ML for Finance - HW1

Amber Walker and Clarice Mottet

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1 EWMA Based Variance

To evaluate if the equation covered in class is equivalent to the recursive formula of estimated weighted moving average-based variance, we used the closing price information for tickers *MXX*, and *VIX* from the dataset *World-Markets9920.RDS* and calculated the EWMA-based variance using both methods and compared results. To start we first calculate the log daily return, Equation 1 based on close price for these tickers.

$$r_t = ln(P_t) - ln(P_{t-1}) \tag{1}$$

Next we created a function in python to calculate the EWMA-based variance using Equation 2 covered in lecture 2 in class. We apply this method to our calculated log daily returns by market with the m value 20 for a 20-day window and 100 for a 100-day window.

$$\sigma^2_{\text{ewma}}(t) = (1 - \lambda) \sum_{k=1}^{m} \lambda^{k-1} r^2_{t-k} \tag{2}$$

We then created another function in python to calculate the EWMA-based variance using the recursive formulation in Equation 3.

$$\sigma^2_{\text{ewma}}(t) = \lambda \sigma^2_{\text{ewma}}(t-1) + (1 - \lambda) r^2_{t-1} \tag{3}$$

Lastly we calculate historical volatility by cumulative sum of squares as in Equation 4 as a benchmark to compare to the above two methods with a 50-day window.

$$\text{Historical Volatility} = \sqrt{\frac{\sum_{i=1}^{t} r_i^2}{t}} \tag{4}$$

As a visual comparison, we include a time series plot of the log daily returns for each ticker and then a plot of the volatility calculations for each ticker (with taking the square root of the EWMA-based variances to give an accurate comparison to the historical volatility) as displayed in Figure 1.
We see in Figure 1 that historical volatility is in red, EWMA equation method is in orange and EWMA recursion method is in green. Using the tickers *MXX*, and *VIX* we see a similar method relationship between the three methods with the historical volatility (with a window of 50 days) not giving a good indication of volatility at points in time, the EWMA recursive method and the EWMA window methods are similar/the same and give us the most granular volatility behavior.

When we define a window size (here two examples, one of 20 days and another of 100 days) and use the EWMA equation covered in class, we see that by prioritizing only the most recent returns in the window, we're better capturing the volatility happening in real time as shown by comparing spikes to the time series plot for the funds. When we compare these calculations to the EWMA recursive formula we get almost the exact same results. The difference for the results most likely being that we initialized our EWMA recursion calculation with the overall sample variance of the full population. If we had used the sample variance of the first maybe 100 days instead, we'd likely see the same results.

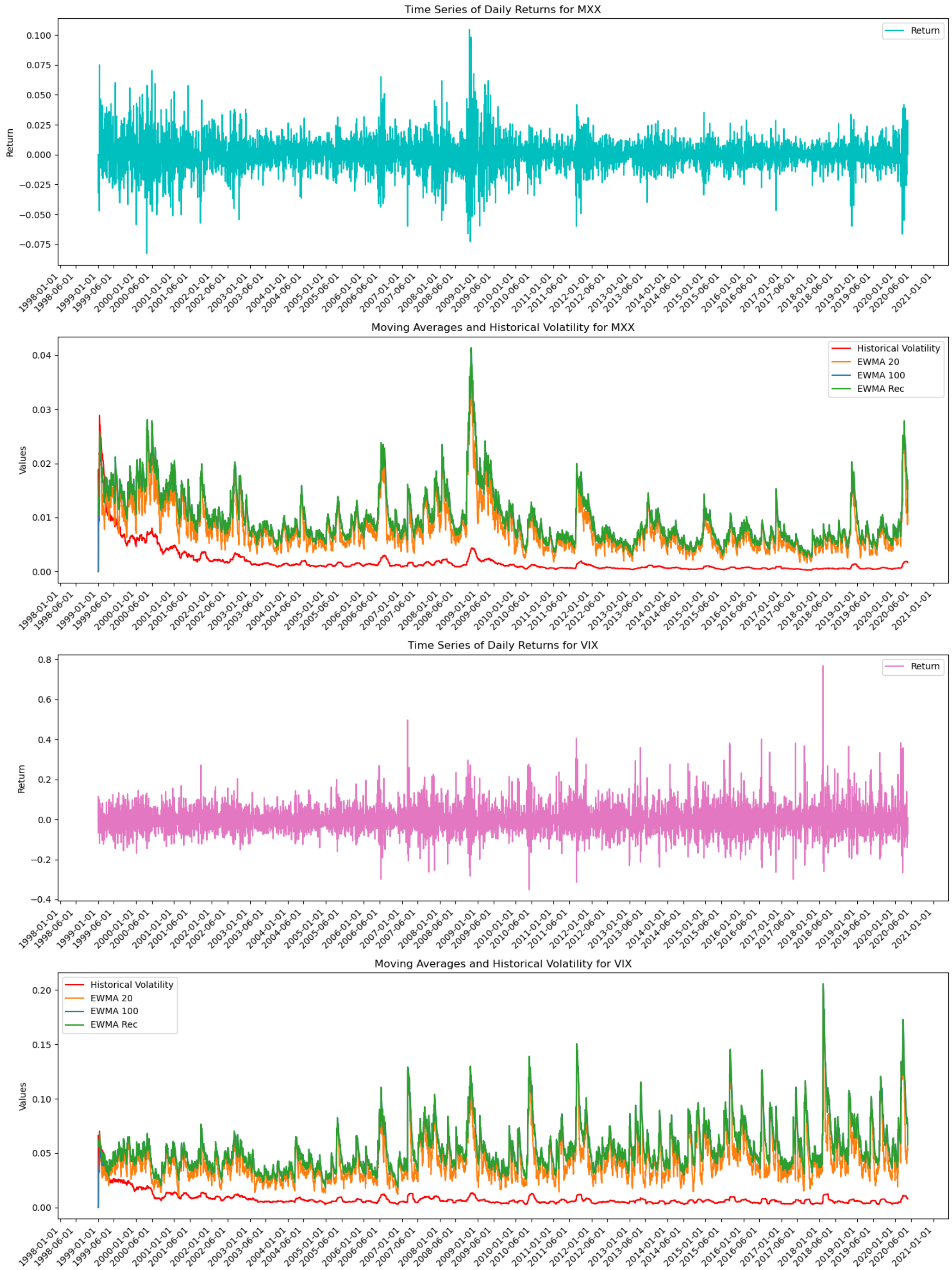


Figure 1: EWMA Method Comparison

Comparing these two methods to a regular historical volatility calculation we see that the historical volatility calculation is not well adapted to the behavior we have in our time series. We expected to see this happen as our regular calculation gives equal weight to all previous log returns in our window of 50 days where as we prioritize returns closer to the time we're estimating in the other two methods.

2 Causality Analysis

Using the dataset *WorldMarkets9920.RDS*, we calculate the log daily return, Equation 1, based on close price for all tickers. We then subset our data to our relevant timeframe 2018-01-01 to 2021-12-31. Before performing a full causality analysis, we check to see if our log daily returns are stationary. Using the augmented Dickey-Fuller test, we have our null hypothesis that the log daily returns are non-stationary and test this. We find that for each individual ticker in our time period, the test statistic is less than all the critical values and the p-value is less than .05. This leads us to reject our null hypothesis (that the time series is non-stationary) and that our log daily returns for every ticker is stationary. We can safely perform a causality analysis now that we know our causality analysis will be dependent upon lag information and not any trend or seasonality behavior. Next we calculate volatility as we did in section 1 using the recursion formulation in Equation 3.

We resample the time series data to the weekly and monthly level by taking the trailing week/trailing month's average of log daily return and EWMA-based variance by ticker. Next we create a lag-1, lag-2, lag-3, lag4 of each of these fields on the week/month level. To perform a full causality analysis, we use the Granger Causality Test on a weekly and monthly level to determine the efficacy of one of these lag fields in predicting their current counterpart and other funds. If the p-value for all four tests; SSR based F-test, SSR based chi-squared test, likelihood ratio test, and parameter F test; were less than .05 then we designate the feature column with a 1 for statistically significance and 0 otherwise in the following tables.

In Figure 2 and Figure 3 we have the results for the log daily returns on the weekly and monthly level respectively. We see that some lag return behavior is indicative of return behavior in other funds on a weekly level but we see very little of this behavior on the monthly level.

In Figure 4 and Figure 5 we have the results for the volatility on the weekly and monthly level respectively. Similarly to the log daily return analysis, we see that volatility in some funds is indicative of volatility behavior in other funds on a weekly level but we see very little of this behavior on the monthly level.

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(1, 0, 1, 0)	(1, 0, 0, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 1, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 1, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 1, 0)
BVSP	(0, 1, 1, 0)	(0, 0, 1, 0)	(0, 1, 1, 0)	(0, 1, 1, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(0, 1, 1, 0)	(0, 0, 0, 1)	(0, 1, 1, 0)	(0, 1, 1, 1)	(0, 0, 1, 0)	(0, 1, 0, 0)	(0, 1, 1, 0)
FTSE	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(1, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 1, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 1, 0)
GDAXI	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(1, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(1, 0, 0, 0)	(0, 0, 1, 0)
GSPC	(0, 0, 1, 0)	(0, 0, 1, 0)	(0, 1, 1, 0)	(0, 1, 1, 0)	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 1, 1, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(0, 1, 0, 0)	(0, 0, 1, 0)
HSCE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 1)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 1)	(0, 0, 0, 0)
IBEX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(1, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)
JKSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
MXX	(0, 1, 1, 0)	(0, 0, 0, 0)	(0, 1, 0, 0)	(0, 1, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 1, 0, 0)	(0, 0, 0, 0)	(0, 1, 0, 0)	(0, 0, 0, 1)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 1, 1, 0)
N225	(0, 0, 1, 0)	(0, 0, 1, 0)	(0, 1, 1, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)
TWII	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 1, 1)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 1, 0)	(0, 0, 0, 0)	(1, 0, 0, 1)	(0, 0, 1, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 1)	(0, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 1)	(0, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 1)	(0, 0, 0, 0)
VLIC	(0, 0, 1, 0)	(1, 0, 1, 0)	(0, 1, 1, 0)	(0, 1, 1, 0)	(1, 0, 1, 0)	(0, 0, 0, 0)	(0, 1, 1, 0)	(1, 0, 0, 0)	(0, 0, 1, 0)	(1, 0, 1, 0)	(0, 0, 1, 0)	(0, 1, 0, 0)	(1, 0, 1, 0)

Figure 2: Causality Analysis: Weekly - Log Daily Returns

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 1)	(0, 0, 0, 1)	(0, 0, 0, 1)	(0, 0, 0, 0)
BVSP	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
FTSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GDAXI	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GSPC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 1, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
HSCE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
IBEX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
JKSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 1, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
MXX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 1, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
N225	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
TWII	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 1, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)

Figure 3: Causality Analysis: Monthly - Log Daily Returns

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(0, 1, 1, 1)	(0, 1, 1, 0)	(1, 1, 1, 1)	(1, 1, 1, 1)	(0, 1, 1, 1)	(0, 1, 1, 1)	(1, 1, 1, 0)	(1, 0, 0, 1)	(1, 1, 1, 1)
BVSP	(1, 1, 0, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(0, 0, 1, 0)	(1, 1, 1, 0)
FTSE	(1, 1, 1, 0)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 0, 0)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 0)	(1, 1, 1, 1)	(1, 1, 0, 0)	(0, 0, 0, 0)	(1, 1, 1, 1)
GDAXI	(1, 1, 1, 0)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 0, 0)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(0, 0, 0, 0)	(1, 1, 1, 1)
GSPC	(1, 1, 1, 0)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 0)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 0, 0)	(0, 0, 1, 0)	(1, 1, 1, 1)
HSCE	(1, 1, 1, 0)	(1, 1, 1, 1)	(0, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 0)	(0, 1, 1, 1)	(1, 1, 1, 0)	(0, 1, 1, 0)	(1, 1, 1, 1)	(1, 1, 0, 0)	(0, 0, 1, 1)	(1, 1, 1, 1)
IBEX	(1, 1, 0, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(0, 0, 1, 0)	(1, 1, 1, 0)
JKSE	(1, 1, 1, 1)	(0, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(0, 1, 1, 1)	(1, 1, 1, 1)	(0, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 0)	(1, 0, 0, 1)	(1, 1, 1, 1)
MXX	(1, 1, 1, 0)	(0, 1, 1, 1)	(0, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 0, 0)	(0, 1, 1, 1)	(1, 1, 1, 0)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 0, 0, 0)	(0, 0, 0, 1)	(1, 1, 1, 1)
N225	(1, 1, 1, 1)	(1, 1, 1, 1)	(0, 1, 1, 1)	(0, 1, 1, 1)	(0, 1, 1, 1)	(0, 1, 1, 0)	(1, 1, 1, 1)	(1, 1, 1, 1)	(0, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 0)	(1, 0, 0, 1)	(0, 1, 1, 1)
TWII	(1, 1, 0, 0)	(1, 1, 1, 0)	(0, 1, 1, 0)	(0, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(0, 1, 1, 0)	(0, 1, 0, 0)	(0, 1, 0, 0)	(0, 1, 0, 0)	(1, 1, 0, 0)	(0, 0, 0, 1)	(1, 1, 1, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(1, 1, 1, 0)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1)	(1, 1, 0, 0)	(1, 1, 1, 1)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(0, 0, 1, 0)	(1, 1, 1, 1)

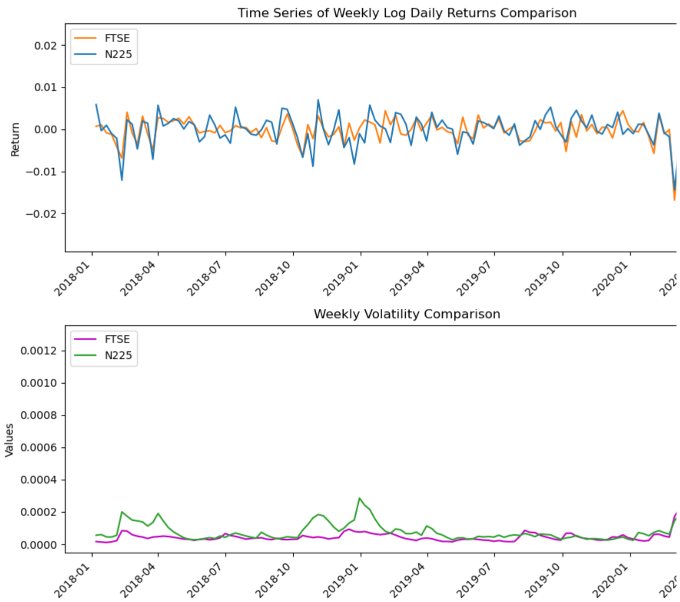
Figure 4: Causality Analysis: Weekly - Volatility

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
BVSP	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
FTSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GDAXI	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GSPC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
HSCE	(0, 0, 0, 1)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
IBEX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
JKSE	(0, 0, 0, 1)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
MXX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
N225	(0, 0, 0, 0)	(0, 1, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)
TWII	(0, 0, 0, 1)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)

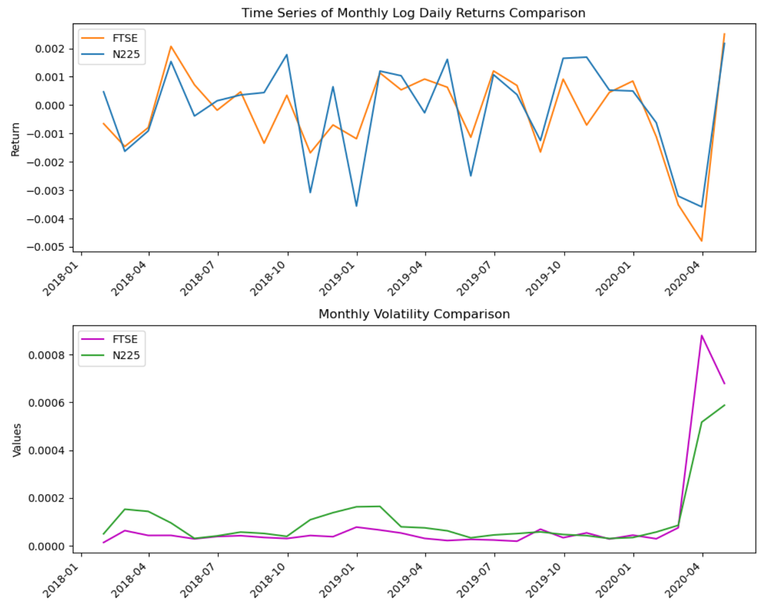
Figure 5: Causality Analysis: Monthly - Volatility

To further dive into these relationships, we create an overlapping time series plot of average log daily return and average volatility on the weekly and monthly level for the tickers *FTSE* and *N225*, specifically chosen as both tickers yielded statistically significant causality results for two of the lag options with regard to weekly log daily return and three or more of the lag options with regard to weekly volatility.

We see in Figure 6a that the average log daily return and average volatility on the week does not differ too greatly so that we could understand that if we were to lag the data by a week, it would perform reasonably as a predictor to the other funds log daily return.



(a) Weekly Comparison



(b) Monthly Comparison

Figure 6: Comparison between *FTSE* and *N225*

On the opposite end of the spectrum we see in Figure 6b that if we were to lag the monthly return or monthly volatility by a month it would not act as a good indicator as this behavior has a larger value gap month to month than on the weekly level. Further giving us insight into why we saw many statistically significant causality results on the weekly level rather than on the monthly level.

Still the table results from the Granger causality tests do not offer a complete assessment of causality. From Figure 6a particularly the comparison of volatility so closely aligning, it is possible that we simply are mistaking correlation as causality. To evaluate whether this is true causality or is in fact correlation, we perform a correlation assessment. We conduct our correlation assessment using a distance correlation calculation. For ease of visualization we create an indicator where an absolute value of correlation greater than .7 is a 1 indicating strong correlation and 0 otherwise.

Continuing with our example of looking specifically at tickers *FTSE* and *N225* our causality tests indicated that using a lag return or lag volatility offered good predictive insight into the other fund's return or volatility on a weekly level. But looking at Figure 7 and Figure 9 we have that the distance correlation values are less than zero for all lag values on the weekly level for log daily returns but not volatility. This indicates to us that there is not in fact true causality between these funds and the lagged returns but there could be when it comes to volatility.

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
BVSP	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
FTSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GDAXI	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GSPC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
HSCE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
IBEX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
JKSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
MXX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
N225	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
TWII	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)

Figure 7: Correlation Analysis: Weekly - Log Daily Returns

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
BVSP	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
FTSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GDAXI	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GSPC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
HSCE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
IBEX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
JKSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
MXX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
N225	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
TWII	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)

Figure 8: Correlation Analysis: Monthly - Log Daily Returns

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(1, 1, 0, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(0, 0, 0, 0)	(1, 1, 1, 0)
BVSP	(0, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)
FTSE	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)
GDAXI	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)
GSPC	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)
HSCE	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)
IBEX	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)
JKSE	(1, 1, 0, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)
MXX	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)
N225	(1, 1, 0, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(1, 1, 1, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)
TWII	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)

Figure 9: Correlation Analysis: Weekly - Volatility

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)
BVSP	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
FTSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GDAXI	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GSPC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
HSCE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
IBEX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
JKSE	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)
MXX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
N225	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)
TWII	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)

Figure 10: Correlation Analysis: Monthly - Volatility

Zooming out at the overall analysis between all tickers we see a similar pattern emerge where we saw many causality indicators on the weekly level for both log daily returns and volatility but we only see correlation supporting this when it comes to volatility behavior. What we are seeing from both the causality test and correlation is spill over volatility, a situation in financial markets where volatility in one fund can impact the volatility of another fund.

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
BVSP	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
FTSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GDAXI	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GSPC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
HSCE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
IBEX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
JKSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
MXX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
N225	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
TWII	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)

Figure 11: Causality Analysis: Rolling Weekly - Log Daily Returns

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
BVSP	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
FTSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GDAXI	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GSPC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
HSCE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
IBEX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
JKSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
MXX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
N225	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
TWII	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)

Figure 12: Causality Analysis: Rolling Monthly - Log Daily Returns

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
BVSP	(0, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
FTSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GDAXI	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GSPC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
HSCE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
IBEX	(0, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
JKSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
MXX	(0, 0, 0, 0)	(0, 1, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
N225	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
TWII	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(1, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 1, 0)	(1, 1, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(1, 1, 0, 0)

Figure 13: Causality Analysis: Rolling Weekly - Volatility

	BSESN	BVSP	FTSE	GDAXI	GSPC	HSCE	IBEX	JKSE	MXX	N225	TWII	VIX	VLIC
BSESN	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
BVSP	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
FTSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GDAXI	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
GSPC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
HSCE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
IBEX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
JKSE	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
MXX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
N225	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
TWII	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VIX	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
VLIC	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)

Figure 14: Causality Analysis: Rolling Monthly - Volatility

Lastly as noted in Part 4 of the homework, we conducted the causality on a rolling weekly basis and a rolling monthly basis with a window of one year. With the above four figures, when we perform the granger causality test on a rolling weekly and rolling monthly level to determine the efficacy of one of these lag fields in predicting their current counterpart and other funds, for ease of presentation and evaluation we aggregate the rolling p-value results to the fund level by taking the average, although it should be noted that this can lead to misrepresentations if p-values are extremely small or there is a p-value that is extremely large in the rolling weekly population. If the average p-value for all four tests; SSR based F-test, SSR based chi-squared test, likelihood ratio test, and parameter F test; were less than .05 then we designate the feature column with a 1 for statistically significance and 0 otherwise in the following tables. We see very different results instead of our original full causality analysis and that our rolling causality analysis more closely aligns with the correlation results.

3 Predictive Analysis of S&P 500 Using Neural Networks and Gaussian Processes

In this section we predict the S&P 500 using Neural Networks and Gaussian Processes. We use historical data from 1927 to 2021 and indicators Dividend Price Ratio, Stock Variance and Book-to-market Ratio as well as lags of each indicator and target variable to forecast price, return and direction of market movements.

3.1 Preprocessing and Analysis of Data

When loading and inspecting the Goyal-Welch monthly dataset, we found it had some missing values for the indicators 'ntis' and 'csp'. Before jumping ahead, we used the Python Package PanelSplit developed by Frey 2023 to apply KNN imputation in a time-series fashion to fill in the missing data. PanelSplit's cross_val_fit_transform takes an object that can implement fit/transform (like the KNN imputer) and iteratively fits on the train and transforms the test for each fold.
The first period was left blank as it was used as the training set, however, since we used the entire dataset, once we dropped data from before 1927, we no longer had any missing values.

3.2 Feature Engineering

After handling the missing data, we then built our feature and target variables. We computed DP as follows

$$DP = log(D_{12}) - log(P)$$

Where D is the 12-month moving sum of Dividends and P is the price level of the stock index.

```
# dividend-price ratio (dp)
sp500_df['dp'] = np.log(sp500_df['d12']) - np.log(sp500_df['index'])
# Book-to-Market (b/m)
sp500_df.rename(columns={'b/m': 'bm'}, inplace=True)
```

Our target variables; price (P_t^{log}), return (R_t^{log}) and direction (D_t) were computed as follows:

$$\begin{aligned}
P_t^{\text{log}} &= log(P_t) \\
R_t^{\text{log}} &= log(P_t) - log(P_{t-1}) \\
R_t^{\text{simp}} &= \frac{P_t - P_{t-1}}{P_{t-1}} \\
D_t &= \begin{cases} 1 & \text{if } \; R_t^{simp} > 0 \\ 0 & \text{if } \; \text{otherwise} \end{cases},
\end{aligned}$$

Where:

- P_t is the price of the index at time t and P_{t-1} is the price of the index at time $t-1$.
- P_t^{log} is the log of the index at time t.
- R_t^{log} and R_t^{simp} represent the log return and the simple return of the index at time t, respectively.

- D_t is a binary indicator (0 or 1) representing the direction of price movement from $t-1$ to t based on the simple return.

```
sp500_df['logret'] = np.log(sp500_df['index']).diff() #log return of sp500
sp500_df['logindex'] = np.log(sp500_df['index']) #log of index
sp500_df['index_return'] = sp500_df['index'].pct_change() #percentage change
sp500_df['direction'] = (sp500_df['index_return'] > 0).astype(int) #direction

print(sp500_df[['date', 'logindex', 'logret', 'direction']].head())
```

date	logindex	logreturn	direction
1926-01-01	2.545	NaN	0
1926-02-01	2.500	-0.045	0
1926-03-01	2.439	-0.061	0
1926-04-01	2.461	0.022	1
1926-05-01	2.469	0.008	1

Taking the log of the price index variable helps to stabilizes variance and reduce the skewness as seen in Figure 15. This will help us when we run our forecast models.

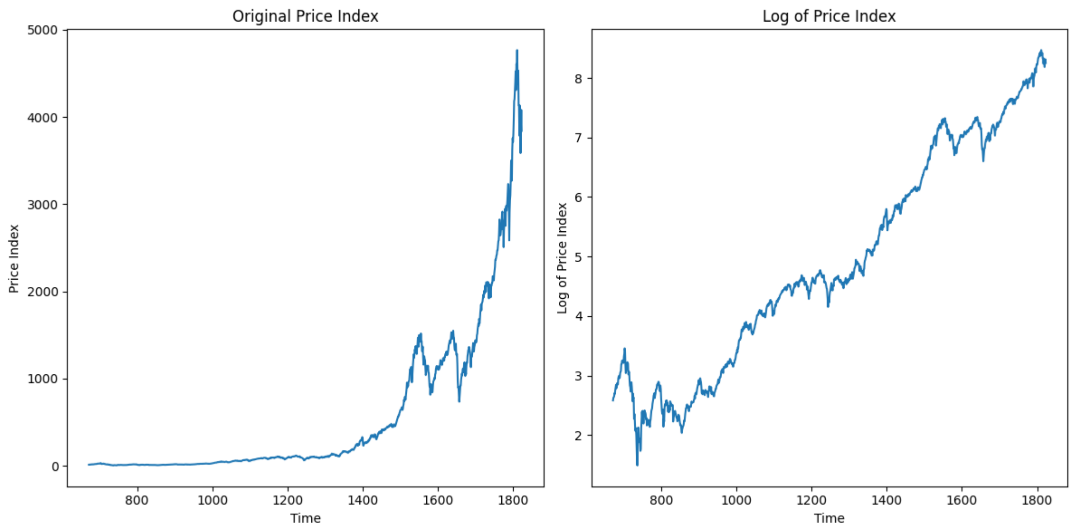


Figure 15: Index and log index comparison

We built our lag variables by using Pandas .shift() method. For each feature and target, we shifted them by 1, 2, 3 and 4 months ahead. This of course produced nulls for the first 4 periods, however, these were dropped when we filtered the dataset to only have data from 1927 and up. Below is a code snippet of how we created the lags for DP, the others were built in a similar fashion.

```
#lags for dp
sp500_df['dp_lag1'] = sp500_df['dp'].shift(1)
sp500_df['dp_lag2'] = sp500_df['dp'].shift(2)
sp500_df['dp_lag3'] = sp500_df['dp'].shift(3)
sp500_df['dp_lag4'] = sp500_df['dp'].shift(4)
```

After handing missing values and building our features and targets, we filtered the data to have data from 1927 and up.

```
sp500_df_filtered = sp500_df[sp500_df['date'] >= '1927-01-01']
```

3.3 Feature Selection

To select the most predictive features for each of our targets, we used Random Forest models. For the continuous targets, logindex and logret, we trained a RandomForestRegressor. For our binary target, direction, we trained a RandomForestClassifier. We then extracted the feature importances from each model and selected the top 7 most important features for each target.

Top 7 important features for logindex:
Index(['logindex_lag1', 'logindex_lag2', 'logindex_lag3', 'logindex_lag4', 'bm', 'dp', 'dp_lag1'], dtype='object')
Top 7 important features for logret:
Index(['svar', 'dp', 'svar_lag1', 'bm_lag2', 'logret_lag3', 'logindex_lag2', 'svar_lag2'], dtype='object')
Top 7 important features for direction:
Index(['svar', 'dp', 'bm', 'logret_lag1', 'svar_lag1', 'dp_lag1', 'logret_lag2'], dtype='object')

3.4 Baseline Model: ARMA(p, q)

Before moving on to the bigger and better forecasting models, we built an Auto Regression and Moving Average model for our continuous targets: price and return. The ARMA model predicts future values in a time series fashion using only its own past values (the auto regressive part) and a moving average of the past forecasting errors (the moving average part). This provides us with a baseline forecast. The formulas below (from class slides) help provide a little more clarity.

$$R_t^{log} = \phi_1 r_{t-1} + \ldots + \phi_p r_{t-p} + a_t + \theta_1 a_{t-1} + \ldots + \theta_q a_{t-q}$$

Where R_t^{log} is the current value of the time series (log of return at tiem t), ϕ_i are the coefficients for the autoregressive (AR) part for lags 1 through p, a_t are the error terms, and θ_i are the coefficients for the moving average (MA) part for lags 1 through q.
Before running the forecast, we first need to check if the time series is stationary as this is a requirement for ARMA. We built a function that uses a Dickey-Fuller test to check for unit roots. Unit roots have a stochastic trend where shocks to the series have a permanent effect on its future values, which would mean that our data is not stationary.

```
def test_stationarity(timeseries):
    result = adfuller(timeseries, autolag='AIC')
    print(f'ADF Statistic: {result[0]}')
    print(f'p-value: {result[1]}')
    print('Critical Values:')
    for key, value in result[4].items():
        print(f'\t{key}: {value}')
    if result[1] > 0.05:
      print("Data is not stationary")
    else:
      print("Data is stationary")
```

We found that the time series for the log of index was not stationary. Below are the results.
ADF Statistic: 0.655
p-value: 0.989
Critical Values:
    1%: -3.436
    5%: -2.864
    10%: -2.568
Data is not stationary

The null hypothesis is that there is a unit root (meaning the data is not stationary). If the ADF statistic is more negative than the critical values, the null hypothesis can be rejected. In our case, the ADF statistic of 0.655 is not more negative than any of the critical values, confirming that the time series can be considered to have a unit root and is non-stationary at the 1%, 5%, and 10% levels. So we cannot reject the null hypothesis. Since the data is not stationary we use differencing to stabilize them. Differencing is done by subtracting the previous observation from the current observation. Since we had to difference the series, we created an ARIMA(p, d, q) model. d is the order of differencing, in our case d would just be 1 since it only took us one difference to make the data stationary.

```
logindex_df['difflogindex'] = logindex_df['logindex'].diff()
logindex_df.dropna(inplace=True)

test_stationarity(logindex_df['difflogindex'])
```

ADF Statistic: -8.091
p-value: 0.000
Critical Values:
  1%: -3.436
  5%: -2.864
  10%: -2.568
Data is stationary

We then used auto_arima function from the pmdarima library to determine the best parameters (p and q) for an ARIMA model, fit to the training data, then predict our future values.

Results for logindexdiff:
Best model: ARIMA(2,0,2)(0,0,0)[0] intercept
Total fit time: 19.221 seconds
Performance metrics:
  Mean Squared Error: 0.002
  Mean Absolute Error: 0.038
  R-squared: -0.02

Results for logret:
Best model: ARIMA(2,0,2)(0,0,0)[0] intercept
Total fit time: 19.259 seconds
Performance metrics:
  Mean Squared Error: 0.002
  Mean Absolute Error: 0.039
  R-squared: -0.021

To level the playing field later on when we compare results and plot, we converted the predictions back to normal values by doing a cumulative sum and adding the last value (reverting the differencing we did earlier).

```
last_known_value = logindex_df['logindex'].iloc[-len(index_predictions['pred_arma'])-1]
restored_predictions = index_predictions['pred_arma'].cumsum() + last_known_value
restored_predictions = pd.concat([pd.Series([last_known_value],
    index=[logindex_df.index[-len(index_predictions)-1]]), restored_predictions])
```

3.5 Gaussian Process Setup, Tuning and Forecast

We then set up a Grid Search using time series cross-validation to find the optimal kernel for the Gaussian Process Regression, for each continuous target. Below are the kernels we checked for each target.

1. The white kernel we used only to help model the noise:

$$k_{\text{white}}(x, x') = \sigma^2 \delta(x, x')$$

where $\sigma^2 = 0.5$ is the noise level, and $\delta(x, x')$ is the Kronecker delta, which equals 1 if $x = x'$ and 0 otherwise.

2. The constant kernel returns a constant value for all pairs of points:

$$k_{\text{constant}}(x, x') = c$$

3. The RBF kernel is the most popular choice of kernel and is special case of radial basis function:

$$k_{\text{RBF}}(x, x') = \exp\left(-\frac{\|x - x'\|^2}{2l^2}\right)$$

where $l = 1$ is the length scale.

4. The Matern kernel a generalization of the RBF kernel and is particularly useful for modeling more conplex processes:

$$k_{\text{Matérn}}(x, x') = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu}\|x - x'\|}{l}\right)^{\nu} K_{\nu}\left(\frac{\sqrt{2\nu}\|x - x'\|}{l}\right)$$

where $l = 1.0$ is the length scale, $\nu = 1.5$, and K_{ν} is the modified Bessel function.

5. The Rational Quadratic (RQ) Kernel can be seen as a scale mixture of RBF kernels with different characteristic length scales:

$$k_{\text{RQ}}(x, x') = \left(1 + \frac{\|x - x'\|^2}{2\alpha l^2}\right)^{-\alpha}$$

where $l = 1.0$ is the length scale and $\alpha = 1.5$.

6. The dot product kernel also known as the polynomial kernel of degree 1, the Dot Product Kernel computes a dot product between the input features:

$$k_{\text{dot}}(x, x') = \sigma_0^2 + x \cdot x'$$

where $\sigma_0 = 1$.

```
#diffkernels
white_kernel = WhiteKernel(noise_level=0.5)
constant_kernel = ConstantKernel(1.0)
rbf_kernel = constant_kernel * RBF(length_scale=1) + white_kernel
matern_kernel = constant_kernel * Matern(length_scale=1.0, nu=1.5)
rq_kernel = RationalQuadratic(length_scale=1.0, alpha=1.5)
dot_product_kernel = DotProduct(sigma_0=1)
```

After finding the best kernel, we then fit the model to the training data and generated predictions. Below are the results:
Best parameters for logindex: {'alpha': 0.0001, 'kernel': DotProduct(sigma_0=1)}
Best cross-validation score for logindex: -0.0003
Performance for logindex:
  Mean Squared Error: 0.00002
  Mean Absolute Error: 0.003
  R-squared: 0.999

Best parameters for logret: {'alpha': 0.0001, 'kernel': RationalQuadratic(alpha=1.5, length_scale=1)}
Best cross-validation score for logret: -0.002

Performance for logret:
    Mean Squared Error: 0.002
    Mean Absolute Error: 0.033
    R-squared: 0.246

We ran a separate Grid Search for our binary target. We used GaussianProcessClassifier, set up the same kernels, instead of searching for the optimal alpha we looked for the best max_iter_predict (specifies the maximum number of iterations allowed for the optimizer in the prediction phase) and we used ROC-AUC as our scoring metric. We got the following results.

Best parameters for direction: {'kernel': DotProduct(sigma_0=1), 'max_iter_predict': 500}
Best cross-validation score for direction: 0.958

For the predictions, we generated the class labels and the probabilities using the best parameters. We computed the ROC-AUC, accuracy score and the log-loss.

```
#class labels for the test set
pred_gp_dir = models[target].predict(X_test_scaled)
#probability estimates for the binary classes
pred_gp_dir_prob = models[target].predict_proba(X_test_scaled)
```

Performance for direction:
    ROC AUC: 0.998
    Accuracy: 0.938
    Log Loss: 0.331

3.6 Neural Network Setup, Tuning and Forecast

For our Neural Network setup and tuning, we also performed a grid search using times series cross-validation. Similar to our GP setup, we built a loop for the continuous target variables and created a separate grid search for our binary target.
We searched different numbers of hidden layers and neurons, different learning rates, alphas, solvers and activation layers.

```
    param_grid = {
        'hidden_layer_sizes': [(20,), (20, 20), (50,), (100,), (100, 100), (50, 50), (100, 50, 25)],
        'activation': ['logistic', 'tanh', 'relu'],
        'solver': ['adam', 'sgd'],
        'alpha': [0.0001, 0.001, 0.01],
        'learning_rate_init': [0.0001, 0.001]
    }
```

Best parameters for logindex: {'activation': 'logistic', 'alpha': 0.001, 'hidden_layer_sizes': (100,), 'learning_rate_init': 0.001, 'solver': 'adam'}
Best cross-validation score for logindex: -0.118
Performance for logindex:
    Mean Squared Error: 0.176
    Mean Absolute Error: 0.413
    R-squared: -4.267

Best parameters for logret: {'activation': 'logistic', 'alpha': 0.0001, 'hidden_layer_sizes': (100, 100), 'learning_rate_init': 0.0001, 'solver': 'sgd'}
Best cross-validation score for logret: -0.002
Performance for logret:

Mean Squared Error: 0.002
Mean Absolute Error: 0.037
R-squared: 0.070

Our grid search for direction involves mostly the same parameters except the activation functions, we chose sigmoid, softmax, relu as these generate probabilities for our classification predictions. Similarly to our GP grid search for direction, we chose ROC-AUC as our scoring metric.

```
param_grid = {
    'hidden_layer_sizes': [(20,), (20, 20), (50,), (100,), (100, 100), (50, 50), (100, 50, 25)],
    'activation': ['sigmoid', 'softmax', 'relu'],
    'solver': ['adam', 'sgd'],
    'alpha': [0.0001, 0.001, 0.01],
    'learning_rate_init': [0.0001, 0.001]
}
```

Best parameters for direction: {'activation': 'relu', 'alpha': 0.001, 'hidden_layer_sizes': (100, 100), 'learning_rate_init': 0.001, 'solver': 'adam'}
Best cross-validation score for direction: 0.731

```
pred_nn[target] = models_nn[target].predict(X_test_scaled)
test_data[f'{target}_nn_pred'] = pred_nn[target]

pred_nn_dir_prob = models_nn[target].predict_proba(x_test_scaled)
test_data[f'{target}_nn_prob'] = pred_nn_dir_prob[:, 1]
```

Performance for direction:
ROC AUC: 0.977
Accuracy: 0.958
Log Loss: 0.253

3.7 Results & Discussion

Below we take a closer look at each target and the performance of each model for that target. To compare each model, we evaluate metrics, analyze plots and examine the top predicting features.

3.7.1 Price (log of index)

Table 1: Performance Metrics for Price (Log of Index)

Model	MSE	MAE	R^2
GP	0.00002	0.003	0.999
NN	0.176	0.413	-4.270
ARMA	0.034	0.141	-0.003

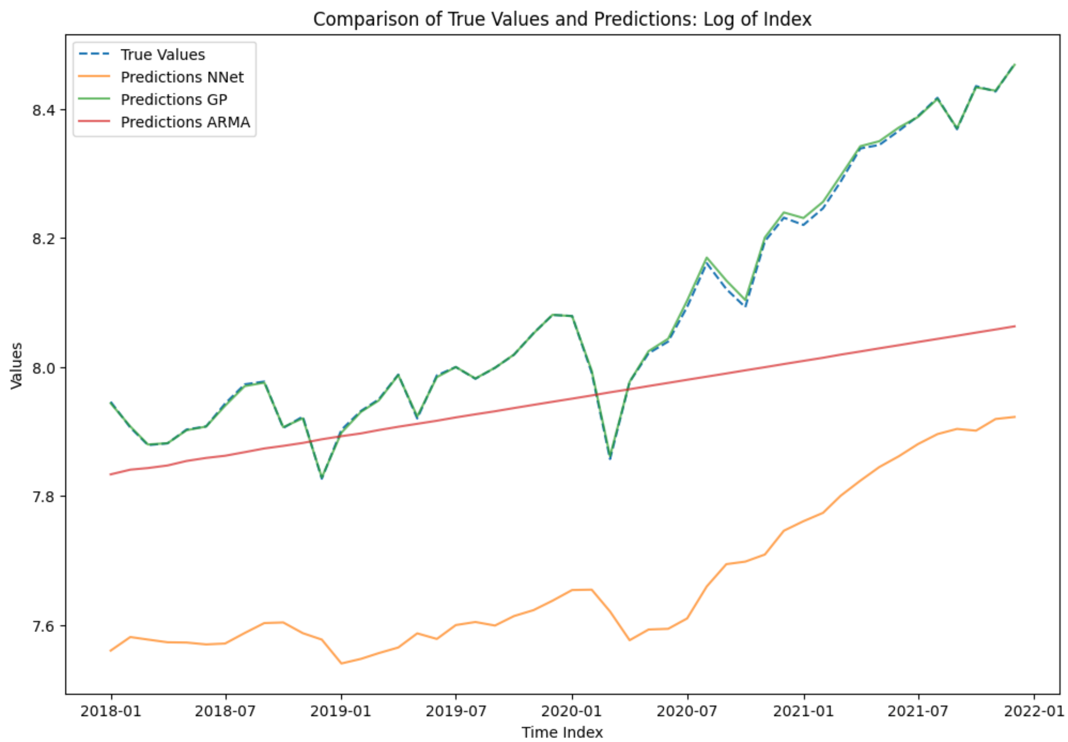


Figure 16: Log of Index Plot Comparison

Looking at the performance metrics and plots for our price target (log of the index), we can see the GP model performed the best with an almost perfect R^2 and super low MSE and MAE. In Figure 16 we visually see just how well the GP model predictions are, they align almost perfectly with the actual values. The NN and ARMA predictions (both with negative R^2 values) are pretty far off from the actual values. The ARMA model shows a higher bias, failing to capture the true trend, while keeping lower variance across predictions. On the other hand, the NN model, despite capturing the general shape of the trend (indicating lower bias), shows higher variance, reflected in its broader spread of predictions around the actual values.

The best kernel found in the grid search was a Dot Product kernel (A.K.A. a liner kernel when σ_0 is included). The success of the model may be largely due to this choice of kernel as it's able to capture the linear relationships of financial data over time. It is even more enhanced by the parameter sigma_0 which adds flexibility in modeling linear trends with a non-zero intercept. This aligns well with financial data characteristics, particularly when the data exhibits clear and persistent trends.

Examining the top features for price, the lagged values (logindex_lag1, logindex_lag2, logindex_lag3, logindex_lag4) being the top 4 features, really highlight the consistency and trend-following behavior of market indices, in other words, the strong predictive power of these lagged features indicates that past price movements of the S&P 500 tend to follow a stable and predictable trend.

Book-to-Market Ratio helps detect when market prices are very high or low compared to their actual value, which most likely helped our model predict when market trends could have changed, especially when used with the past data.

Dividend-Price Ratio (DP and DP_lag1) reflect both the current and past market sentiment regarding earnings yield, which is important for understanding how expensive the stocks are and predicting where the market might go next.

3.7.2 Log of Returns

Table 2: Performance Metrics for Log of Returns

Model	MSE	MAE	R^2
GP	0.002	0.033	0.246
NN	0.002	0.036	0.110
ARMA	0.002	0.039	-0.021

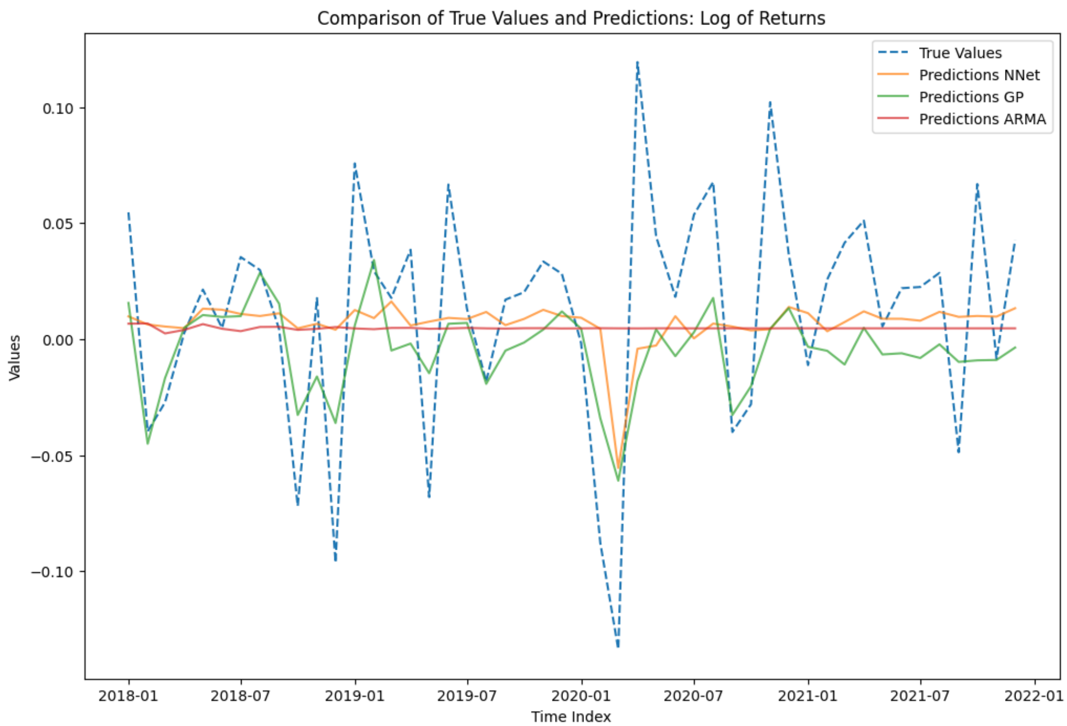


Figure 17: Log of Returns Plot Comparison

For predicting the log of returns, the GP model performs relatively better compared to NNet and ARMA. While all models have similar MSE and MAE, the GP model has a significantly higher R^2 value (0.246) compared to NNet (0.110) and ARMA (-0.021). This shows that GP can explain a higher proportion of variance in the returns compared to the other models. This can be seen in Figure 17 where the GP model (green line) fits the data better than NN (orange line) and ARMA (red line).

The best kernel found for the GP model was the RationalQuadratic. The kernel's key parameter, alpha at 1.5, allows it to handle variations in market data at different scales, making it better at adapting to the complex behaviors seen in log returns, such as volatility. The length scale of 1 allow it to pick up both short-term changes and longer-term trends in the market. The combination of features most likely contributed to the predictive ability of the model.

Looking at the top features for returns:
Stock Market Variance (SVAR, SVAR_lag1, SVAR_lag2) measure volatility and its lags are key predictors for returns, as higher volatility is often related with higher risk and higher returns. This shows that recent market instability could play a role in forecasting short-term return fluctuations.

The lagged BM (BM_lag2) ratio helps the model spot times when the market might correct itself or change direction after being over- or undervalued, which are essential for predicting returns.

The DP ratio gives us clues about what investors expect to earn from dividends relative to the stock price, helping the model predict future profits.

Lagged Returns (logret_lag3, logret_lag2) most likely help the models to understand trends and patterns in how returns move over time.

3.7.3 Direction

Table 3: Performance Metrics for Direction

Model	Accuracy	ROC-AUC
GP	0.938	0.998
NN	0.958	0.978

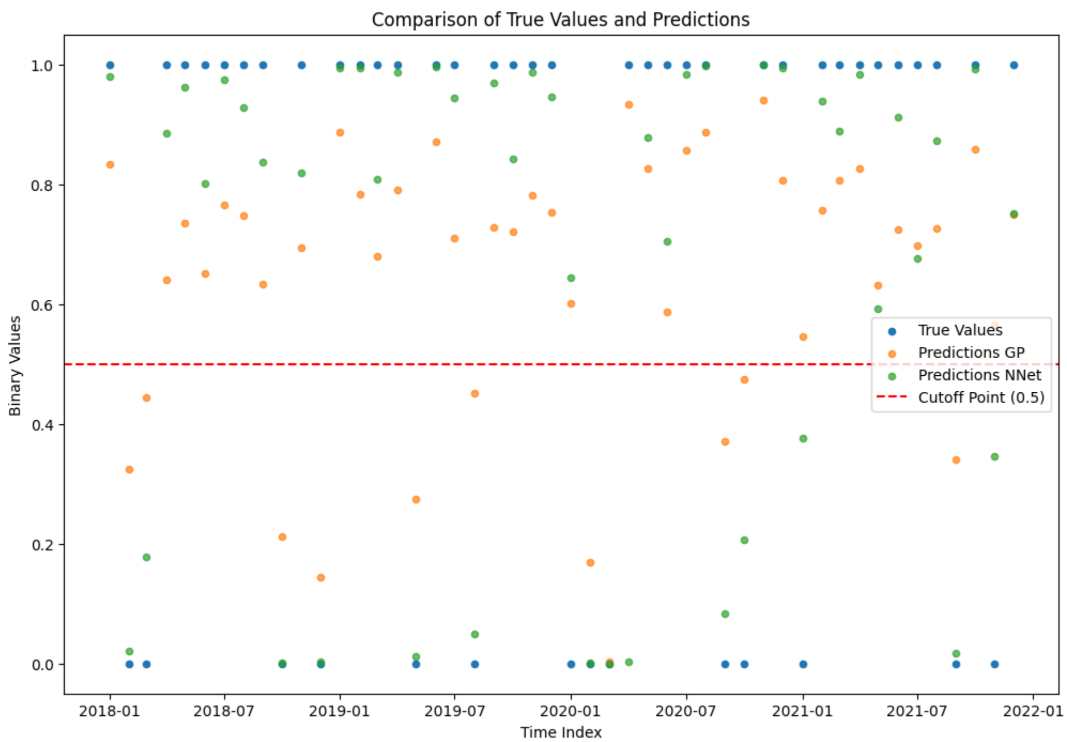


Figure 18: Direction Scatter Plot Comparison

Both the GP and NNet models show pretty high effectiveness in predicting the direction of movement, with NNet (accuracy of 95.8%, ROC-AUC of 97.8%) slightly outperforming GP (accuracy of 93.8%, ROC-AUC of 99.8%). NNet shows a stronger balance between accuracy and the ability to differentiate between classes (ROC-AUC), making it slightly more reliable for predicting market direction. As seen in Figure 18, the scatter plot shows the predicted probabilities and the cutoff threshold at 0.5, where we can visually see the NN model did slightly better at predicting the values. **Note:** We did not run the ARMA model for this target because it's designed to predict continuous outcomes based on past values and is not geared towards binary targets.

The top feature stock Market Variance (SVAR & SVAR_lag1) shows its role in predicting the likelihood of market upturns or downturns. Dividend-Price Ratio (DP & DP_lag1) reflects the market's expectations, influencing predictions on whether the market will go up or down.

Book-to-Market Ratio (BM) might have helped the model helps predict when the market might adjust or recover after being undervalued or overvalued.

Lagged Returns (logret_lag1, logret_lag2) could help the models forecast the short-term direction of the market.

3.8 Conclusion

Based on the metrics, our Gaussian Process (GP) model overall shows the strongest performance. It ranks best in terms of fitting the data for both the log of returns and log of index, and is also super accurate in the direction prediction. The GP model's ability to adapt its fit across different types of financial data—thanks to its flexible kernel choice—seems to provide it an edge in handling the complexities in financial time series predictions.

The Gaussian Process model and financial indicators perform best with the target of the price (log of index) as the model achieved near-perfect metrics and an R^2 score of 0.999.