

CS 295A/395D: Artificial Intelligence

Elementary Game Theory

Prof. Emma Tosch

30 March 2022



The University of Vermont

Agenda

Review decision theory

Elementary game theory

- Optimum vs. optimal solutions
- Strategies
- Vocabulary

Logistics

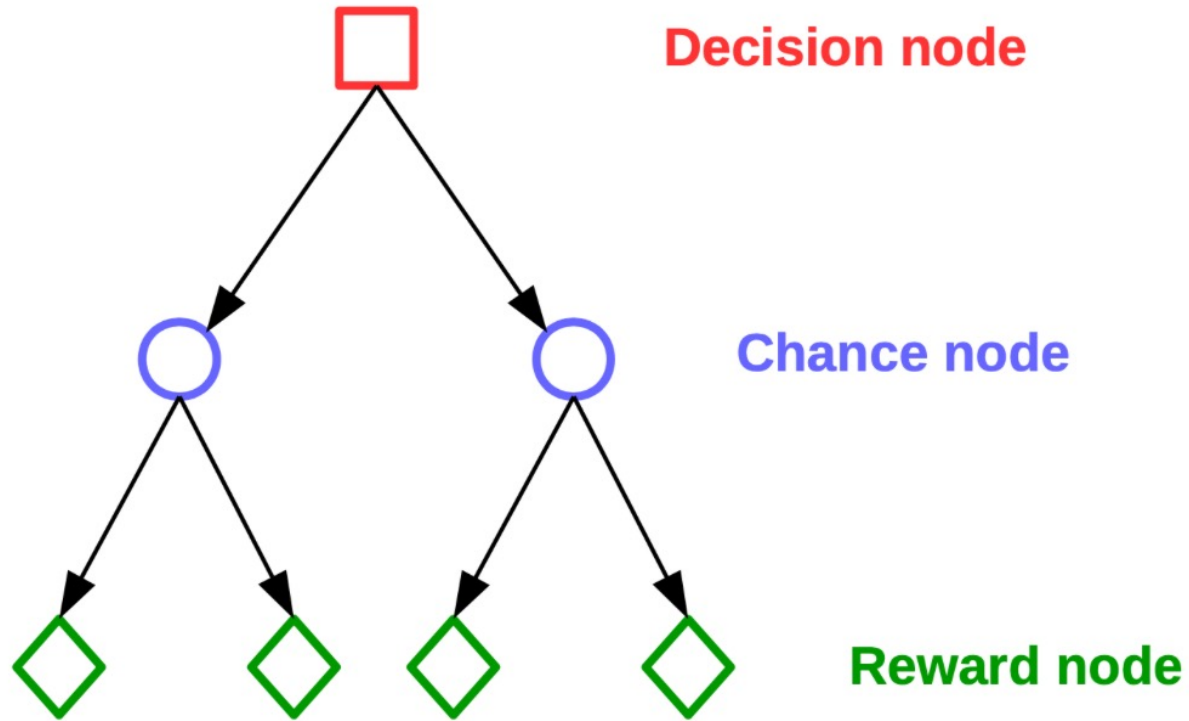
- BB theory assignment out today
- Exam through game theory
 - Temporal reasoning pushed to next unit
 - Exam next Friday (April 8)
- Next unit: temporal reasoning and program synthesis
 - Removed machine learning

Recap: Decision Theory

We can express taking actions in a world with uncertainty via *decision trees*

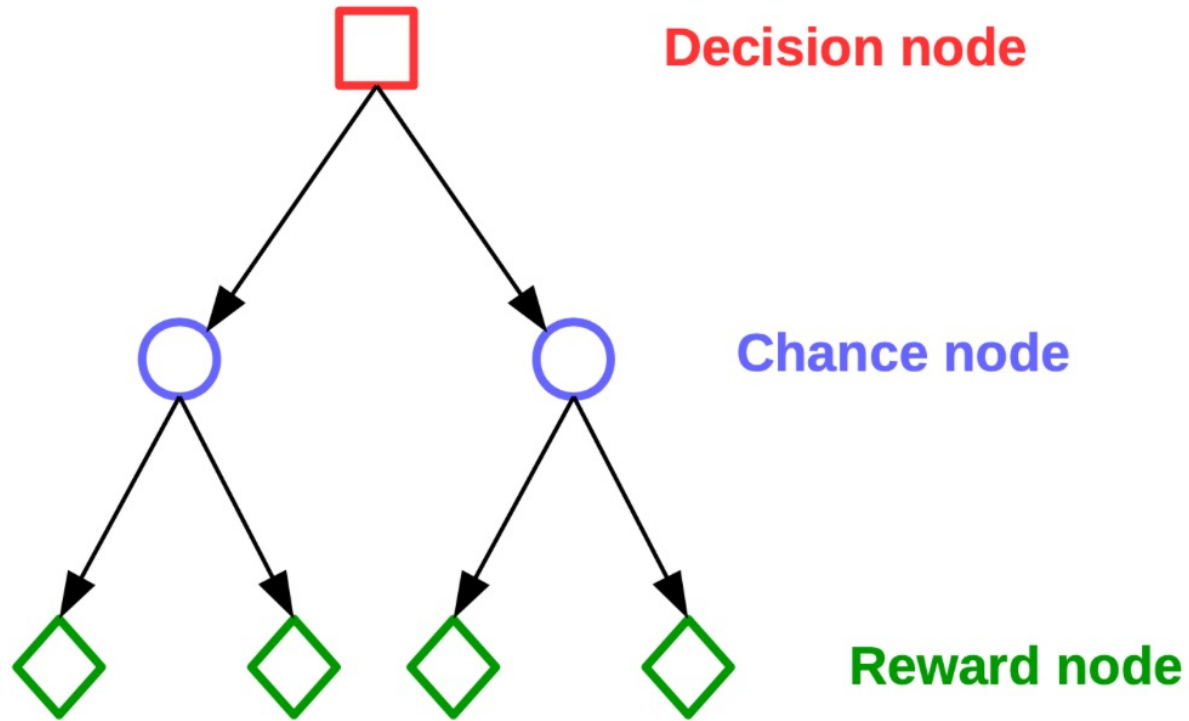
Decision trees are temporally-ordered nodes where each level corresponds to alternating:

- **Decision nodes** – state of the system; outgoing edges represent different actions
- **Chance nodes** – probability distributions over outcomes; outgoing edges represent reachable states with some probability
- **Reward nodes** – utility obtained from following the path



Note: "decision tree" also refers to a classification algorithm in machine learning and is completely different from the type of decision tree we will talk about here.

Recap: Maximize expected utility



$$EU[a \mid e_1, e_2, \dots] = \sum_{s'} P(S_{t+1} = s' \mid a, e_1, e_2, \dots) U(s')$$

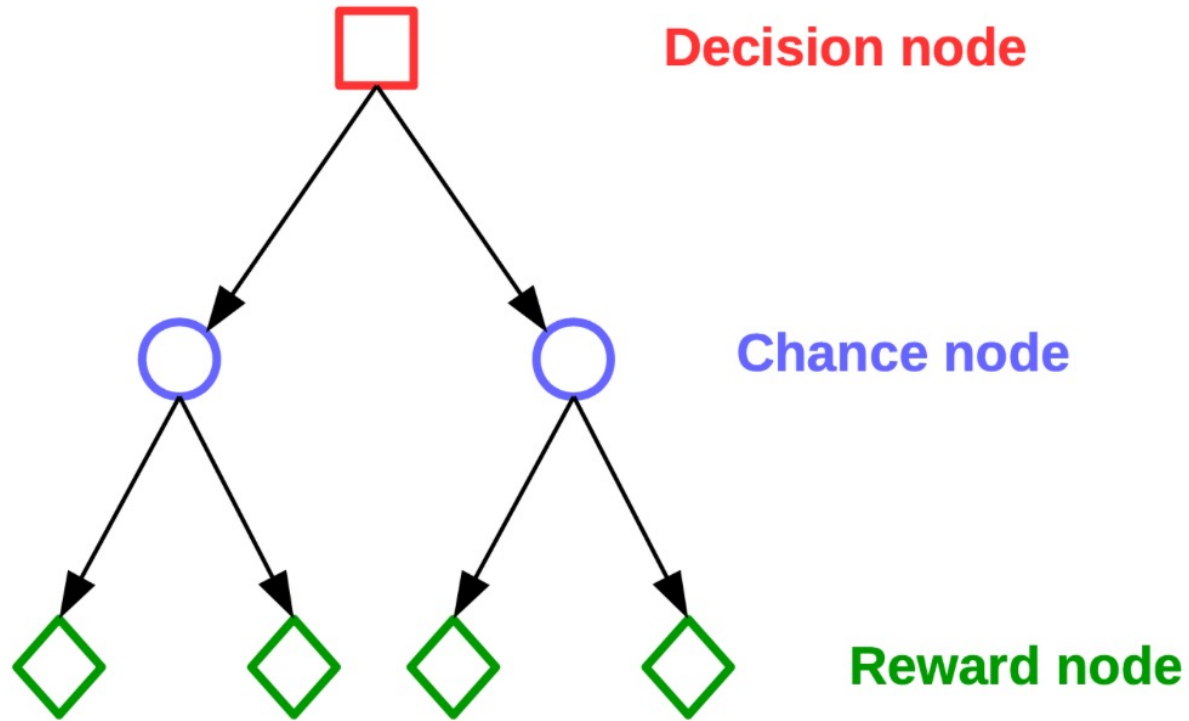
Best action is the action a that maximizes

$$EU[a \mid e_1, e_2, \dots]$$

Sum of the utility of actions taken.

Note: “decision tree” also refers to a classification algorithm in machine learning and is completely different from the type of decision tree we will talk about here.

Recap: Comparing actions



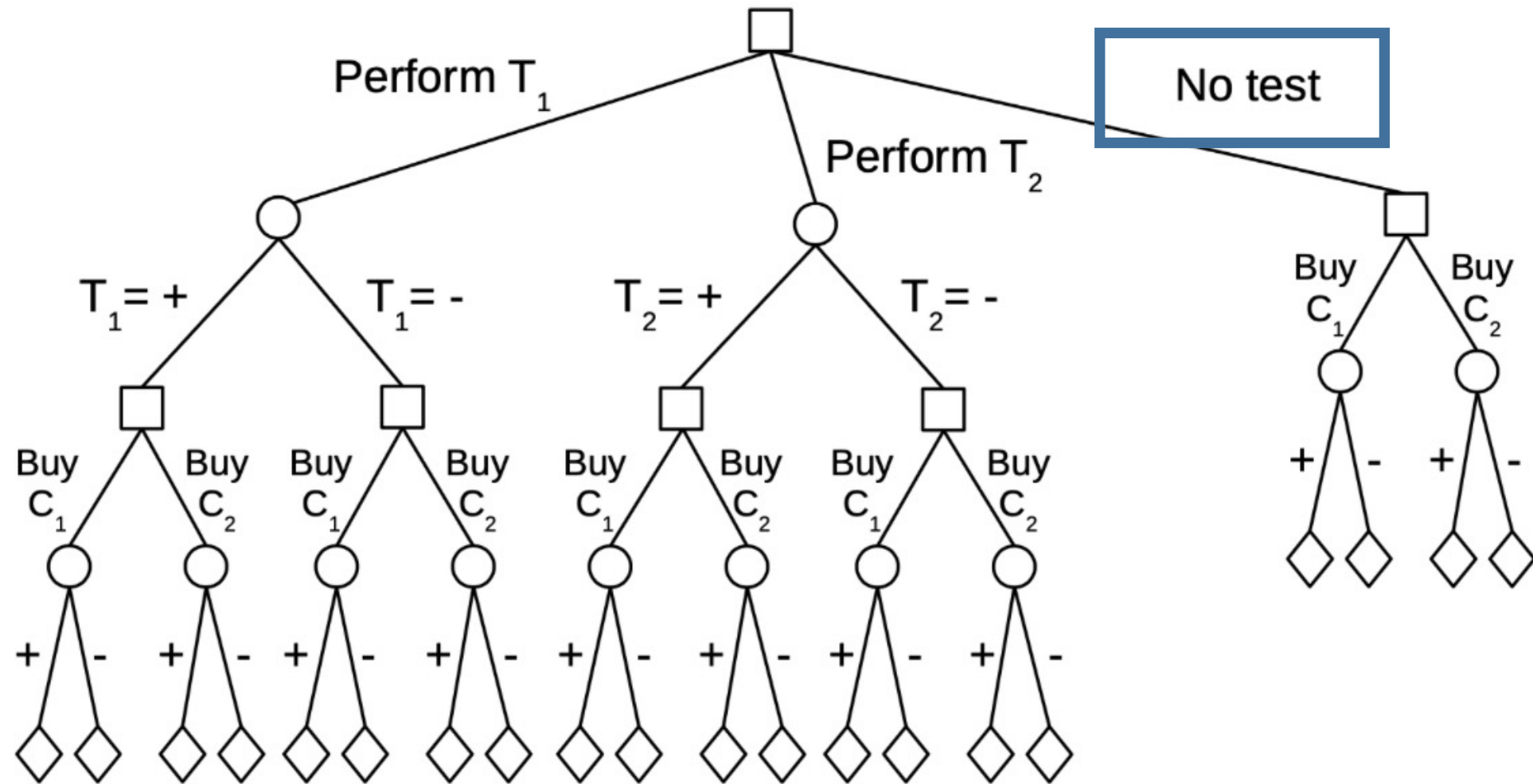
Note: “decision tree” also refers to a classification algorithm in machine learning and is completely different from the type of decision tree we will talk about here.

$$EU[a \mid e_1, e_2, \dots] = \sum_{s'} P(S_{t+1} = s' \mid a, e_1, e_2, \dots) U(s')$$

Probability mass function – over all sources of uncertainty associated with this action.

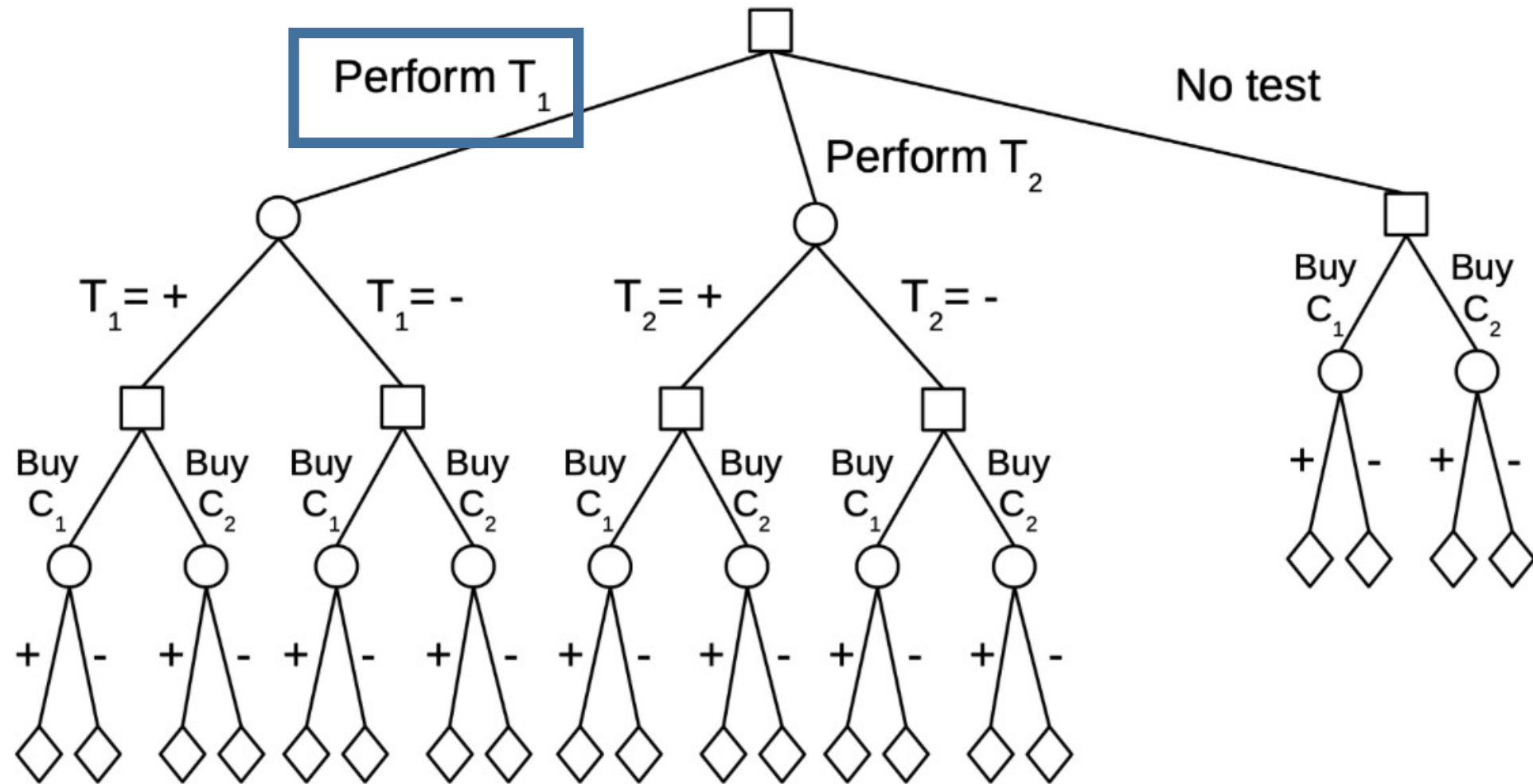
Utility function

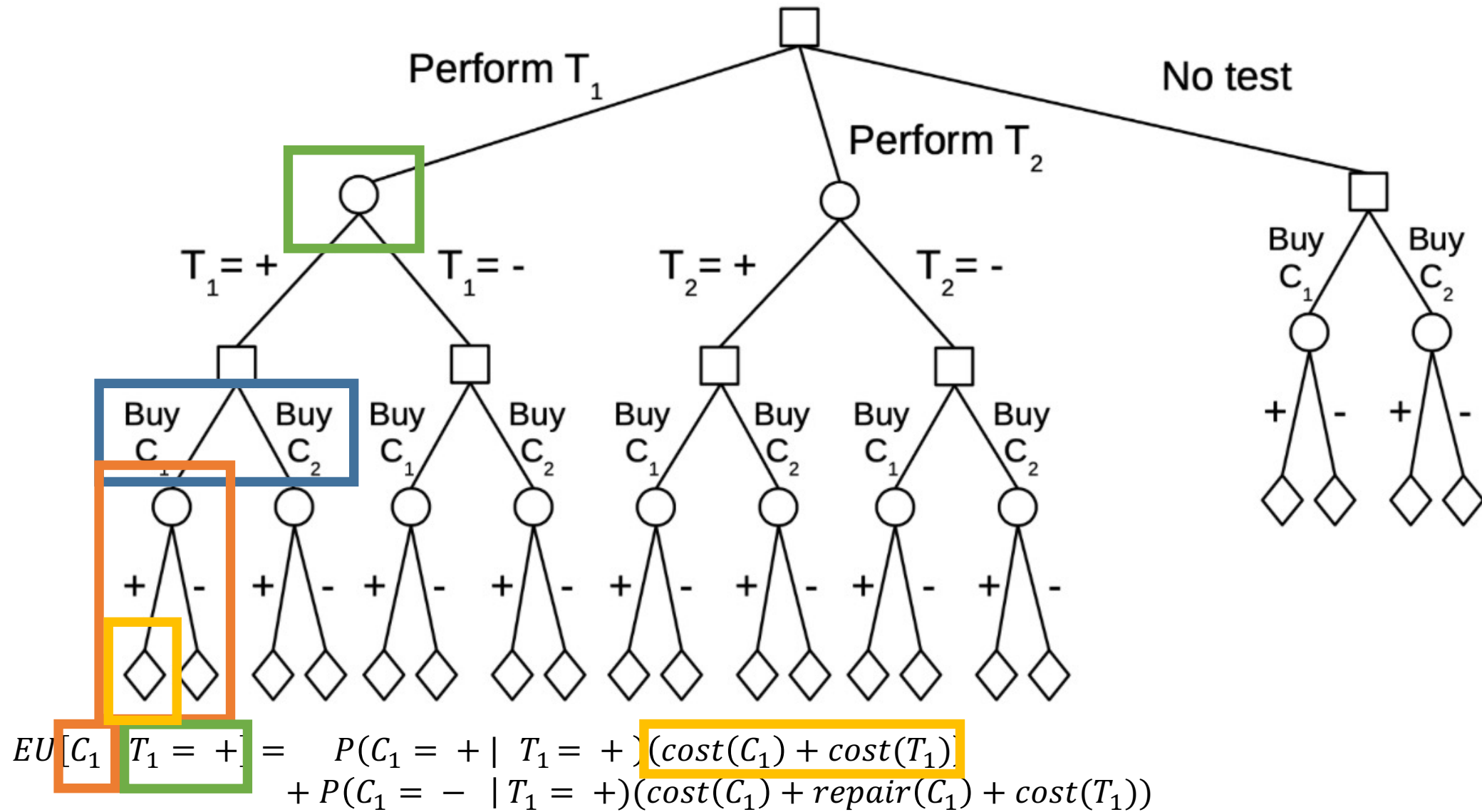
- Basic actions: reward
- Actions with subsequent actions with uncertain outcomes: EU of those actions



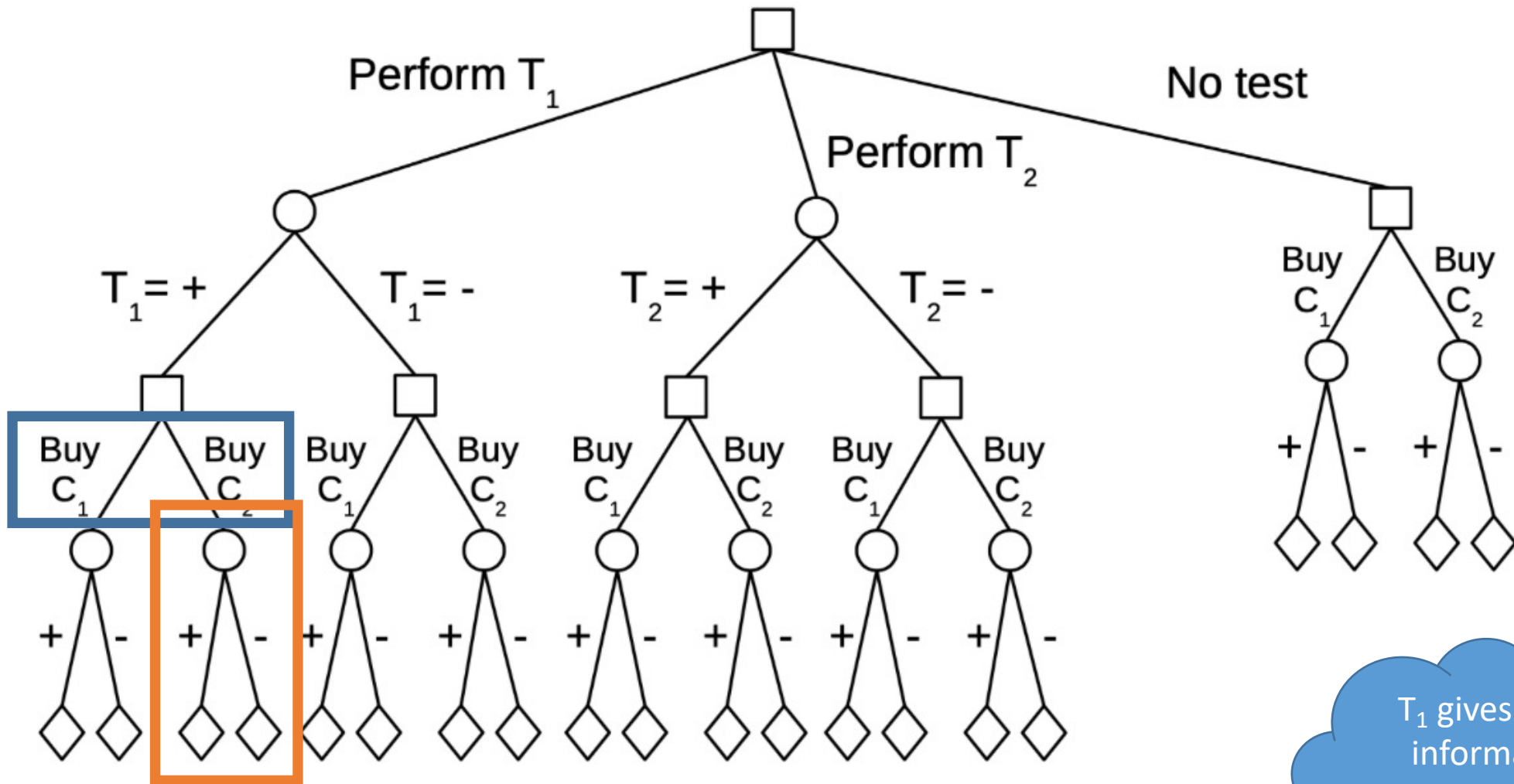


$$EU[C_2] = P(C_2 = +)(cost(C_2)) + P(C_2 = -)(cost(C_2) + repair(C_2))$$





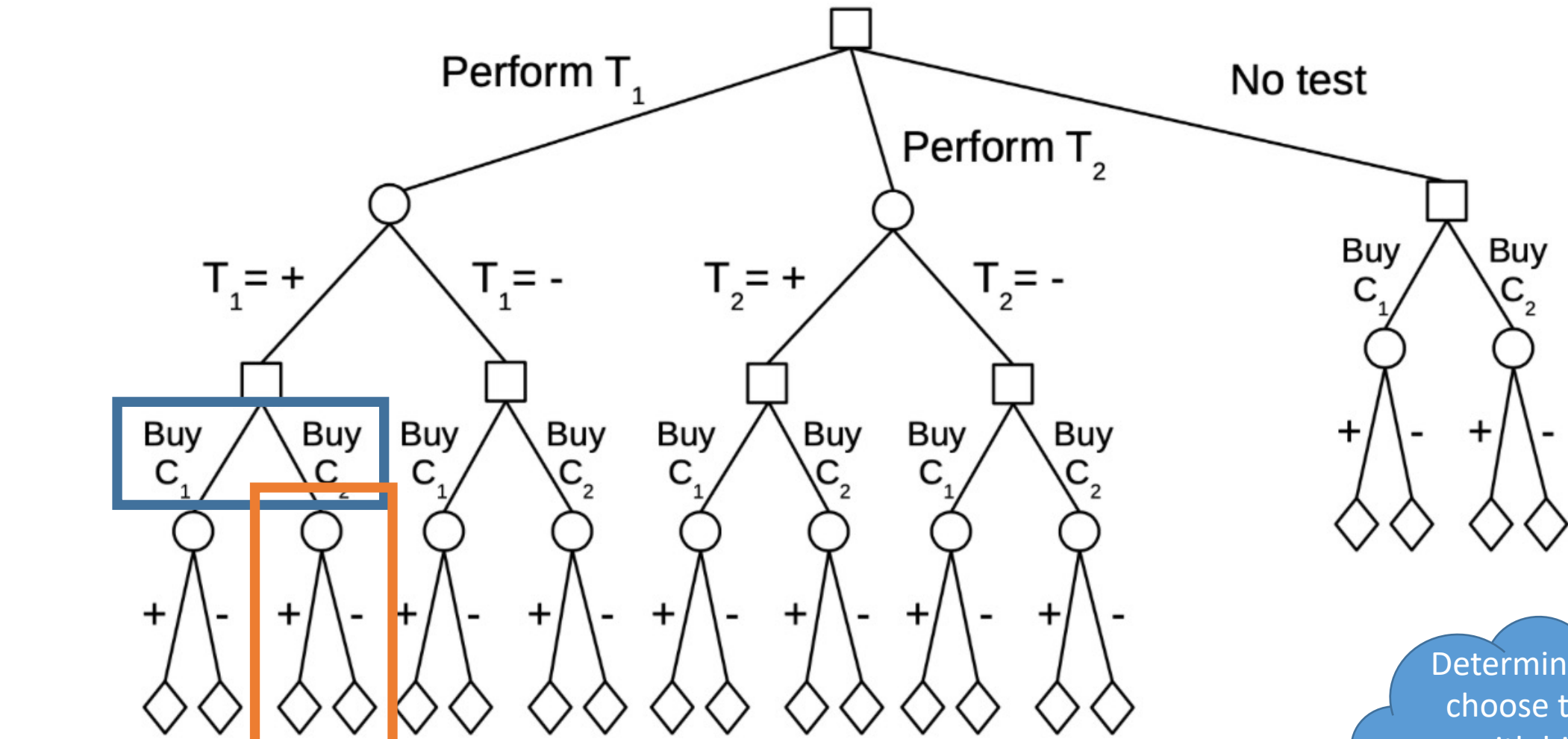
$$EU[C_1 | T_1 = +] = P(C_1 = + | T_1 = +) (cost(C_1) + cost(T_1)) + P(C_1 = - | T_1 = +) (cost(C_1) + repair(C_1) + cost(T_1))$$



$$EU[C_1 | T_1 = +] = P(C_1 = + | T_1 = +)(cost(C_1) + cost(T_1)) \\ + P(C_1 = - | T_1 = +)(cost(C_1) + repair(C_1) + cost(T_1))$$

$$EU[C_2 | T_1 = +]$$

T_1 gives us no
information
about $C_2 \rightarrow$
independent

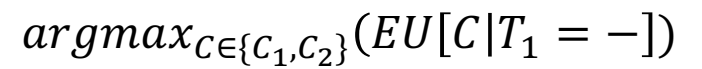


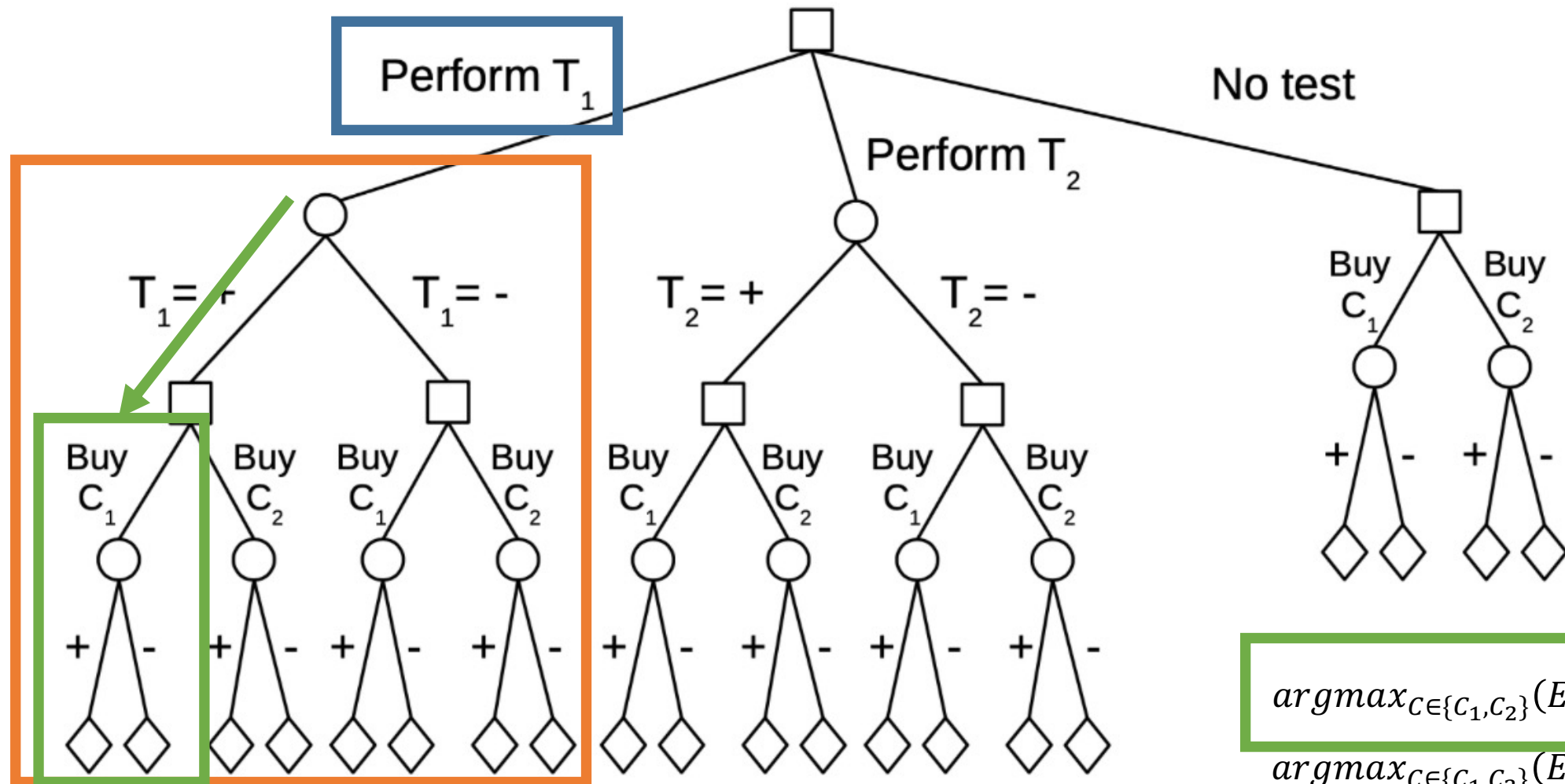
Deterministically
choose the car
with higher
expected utility
given $T_1 = +$

$$EU[C_1 | T_1 = +] = P(C_1 = + | T_1 = +)(cost(C_1) + cost(T_1)) + P(C_1 = - | T_1 = +)(cost(C_1) + repair(C_1) + cost(T_1))$$

$$EU[C_2 | T_1 = +] = P(C_2 = +)(cost(C_2) + cost(T_1)) + P(C_2 = -)(cost(C_2) + repair(C_2) + cost(T_1))$$

$$\operatorname{argmax}_{C \in \{C_1, C_2\}} (EU[C | T_1 = +])$$

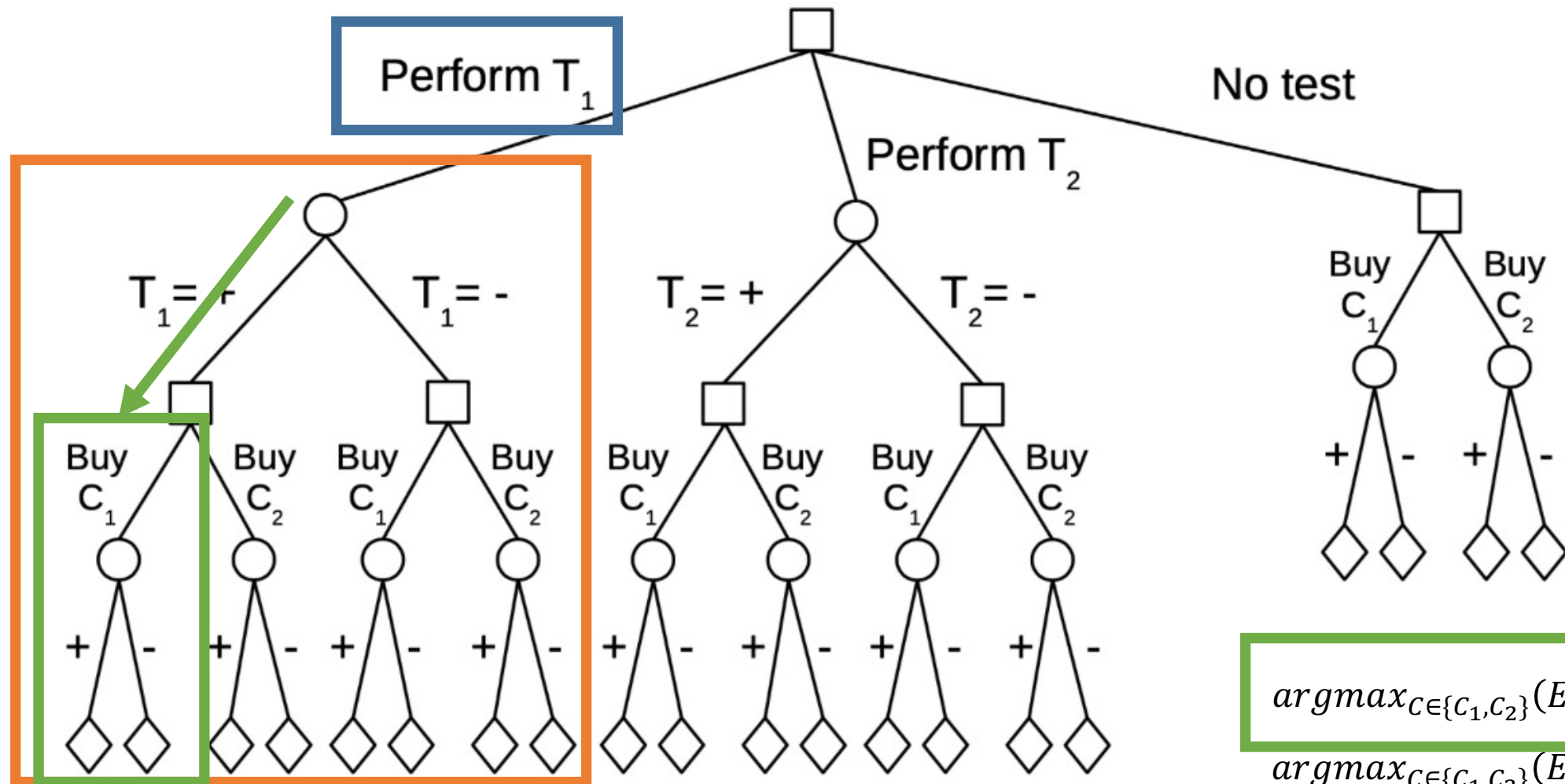




$$EU[T_1] = P(T_1 = +) U(T_1 = +) + P(T_1 = -) U(T_1 = -)$$

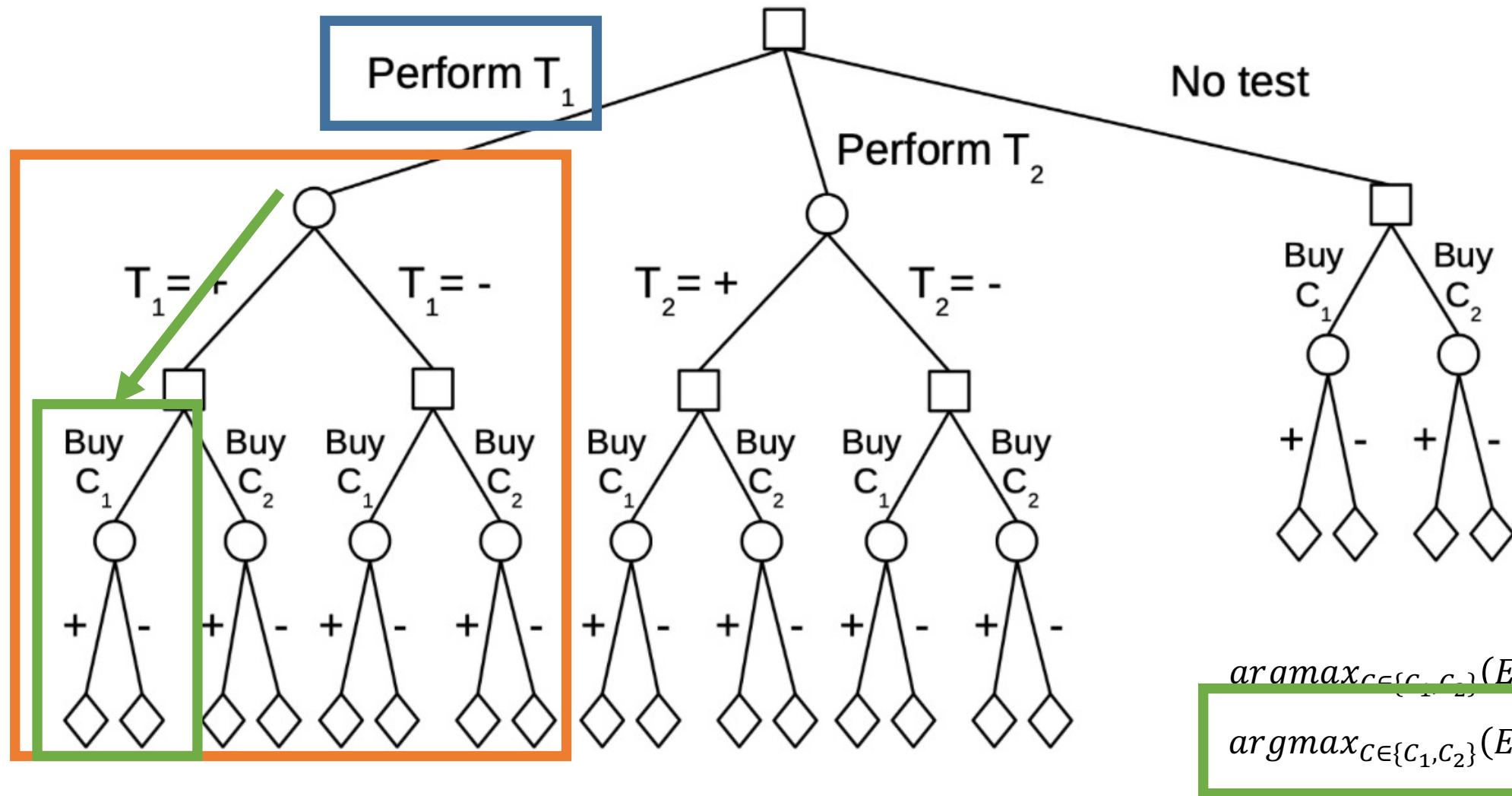
????

If we deterministically choose C_1 when $T_1 = +$, ...



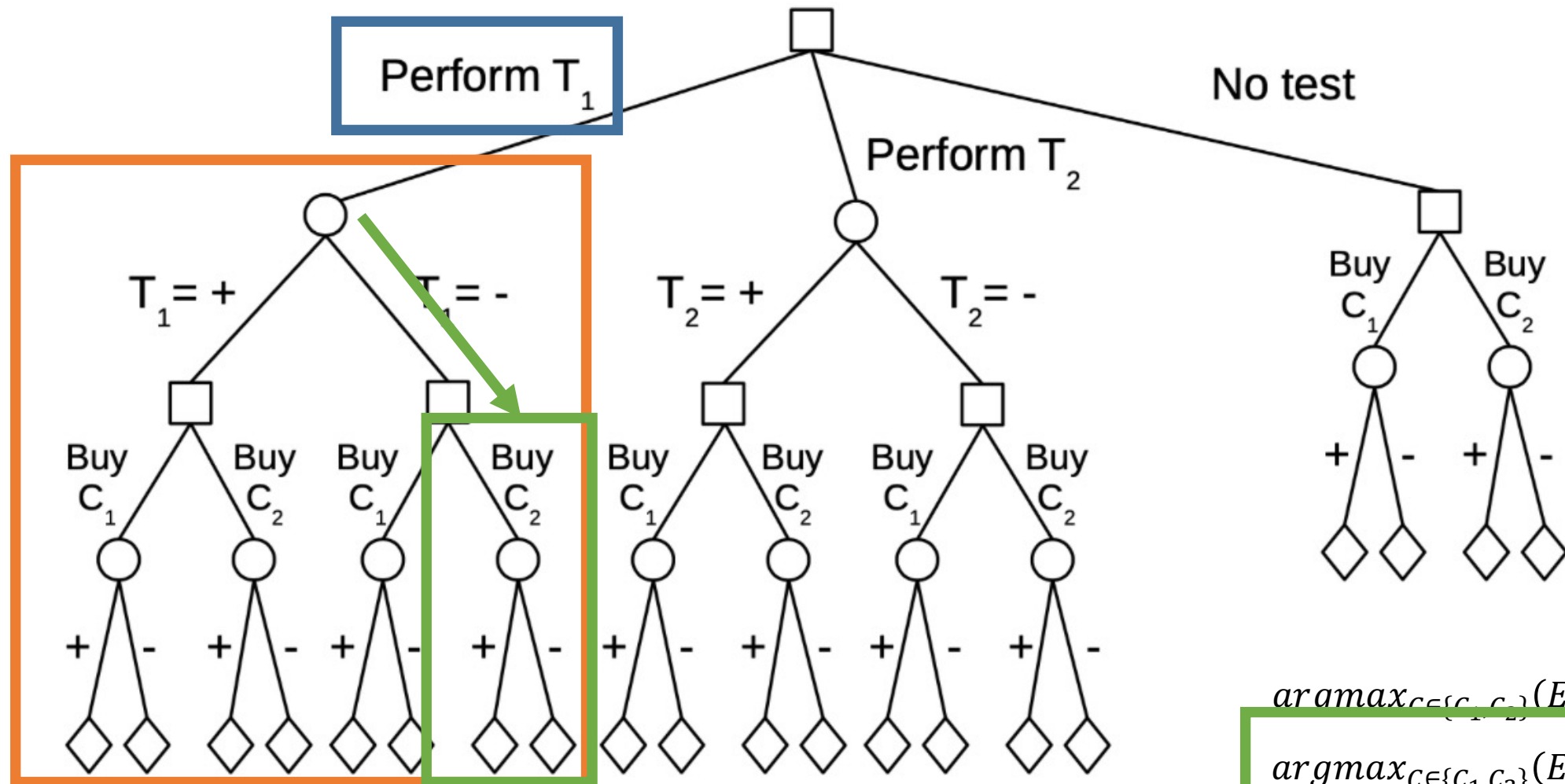
$$EU[T_1] = P(T_1 = +)EU(C_1|T_1 = +) + P(T_1 = -)U(T_1 = -)$$

If we deterministically choose C_1 when $T_1 = +$, ...



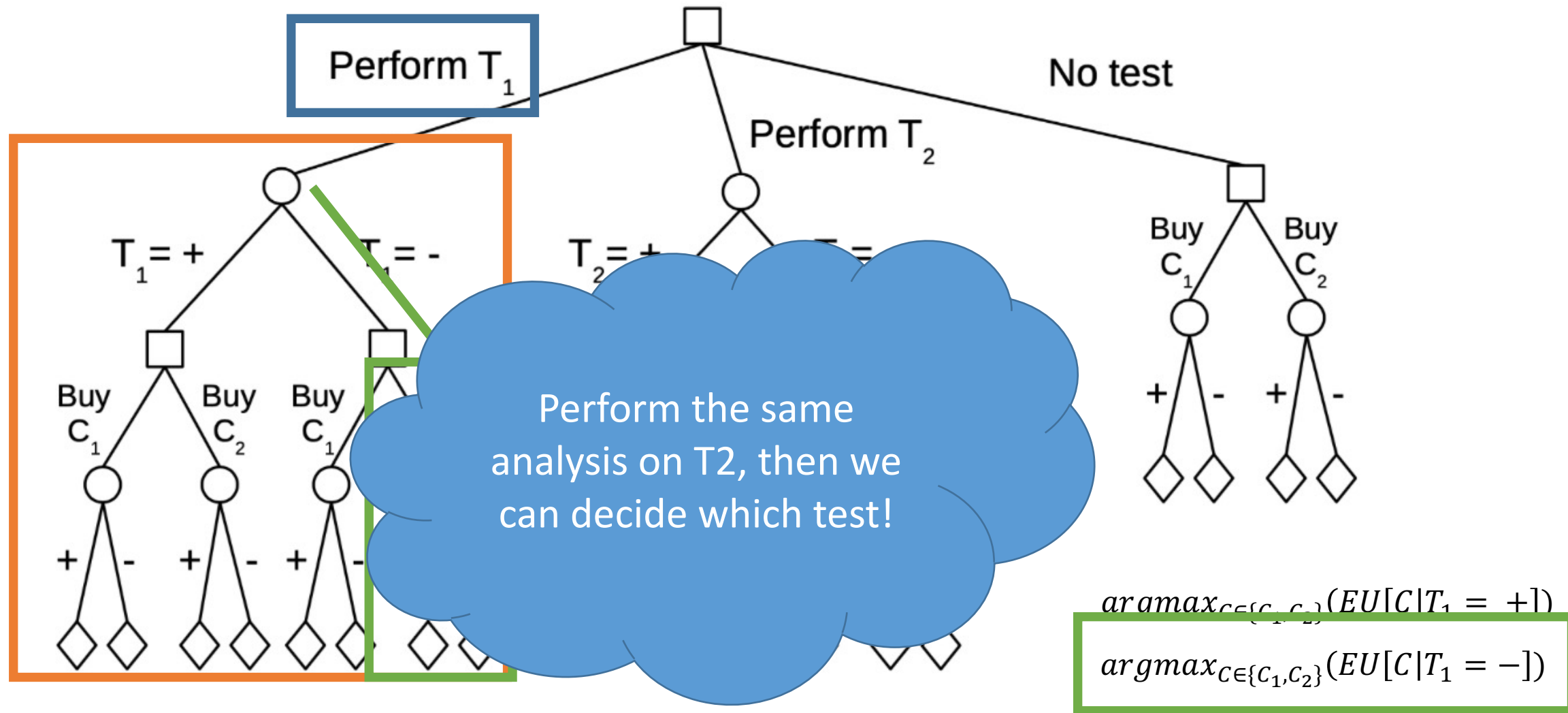
$$EU[T_1] = P(T_1 = +)EU(C_1|T_1 = +) + P(T_1 = -)U(T_1 = -)$$

If we deterministically choose C_2 when $T_1 = -$, ...



$$EU[T_1] = P(T_1 = +)EU(C_1|T_1 = +) + P(T_1 = -)U(T_1 = -)$$

If we deterministically choose C_2 when $T_1 = -$, ...



$$EU[T_1] = P(T_1 = +)EU(C_1|T_1 = +) + P(T_1 = -)EU(C_2|T_1 = -)$$

If we deterministically choose C_2 when $T_1 = -$, ...

When uncertainty comes from another agent's actions

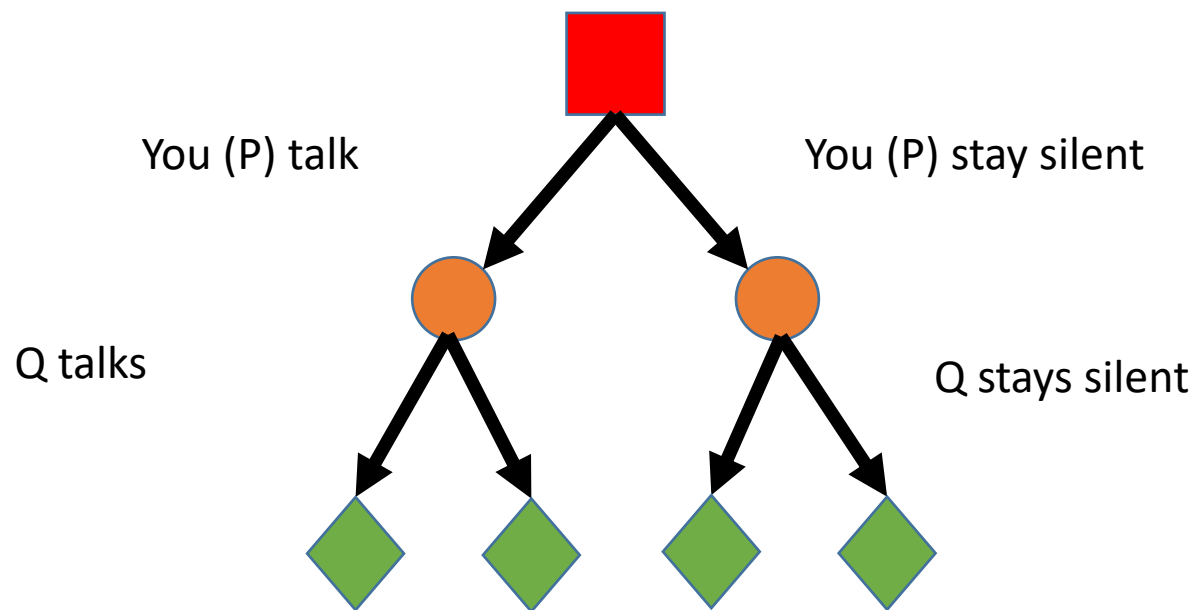
Car example: taking an action in one branch closes off possibilities in another

- Randomness comes from
 - Epistemic uncertainty about effects of past actions (e.g., accuracy of test results)
 - Epistemic uncertainty about future state (e.g., quality of car)

Consider the case when randomness comes from another agent's actions...

Example: Prisoner's Dilemma

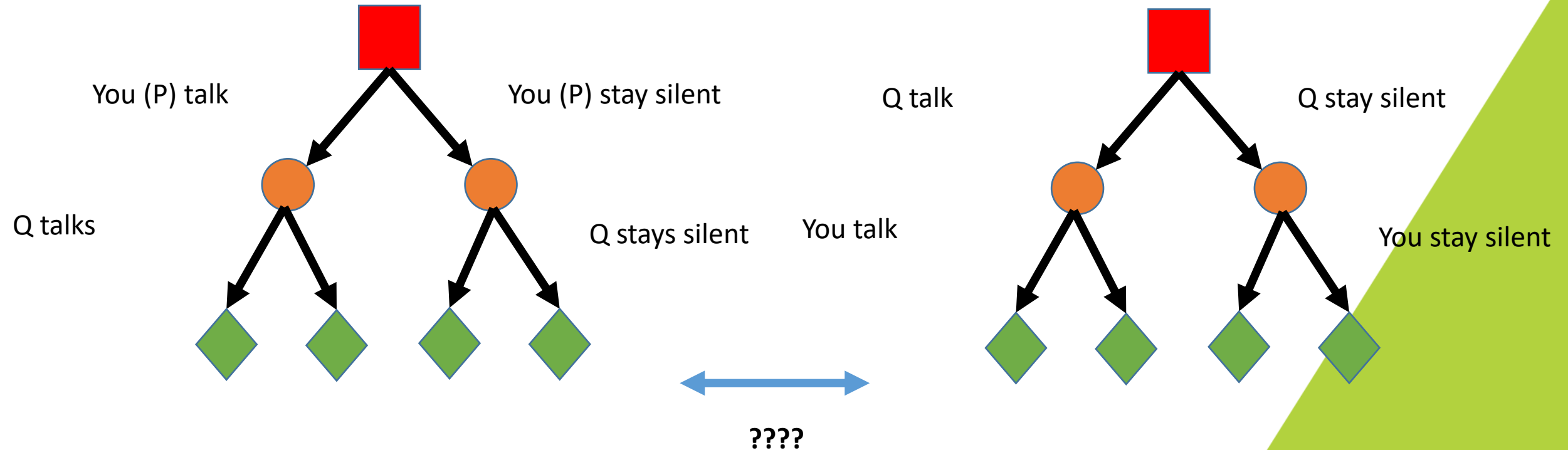
You (agent P) and an accomplice (agent Q) have been arrested for a crime...



But Q also knows all this and must make the same choices...

Example: Prisoner's Dilemma

You (agent P) and an accomplice (agent Q) have been arrested for a crime...



Both parties know this

Example: Prisoner's Dilemma

P and Q have been arrested for a crime and separated for interrogation. Each has the choice of whether or not to confess and each action is assumed to be rational. You don't know how your accomplice will act. What do you do?

Utility function:
Collective cost?

	P silent	P talks
Q silent	(0, 0)	(-2, -5)
Q talks	(-5, -2)	(-10, -10)

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

P and Q have been arrested for a crime and separated for interrogation. Each is given the choice of whether or not to confess and each action is assigned a utility value. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	0	-7
Q talks	-7	-20

Utility function:
Collective cost?

Both choose silent if
both are using the
same utility function

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

P and Q have been arrested for a crime and separated for interrogation. Each is given the choice of whether or not to confess and each action is assigned a utility value. You don't know how your accomplice will act. What do you do?

		P silent	P talks
Q	silent	0	-7
	talks	-7	-20

Utility function:
Collective cost?

What if one uses a
different utility
function?

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are undergoing interrogation. You have the choice of whether or not to confess. Confessing is associated with a cost. You don't know how your accomplice will choose to confess or not. What do you do?

Utility function:
Individual Cost?

	P silent	P talks
Q silent	(0, 0)	(-2, -5)
Q talks	(-5, -2)	(-10, -10)

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess and your accomplice does not, you will receive a 10-year sentence, while your accomplice will receive a 1-year sentence. If you both confess, you will each receive a 5-year sentence. If you both remain silent, you will each receive a 0-year sentence. You don't know how your accomplice will choose to confess or remain silent. What do you do?

Utility function:
Individual cost?

	P silent	P talks
Q silent	0	-2
Q talks	-5	-10




Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess and your accomplice does not, you will receive a 10-year sentence, while your accomplice will receive a 3-year sentence. If you both confess, you will each receive a 5-year sentence. If you both remain silent, you will each receive a 1-year sentence. You don't know how your accomplice will choose to act, and you don't know if your accomplice knows your choice. What should you do?

Utility function:
Individual cost?

		 P silent		P talks	
 Q	silent	<div></div> 0		-5	
	talks	-2		-10	

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess, you will receive a sentence associated with a cost. You don't know how your accomplice will choose to confess or not confess. What should you do?

Utility function:
Individual Cost?

	 P silent	P talks
 Q silent	(0, 0)	(-2, -5)
Q talks	(-5, -2)	(-10, -10)

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess and your accomplice confesses, you both get a 5-year sentence. If you confess and your accomplice stays silent, you get a 2-year sentence and your accomplice gets a 5-year sentence. If you stay silent and your accomplice confesses, you get a 5-year sentence and your accomplice gets a 2-year sentence. If you both stay silent, you both get a 0-year sentence. You don't know how your accomplice will choose, and you must choose your action before your accomplice does?

Local reasoning,
rather than global

Uncertainty

Q silent
Q talks

P silent

(0, 0)

P talks

(-2, -5)

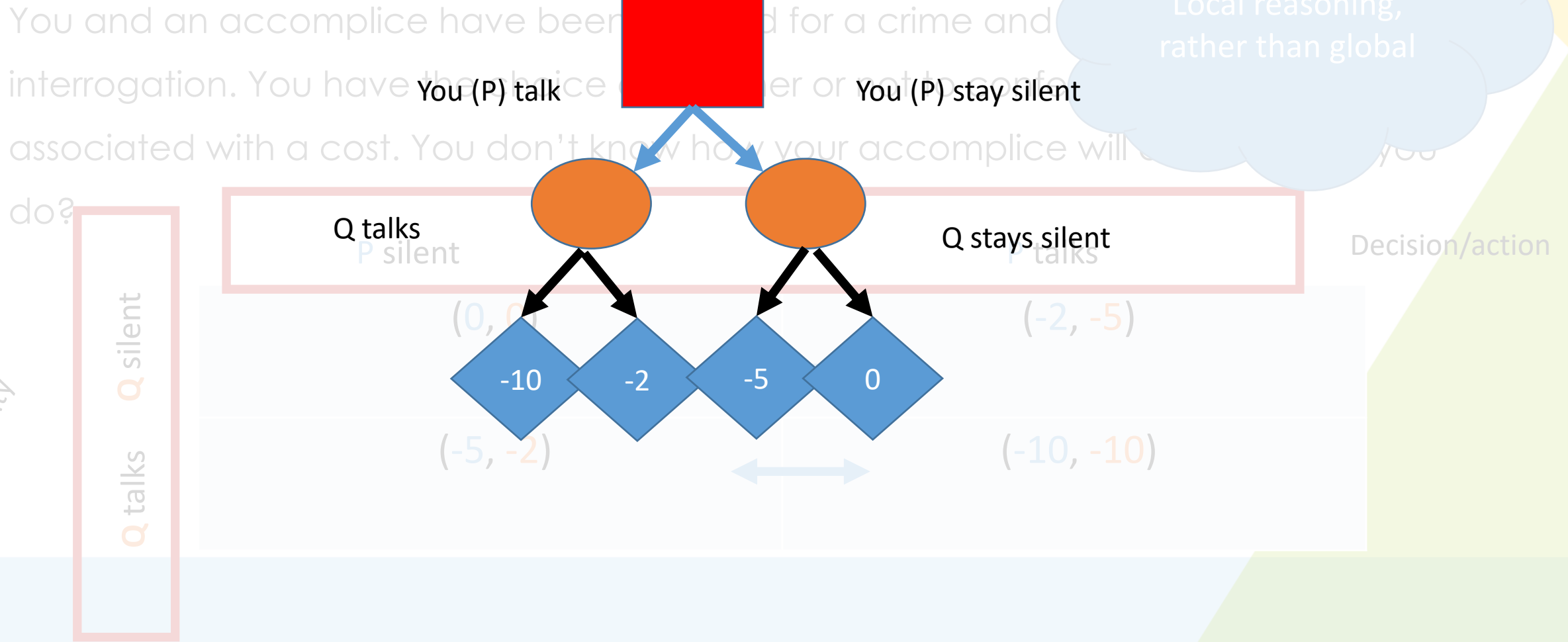
(-5, -2)

(-10, -10)

Decision/action

Weak case

Example: Prisoner's Dilemma



Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess and your accomplice confesses, you both get a 5-year sentence. If you confess and your accomplice stays silent, you get a 1-year sentence and your accomplice gets a 3-year sentence. If you stay silent and your accomplice confesses, you get a 3-year sentence and your accomplice gets a 1-year sentence. If you both stay silent, you both get a 0-year sentence. You don't know how your accomplice will choose, and you must choose your action before your accomplice does?

Local reasoning, rather than global

		Decision/action	
		P silent	P talks
Uncertainty	Q silent	(0, 0)	(-2, -5)
	Q talks	(-5, -2)	(-10, -10)
Weak case			

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

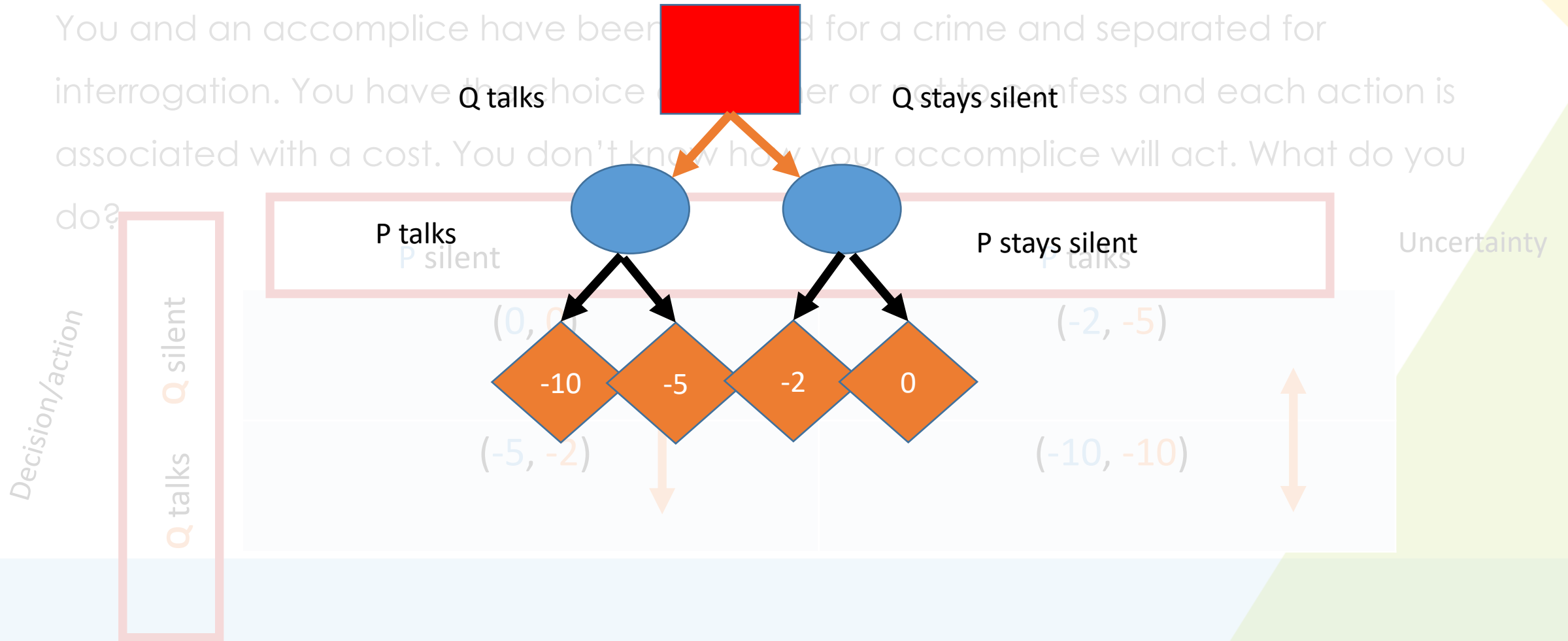
do?

		P silent		P talks		Uncertainty
Decision/action	Q silent	(0, 0)	↕	(-2, -5)	↕	
	Q talks	(-5, -2)		(-10, -10)		

Weak case


Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice to confess or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?




Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

		P silent	P talks
Q	silent	<div><div>(0, 0)</div><div></div></div>	<div><div>(-2, -5)</div></div>
	talks	<div><div>(-5, -2)</div></div>	<div><div>(-10, -10)</div></div>

Example: Prisoner's Dilemma

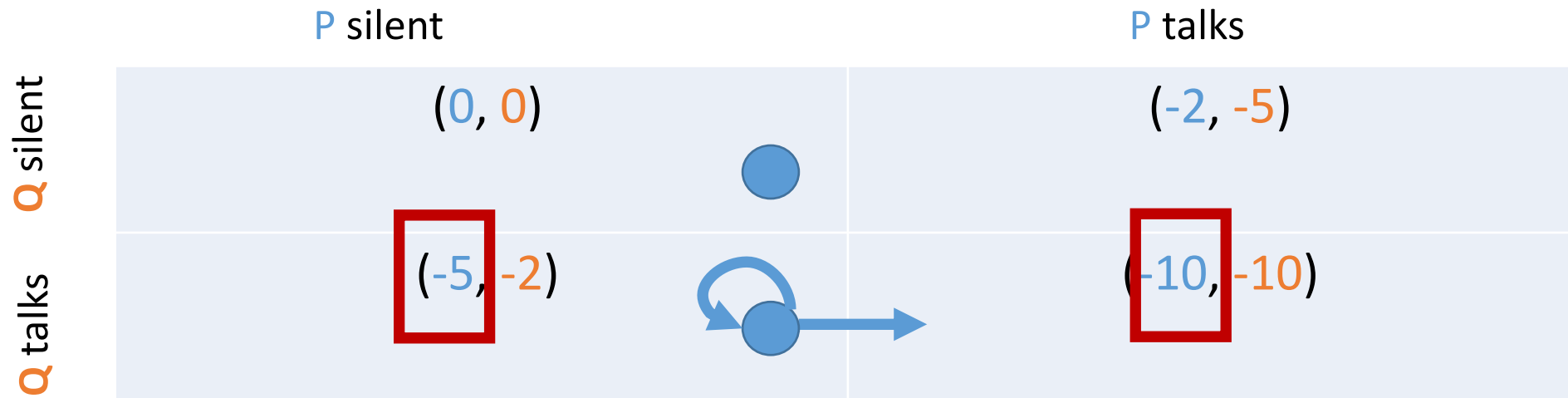
You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

		P silent	P talks
Q	silent	<div><div>(0, 0)</div></div>	<div><div>(-2, -5)</div></div>
	talks	<div><div>(-5, -2)</div></div>	<div><div>(-10, -10)</div></div>

Example: Prisoner's Dilemma



You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	$(0, 0)$	$(-2, -5)$
Q talks	$(-5, -2)$	$(-10, -10)$



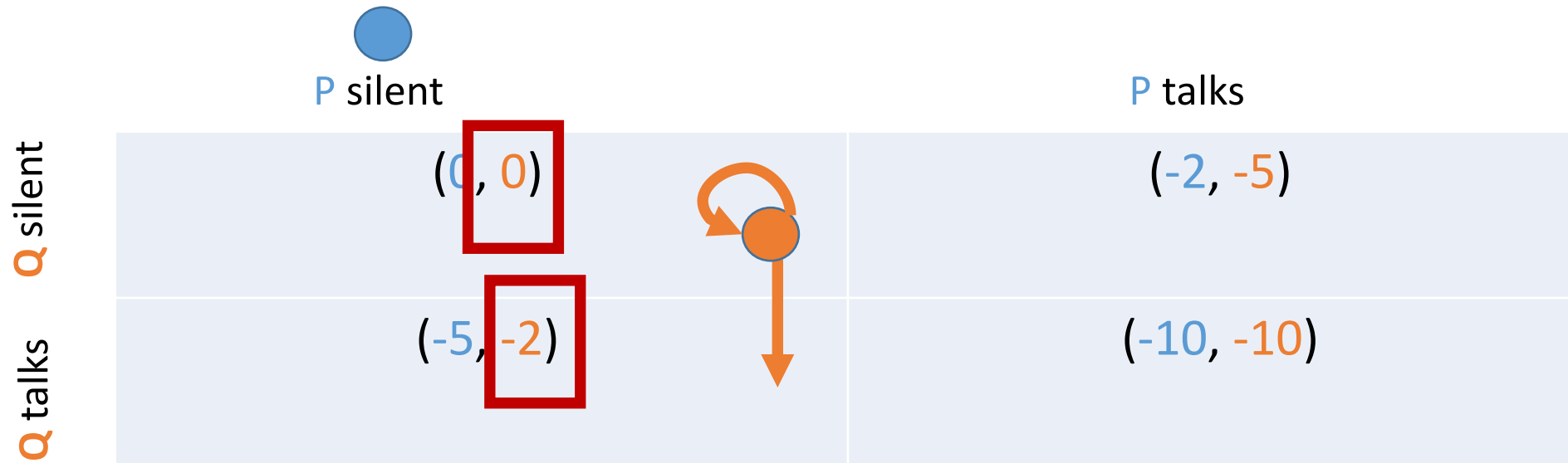
Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

		P silent	P talks
Q	silent	$(0, 0)$ 	$(-2, -5)$
	talks	$(-5, -2)$ 	$(-10, -10)$

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?





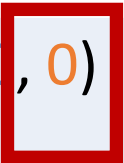

A diagram of a Prisoner's Dilemma payoff matrix. The matrix is a 2x2 grid with light blue cells. Above the grid, a blue circle is positioned above the 'P silent' column header, and an orange circle is positioned above the 'Q silent' row header. An orange arrow starts from the orange circle, loops around, and points down to the 'Q talks' row header. The payoffs are as follows:

	P silent	P talks
Q silent	(0, 0)	(-2, -5)
Q talks	(-5, -2)	(-10, -10)

The payoffs (0, 0) and (-5, -2) are highlighted with red boxes.




Example: Prisoner's Dilemma

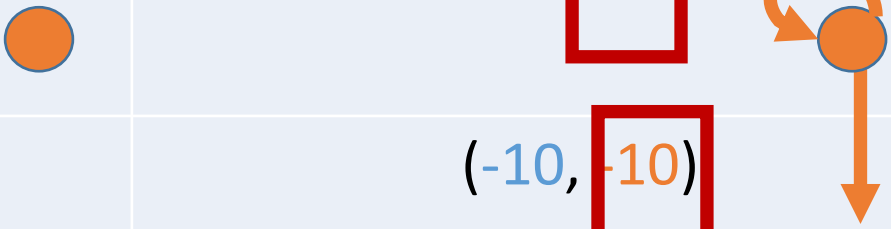
You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

		 P silent		P talks	
 Q	silent	 (0, 0)		(-2, -5)	
	talks	 (-5, -2)		(-10, -10)	

Example: Prisoner's Dilemma



You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	 P silent	P talks
 Q silent	(0, 0)	(-2, -5)
 Q talks	(-5, -2)	(-10, -10)



Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	 P silent	 P talks
 Q silent	(0, 0) 	(-2, -5) 
 Q talks	(-5, -2)	(-10, -10) 

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

		 P	
		silent	talks
 Q	silent	(0, 0)	(-2, -5)
	talks	(-5, -2)	(-10, -10)

Weak case

Same as globally optimal case for both utility functions

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and sent to prison for interrogation. You have the choice of whether or not to confess. Confessing is associated with a cost. You don't know how your accomplice will choose to do you.

Consider a different payoff matrix

	P silent	P talks
Q silent	$(-2, -2)$	$(0, -15)$
Q talks	$(-15, 0)$	$(-10, -10)$

Incentivize talking

Still globally optimal for collective utility?

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?



A 2x2 payoff matrix for a Prisoner's Dilemma. The columns represent Player P's choices: 'P silent' and 'P talks'. The rows represent Player Q's choices: 'Q silent' and 'Q talks'. The payoffs are: (Q silent, P silent) = -4, (Q silent, P talks) = -15, (Q talks, P silent) = -15, and (Q talks, P talks) = -20. The cell for (Q silent, P silent) is highlighted with a red border. Above the columns are blue circles with 'P' and below the rows are orange circles with 'Q'.

	P silent	P talks
Q silent	-4	-15
Q talks	-15	-20

Incentivize talking

Still globally optimal for collective utility?

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and sent to prison for interrogation. You have the choice of whether or not to confess. Confessing is associated with a cost. You don't know how your accomplice will choose to do or not to do. What do you do?

Assume Q uses collective utility, but P uses individual utility...

		P silent	P talks
Q	silent	-2	0
	talks	-15	-10

Incentivize talking

Still globally optimal for individual utility for P?

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and sent to separate interrogation. You have the choice of whether or not to confess to the crime, which is associated with a cost. You don't know how your accomplice will act. What do you do?

What if Q knows P's utility function and decides to mimic?

		P	
		silent	talks
Q	silent	-2	-15
	talks	0	-10

Incentivize talking

Still globally optimal for individual utility?

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and sent to prison for interrogation. You have the choice of whether or not to confess to the crime, which is associated with a cost. You don't know how your accomplice will act. What do you do?

What if Q knows P's utility function and decides to mimic?



		P	
		silent	talks
Q	silent	-2	-15
	talks	0	-10

Incentivize talking

Still globally optimal for individual utility?

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

			
		P silent	P talks
	Q silent	$(-2, -2)$	$(0, -15)$
	Q talks	$(-15, 0)$	$(-10, -10)$

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

			
		P silent	P talks
	Q silent	-4	-15
	Q talks	-15	-20

Worst possible global outcome!

Incentivize talking

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you both confess, you will both receive a 15-year sentence, which is associated with a cost. You don't know how your accomplice will choose to act. What should you do?

Local reasoning,
rather than global

	P silent	P talks
Q silent	<div>$(-2, -2)$</div>	<div>$(0, -15)$</div>
Q talks	$(-15, 0)$	$(-10, -10)$

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	<div><div></div><div>$(-2, -2)$</div></div>	<div><div></div><div>$(0, -15)$</div></div>
Q talks	<div><div></div><div>$(-15, 0)$</div></div>	<div><div></div><div>$(-10, -10)$</div></div>

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
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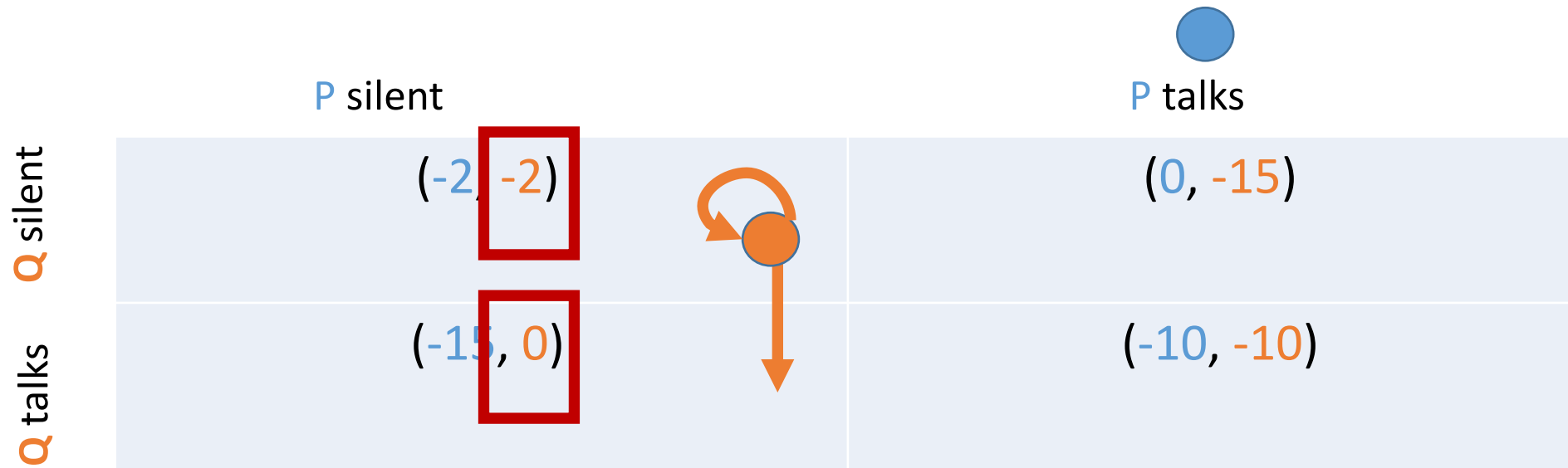
Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

		P silent	P talks
Q	silent	$(-2, -2)$	$(0, -15)$
	talks	$(-15, 0)$	$(-10, -10)$

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?




A diagram of a Prisoner's Dilemma payoff matrix. The matrix is a 2x2 grid with a light blue background. The columns are labeled 'P silent' and 'P talks' at the top. The rows are labeled 'Q silent' and 'Q talks' on the left. The payoffs are shown as (P's payoff, Q's payoff) in each cell. The top-left cell (P silent, Q silent) contains (-2, -2) and is highlighted with a red square. The bottom-left cell (P silent, Q talks) contains (-15, 0) and is also highlighted with a red square. The top-right cell (P talks, Q silent) contains (0, -15). The bottom-right cell (P talks, Q talks) contains (-10, -10). An orange dot is located in the top-left cell, with an orange arrow pointing down to the bottom-left cell. Another orange dot is located in the top-right cell, with an orange arrow pointing down to the bottom-right cell. A blue dot is located above the 'P talks' column header.

	P silent	P talks
Q silent	$(-2, -2)$	$(0, -15)$
Q talks	$(-15, 0)$	$(-10, -10)$

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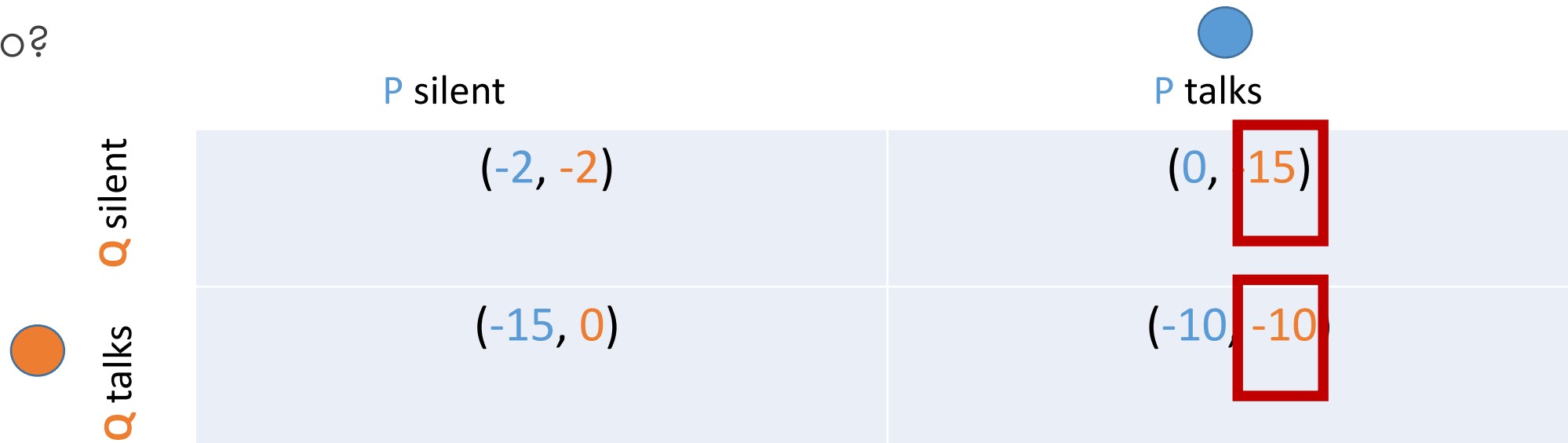
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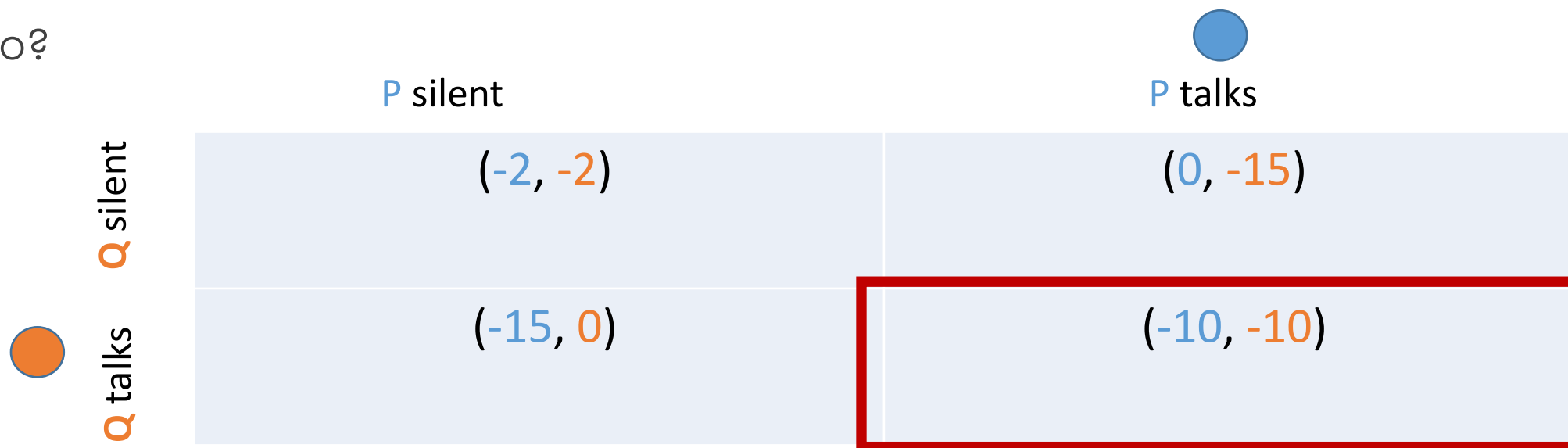


A 2x2 payoff matrix for a Prisoner's Dilemma. The rows represent Player Q's choices (silent or talks) and the columns represent Player P's choices (silent or talks). Payoffs are shown as (P, Q). The cell for (P talks, Q talks) contains the values (-10, -10), with the -10 for Q highlighted by a red box. The cell for (P talks, Q silent) contains the values (0, 15), with the 15 for Q highlighted by a red box. A blue circle is positioned above the 'P talks' column header, and an orange circle is positioned to the left of the 'Q talks' row header.

	P silent	P talks
Q silent	$(-2, -2)$	$(0, 15)$
Q talks	$(-15, 0)$	$(-10, -10)$

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?



A 2x2 payoff matrix for a Prisoner's Dilemma. The columns represent Player P's choices: 'P silent' and 'P talks'. The rows represent Player Q's choices: 'Q silent' and 'Q talks'. Each cell contains a pair of payoffs (Q's payoff, P's payoff). The cell for (Q talks, P talks) with payoffs (-10, -10) is highlighted with a red border. Above the 'P talks' column is a blue circle, and to the left of the 'Q talks' row is an orange circle.

	P silent	P talks
Q silent	$(-2, -2)$	$(0, -15)$
Q talks	$(-15, 0)$	$(-10, -10)$

Incentivize talking

Same result!

Example: 2-finger Morra

Choose between 1 and 2 fingers. P wins if sum is even. Q wins if sum is odd. Loser pays the winner.

		P plays 1	P plays 2
Q plays 1	Q plays 1	$(+2, -2)$	$(-3, +3)$
	Q plays 2	$(-3, +3)$	$(+4, -4)$

Example: 2-finger Morra

Choose between 1 and 2 fingers. P wins if sum is even. Q wins if sum is odd. Loser pays the winner.

		P plays 1	P plays 2
Q plays	1	0	0
	2	0	0

Example: 2-finger Morra

Choose between 1 and 2 fingers. P wins if sum is even. Q wins if sum is odd. Loser pays the winner.

	P plays 1	P plays 2
Q plays 1	+2	-3
Q plays 2	-3	+4

Example: 2-finger Morra

Choose between 1 and 2 fingers. P wins if sum is even. Q wins if sum is odd. Loser pays the winner.





		P plays 1	P plays 2
Q plays	1	-2	+3
	2	+3	-4

Zero-sum

Example: 2-finger Morra

Choose between 1 and 2 fingers. P wins if sum is even. Q wins if sum is odd. P pays the winner.

Choose
probabilistically

		P plays 1	P plays 2
Q plays 1	Q plays 1	$(+2, -2)$ 	$(-3, +3)$ 
	Q plays 2	$(-3, +3)$ 	$(+4, -4)$ 

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	$(-4, -4)$	$(0, -6)$
Q talks	$(-5, -2)$	$(-10, -10)$

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	-8	-6
Q talks	-7	-20


Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
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Example: Prisoner's Dilemma


You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?



	P silent	P talks
Q silent	-4	0
Q talks	-5	-10

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

			
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Example: Prisoner's Dilemma

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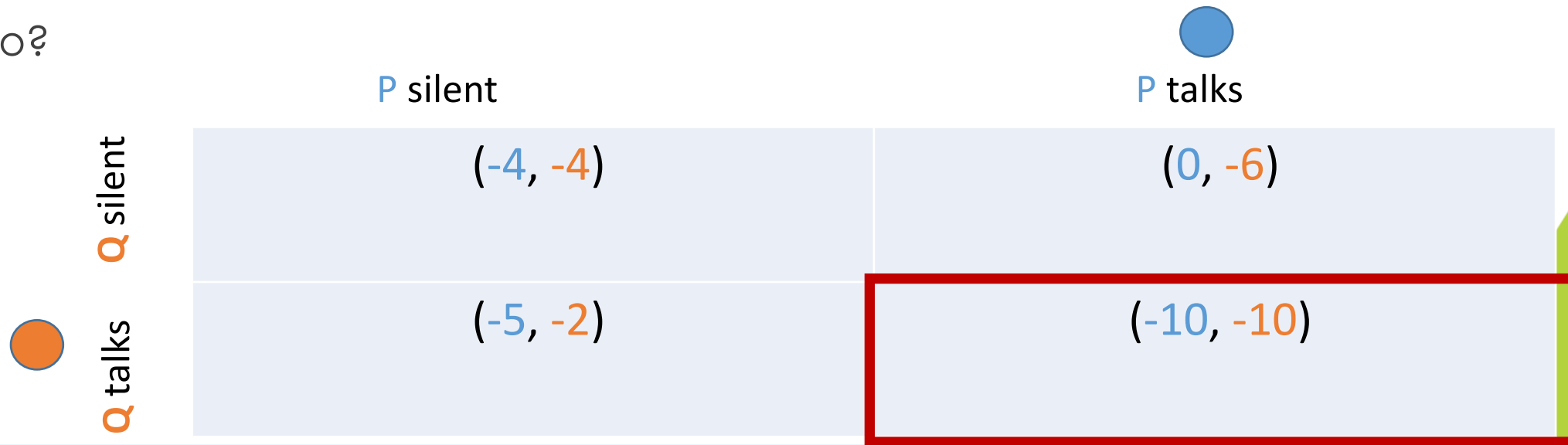


A 2x2 payoff matrix for the Prisoner's Dilemma. The columns represent the accomplice's (P) choices: 'P silent' and 'P talks'. The rows represent the player's (Q) choices: 'Q silent' and 'Q talks'. The payoffs are shown in orange numbers within light blue cells. The cell for (Q talks, P silent) with a payoff of -2 is highlighted with a red border. A blue circle is positioned above the 'P talks' column header, and an orange circle is positioned to the left of the 'Q talks' row header.

	P silent	P talks
Q silent	-4	-6
Q talks	-2	-10

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?




A 2x2 payoff matrix for a Prisoner's Dilemma game. The rows represent Player Q's choices (silent or talks) and the columns represent Player P's choices (silent or talks). Each cell contains a pair of payoffs (Q, P). The cell for (Q talks, P talks) with payoffs (-10, -10) is highlighted with a red border. Above the matrix, a blue circle is positioned above the 'P talks' column header, and below the matrix, an orange circle is positioned to the left of the 'Q talks' row header.

	P silent	P talks
Q silent	$(-4, -4)$	$(0, -6)$
Q talks	$(-5, -2)$	$(-10, -10)$

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

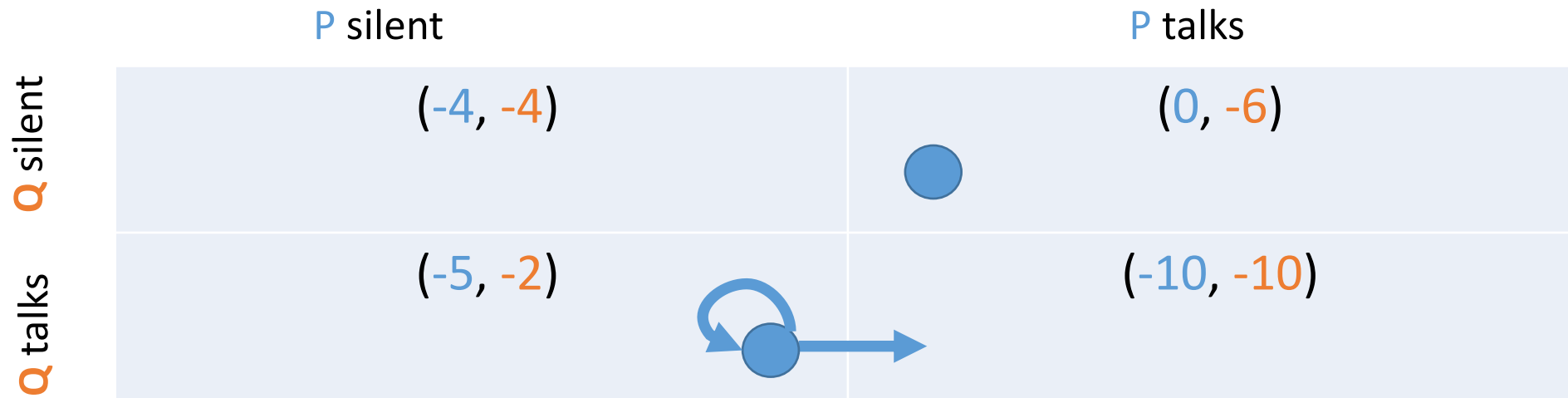
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Example: Prisoner's Dilemma





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Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	$(-4, -4)$ 	$(0, -6)$ 
Q talks	$(-5, -2)$ 	$(-10, -10)$ 

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and sent to prison for interrogation. You have the choice of whether or not to confess. If you confess and your accomplice does not, you will be sentenced to 10 years in prison, while your accomplice will be released. If you both confess, you will both be sentenced to 5 years in prison. If neither of you confesses, you will both be sentenced to 4 years in prison. You don't know how your accomplice will choose to confess or not to confess. What do you do?

Suppose we model Q's choice probabilistically...

	P silent	P talks
Q silent	$(-4, -4)$	$(0, -6)$
Q talks	$(-5, -2)$	$(-10, -10)$

Vocabulary & Concepts

- Always assume local decision making (all players maximizing individual utility)
- **Zero sum** – every entry in global collective payoff is 0
- **Pure strategy** – always pick the same action no matter what
- **Mixed strategy** – pick an action probabilistically
- **Dominant strategy** – one action is strictly better no matter what the other plays does