CENG 215

Circuits and Electronics Lecture Notes

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Chapter 1

Introduction - October 15, 2020

1.1 Abstractions

Recall the Newton's formula $F = ma$, which defines the relationship between force, mass and acceleration. This formula modals acceleration using force and mass. However, according to this model, there is no connection between mass and speed. Consider now, the Einstein's equation:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.1}$$

As this equation shows, speed affects mass. The abstractions ignore certain connections for the sake of simplicity. Likewise, electrical engineering, based on Maxwell's Equations, create abstractions, notably, this lecture deals with the *Lumped Circuit Abstraction*.

Consider a statement in a high level programming language `int n = 3;`, this basic statment goes through many abstractions eventually reaching circuitery.

1.2 Circuits

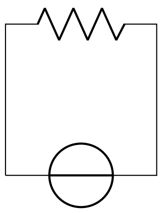
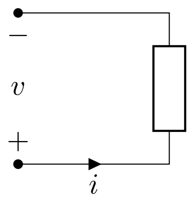


Figure 1.1: A simple cirucit abstraction

From this abstraction, arises the **Ohm's Law**

$$v = iR \tag{1.2}$$

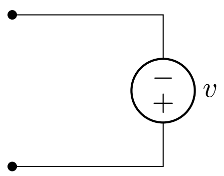
1.2.1 Two Terminal Element



Two terminal elements include batteries, resistors, capacitors, etc...

Battery

Batteries provide voltage and can be bind into serial or paralel.



Below are power (in watts) and energy (in Jouless or watt-seconds) for batteries.

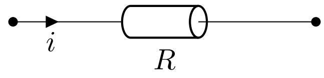
$$P = vi \tag{1.3}$$

$$w = Pt \tag{1.4}$$

Enery formula can also be represented as:

$$w = \int_{t_1}^{t_2} v(t)i(t)\mathrm{d}t \tag{1.5}$$

Resistance



Imagine a generic tube with length l, resistivity ρ and cross sectional area a, in this case, the Resistance of the element R is

$$R = \rho\frac{l}{a} \tag{1.6}$$

The resistance can be showed as:

Where the Ohm's Law state:

$$v = Ri \tag{1.7}$$

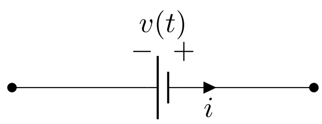
or alternativelly

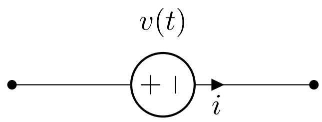
$$i = Gv \tag{1.8}$$

Where G is conductance, whose SI unit is siemens and defined as $\frac{1}{R}$

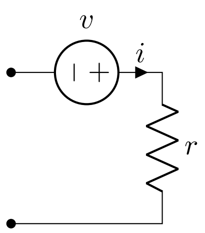
Ideal Voltage Source

Ideal Voltage source can be represnted by:





In general, any voltage soucre can be drawn as



Where r is the internal resistence that arise from the material itself. An ideal voltage source would be able to provide the same current no matter what the voltage is, however this is not possible in real life, where any voltage source has a r

Chapter 2

Resistive Networks - 22 October, 2020

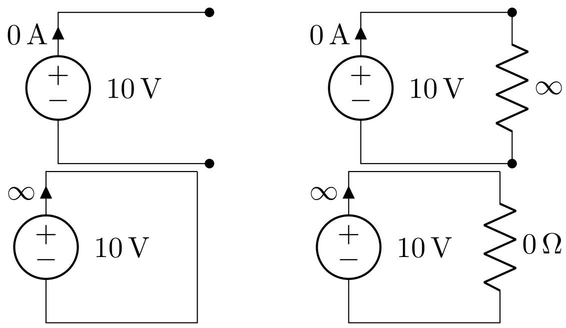


Figure 2.1: An open circuit is equivalent to a circuit with a resistor with an infinite resistance. Whereas a short cirucit can be modelled as a circuit with zero resistence.

The perfect current source is a current source that can supply current in any voltage.

2.1 Signals

Signals can be analog or digital. In Figure 2.2, the sinosodial signals, which has continous values is an analog signal. Where it is represented via the $v(t) = A\sin(\omega t + \phi)$, where A is its amplitude, ω is its frequency and ϕ is its phase, it is analog because it has *continous* values. In the meantime, the second signal is a digital signal as it has *discrete* and quantised values.

Digital signals trade precision about the signal with *immunity towards the noise.*

Resistance A measure of the ability of the device to consume energy.

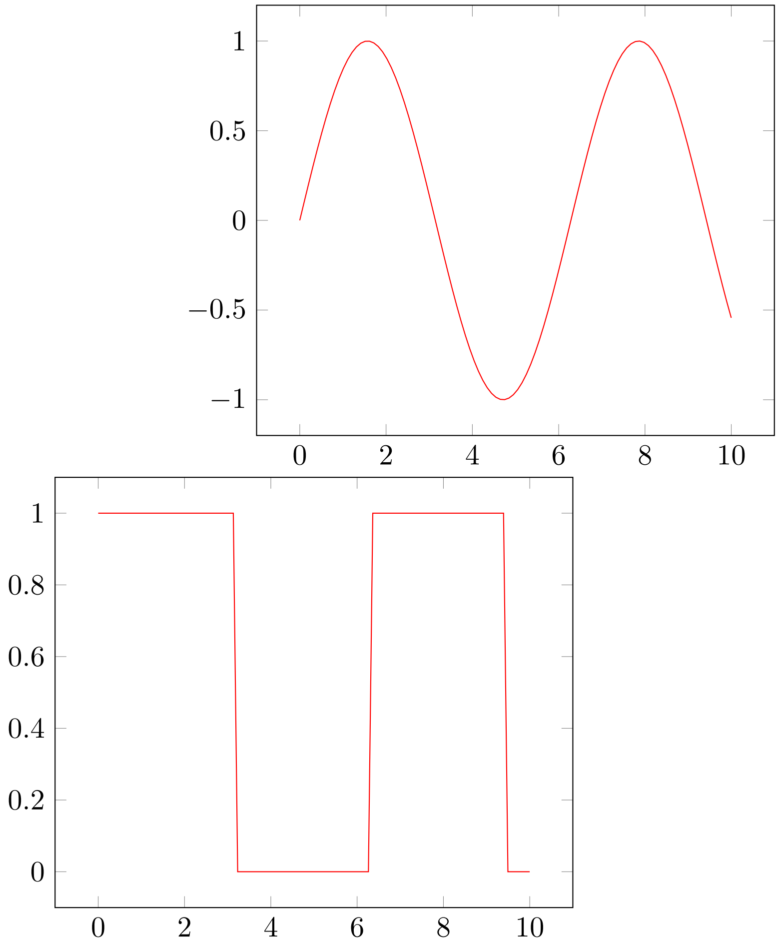
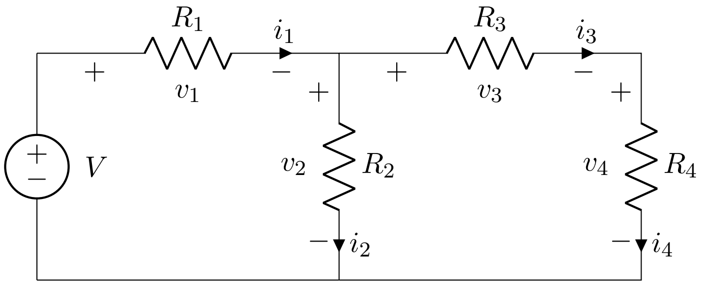


Figure 2.2: Two signals.

Capacitence A measure of the ability of the device to store energy in the form of potential energy. (voltage).

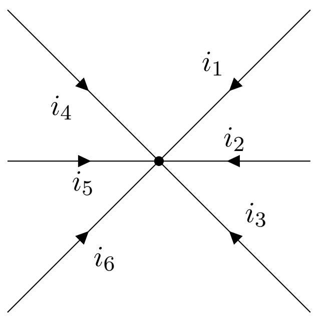
Inductance A measure of the ability of the device to store energy as the moving charge (current).

2.2 Resistive Networks



This sort of circuits can be analysed using two laws, **Kirchoff's Current Law** (KCL) and **Kirchoff's Voltage Law** (KVL).

2.2.1 Kirchoff's Current Law

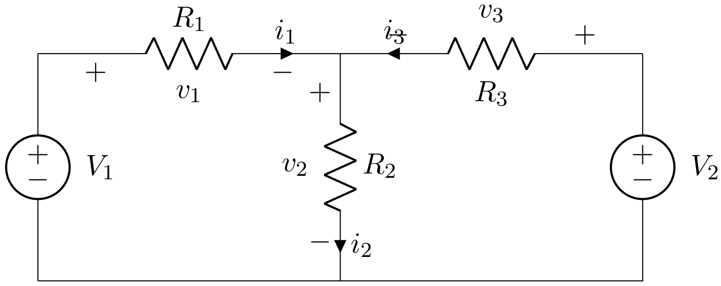


Kirchoff's current law state that the sum of currents entering a node must equal zero.

$$\sum_{n=1}^{6} i_n = 0 \tag{2.1}$$

When one takes the directions of the currents into account, this means that the *currents entering a node must equal the curents exiting a node.*

2.2.2 Kirchoff's Voltage Law



Consider three loops, if clockwise, starting from the battery's top, each node is called a, b, c, d then for loop abcda:

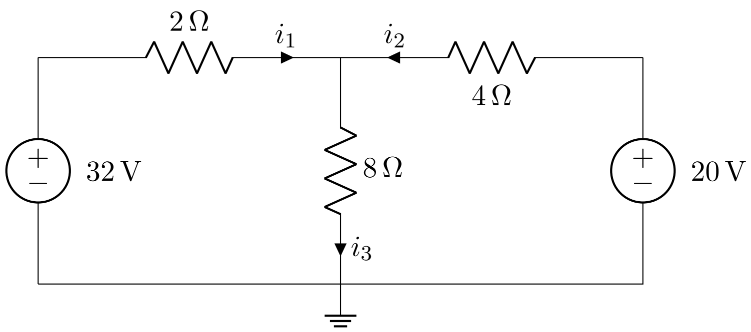
$$
\begin{aligned}
v_{ba} + v_{bc} + v_{cd} + v_{da} &= 0 \\
v_{ab} = v_1 &= R_1 i_1 \\
v_{bc} = v_3 &= -R_3 i_3 \\
v_{cd} &= V_2 \\
v_{da} &= -V_1 \\
V_2 - V_1 + R_1 i_1 - R_3 i_3 &= 0
\end{aligned}
$$

In general, KVL states that, for a closed loop L:

$$\sum^{L} v_{L_i} = 0 \tag{2.2}$$

That is, sum of voltages in a closed loop equals to zero.

2.2.3 Node Voltage Method



By denoting voltages at nodes as v_a, v_b, v_c and v_d and connect v_d at the ground, making it effectively zero.

$$
\begin{aligned}
i_1 + i_2 - i_3 &= 0 (\text{ KCL at node b.}) && (2.3) \\
i_1 = \frac{v_a - v_b}{2} &= \frac{32 - v_b}{2} && (2.4) \\
i_2 = \frac{v_c - v_b}{4} &= \frac{20 - v_b}{4} && (2.5) \\
i_3 = \frac{v_b - 0}{8} &= \frac{v_b}{8} && (2.6) \\
& && (2.7)
\end{aligned}
$$

And therefore subsituting values of i_1, i_2 and i_3 at Equation 2.3

$$\begin{aligned}
\frac{32 - v_b}{2} + \frac{20 - v_b}{4} - \frac{v_b}{8} &= 0 \\
128 - 4v_b + 40 - 2v_b - v_b &= 0 \\
v_b &= 24\text{V}
\end{aligned}$$

And from here, one can calculate the currents.

$$\begin{aligned}
i_1 = \frac{32 - 24}{2} &= 4\text{A} \\
i_2 = \frac{20 - 24}{4} &= 1\text{A} \\
i_3 &= 3\text{A}
\end{aligned}$$