

## ASSIGNMENT 4

IS 605 FUNDAMENTALS OF COMPUTATIONAL MATHEMATICS - 2014

### 1. PROBLEM SET 1

In this problem, we'll verify using R that SVD and Eigenvalues are related as worked out in the weekly module. Given a  $3 \times 2$  matrix  $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \end{bmatrix} \quad (1)$$

write code in R to compute  $\mathbf{X} = \mathbf{A}\mathbf{A}^T$  and  $\mathbf{Y} = \mathbf{A}^T\mathbf{A}$ . Then, compute the eigenvalues and eigenvectors of  $\mathbf{X}$  and  $\mathbf{Y}$  using the built-in commands in R.

Then, compute the left-singular, singular values, and right-singular vectors of  $\mathbf{A}$  using the *svd* command. Examine the two sets of singular vectors and show that they are indeed eigenvectors of  $\mathbf{X}$  and  $\mathbf{Y}$ . In addition, the two non-zero eigenvalues (the 3rd value will be very close to zero, if not zero) of both  $\mathbf{X}$  and  $\mathbf{Y}$  are the same and are squares of the non-zero singular values of  $\mathbf{A}$ .

Your code should compute all these vectors and scalars and store them in variables. Please add enough comments in your code to show me how to interpret your steps.

### 2. PROBLEM SET 2

Using the procedure outlined in section 1 of the weekly handout, write a function to compute the inverse of a well-conditioned full-rank square matrix using co-factors. In order to compute the co-factors, you may use built-in commands to compute the determinant. Your function should have the following signature:

`B = myinverse(A)`

where  $\mathbf{A}$  is a matrix and  $\mathbf{B}$  is its inverse and  $\mathbf{A}\mathbf{B} = \mathbf{I}$ . The off-diagonal elements of  $\mathbf{I}$  should be close to zero, if not zero. Likewise, the diagonal elements should be close to 1, if not 1. Small numerical precision errors are acceptable but the function *myinverse* should be correct and must use co-factors and determinant of  $\mathbf{A}$  to compute the inverse.

Please submit PS1 and PS2 in an R-markdown document with your first initial and last name.