:	Mon, 15 Mar 2021	Prob (F-statistic):	1.06e-281
Time:	16:14:42	Log-Likelihood:	-3056.1
No. Observations:	1000	AIC:	6120.
Df Residuals:	996	BIC:	6140.
Df Model:	3		
Covariance Type:	nonrobust		

========		=========		=========	========
coef	std err	t	P> t	[0.025	0.975]
1.9431	0.450	4.319	0.000	1.060	2.826
3.7654	0.442	8.522	0.000	2.898	4.633
-6.0082	0.367	-16.367	0.000	-6.729	-5.288
0.0511	0.330	0.155	0.877	-0.596	0.698
=======	 1.	063 Durbii	======= n-Watson:		2.050
):	0.	588 Jarque	e-Bera (JB):		1.094
	-0.	016 Prob(.	JB):		0.579
	2.	841 Cond.	No.		13.4
	1.9431 3.7654 -6.0082 0.0511	1.9431 0.450 3.7654 0.442 -6.0082 0.367 0.0511 0.330 	1.9431	1.9431 0.450 4.319 0.000 3.7654 0.442 8.522 0.000 -6.0082 0.367 -16.367 0.000 0.0511 0.330 0.155 0.877  1.063 Durbin-Watson: 0.588 Jarque-Bera (JB): -0.016 Prob(JB):	1.9431

### Warnings:

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[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

## Question 1.g. Explain.

Compared to the OLS conducted in part e, we observe that the accuracy for every term, including coefficient, X1, X2, and X3, decreases. However, this OLS still captures the true parameter in the confidence intervals produced. Therefore, we concliude that this OLS is less accurate than that from part e but is more accurate than that from part d. However, it is worth noting that this OLS generates the greater standard error among all three OLSs. This feature should be attribute to the imperfect multicollinearity because X3 is composed as the linear combination of X1 and X2. This means that its variations can also be explained the linear combination of X1 and X2. R2 and the adjusted R2 remains the same.

Question 2.a. Run a regression of course\_eval on beauty using robust standard errors. What is the estimated slope? Is it statistically significant?

This question is for your code, the next is for your explanation.

```
[36]: y_2a = ratings['course_eval']
X_2a = ratings['beauty']
model_2a = sm.OLS(y_2a, sm.add_constant(X_2a))
results_2a = model_2a.fit(cov_type='HC1')
results_2a.summary()
```

## [36]: <class 'statsmodels.iolib.summary.Summary'>

## OLS Regression Results

Dep. Variable:	course_eval	R-squared:	0.036
Model:	OLS	Adj. R-squared:	0.034
Method:	Least Squares	F-statistic:	16.94
Date:	Mon, 15 Mar 2021	Prob (F-statistic):	4.58e-05
Time:	16:14:44	Log-Likelihood:	-375.32
No. Observations:	463	AIC:	754.6
Df Residuals:	461	BIC:	762.9
Df Model:	1		
Covariance Type:	HC1		

	coef	std err	z	P> z	[0.025	0.975]
const	3.9983	0.025	157.727	0.000	3.949	4.048
beauty	0.1330	0.032	4.115	0.000	0.070 	0.196
Omnibus:		15	.399 Dur	oin-Watson:		1.410
Prob(Omnibus)	:	0	.000 Jar	que-Bera (JB	):	16.405
Skew:		-0	.453 Prol	o(JB):		0.000274
Kurtosis:		2	.831 Cond	d. No.		1.27

## Warnings:

[1] Standard Errors are heteroscedasticity robust (HC1)

### Question 2.b. Explain.

The estimated slope here is 0.1330 and it is statistically significant on both the 0.05 level and 0.01 level. Therefore, this estimated slope may reflect a positive relationship between the beauty of the teachers and the ratings they received.

Question 2.c. Run a regression of course\_eval on beauty, including some additional variables to control for the type of course and professor characteristics. In particular, include as additional regressors intro, onecredit, female, minority, and nnenglish. What is the estimated effect of beauty on course\_eval? Does the regression in (a) suffer from important omitted variable bias (OVB)? What happens with the  $R^2$ ? Based on the confidence interval from the regression, can you reject the null hypothesis that the effect of beauty is the same as in part (a)? What can you say

about the effect of the new variables included?

This question is for your code, the next is for your explanation.

```
[37]: y_2c = ratings['course_eval']
      X_2c_beauty = ratings['beauty']
      X_2c_intro = ratings['intro']
      X_2c_onecredit = ratings['onecredit']
      X_2c_female = ratings['female']
      X_2c_minority = ratings['minority']
      X_2c_nnenglish = ratings['nnenglish']
      X_2c_const = sm.add_constant(np.stack([X_2c_beauty, X_2c_intro,_
      →X_2c_onecredit,X_2c_female, X_2c_minority, X_2c_nnenglish], axis=1))
      model_2c = sm.OLS(y_2c, X_2c_const)
      results_2c = model_2c.fit(cov_type='HC1')
      results_2c.summary()
```

## [37]: <class 'statsmodels.iolib.summary.Summary'>

## OLS Regression Results

Dep. Variable:	course_eval	R-squared:	0.155
Model:	OLS	Adj. R-squared:	0.144
Method:	Least Squares	F-statistic:	17.03
Date:	Mon, 15 Mar 2021	Prob (F-statistic):	8.67e-18
Time:	16:14:45	Log-Likelihood:	-344.85
No. Observations:	463	AIC:	703.7
Df Residuals:	456	BIC:	732.7
Df Model:	6		
Covariance Type:	HC1		

HC1 Covariance Type:

========	:========	:=======	========	========	=======	=======
	coef	std err	z	P> z	[0.025	0.975]
const	4.0683	0.037	109.926	0.000	3.996	4.141
x1	0.1656	0.032	5.246	0.000	0.104	0.227
x2	0.0113	0.056	0.202	0.840	-0.099	0.121
x3	0.6345	0.108	5.871	0.000	0.423	0.846
x4	-0.1735	0.049	-3.505	0.000	-0.270	-0.076
x5	-0.1666	0.067	-2.472	0.013	-0.299	-0.034
x6	-0.2442	0.094	-2.608	0.009	-0.428	-0.061
Omnibus:		 22.	413 Durbi	======== n-Watson:	========	1.516
Prob(Omnib	ous):	0.	000 Jarque	e-Bera (JB):		24.406
Skew:		-0.	555 Prob(	JB):		5.02e-06
Kurtosis:		3.	179 Cond.	No.		5.81

### Warnings:

```
[1] Standard Errors are heteroscedasticity robust (HC1)
```

## Question 2.d. Explain.

The estimated effect of beauty on course\_eval increases from 0.1330 to 0.1656. However, we cannot reject the hypothesis that the effect of beauty is the same as in part (a) because the confidence interval conducted in c for beauty is (0.104, 0.227), which includes 0.1330. According to this result, we may conclude that the regression in (a) does seemingly suffers from important omitted variable bias (OVB). Among all other factors, x3, which is whether this class is taken as a one credit class, has the greatest weight. This means that instructors who have students taking classes in one credit would receive a higher rating on average. Approaching these coefficient for factors through a similar way, we find some outstanding results. On average, female instructors receive lower ratings than male, indicated by the negative sign in front of the coefficient. Similarly, instructors who is minority or non-English speakers have the same pattern compared with majority or English speaking instructors. On class level, instructors who teach introductary level class on average have higher rating. Incorporating all these factors into our model, we find that our R2 increases from 0.036 to 0.156. This means that the including all these factors make our OLS more accurate.

Question 2.e. Estimate the coefficient on beauty for the multiple regression model in (c) using the three-step process in Appendix 6.3 (the Frisch-Waugh theorem). Verify that the three-step process yields the same estimated coefficient for beauty as that obtained in (c). Comment.

Hint: Recall that if your regression results are called results, you could get the residuals using results.resid.

This question is for your code, the next is for your explanation.

```
[38]: # Do the first step here (regress the outcome variable on covariates)
      course eval = ratings['course eval']
      covariates = sm.add_constant(np.stack([X_2c_intro, X_2c_onecredit,_
       →X_2c_female,X_2c_minority, X_2c_nnenglish], axis=1))
      model_eval_on_covariates = sm.OLS(course_eval, covariates)
      results_eval = model_eval_on_covariates.fit(cov_type='HC1')
      eval_residuals = results_eval.resid
      # Do the second step here (regress the explanatory variable on covariates)
      beauty = ratings['beauty']
      model_beauty_on_covariates = sm.OLS(beauty, covariates)
      results_beauty = model_beauty_on_covariates.fit(cov_type='HC1')
      beauty_residuals = results_beauty.resid
      # Do the last step here (regress the outcome variable's residuals on the
       → explanatory variable's residuals)
      model_fw = sm.OLS(eval_residuals, beauty_residuals)
      results_fw = model_fw.fit(cov_type='HC1')
      results_fw.summary()
```

```
[38]: <class 'statsmodels.iolib.summary.Summary'>
```

### OLS Regression Results

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Dep. Variable: y R-squared (uncentered):

0.060

Model: OLS Adj. R-squared (uncentered):

0.058

Method: Least Squares F-statistic:

27.88

Date: Mon, 15 Mar 2021 Prob (F-statistic):

1.99e-07

Time: 16:14:46 Log-Likelihood:

-344.85

No. Observations: 463 AIC:

691.7

Df Residuals: 462 BIC:

695.8

Df Model: 1
Covariance Type: HC1

	coef	std err	z 	P> z	[0.025	0.975]
x1	0.1656	0.031	5.280	0.000	0.104	0.227
Omnibus: Prob(Omnibus): Skew: Kurtosis:		22.413 0.000 -0.555 3.179	Jarqı Prob	• •		1.516 24.406 5.02e-06 1.00

#### Warnings:

[1] Standard Errors are heteroscedasticity robust (HC1)

## Question 2.f. Explain.

Estimating the coefficient on beauty for the multiple regression model using the Frisch-Waugh theorem, we could potentially isolate/minimize the effect of other factors, such as nnenglish, minority, onecredit, etc, which is counted as the marginal effect in this model and get the coefficient just for the factor beauty. The result we get using the Frisch-Waugh theorem is 0.1656, which is exactly the same as part(c)

**Question 2.g.** Professor Smith is a black male with average beauty and is a native English speaker. He teaches a three-credit upper-division course. Predict Professor Smith's course evaluation.

4.0683 + -0.1666 \* 1

minority = 1 female = 0 nnenglish = 0 onecredit = 0 intro = 0 beauty = average = 0 Therefore, predicted rating = 4.0683 + -0.1666 \* 1 = 3.9017

Question 3.a. What do you expect for the sign of the relationship and what mechanism can you think about to explain it?

I expect the sign of relationship to be negative. As on overage, the closer students's from high school to nearest four-year college, the larger the number of completed years of education.

Question 3.b. Run a regression of years of completed education (yrsed) on distance to the nearest college (dist), measured in tens of miles (For example, dist = 2 means that the distance is 20 miles). What is the estimated slope? Is it statistically significant? Does distance to college explain a large fraction of the variance in educational attainment across individuals? Explain.

This question is for your code, the next is for your explanation.

```
[40]: y_3b = dist['yrsed']
X_3b = sm.add_constant(dist['dist'])
model_3b = sm.OLS(y_3b, X_3b)
results_3b = model_3b.fit(cov_type = "HC1")
results_3b.summary()
```

[40]: <class 'statsmodels.iolib.summary.Summary'>

### OLS Regression Results

===========			=========
Dep. Variable:	yrsed	R-squared:	0.007
Model:	OLS	Adj. R-squared:	0.007
Method:	Least Squares	F-statistic:	29.83
Date:	Mon, 15 Mar 2021	Prob (F-statistic):	5.01e-08
Time:	16:14:51	Log-Likelihood:	-7632.2
No. Observations:	3796	AIC:	1.527e+04
Df Residuals:	3794	BIC:	1.528e+04
Df Model:	1		
Covariance Type:	HC1		

========						
	coef	std err	z	P> z	[0.025	0.975]
const	13.9559	0.038	369.093	0.000	13.882	14.030

const	13.9559	0.038	369.093	0.000	13.882	14.030
dist	-0.0734	0.013	-5.462	0.000	-0.100	-0.047
========	========	=======				=======
Omnibus:		7187.	794 Durbi	in-Watson:		1.769
Prob(Omnibus):		0.	000 Jarqı	ie-Bera (JB):		361.676
Skew:		0.	410 Prob	Prob(JB):		2.90e-79
Kurtosis:		1.	729 Cond	No.		3.73
=========		=======	========	.========	========	=======

Warnings:

[1] Standard Errors are heteroscedasticity robust (HC1)

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## Question 3.c. Explain.

The estimated slope is -0.0734, it is statistically significant at 1% level. According to  $R^2$ , distance to college explain a small fraction of the variance in educational attainment across individuals.

Question 3.d. Now run a regression of yrsed on dist, but include some additional regressors to control for characteristics of the student, the student's family, and the local labor market. In particular, include as additional regressors: bytest, female, black, hispanic, incomehi, ownhome, dadcoll, cue80, and stwmfg80. What is the estimated effect of dist on yrsed? Is it substantively different from the regression in (b)? Based on this, does the regression in (b) seem to suffer from important omitted variable bias?

This question is for your code, the next is for your explanation.

# [41]: <class 'statsmodels.iolib.summary.Summary'>

## OLS Regression Results

Dep. Variable:	yrsed	R-squared:	0.279			
Model:	OLS	Adj. R-squared:	0.277			
Method:	Least Squares	F-statistic:	197.7			
Date:	Mon, 15 Mar 2021	Prob (F-statistic):	0.00			
Time:	16:14:52	Log-Likelihood:	-7025.9			
No. Observations:	3796	AIC:	1.407e+04			
Df Residuals:	3785	BIC:	1.414e+04			
Df Model:	10					
Covariance Type:	HC1					

========						
	coef	std err	z	P> z	[0.025	0.975]
const	8.8275	0.241	36.583	0.000	8.355	9.300
dist	-0.0315	0.012	-2.705	0.007	-0.054	-0.009
bytest	0.0938	0.003	31.479	0.000	0.088	0.100
female	0.1454	0.050	2.885	0.004	0.047	0.244
black	0.3680	0.068	5.449	0.000	0.236	0.500
hispanic	0.3985	0.074	5.394	0.000	0.254	0.543
incomehi	0.3952	0.062	6.382	0.000	0.274	0.517
ownhome	0.1521	0.065	2.343	0.019	0.025	0.279
dadcoll	0.6961	0.071	9.838	0.000	0.557	0.835
cue80	0.0232	0.009	2.493	0.013	0.005	0.041

stwmfg80	-0.0518	0.020	-2.632	0.008	-0.090	-0.013
Omnibus: Prob(Omnibus):		118.20		Durbin-Watson: Jarque-Bera (JB):		1.924 97.867
Skew:		0.3	20 Prob	(JB):		5.60e-22
Kurtosis:		2.54	43 Cond	. No.		539.

### Warnings:

[1] Standard Errors are heteroscedasticity robust (HC1)

## Question 3.e. Explain.

The estimated effect of dist on yrsed is -0.0315, it's statistically significant at 1% level.  $R^2$  increased dramatically compared with part(a), and coefficients also decreased a lot. The regression in (b) did seem to suffer from important omitted variable bias.

Question 3.f. The value of the coefficient on dadcoll is positive. What does this coefficient measure? Interpret this effect.

Holding other variables constant, on overage, students with dadcoll = 1 (which means their father did go to college) have about 0.6961 more years of education compared with those with dadcoll = 0 (which means their father did not go to college).

Question 3.g. Explain why cue80 and stwmfg80 appear in the regression. Are the signs of their estimated coefficients what you would have believed? Explain.

Those reflect the opportunity cost of going to college. An increase in cue80 will lead to increase in years of schooling, as it is harder to find jobs. An increase in cue80 will lead to decrease in years of education.

Question 3.h. Bob is a black male. His high school was 20 miles from the nearest college. His base-year composite test score (bytest) was 58. His family income in 1980 was \\$26,000, and his family owned a home. His mother attended college, but his father did not. The unemployment rate in his county was 7.5%, and the state average manufacturing hourly wage was \\$9.75. Predict Bob's years of completed schooling using the regression in (d).

```
-0.0315 * 2 + 0.0938 * 58 + 0.1454 * 0 + 0.3680 * 1 + 0.3985 * 0 + 0.3952 * 1 + 0.1521 * 1 + 0.6961 * 0 + 0.0232 * 7.5 - 0.0518 * 9.75 + 8.8275 = 14.79
```

**Question 4.a.** Why do you think Jaeger and Page estimate their model using only people of a single race and gender (in this particular case the sample consists of white males)?

Because there might be racial and gender discrimination issues, thus it will be more reliable to use people of a single race and gender.

**Question 4.b.** Look at column (3) of the table. In words, interpret the coefficient on the dummy variable "9".

Hint: Note that "12" is the omitted category.

On average, people with 9 years of education earns 0.227 less in log(hourly wages) compared with those with 12 years of education, while holding all other variables constant.

**Question 4.c.** Why do you think the effect of the 14th year of education is larger than that of the 15th?

14th year of education might corresponds to those students graduating from a junior college (two-year college), and 15th year of education might corresponds to those who could not finish college and got dropped off. Compared with 15th year, employers may prefer 14th year.

Question 4.d. Now look at column (4). Think about a student who is currently a senior. What is the average difference in the student's wage now and the one that the student could get at the end of the year following graduation?

Now: 0.052 + 0.083 = 0.135. After: 0.178 + 0.245 = 0.423. 0.423 - 0.135 = 0.288. Thus, there will be approximately a 28.8% increase.

**Question 4.e.** Based on the results presented in this column, would you rather choose to complete a PhD or a professional degree? Explain.

Complete a professional degree other than PhD. Marginal effect of professional degrees is 0.289, while marginal effect of PhD degree is 0.067.

**Question 4.f.** Using the results from columns (3) and (4), how would you test the presence of a "diploma effect"? Carry out the test at a 5% significance level.

Hint: You may find some of the information you need in the footnote of the table.

Null hypothesis: all diploma effect in (4) are zero. Alternative: at least one in (4) is not zero.

F = ((0.154-0.147)/8) / ((1-0.154)/(8957-28-1)) = 9.234

9.234 > 1.93, we reject the null.