Section 4.5 Odds and Expectation

Definition of Odds

In a uniform sample space, suppose E is an event containing m outcomes, and that n outcomes are not in E. Then we say that the ***odds in favor of*** E are $m : n$. The ***odds against*** E are $n : m$.

E.g., throw a single fair die, and let E be the event that a number greater than 4 turns up. There are two outcomes in E and four not in E, so the odds in favor of E are two to four (written 2:4).

If we have an expression for odds, we can multiply both numbers in it by the same nonzero number, so 2:4 is the same as 1:2 or even 37:74. Normally we express odds in lowest possible terms.

E.g., two fair dice are thrown. What are the odds against getting doubles?

Converting Between Odds and Probability

If you know $p(E)$, the probability of the event E, then the odds in favor of E are $p(E) : 1 - p(E)$.

If the odds in favor of E are $m : n$, then the probability of E is $p(E) = \frac{m}{m+n}$.

E.g., a horse is given a probability of $\frac{5}{8}$ of winning her race. What are the odds against her winning it?

E.g., a five-card hand is dealt from a standard deck. What are the odds that it contains exactly one ace?

Mathematical Expectation

Mathematical expectation can be computed whenever events have been given values (or, as they're sometimes called, payoffs). To calculate the mathematical expectation for an event, multiply the probability of the event by the payoff for that event. To calculate the mathematical expectation for several events, just multiply the probability of each event by the payoff for that event, and then add up the results.

The mathematical expectation for a game is the expected winnings.

E.g.: Suppose you meet an idiot who offers to play the following game with you. You will flip a fair coin; if you get heads, she'll pay you \$2. (If you get tails, nothing happens.) How much would you expect to win per game, on average, over a long series of games? Here there are two possible events, heads (H) and tails (T). The mathematical expectation for this game is $p(H)\cdot$(payoff for $H) + p(T)\cdot$(payoff for $T) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot (0) = 1$. That is, you expect to win \$1 per game, on average, over a long series of games.

E.g., a roulette wheel has 38 slots labeled 00, 0, 1, ..., 36. Slots 2, 4, ..., 36 are black, slots 1, 3, ..., 35 are red, and slots 00 and 0 are green. Red and Black pay one for one—i.e., if you bet \$1 and win, you receive your \$1 back and an extra \$1 (so you win \$2). Individual numbers pay 36 to 1. What are your expected winnings if you bet \$10 (a) on Red

---

(b) on the number 7

E.g., a game consists of throwing one fair die once. You receive twice as many dollars as the number that turns up. How much would you be willing to pay to play each game?