CENG 215
Circuits and Electronics Lecture Notes

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December 3, 2020

Chapter 1

Introduction - October 15, 2020

1.1 Abstractions

Recall the Newton's formula $F = ma$, which defines the relationship between force, mass and acceleration. This formula modals acceleration using force and mass. However, according to this model, there is no connection between mass and speed. Consider now, the Einstein's equation:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.1}$$

As this equation shows, speed affects mass. The abstractions ignore certain connections for the sake of simplicity. Likewise, electrical engineering, based on Maxwell's Equations, create abstractions, notably, this lecture deals with the *Lumped Circuit Abstraction*.

Consider a statement in a high level programming language `int n = 3;`, this basic statment goes through many abstractions eventually reaching circuitery.

1.2 Circuits

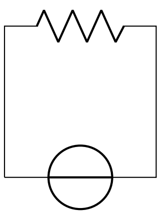
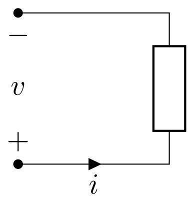


Figure 1.1: A simple cirucit abstraction

From this abstraction, arises the **Ohm's Law**

$$v = iR \tag{1.2}$$

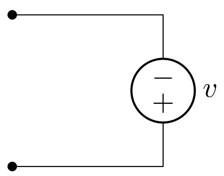
1.2.1 Two Terminal Element



Two terminal elements include batteries, resistors, capacitors, etc...

Battery

Batteries provide voltage and can be bind into serial or paralel.



Below are power (in watts) and energy (in Jouless or watt-seconds) for batteries.

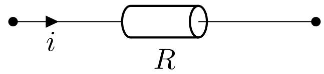
$$P = vi \tag{1.3}$$

$$w = Pt \tag{1.4}$$

Enery formula can also be represented as:

$$w = \int_{t_1}^{t_2} v(t)i(t)\mathrm{d}t \tag{1.5}$$

Resistance



Imagine a generic tube with length l, resistivity ρ and cross sectional area a, in this case, the Resistance of the element R is

$$R = \rho\frac{l}{a} \tag{1.6}$$

The resistance can be showed as:

Where the Ohm's Law state:

$$v = Ri \tag{1.7}$$

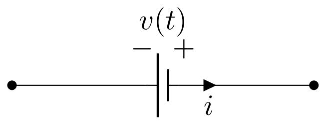
or alternativelly

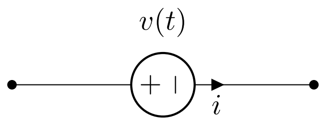
$$i = Gv \tag{1.8}$$

Where G is conductance, whose SI unit is siemens and defined as $\frac{1}{R}$

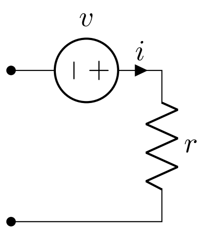
Ideal Voltage Source

Ideal Voltage source can be represnted by:





In general, any voltage soucre can be drawn as



Where r is the internal resistence that arise from the material itself. An ideal voltage source would be able to provide the same current no matter what the voltage is, however this is not possible in real life, where any voltage source has a r

Chapter 2

Resistive Networks - 22 October, 2020

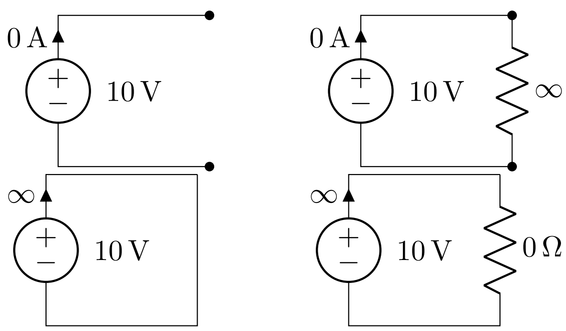


Figure 2.1: An open circuit is equivalent to a circuit with a resistor with an infinite resistance. Whereas a short cirucit can be modelled as a circuit with zero resistence.

The perfect current source is a current source that can supply current in any voltage.

2.1 Signals

Signals can be analog or digital. In Figure 2.2, the sinosodial signals, which has continous values is an analog signal. Where it is represented via the $v(t) = A\sin(\omega t + \phi)$, where A is its amplitude, ω is its frequency and ϕ is its phase, it is analog because it has *continous* values. In the meantime, the second signal is a digital signal as it has *discrete* and quantised values.

Digital signals trade precision about the signal with *immunity towards the noise.*

Resistance A measure of the ability of the device to consume energy.

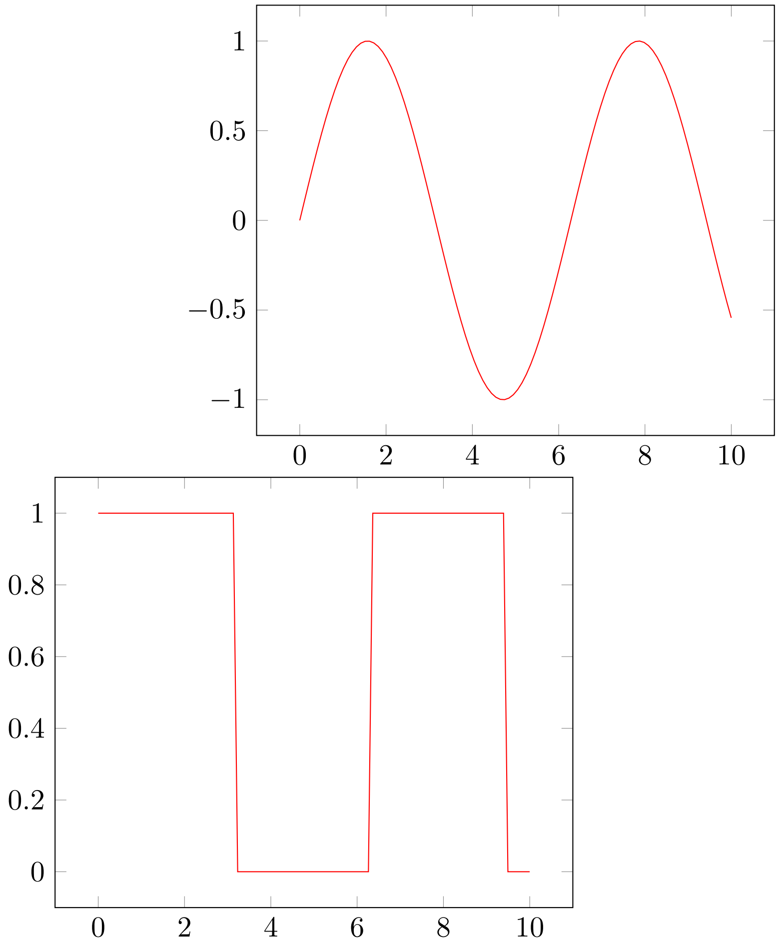
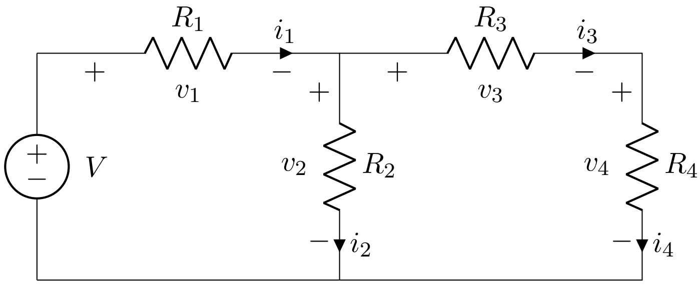


Figure 2.2: Two signals.

Capacitence A measure of the ability of the device to store energy in the form of potential energy. (voltage).

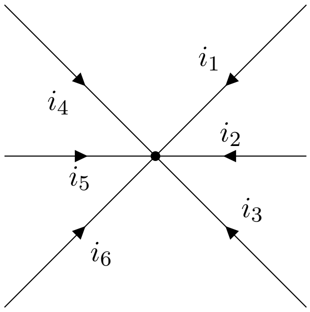
Inductance A measure of the ability of the device to store energy as the moving charge (current).

2.2 Resistive Networks



This sort of circuits can be analysed using two laws, **Kirchoff's Current Law** (KCL) and **Kirchoff's Voltage Law** (KVL).

2.2.1 Kirchoff's Current Law

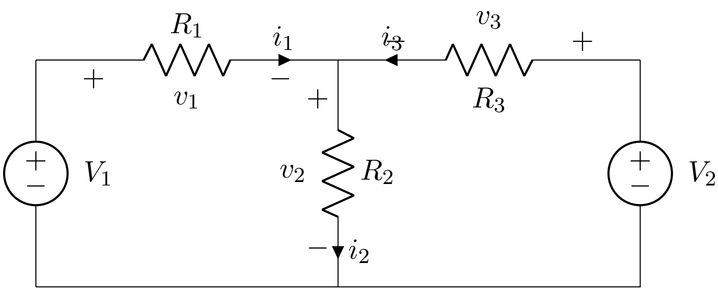


Kirchoff's current law state that the sum of currents entering a node must equal zero.

$$\sum_{n=1}^{6} i_n = 0 \tag{2.1}$$

When one takes the directions of the currents into account, this means that the *currents entering a node must equal the curents exiting a node.*

2.2.2 Kirchoff's Voltage Law



Consider three loops, if clockwise, starting from the battery's top, each node is called a, b, c, d then for loop abcda:

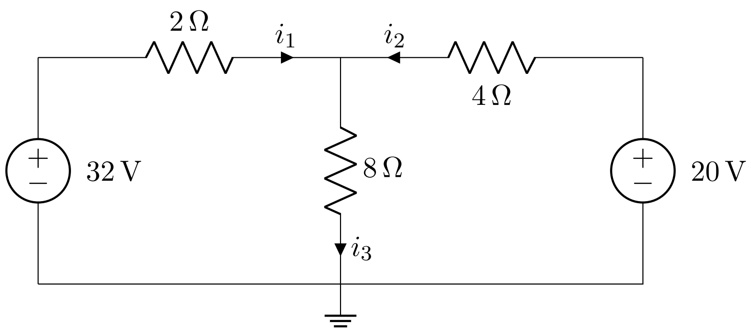
$$
\begin{aligned}
v_{ba} + v_{bc} + v_{cd} + v_{da} &= 0 \\
v_{ab} = v_1 &= R_1 i_1 \\
v_{bc} = v_3 &= -R_3 i_3 \\
v_{cd} &= V_2 \\
v_{da} &= -V_1 \\
V_2 - V_1 + R_1 i_1 - R_3 i_3 &= 0
\end{aligned}
$$

In general, KVL states that, for a closed loop L:

$$\sum^{L} v_{L_i} = 0 \tag{2.2}$$

That is, sum of voltages in a closed loop equals to zero.

2.2.3 Node Voltage Method



By denoting voltages at nodes as v_a, v_b, v_c and v_d and connect v_d at the ground, making it effectively zero.

$$i_1 + i_2 - i_3 = 0(\text{ KCL at node b.}) \tag{2.3}$$

$$i_1 = \frac{v_a - v_b}{2} = \frac{32 - v_b}{2} \tag{2.4}$$

$$i_2 = \frac{v_c - v_b}{4} = \frac{20 - v_b}{4} \tag{2.5}$$

$$i_3 = \frac{v_b - 0}{8} = \frac{v_b}{8} \tag{2.6}$$

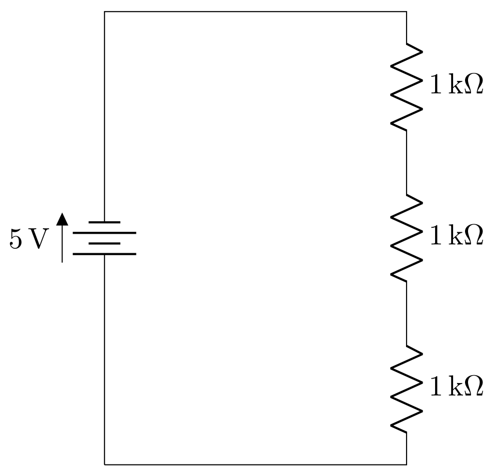
$$\tag{2.7}$$

And therefore subsituting values of i_1, i_2 and i_3 at Equation 2.3

$$\begin{aligned}\frac{32-v_b}{2}+\frac{20-v_b}{4}-\frac{v_b}{8}&=0\\128-4v_b+40-2v_b-v_b&=0\\v_b&=24\text{V}\end{aligned}$$

And from here, one can calculate the currents.

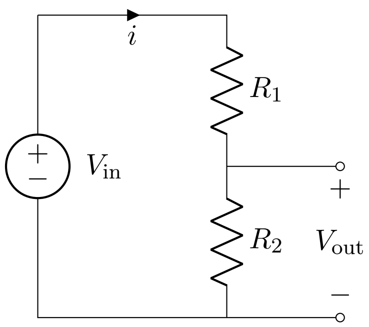
$$\begin{aligned}i_1=\frac{32-24}{2}&=\ 4\text{A}\\i_2=\frac{20-24}{4}&=\ 1\text{A}\\i_3&=\ 3\text{A}\end{aligned}$$



Chapter 3

Resistive Networks (Cont‘d) - November 12, 2020

3.1 Voltage Divider



The above circuit is refeered to as a Voltage divider cricuit, taking into account that V_{out} is the voltage accross the R_2 resister, using the Ohm's law, we can find:

$$
\begin{aligned}
i &= \frac{V_{\text{in}}}{R_1 + R_2} \\
V_{\text{out}} &= iR_2 \\
V_{\text{out}} &= R_2 \frac{V_{\text{in}}}{R_1 + R_2}
\end{aligned}
$$

And hence:

$$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2} \tag{3.1}$$

Chapter 4

Energy Storage Elements - November 12, 2020

Capacitive and Inductive effects are used to store energy.

4.1 Capacitor

Consisting of two plates, seperated by an insulator, whose dielectric permitivity is ε, the area of the plates is A, and the distance between them is l, when a voltage source is applied on these plates, positive charge q will accumulate on the one side, and the negative on the other side creating an electrical field E, where v is the voltage between them.

$$E(t) = \frac{q(t)}{\varepsilon A(t)}$$
$$v(t) = l(t) \times E(t)$$

Therefore

$$q(t) = \frac{\varepsilon A(t)}{l(t)} v(t)$$

Here, we say that:

$$C(t) = \frac{\varepsilon A(t)}{l(t)}$$

Therefore, unifying these formulas

$$q(t) = C(t) \times v(t) \tag{4.1}$$

Where C is the capacitance, whose value is Coulombs/Volt or Farad (F). Capacitor exibits proportional relation between voltage and stored charge. since rate of charge transfer is current, that is $\frac{\mathrm{d}q(t)}{\mathrm{d}t} = i(t)$ We can say that

$$i(t) = C\frac{\mathrm{d}}{\mathrm{d}t}\left(v(t)\right) \tag{4.2}$$

And likewise, from the same relation:

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\mathrm{d}t \tag{4.3}$$

When n capacitors are connected in series, their equivalent capacitance is:

$$\frac{1}{C_{\mathrm{eq}}} = \sum_{i=1}^{n}\frac{1}{C_i} \tag{4.4}$$

When n capacitors are connected in parallel, their equivalent capacitance is:

$$C_{\mathrm{eq}} = \sum_{i=1}^{n} C_i \tag{4.5}$$

4.2 Inductor

Inductor is a wire wrapped around a ring of crossectional area A, and with magnetic permatibility μ of circumferance l, wrapped around N times, and a current i is applied, a magnatic flux Φ of density B is generated. This magnetic field can then be used to store energy, similar to how a capacitor uses electric field to store energy.

$$B(t) = \frac{\mu N i(t)}{l(t)}$$

$$\Phi(t) = A(t) \times B(t)\lambda(t) = N\Phi(t) = \frac{\mu N^2 A(t)}{l(t)}i(t)$$

Where, λ is the total flux.

The Inductance (L) is generated, it is defined as the ratio of fthe voltage to the rate of change of the current. Furthermore, $L(t)$ is said to be

$$L(t) = \frac{\mu N^2 A(t)}{l(t)} \tag{4.6}$$

And hence

$$\lambda(t) = L(t)i(t) \tag{4.7}$$

Also, voltage and current of the inductor is defined as:

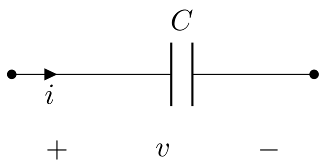
$$v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t} \tag{4.8}$$

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)\mathrm{d}t \tag{4.9}$$

Inductance has the unit Henry, (H) is the SI unit system, their equivalence follows the rules of the resistence.

4.3 Summary

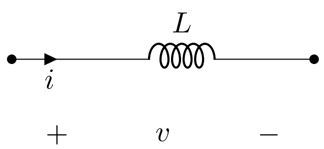
Capacitor



$$i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t} \tag{4.10}$$

If a voltage source is connected to a capacitor without nothing else, the current and capacitance will be zero.

Inductor

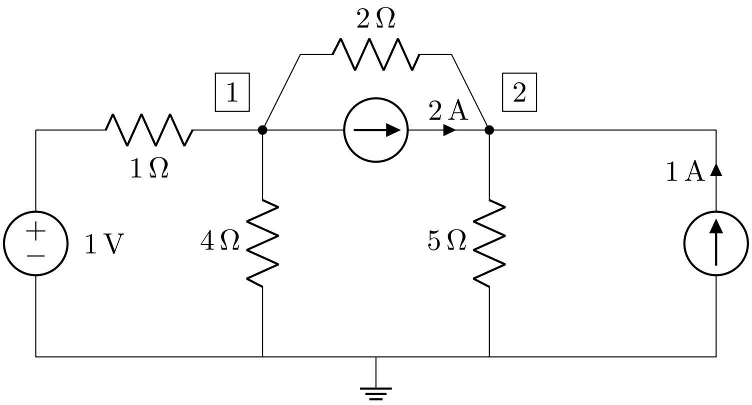


$$v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t} \tag{4.11}$$

If a current source is connected to a inductor without nothing else, the current and inductance will be zero.

Chapter 5

Network Theorems - November 13, 2020

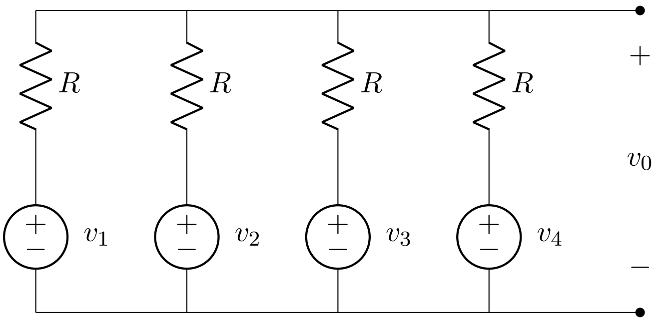


Here, after grounding at the bottom, hence giving it zero volts voltage, we can perform KVL at Nodes 1 and 2.

$$
\begin{aligned}
\frac{V_1 - 1}{1} + \frac{V_1}{4} + \frac{V_1 - V_2}{2} + 2 = 0 \\
-2 + \frac{V_2}{5} + \frac{V_2 - V_1}{2} - 1 = 0 \\
V_1 = 0.65\text{V} \\
V_2 = 4.75\text{V} \\
i = 0.95\text{A}
\end{aligned}
$$

The current formulas above arise from the node-voltage method being performed at both nodes.

Resistive Adder



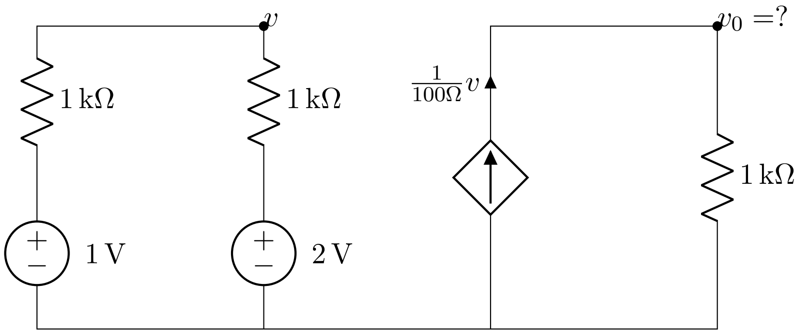
Where we can conclude, ısing the node voltage method:

$$\frac{v_0 - v_1}{R} + \frac{v_0 - v_2}{R} + \frac{v_0 - v_3}{R} + \frac{v_0 - v_4}{R} \tag{5.1}$$

$$v_0 = \frac{1}{4}\left(v_1 + v_2 + v_3 + v_4\right) \tag{5.2}$$

Dependant Sources

Dependant sources are sources whose values depend on the values of other points. There are dependant voltage sources and dependant current sources.

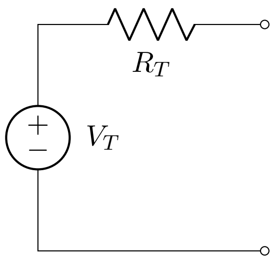


Here we start by calculating v, the current of the dependant current source depends on this value.

$$\begin{aligned} v &= \frac{1}{2}(1+2) = 1.5\text{V} \\ v_0 &= I.1\text{k}\Omega \\ &= \frac{1}{100} \times 1.5 \times 1\text{k}\Omega \\ &= 15\text{V} \end{aligned}$$

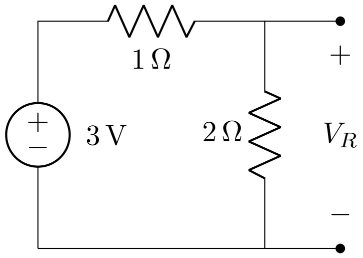
5.1 Thevenin Network Theorems

If the system is linear, any network can be represented by a sinlge voltage source and a resistor.



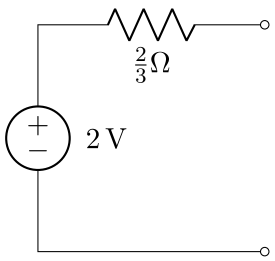
Example

Given an example circuit of the form:

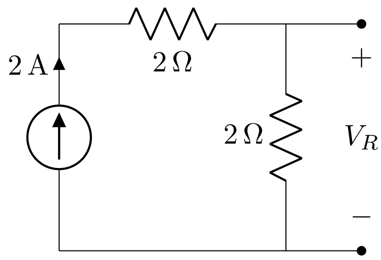


Where V_R is taken the thevenin voltage, V_T. To calculate V_R, a simple KVL would do, where i the main current of the circuit $i = 1\text{A}$ and hence, $V_R = 1\text{A} \times 2\Omega = 2\text{V}$ and hence, $V_T = 2\text{V}$.

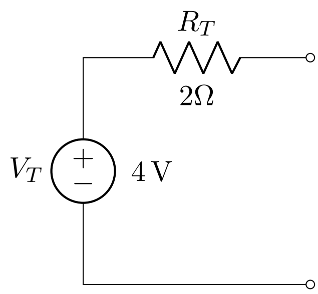
Then, short-circuit between the output terminals, and calculate the i again, the resistor will be short circuited, with $i_2 = 3\text{A}$ And hence, we can now calculate the R_T by saying that the current in the thevenin equivalent circuit will equal i_2. Calculating this, yields $R_T = \frac{2}{3}\Omega$. And hence, the thevenin equivalent circuit is:



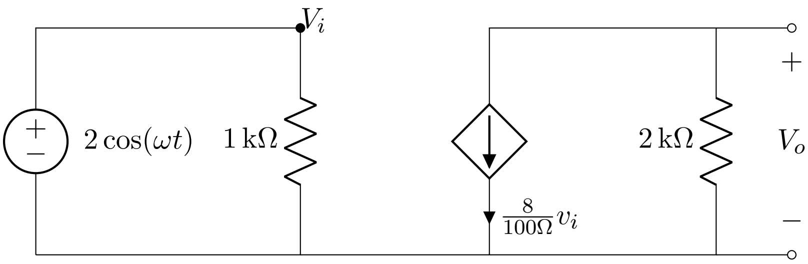
Example



Start by finding the voltage accross the resistor once more, which is $V_R = 2\Omega \times 2\text{A} = 4\text{V}$ Hence, so is V_T, shortcircuiting the resistor once more, since this is a current source, i is still the same (2A) and can be used to calculate R_T again, which is $R_T = 2\Omega$

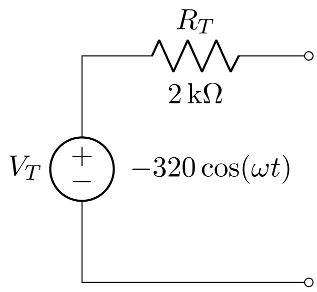


Example



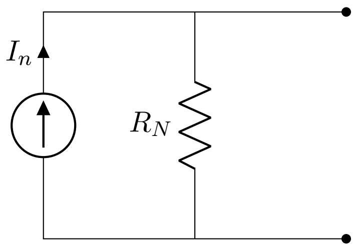
Here, to calculate V_o, we first need to calculate V_i, V_i is of course equivalent to the voltage source at the left side, and hence $V_i = 2\cos(\omega t)$ and hence the current at the left hand side, $I = -\frac{8}{100} 2\cos(\omega t) \times 2000$, and hence, $V_o = -320\cos(\omega t)$.

To find the equivalent resistor, we short cirucit the open ports, and once again find the current, which equals to the current source, and then, we must use the resulting current in the equivalent circuit to determine the resistance, which is $R_T = 2\text{k}\Omega$.



5.2 Norton Theorem

Norton theorem states that any linear circuit can be represented using a single current source, and a resistance.



Chapter 6

First Order Systems - November 19, 2020

In an RC circuit, if we were to solve the circuit using KVL and differential equations, we will find:

$$i(t) = Ae^{-\frac{1}{RC}t} \tag{6.1}$$

Since $V_c = V_0$ at time $t = 0$, we say that:

$$i(t) = \frac{V_0}{R}e^{-\frac{t}{RC}} \tag{6.2}$$

6.1 Time Constant

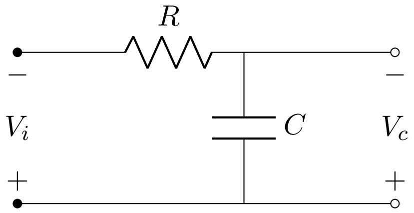
Here, the term $s + \frac{1}{RC} = 0$ is called the characteristic equation of the function, with larger RC resulting in the current discharging slower and lower RC causing it to discharge faster, therefore RC is called the *time constant*, and is sometimes written as τ.

For any system of type $x = Ae^{-\frac{t}{\tau}}$, $x(0) = A$ and $x(\tau) = \frac{A}{e}$, where e is the natural number. (More formally, $f(x) = e^x$ is the function with the characteristic $f'(x) = f(x)$ and $f(x) \neq 0$).

Theoratically, the current never reaches zero. However, it is accepted in engineering that once $t = 5\tau$, the current is zero. This divides the states of the RC circuit into two, where $t \leq 5\tau$, the system is said to be in the **transient state**, where meaningful change still occurs, where $t > 5\tau$, the system is said to be in the **steady state**, or the **final state**.

6.2 Step Response

In a circuit of the type:



If we were to apply a **step voltage** to the input terminal of the system, such that:

$$V_i = V \cdot u(t)$$
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

After solving this circuit, we will end up with an equation of the form

$$V_c = v + Ae^{-\frac{t}{RC}}$$

And after plugging in A

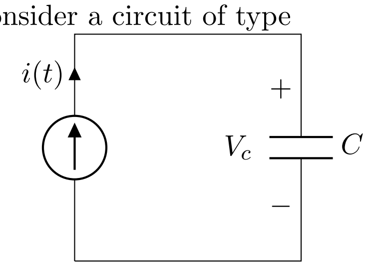
$$V_c = Vt(V_0 - V)e^{-\frac{t}{RC}} \tag{6.3}$$

And, the current of the system shall be:

$$i_c = C\frac{\mathrm{d}V_c}{\mathrm{d}t} \tag{6.4}$$

6.3 State

Consider a circuit of type



Here, $q(t)$ is dependant on the values of the previous currents:

$$q(t) = \int_{-\infty}^{t} i(t)\mathrm{d}t \tag{6.5}$$

Hence:

$$q(t_2) = \int_{-\infty}^{t_1} i(t)\mathrm{d}t + \int_{t_1}^{t_2} i(t)\mathrm{d}t = q(t_1) + \int_{t_1}^{t_2} i(t)\mathrm{d}t \tag{6.6}$$

6.3.1 State Equation

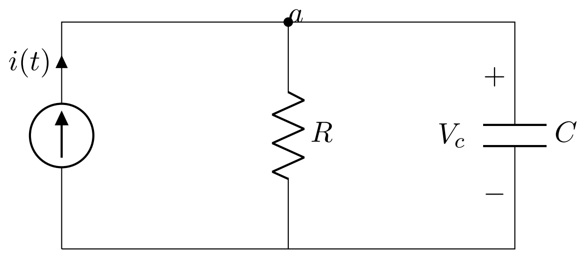
A state equation is an equation of the type:

$$\frac{\mathrm{d}}{\mathrm{d}t}(S) = K_1 + K_2 \tag{6.7}$$

Where S is the state variable, K_1 is the present value of the state variable and K_2 is the input variable.

6.3.2 Computer Simulation

For example, consider a circuit of the type:



Now we can say that, using the a node and KCL:

$$i(t) = \frac{V_c}{R} + C\frac{\mathrm{d}V_c}{\mathrm{d}t}$$
$$\frac{\mathrm{d}V_c}{\mathrm{d}t} = -\frac{V_c}{RC} + \frac{i(t)}{C}$$

Now using the Euler's method to get rid of the derivative:

$$\frac{\mathrm{d}V_c(t)}{\mathrm{d}t} \approx \frac{V_c(t+\Delta t) - V_c(t)}{\Delta t}$$

And hence:

$$V_c(t_0 + \Delta t) = V_c(t_0) - \frac{V_c(t_0)}{RC}\Delta t + \frac{i(t_0)}{C}\Delta t$$

Now, if we defined t_0 as the kth state of the system, and $t_0 + \Delta t$ as the $k+1$th state of the system:

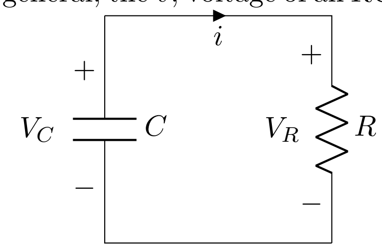
$$V_c(k+1) = V_c(k) - \frac{V_c(k)}{RC}\Delta t + \frac{i(k)}{C}\Delta t$$

This formula can easily be plotted and simulated in a computer.

Chapter 7

First Order Systems (Cont‘d) - November 26, 2020

In general, the v, voltage of an RC circuit tends to be of the form $v = Ae^{-\frac{t}{RC}}$.



In the above diagram, using the KVL, we find that $-V_C + V_R = 0$, where $V_R = R \cdot i$ and where normally $i = C\frac{\mathrm{d}V_C}{\mathrm{d}t}$, however since the current enters the capacitor through its negative side, it is instead $i = -C\frac{\mathrm{d}V_C}{\mathrm{d}t}$. After substituting the values in the KVL and taking the derivate of both sides with respect to time, results in:

$$R\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{C}i = 0R\frac{\mathrm{d}i}{\mathrm{d}t} = -\frac{1}{C}i$$

Hence here, $f(x) = f'(x)$, we can see that $i(t) = Ae^{st}$ (where s is the decaying factor).

$$i(t) = Ae^{st}\frac{\mathrm{d}i}{\mathrm{d}t} = Ase^{st}$$

And hence, when substituted in the equation:

$$\begin{aligned} R \cdot Ase^{st} + \frac{1}{C}Ae^{st} &= 0 \\ Ae^{st}\left(sR + \frac{1}{C}\right) &= 0 \end{aligned}$$

Here, either $Ae^{st} = 0$, which would imply $i = 0$, and hence is not a solution we are looking for, or:

$$sR + \frac{1}{C} = 0$$
$$s = -\frac{1}{RC}$$

And henceforth:

$$i = Ae^{-\frac{t}{RC}} \tag{7.1}$$

Since the voltage of the capacitor, V_0, and the $i(0)$ at $t = 0$ for this circuit is:

$$i(0) = \frac{V_0}{R} \tag{7.2}$$

If we equate this to the equation we found at $t = 0$:

$$i(0) = \frac{V_0}{R} = Ae^{-\frac{0}{RC}} = A$$
$$A = \frac{V_0}{R}$$

and hence:

$$i(t) = \frac{V_0}{R}e^{-\frac{t}{RC}} \tag{7.3}$$

Where RC is called the **time constant**, τ. As the τ increases, time it will take for the signal to decay will increase as well.

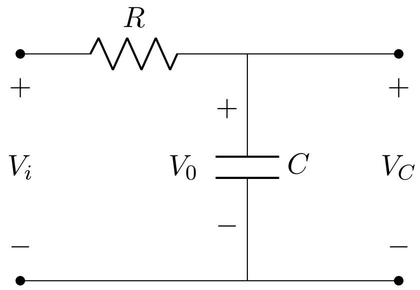
$$S + \frac{1}{RC} = 0 \tag{7.4}$$

Is called the characteristic equation of this system. In general, for any first order system, there a will be solution of type.

$$x = Ae^{-\frac{t}{\tau}} \tag{7.5}$$

The area before the $t = 5\tau$ ($t < 5\tau$)is called the transient response, the area after ($t > 5\tau$) is called the steady state response, and it is accepted that at this point the output is zero.

Step Response



Initially, $V_C(0) = V_0$ and $V_i = V \cdot u(t)$ where $u(t) = 1$ for $t \geq 0$ and $u(t) = 0$ for $t < 0$. Using KVL on the left side:

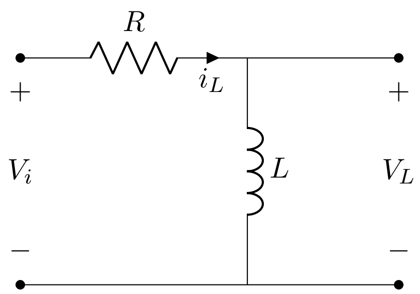
$$\frac{V_C - V_i}{R} + C\frac{\mathrm{d}V_C}{\mathrm{d}t} \tag{7.6}$$

Here, rather than solving the equation, we know two things, one is that $V_C(0) = V_0$ but also, we know that $V_C(\infty) = V$ and $V_C = V + (V_0 - V)e^{-\frac{t}{RC}}$

In general, for V_i, V_f for initial and final voltage respectively:

$$V_C = V_i e^{-\frac{t}{RC}} + V_f\left(1 - e^{-\frac{t}{RC}}\right) \tag{7.7}$$

Analysis of RL



Using KVL

$$-V_i + i_L \cdot R + L\frac{\mathrm{d}i}{\mathrm{d}t} = 0$$

Here, using Laplace transforms (§Appendix A), we can transform this equation to:

$$\begin{aligned}
-\frac{V}{s} + I_L(s) \cdot R + L\left(SI_L(s) - i_L(0)\right) = 0 \\
s \cdot I_L\left(R + SL\right) = V \\
I_L(s) = \frac{V}{s(R + SL)}
\end{aligned}$$

Using partial fractions:

$$
\begin{aligned}
I_L(s) &= \frac{V}{s(R+SL)} \\
&= \frac{A}{s} + \frac{B}{R+SL} \\
& s\,(AL+B) + AR = V \\
A &= \frac{V}{R} B = -\frac{VL}{R}
\end{aligned}
$$

where we accepted $AL + B = 0$

$$
\begin{aligned}
I_L(s) &= \frac{\frac{V}{R}}{s} - \frac{\frac{VL}{R}}{R+SL} \\
&= \frac{\frac{V}{R}}{S} - \frac{\frac{V}{R}}{\frac{R}{L}+S}
\end{aligned}
$$

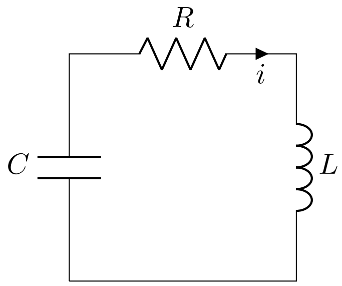
Using reverse Laplace transforms:

$$
i_L(t) = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t} \tag{7.8}
$$

Chapter 8

2nd Degree Circuits - December 3, 2020

8.1 Undriven RLC Circuit



$$i'' + \frac{R}{L}i' + \frac{1}{LC}i = 0 \tag{8.1}$$

If one writes down $i = Ae^{st}$, we can write the following:

$$A\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) = 0 \tag{8.2}$$

Without A this equation is called **the Characteristic Equation**, and is also written as:

$$s^2 + 2\alpha s + \omega_0^2 = 0 \tag{8.3}$$

Where

$$\alpha \equiv \frac{R}{2L} \tag{8.4}$$

$$\omega_0 \equiv \frac{1}{\sqrt{LC}} \tag{8.5}$$

Where the roots of the equation is:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \tag{8.6}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \tag{8.7}$$

And hence:

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \tag{8.8}$$

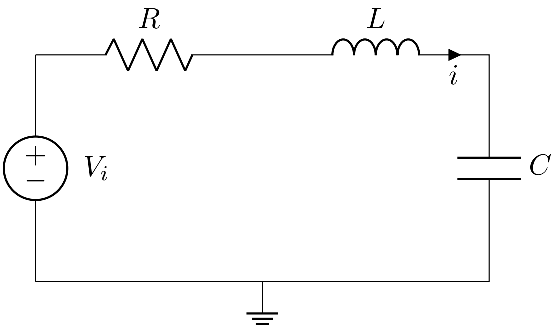
Where, the relation of α and ω affect the nature of the circuits behaviour.

	Circuit Behaviour
$\alpha < \omega_0$	Under-damped Dynamics
$\alpha = \omega_0$	Critically-damped Dynamics
$\alpha > \omega_0$	Over-damped Dynamics

This behaviour arises from the very nature of the exponential function, simply put, when $\alpha < \omega_0$, the euler's identity comes into effect. Turning the equation into *some* sort of $\gamma e^{c\sqrt{\omega_0^2 - \alpha^2} jt}$ where γ is just a real value, and $j^2 = -1$, hence this circuit will show sinusoidal behaviour.

Furthermore, ω_0 is called the **natural frequency**, and α is the **damping constant**.

8.2 Driven RLC Circuit



$$V_c'' + \frac{R}{L} V_c' + \frac{1}{LC} V_c = \frac{1}{LC} V_i \tag{8.9}$$

$$i = C \frac{\mathrm{d}V_c(t)}{\mathrm{d}t} \tag{8.10}$$

Appendix A

Laplace Transforms

General Laplace Transform Formula

$$F(s) = \int_0^\infty f(t)e^{-st}\mathrm{d}t \tag{A.1}$$

Table of Common Formulas

$$Au(t) \rightarrow \frac{A}{s} \tag{A.2}$$

$$f'(t) \rightarrow sF(s) - f(0) \tag{A.3}$$

$$e^{-at} \rightarrow \frac{1}{s+a} \tag{A.4}$$

$$f''(t) \rightarrow s^2F(s) - sf(0) - f'(0) \tag{A.5}$$

$$\sin(at) \rightarrow \frac{a}{s^2+a^2} \tag{A.6}$$

$$\cos(at) \rightarrow \frac{s}{s^2+a^2} \tag{A.7}$$

$$atu(t) \rightarrow \frac{A}{s^2} \tag{A.8}$$