

# Modelling supernova line profile asymmetries to determine ejecta dust masses: SN 1987A from days 524 to 3640

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## ABSTRACT

Unexpectedly large masses of dust up to  $0.7M_{\odot}$  have been discovered in the ejecta of SN 1987A. These dust masses are based on fitting the observed mid-IR and far-IR continuum. In many supernovae, line profiles in the optical and near-IR exhibit a red-blue asymmetry as a result of greater extinction to radiation emitted from the receding part of the supernova. We present here a new code, DAMOCLES, that models the effects of dust on the line profiles of core-collapse supernovae in order to determine the masses of newly formed dust. We have tested the code against previously published models and against analytical results. We find that the presence of a shoulder near the peak or an extended red scattering wing in a line profile may also be suggestive of dust formation. We also note that dust-affected line profiles need not necessarily be flux-biased towards to the blue. We have collated optical spectra of SN 1987A from a variety of archives and sources and have modelled the evolution of the H $\alpha$  line from day 714 to day 3604. We also present models of the blue-shifted [O I]  $\lambda$ 6300,6363 Å doublet between days 714 and 1478. We conclude that a significant dust mass ( $\sim 0.1M_{\odot}$ ) had formed by day 3604 and infer that the majority of the present dust mass must have formed after this epoch. For pure amorphous carbon models, large grain sizes ( $\geq 0.6\mu\text{m}$ ; capable of surviving the reverse shock) are required to fit the data even at early epochs (day 714). Our findings are in strong agreement with recent dust mass estimates for SED fits and confirm the suggestion that the majority of SN 1987A dust formed many years after the initial explosion.

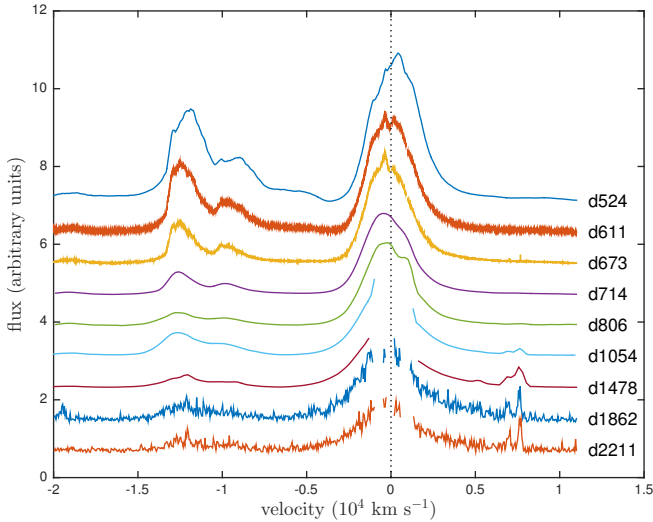
**Key words:** supernovae: general - supernovae: individual: SN 1987A - ISM: supernova remnants - radiative transfer

## 1 INTRODUCTION

Core-collapse supernovae (CCSNe) have long been thought to be potential dust factories (Hoyle & Wickramasinghe 1970; Kozasa et al. 1991; Todini & Ferrara 2001). However over the past decade observations at mid-infrared (mid-IR) wavelengths of warm dust emission from CCSNe had suggested that the quantities of dust produced, typically  $\leq 10^{-3} M_{\odot}$  during the first 1000 days (Sugerman et al. 2006; Meikle et al. 2007; Kotak et al. 2009; Andrews et al. 2010; Fabbri et al. 2011) were much less than the  $0.1\text{--}1.0 M_{\odot}$  of dust per CCSN that had been estimated to be needed (??) in order to account for the very large dust masses measured in some high redshift galaxies (Omont et al. 2001; Bertoldi et al. 2003; Watson et al. 2015). However, recent *Herschel* far-IR and sub-mm observations of cold dust masses as high as  $0.2\text{--}0.8M_{\odot}$  in several young supernova remnants have resulted in a re-evaluation of the rate of dust production by CCSNe (Barlow et al. 2010; Matsuura et al. 2011; Gomez

et al. 2012). The *Herschel* dust mass estimates were based on fitting dust spectral energy distributions (SEDs) that peaked at far-IR wavelengths. Following the end of the *Herschel* mission in 2013 there is likely to be a long wait for far-IR facilities with comparable or better sensitivities than *Herschel* to become available, providing an incentive to make use of alternative methods to estimate the dust masses that form in supernova ejecta.

The absorption and scattering of optical or near-IR radiation by newly-formed dust within the ejecta of supernovae can result in an asymmetry between the red and blue shifted components, with redwards emission from the far side of the ejecta undergoing greater absorption. Lucy et al. (1989) identified a progressive blue-shifting of the [O I]  $\lambda$ 6300,6363 Å doublet from SN 1987A between days 529 and 739 after outburst, with the doublet in the later spectrum being blue-shifted by  $\sim 600 \text{ km s}^{-1}$ . Since then, such red-blue asymmetries have been frequently observed in the late-time ( $> 400$  days) spectra of supernova ejecta and



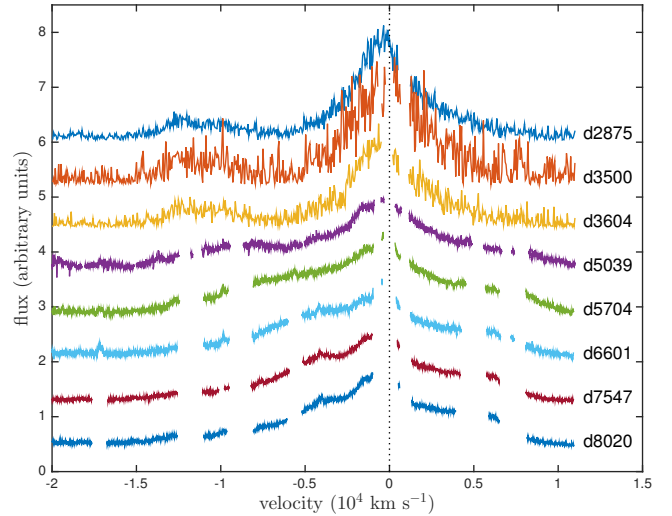
**Figure 1.** Archival data showing the evolution of the H $\alpha$  and [O I] line profiles from SN 1987A at the earlier epochs. The spectral gaps at the last two epochs correspond to where narrow line emission from the equatorial ring has been removed. The spectra have been continuum-subtracted and offsets applied for display purposes.

there is now a growing database of such observations (Lucy et al. 1989; Fabbri et al. 2011; Mauerhan & Smith 2012; Milisavljevic et al. 2012).

SN 1987A as an archetypal object is critical to our growing understanding of the formation and evolution of dust in CCSNe. Since its outburst there have been numerous observations at many wavelengths and many epochs. Mid-infrared emission from warm dust ( $T \sim 400$  K) was observed by day  $\sim 450$  (Roche et al. 1989; Bouchet et al. 1991; Wooden et al. 1993) and by day 775 the emitting dust mass was estimated to have been  $\sim 5 - 20 \times 10^{-4} M_{\odot}$  (Wooden et al. 1993; Ercolano et al. 2007; Wesson et al. 2015). Beginning from 23 years after outburst, the *Herschel Space Observatory* detected much larger quantities ( $0.4-0.8 M_{\odot}$ ) of  $T \sim 20$  K cold dust emitting at far-IR and submillimetre wavelengths (Matsuura et al. 2011, 2015). This emission has been confirmed by ALMA observations to originate from the ejecta of SN 1987A (Indebetouw et al. 2014).

We here seek to model formally the effects of dust on line profiles with a view to providing both an alternative way of determining dust masses formed in the ejecta of core-collapse supernovae and in order to investigate the effects of dust on the shape of line profiles emitted from these objects. We present a new code, DAMOCLES (Dust Affected Models Of Characteristic Line Emission in Supernovae), that utilises a Monte Carlo methodology in order to model line profiles in expanding atmospheres. The code can treat dust composed of multiple species and grain sizes with variable density and velocity distributions. Both clumped and smooth geometries may be modelled.

In this paper we collate optical spectra from the archives of four different telescopes in order to study the effects of dust formation on the H $\alpha$  line and the [O I]  $\lambda 6300, 6363$  Å doublet. We model five different epochs spanning a range of approximately 8 years from the first indications of blue-



**Figure 2.** Archival data showing the evolution of the H $\alpha$  line profile from SN 1987A at the later epochs. The spectral gaps correspond to where narrow line emission from the equatorial ring has been removed. The spectra have been continuum-subtracted and offsets applied for display purposes.

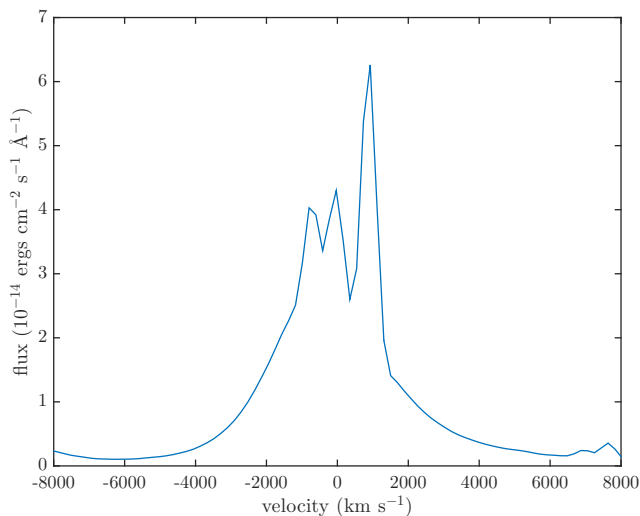
shifting in the H $\alpha$  line at  $\sim$ day 700, using both smooth and clumped geometries. We compare the derived dust masses to those obtained by W15 and consider the implied formation rate. We present our testing of the new code against analytical cases and previously published optically thick models from the literature (Lucy et al. 1989). We also present our investigation into the sensitivity of each of the various variable parameters and consequently note the signatures that observed line profiles may exhibit in the presence of dust.

In Section 2 we detail the spectra that we obtained for our modelling. In Section 3 we discuss the details of the DAMOCLES code and in Section 4 we present our testing of the code and our parameter sensitivity analyses. Our modelling of the H $\alpha$  and [O I]  $\lambda 6300, 6363$  Å lines is presented in Section 5 and finally we discuss our findings in Section 6.

## 2 ARCHIVAL SPECTRA OF SN 1987A

SN 1987A has been the most intensively observed supernova in history, with a wealth of both spectral and photometric data available to model. From the archives of a number of different telescopes we have collated optical spectra acquired over a wide range of epochs. At the earlier epochs we use spectra obtained by the Anglo-Australian Telescope (AAT) and the Cerro Tololo Inter-American Observatory (CTIO) and at later epochs we use spectra from the archives of the Hubble Space Telescope (HST) and Very Large Telescope (VLT). An explosion date of 23 February 1987 is adopted throughout and epochs are measured relative to this date. Full details of all observations may be found in Table 1. The spectral resolutions of the grating spectrograph observations are listed in column 7, while column 8 lists the spectral resolving powers of the echelle spectrograph observations.

Wavelength ranges encompassing the H $\alpha$  line and [O I]  $\lambda 6300, 6363$  Å doublet were selected in order to trace



**Figure 3.** The  $H\alpha$  line at day 1054 showing severe contamination by unresolved nebular lines from the equatorial ring

their evolution from day 524, near the time of the first indications of dust formation (Wooden et al. 1993) to day 8020, near the current era. Optical spectroscopy obtained at the AAT using the Faint Object Red Spectrograph (FORS) during the first two years after outburst was kindly supplied by Raylee Stathakis (Spyromilio et al. 1991, 1993; Hanuschik et al. 1993) and optical spectra from the CTIO donated by Mark Phillips (Suntzeff et al. 1991).

The evolution of the  $H\alpha$  and  $[O\text{ I}]$  line profiles is presented in Figures 1 and 2. At later epochs, the broad  $H\alpha$  profile emitted by the ejecta becomes contaminated by narrow line emission from the equatorial ring. These lines have been removed for the purposes of modelling the broad line. A continuum fit has been subtracted from each spectrum and a velocity correction has been applied for a recession velocity of  $287\text{ km s}^{-1}$  (Gröningson et al. 2008).

### 2.1 Contamination of the $H\alpha$ profiles

The  $H\alpha$  profile at day 714 exhibits a very slight corner visible at  $V \approx 900\text{ km s}^{-1}$ . By day 806, this slight corner has developed into a noticeable shoulder in the line profile of  $H\alpha$  at a similar velocity (see Figure 12).

Although these features are similar in nature to features produced by dust absorption in the flat-topped region (as discussed in Section 4.3.5), we assume this shoulder to be the first appearance of the unresolved  $[N\text{ II}]$   $\lambda 6583\text{ Å}$  line from the equatorial ring (Kozma & Fransson 1998). Unresolved nebular  $[N\text{ II}]$  lines at  $\lambda = 6583\text{ Å}$  and  $\lambda = 6548\text{ Å}$  either side of the  $H\alpha$  rest frame velocity at  $6563\text{ Å}$  are certainly seen by day 1054 and have to be removed in order to consider the evolution of the broad  $H\alpha$  profile. It is unfortunate that the theoretical minimum velocity falls at a similar velocity to the  $6583\text{ Å}$  line as any dust-induced features near this wavelength are overwhelmed by the nebular line. We do not remove this line at earlier epochs but try to fit the broad line profiles around it.

By day 1054, all three of the narrow nebular lines are

strong. They remain unresolved for the CTIO data at days 1054 and 1478 and therefore contaminate the entire central region of the  $H\alpha$  line profile (see Figure 3). Their removal therefore renders the profiles unusable for modelling purposes. The  $H\alpha$  profiles at later epochs ( $\geq 1862$  days), however, have a higher spectral resolution and it is therefore possible to remove these narrow  $[N\text{ II}]$  and  $H\alpha$  lines from the broad profiles. Although this does remove a critical section of the line ( $+500\text{ km s}^{-1} < v < +1500\text{ km s}^{-1}$ ) that would be potentially informative for  $V_{\min}$ , we achieve good fits to the overall line profiles at these epochs.

## 3 THE DAMOCLES CODE

Monte Carlo methods have long been used to model radiative transfer problems in diverse environments and there are several examples of codes which utilise the technique in application to supernovae (for example Maeda et al. (2003); Lucy (2005); Jerkstrand et al. (2012); Owen & Barlow (2015)). Whilst there are numerous codes that treat dust or gas or both in order to produce an overall spectral energy distribution (SED), there is a dearth of codes designed to focus on the shapes of single line profiles. Although a velocity field is naturally considered in codes that seek to reproduce the spectra of supernovae, absorption and scattering by dust is not and thus the resulting shapes of line profiles are potentially unrepresentative of those emerging from dusty ejecta at late times.

In this work we aim to model single line profiles or doublets produced by a moving atmosphere in a dusty medium. Since a comparatively small wavelength range is considered, a fully self-consistent radiative transfer model is unnecessarily expensive. Instead any energy packet that is absorbed during the simulation may simply be removed on the grounds that it would be reemitted outside the wavelength range of interest. The extinction due to dust is assumed to be temperature-independent and it is therefore unnecessary to iteratively calculate the temperature of the ejecta as in a fully self-consistent calculation of the SED. Though clearly the total energy transferred through the medium is not conserved, the signature of the normalised line profile is preserved.

The DAMOCLES code builds on the work of Lucy et al. (1989) who employed a similar approach to model the broad  $[O\text{ I}]$   $\lambda 6300, 6363\text{ Å}$  doublet seen in SN 1987A at early epochs (up to  $\sim$  day 775). It models the transport of initially monochromatic energy packets through a smooth or clumped dusty medium having a smooth velocity field. The velocity field and the inner and outer ejecta radii are free parameters. The late-time ( $> 400$  days) line emission is assumed to be optically thin, with an emissivity distribution proportional to the square of the local gas density, i.e. proportional to the product of the recombining proton and electron densities in the case of  $H\alpha$  and to the product of the neutral oxygen and electron densities in the case of collisionally excited  $[O\text{ I}]$  emission.

### 3.1 The energy packet formalism

The initial radiation field is inherently tied to the distribution of gas throughout the supernova ejecta which is declared

**Table 1.** Details of the archival data for SN 1987A.

Date	Age (days)	Telescope	Inst	$\lambda_{min}$ (Å)	$\lambda_{max}$ (Å)	Res. (Å)	Res. Power	Reference
31 Jul 1988	524	AAT	FORS	5500	10190	20		<a href="#">Spyromilio et al. (1991)</a>
26 Oct 1988	611	AAT	UCLES	6011	7336		30000	<a href="#">Hanuschik et al. (1993)</a> ; <a href="#">Spyromilio et al. (1993)</a>
27 Dec 1988	673	AAT	UCLES	5702	10190		30000	<a href="#">Hanuschik et al. (1993)</a> ; <a href="#">Spyromilio et al. (1993)</a>
06 Feb 1989	714	CTIO-1.5m	Cass.	6420	10380	16		<a href="#">Phillips et al. (1990)</a>
09 May 1989	806	CTIO-1.5m	Cass.	6430	10330	16		<a href="#">Phillips et al. (1990)</a>
12 Jan 1990	1054	CTIO-4m	RC	3565	10000	11		<a href="#">Suntzeff et al. (1991)</a>
12 Mar 1991	1478	CTIO-4m	RC	3245	9175	11		
30 Mar 1992	1862	HST	STIS	4569	6818	4.4		<a href="#">Wang et al. (1996)</a>
14 Mar 1993	2211	HST	STIS	4569	6818	4.4		<a href="#">Wang et al. (1996)</a>
07 Jan 1995	2875	HST	STIS	4569	6818	4.4		<a href="#">Chugai et al. (1997)</a>
23 Sep 1996	3500	HST	STIS	4569	6818	4.4		
05 Jan 1997	3604	HST	STIS	4569	6818	4.4		
10 Dec 2000	5039	VLT	UVES	4760	6840		50000	<a href="#">Gröningsson et al. (2006, 2007)</a>
06 Oct 2002	5704	VLT	UVES	4760	6840		50000	<a href="#">Gröningsson et al. (2006, 2007, 2008)</a>
21 Mar 2005	6601	VLT	UVES	4760	6840		50000	<a href="#">Gröningsson et al. (2006, 2007)</a>
23 Oct 2007	7547	VLT	UVES	4760	6840		50000	<a href="#">Gröningsson et al. (2007)</a>
07 Feb 2009	8020	VLT	UVES	4800	6800		50000	<a href="#">Tziamtzis et al. (2010)</a>

as a power law  $\rho(r) \propto r^{-\beta}$  between  $R_{in}$  and  $R_{out}$ .  $R_{out}$  is calculated directly from the epoch of the line to be modelled and the declared maximum line velocity. The emissivity distribution is also specified as a power law with  $i(\rho) \propto \rho^k$ . However this is generally taken to be  $i(r) \propto r^{-2\beta}$  since the majority of lines modelled are optically thin recombination lines or collisionally excited lines and therefore  $i(\rho) \propto \rho^2$ . The radiation is quantised into monochromatic packets with equal energy  $E_0 = nh\nu_0$ . In Monte Carlo simulations (that model non-moving media) packets are usually taken to be of constant energy. When the frequency of a packet is altered after an event, the energy of that packet is kept constant and the number of real photons contained within it is assumed to change. However, in the case of dust scattering, the number of real photons is conserved and thus the energy of the packet is altered. This is most easily achieved by weighting each packet over all scattering events as

$$w = \prod_{scat} \frac{\nu'}{\nu} \quad (1)$$

where  $w$  is the weight of the packet and  $\nu$  and  $\nu'$  are the frequencies of the packet before and after the scattering event respectively. The final energy of each packet is then  $E = wE_0$ , where  $E_0$  is the initial energy of the packet.

The supernova ejecta is divided into shells between  $R_{in}$  and  $R_{out}$  and the number of packets to be emitted isotropically in each shell calculated according to the emissivity distribution. For each packet a location within that shell and an initial trajectory is randomly sampled from an isotropic distribution such that

$$\phi = 2\pi\eta \quad (2)$$

$$\cos\theta = 2\xi - 1 \quad (3)$$

where  $0 < \eta < 1$  and  $0 < \xi < 1$  are random numbers,  $\phi$  is the azimuthal angle and  $\cos\theta$  is the radial direction cosine. At emission and at each scattering event the frequency of

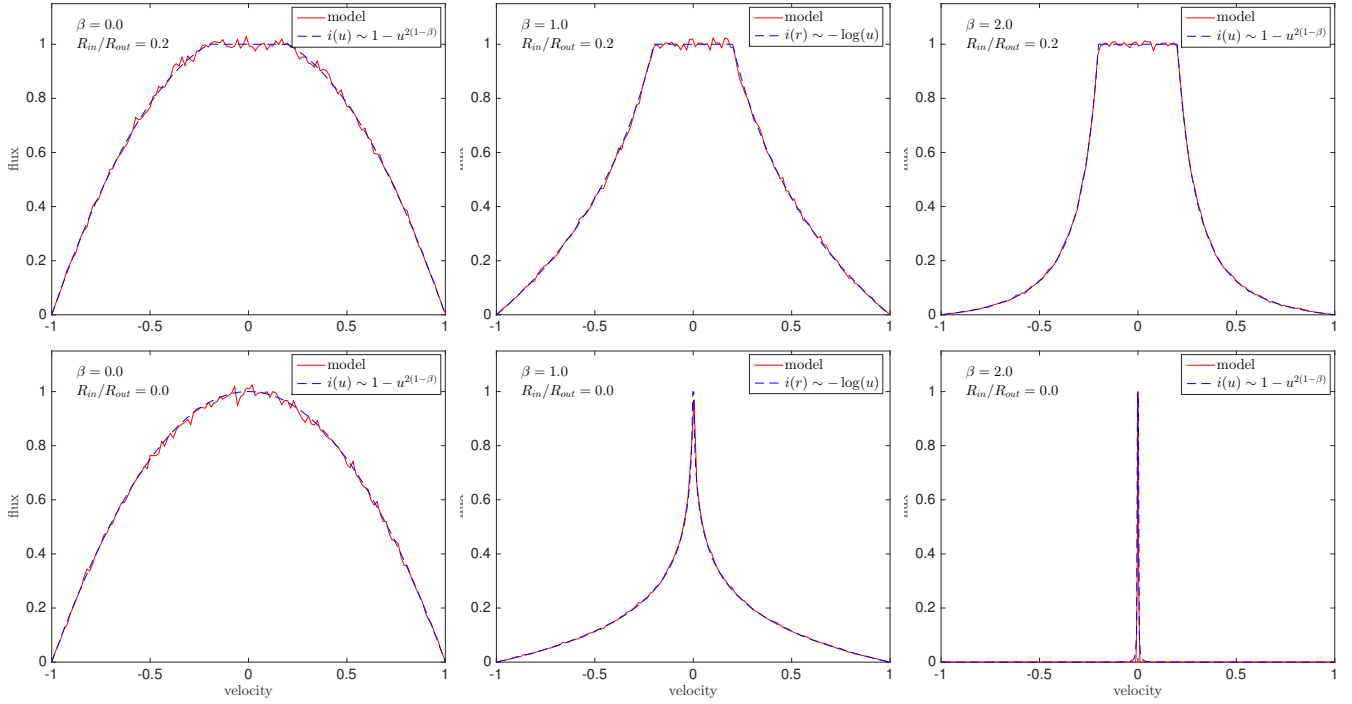
the packet is recalculated according to the specified radial velocity field  $v(r) \propto V_{max}r^\alpha$  (see Section 3.3).

### 3.2 The geometry of the ejecta and the grid

The supernova ejecta is approximated by a three-dimensional cartesian grid, each cell of which is assumed to have uniform density and composition. The grid is a cube with sides of width  $2R_{out}$  and a declarable number of divisions. After the initial emission of energy packets, the gas plays no further role in the simulation and thus only dust properties are considered. By default, the dust is coupled to the gas (although it may be decoupled) and thus follows the smooth distribution described above ( $\rho \propto r^{-\beta}$ ). The dust density in each cell is therefore calculated accordingly and any cell whose centre falls outside of the bounds of the supernova ejecta has density set to zero.

It is worth noting that if a constant mass loss rate is required, the exponent of the velocity profile and the exponent of the density profile are not independent. A constant mass loss rate implies that  $4\pi\rho v r^2 \propto k$ , where  $k$  is a constant, and thus for  $v \propto r^\alpha$  and  $\rho \propto r^{-\beta}$ , we require that  $\beta - \alpha = 2$ . However, it is possible that the supernova event may have induced a mass-flow rate that is not constant with radius and thus both exponents may be declared independently.

It is known from SED modelling that clumped environments produce very different results to environments assumed to have a smooth distribution of dust and gas. Specifically, clumped models tend to require a higher dust mass in order to reproduce a similar level of infrared dust emission compared to a smoothly distributed model. The capacity for modelling a clumped dusty medium is therefore included in the code. The fraction of the dust mass that is in clumps is declared ( $m_{frac}$ ) and the total volume filling factor of the clumps ( $f$ ) is also specified. Dust that is not located in clumps is distributed according to a smooth radial profile. The clumps occupy a single grid cell and their size can therefore be varied by altering the number of divisions in the grid. The clumps are distributed stochastically with the



**Figure 4.** Red: Benchmark models for optically thin ( $\tau = 0$ ) line profiles with fractional velocity  $v \propto r$ . Left to right: initial emissivity profiles  $i(r) \propto r^{-2\beta}$  with  $\beta = 0.0$ ,  $\beta = 1.0$  and  $\beta = 2.0$ . Cases with  $R_{in}/R_{out} = 0.2$  are on the top and with  $R_{in}/R_{out} = 0.0$  on the bottom. The presence of a plateau in the upper plots is due to the finite inner radius (detached shell). Blue: The analytical case with  $i(u) \sim 1 - u^{2(1-\beta)}$  except in the case of  $\beta = 1$  where  $i(u) \sim -\log u$ .

probability of a given cell being a clump proportional to the smooth density profile (i.e.  $p(r) \propto r^{-\beta}$ ). The density within all clumps is constant and is calculated as

$$\rho_{clump} = \frac{M_{clumps}}{V_{clumps}} = \frac{m_{frac} M_{tot}}{\frac{4}{3} \pi (R_{out}^3 - R_{in}^3)} \quad (4)$$

where  $M_{tot}$  is the total dust mass,  $M_{clumps}$  is the total dust mass in clumps and  $V_{clumps}$  is the total volume occupied by clumps.  $m_{frac}$  and  $f$  are defined as above.

### 3.3 The radiative transport mechanism

Following emission, a packet must be propagated through the grid until it escapes the outer bound of the ejecta  $R_{out}$ . The probability that the packet travels a distance  $l$  without interacting is  $p(l) = e^{-n\sigma l} = e^{-\tau}$  where  $n$  is the grain number density,  $\sigma$  is the cross-section for interaction and  $\tau = n\sigma l$  for constant  $n$  and  $\sigma$  (as in a grid cell). Noting that the probability that a packet will interact within a distance  $l$  is  $1 - e^{-\tau}$ , we may sample from the cumulative probability distribution to give:

$$\xi = 1 - e^{-\tau} \implies \tau = -\ln(1 - \xi) \quad (5)$$

where  $0 < \xi < 1$  is a random number sampled from a uniform distribution. The frequency of the photon packet and the mass density of the cell are then used to calculate the opacity of that cell. Using the fact that  $n\sigma = \kappa\rho$ , the distance  $l$  that the packet travels before its next interaction is calculated. If this value is greater than the distance from

its position to the edge of the cell then the packet is moved along its current trajectory to the cell boundary and the process is repeated. If the distance is less than the distance to the boundary then an event occurs and the packet is either scattered or absorbed with the probability of scattering equal to the albedo of the cell

$$\omega = \frac{\sigma_{sca}}{\sigma_{sca} + \sigma_{abs}} \quad (6)$$

If the packet is absorbed then it is simply removed from the simulation as discussed above. If the packet is scattered then a new trajectory is sampled from an isotropic distribution in the comoving frame of the dust grain and the frequency of the packet is recalculated using Lorentz transforms subject to the velocity at the radius of the interaction (see Appendix A for further details). This process is repeated until the packet has either escaped the outer boundary of the supernova ejecta or has been absorbed.

Escaped photon packets are added to frequency bins, weighted by  $w$ , in order to produce an overall emergent line profile.

### 3.4 Properties of the Dusty Medium

Dust of any composition for which optical data is available may be used and the relative abundances of the species may be declared by the user. A grain size may be specified for each species. Since a full radiative transfer calculation is not performed, it is not useful to specify a grain size distribution since the extinction due to dust is only dependent on the



cross-sectional area of the grains and not on the overall distribution. The capacity to declare a size distribution is however included for the sake of ease of comparison with SED models. Mie theory and optical properties are used to calculate the overall  $Q_{abs}(\nu)$  and  $Q_{sca}(\nu)$  for each species and the derived opacities are summed over each species weighted according to their relative abundances.

As will be discussed in Section 4, the effects of scattering on the shapes of line profiles can potentially be quite pronounced and it is therefore important to consider the potential effects of electron scattering as well as those of dust scattering. Electron densities are calculated using an estimated average temperature of 10,000K and observed luminosity of H $\alpha$  and the optical depth to electrons calculated from this. Electron scattering is treated in an identical manner to dust scattering, with  $\tau = \tau_{dust} + \tau_e$  in each cell. If, for a given packet, an event occurs, it is first calculated whether this is an electron scattering event or a dust event (either scattering or absorption) by considering the ratio of the optical depths to each. If the packet is scattered by an electron then the velocity of that electron is calculated by considering the bulk velocity at that radius and adding a thermal velocity component following the formalism described by Hillier (1991). The scattering process is then identical to that for dust. If the event is a dust event then the process continues as described above.

In the majority of cases the electron scattering optical depths are not high enough to discernibly affect the overall shape of the profile. However, there may be a few early epoch cases (the concept is discussed for SN 2010jl by Fransson et al. (2014)) where the electron scattering optical depths are high enough to become significant in the observed profiles. Whilst the inclusion of electron scattering in the code is an approximation, it gives a good representation of the potential effects.

#### 4 COMPARISON OF DAMOCLES MODELS WITH ANALYTICAL AND PREVIOUSLY PUBLISHED RESULTS

There is a general lack of published models in the literature that consider dust absorption-affected asymmetric line profiles. We therefore test the code by comparing the results to optically thin profiles that may be derived analytically. We then test the absorption and scattering components of the code by comparing our results in the case of an optically thick medium with those derived by Lucy et al. (1989) in their Model II and Model III scenarios.

##### 4.1 Comparison of DAMOCLES models with analytical results

Analytical profiles may be calculated in the dust-free case. We ran a number of models based on the methods of Gerasyimovic (1933) who derived equations for line profiles emitted from a transparent expanding shell.

Describing the fractional expansion velocity of the shell as  $v(r) = r^\alpha$  with  $\alpha \neq 0$  such that  $v(r) = \frac{V(r)}{V_{max}}$  where  $V(r)$  and  $V_{max}$  represent physical velocities and  $v_{max} = 1$ ,

the energy emitted by the nebula between radial velocities  $v$  and  $v + dv$  is proportional to

$$\int_{\tau} i(r) r \sin(\theta) r d\theta dr \quad (7)$$

where  $i(r)$  represents the emission per unit volume at radius  $r$  and  $\theta$  is the angle to the observer's line of sight. We adopt inner radius  $R = q$  and outer radius  $R = 1$ .

Setting  $i(r) \propto r^{-2\beta}$  (for a recombination or collisionally excited line emitted from a medium with an assumed density profile for the emitter  $\rho \propto r^{-\beta}$ ) then gives

$$\begin{aligned} i(v) dv &\sim \frac{dv}{\alpha v^{\frac{2\beta-3+\alpha}{\alpha}}} \int_{\theta_0}^{\theta_1} \cos^{\frac{2\beta-3}{\alpha}} \theta \sin \theta d\theta \\ &\sim \frac{dv}{v^{\frac{2\beta-3+\alpha}{\alpha}}} \left[ \frac{\cos^{\frac{2\beta-3+\alpha}{\alpha}} \theta}{2\beta-3+\alpha} \right]_{\theta_0}^{\theta_1} \end{aligned} \quad (8)$$

for  $\frac{2\beta-3}{\alpha} \neq -1$  where  $i(v) dv$  is the energy emitted in a volume element and  $\theta_0$  and  $\theta_1$  are the bounds of this element. The case  $\frac{2\beta-3}{\alpha} = -1$  results in a logarithmic relationship.

In the case of a “complete” nebula, i.e. one where the inner radius is vanishingly small in comparison to the outer radius, we obtain

$$i(v) dv \sim \pm \frac{du}{(2\beta-3+\alpha)v^{\frac{2\beta-1+\alpha}{\alpha}}} \left( 1 - v^{\frac{2\beta-3+\alpha}{\alpha}} \right) \quad (9)$$

If the nebula is not “complete”, that is to say, the inner radius is some fraction of the outer radius and the remnant is a detached shell, the above formula becomes valid only from  $v = 1$  to some critical value  $v' = q^\alpha$ . For  $v < v'$ , we obtain

$$i(v) dv \sim \pm \frac{dv}{(2\beta-3+\alpha)} \left( \frac{1}{q^\alpha} - 1 \right) \quad (10)$$

and therefore the top of the line is flat while the sides are sloping.

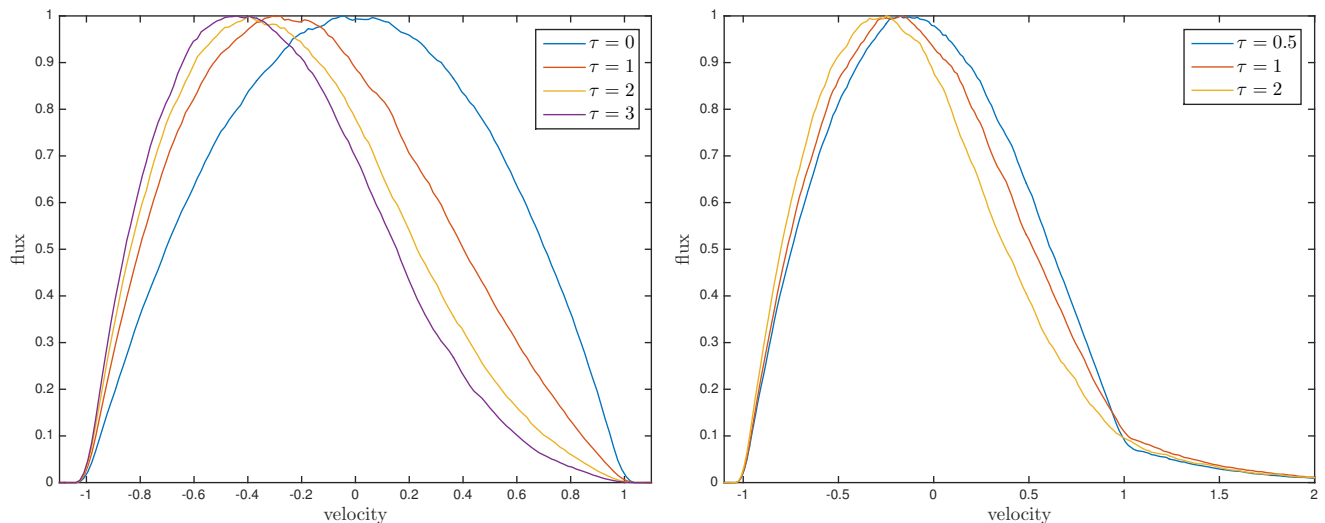
Crucially, the width of the flat section is determined by  $v' = q^\alpha$  or simply  $v' = q$  in the case where  $v \propto r$ , whilst the shape of the profile outside of the flat top is described by equation 9.

Profiles with a variety of shapes may be derived from these formulae depending on the relative values of  $\alpha$  and  $\beta$ . Here we consider three main families of curves:

- (i)  $i(v) \sim v^{-\gamma} - 1$  ( $\alpha > 0, 2\beta - 3 + \alpha > 0$ )
- (ii)  $i(v) \sim 1 - v^\gamma$  ( $\alpha > 0, 2\beta - 3 + \alpha < 0$ )
- (iii)  $i(v) \sim -\log v$  ( $\alpha > 0, 2\beta - 3 + \alpha = 0$ )

where  $\gamma$  is defined as  $\gamma = |\frac{2\beta-3+\alpha}{\alpha}|$ .

Models are presented for each of these cases, both for a complete nebula and for a shell structure with  $R_{in}/R_{out} = 0.2$ . A velocity profile  $v \propto r$  appropriate for supernova ejecta in the free expansion phase is used throughout. Values of  $\beta = 0, 1$  and  $2$  are adopted. Figure 4 illustrates the excellent agreement between the analytical case and the models. All fluxes are scaled to unity at the peak.



**Figure 5.** Benchmark models for line profiles with  $v \propto r$ ,  $i(r) \propto \text{constant}$  and a filled sphere with  $R_{in}/R_{out} = 0$ . Pure dust absorption models ( $\omega = 0$ ) are presented in the left-hand plots, whilst partially scattering models are presented on the right ( $\omega = 0.6$ ) as per [Lucy et al. \(1989\)](#) Models II and III. All resulting profiles have been scaled to unit flux at their peaks.

## 4.2 Comparison of DAMOCLES models with previously published results

In addition to the tests for optically thin lines described above, we also compared our outputs to those derived by [Lucy et al. \(1989\)](#) in order to assess the accuracy of the scattering and absorption aspects of the code. We consider two similar cases, equivalent to Models II and III of [Lucy et al. \(1989\)](#). In the first case, dust with zero albedo is uniformly distributed throughout a completely filled nebula with a velocity profile  $v \propto r$ . In the second case, the same scenario is considered but a medium of dust with albedo  $\omega = 0.6$  is considered.

In the first case, the profile may once again be derived analytically from the basic geometry using the fact that radiation will be attenuated by a factor  $e^{-2\tau_\nu v}$  between points with line of sight fractional velocities  $-v$  and  $+v$  where  $\tau_\nu$  is optical depth at frequency  $\nu$  from the centre to the outer edge of the ejecta. The line profile is therefore given by

$$\frac{I(v)}{I(-v)} = \exp(-2\tau_\nu v) \quad (11)$$

[Lucy et al. \(1989\)](#) presented several examples for both the analytical case of the perfect absorber and a Monte Carlo model for grains with  $\omega = 0.6$ . We present the same cases in Figure 5 and note that the resulting profiles exhibit the same features and shape. Of particular interest is the scattering wing that appears beyond the maximum velocity ( $v_{max} = 1$ ) on the red side of profiles in the case of the partial scatterer as a result of the packets doing work on the expanding sphere. This was noted by [Lucy et al. \(1989\)](#) as a potential diagnostic for the presence of dust in the ejecta of a supernova and we will discuss this further in Section 4.3.

## 4.3 The sensitivity of the variable parameters

It is of general interest to establish potential diagnostic signatures in the line profiles of supernovae and their rem-

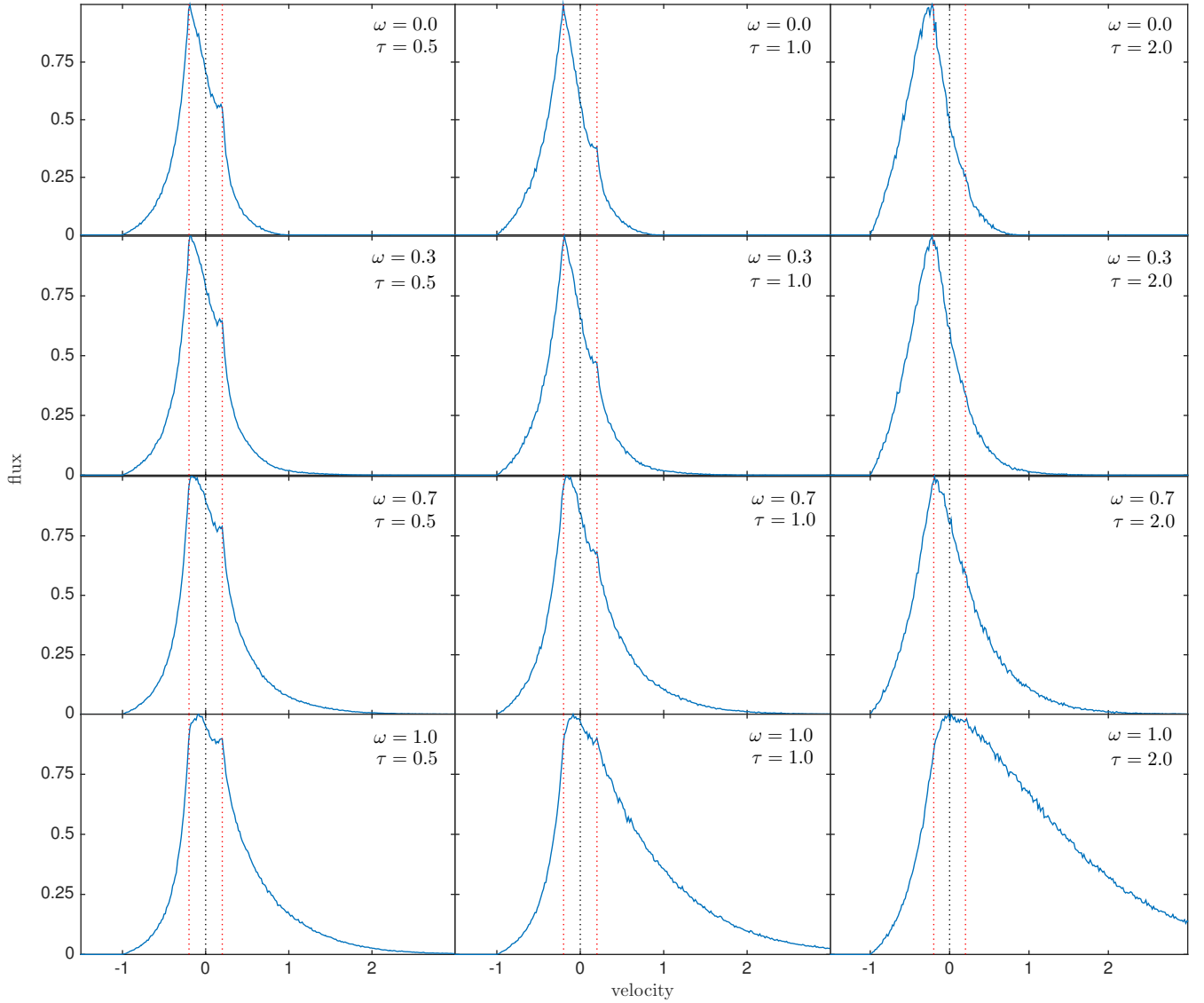
nants in order to trace dust formation more effectively. We here discuss the effects of the main parameters of interest, namely:

- the maximum line velocity  $V_{max}$
- the ejecta radius ratio  $R_{in}/R_{out}$
- the dust optical depth  $\tau$
- the dust albedo  $\omega$
- the density profile index  $\beta$ , where  $\rho \propto r^{-\beta}$

### 4.3.1 The maximum line velocity $V_{max}$

The maximum velocity is defined as the velocity at the outer edges of the line emitting region for a given line. The maximum velocity may vary between different spectral lines or doublets due to different locations of species having differing ionization thresholds. Clearly, the larger the maximum velocity used the wider the profile becomes. To some extent therefore the steepness of the density profile and the maximum velocity can act to counter each other since a steeper density distribution narrows the profile (see Section 4.3.5). The shape of the wings of the profiles, however, generally preclude much degeneracy in this aspect - the overall shape of the line profile can be used to determine the exponent of the density distribution to within a relatively small range.

More important is the effect that the maximum velocity has on the overall optical depth. Since the outer radius is calculated directly from the maximum velocity, the overall volume of the ejecta is determined solely by this value and the ratio of the inner and outer radii. The total dust optical depth to which the radiation is exposed can therefore be greatly affected by even a relatively small change in the maximum velocity for fixed values of the other parameters. Practically, however, the maximum velocity can usually be fairly well determined from the observations (identified as the point where the flux vanishes on the blue side) and may be further constrained through modelling.



**Figure 6.** Set of models with  $i(r) \propto r^{-4}$  (i.e.  $\beta = 2.0$ ),  $R_{in}/R_{out} = 0.2$ ,  $v(r) \propto r$  and  $v_{max} = 1$  illustrating the effects of varying  $\tau$  and  $\omega$ . Peak fluxes are scaled to unity.

#### 4.3.2 The ejecta radius ratio $R_{in}/R_{out}$

As already discussed in Section 4.1, the width of the flat top is determined solely by the ratio of the inner and outer radii, the exponent of the velocity profile and the maximum velocity. We assume that the velocity profile takes the form  $v \propto r$  from just a few months after the explosion as the supernova is in free expansion. For this case,  $R_{in}/R_{out}$  is given by

$$\frac{R_{in}}{R_{out}} = \frac{V_{min}}{V_{max}} \quad (12)$$

where it is often possible to constrain  $V_{min}$  and  $V_{max}$  to a relatively narrow range simply from the observed line profile.

The majority of spectral lines emitted from supernovae

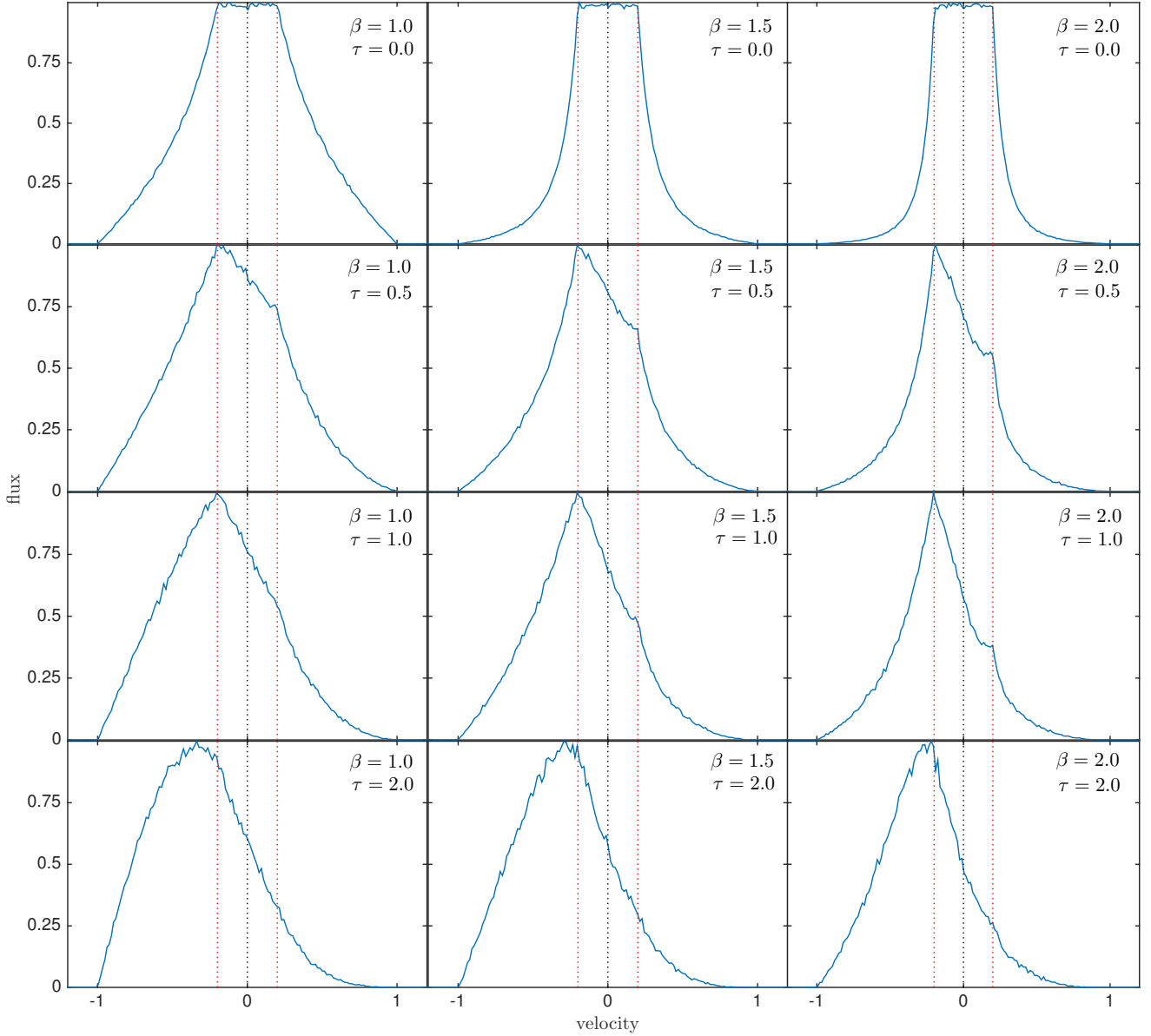
and supernova remnants are expected to have a flat top before dust attenuation effects since it is rare for these objects to form a completely filled nebula. However, even a very small amount of dust attenuation may result in the line profile appearing to be smoothed at its peak.

The effects of absorption by dust on a line profile for a complete nebula with  $R_{in}/R_{out} = 0$ , as opposed to a detached shell, are shown in Figure 5. All profiles have been scaled to unit flux at their peaks.

#### 4.3.3 The dust optical depth $\tau$ (detached shell case)

As expected, greater attenuation of the original line profile is seen on the red side (see Figures 7 and 6). The profiles are most revealing at lower dust optical depths since the



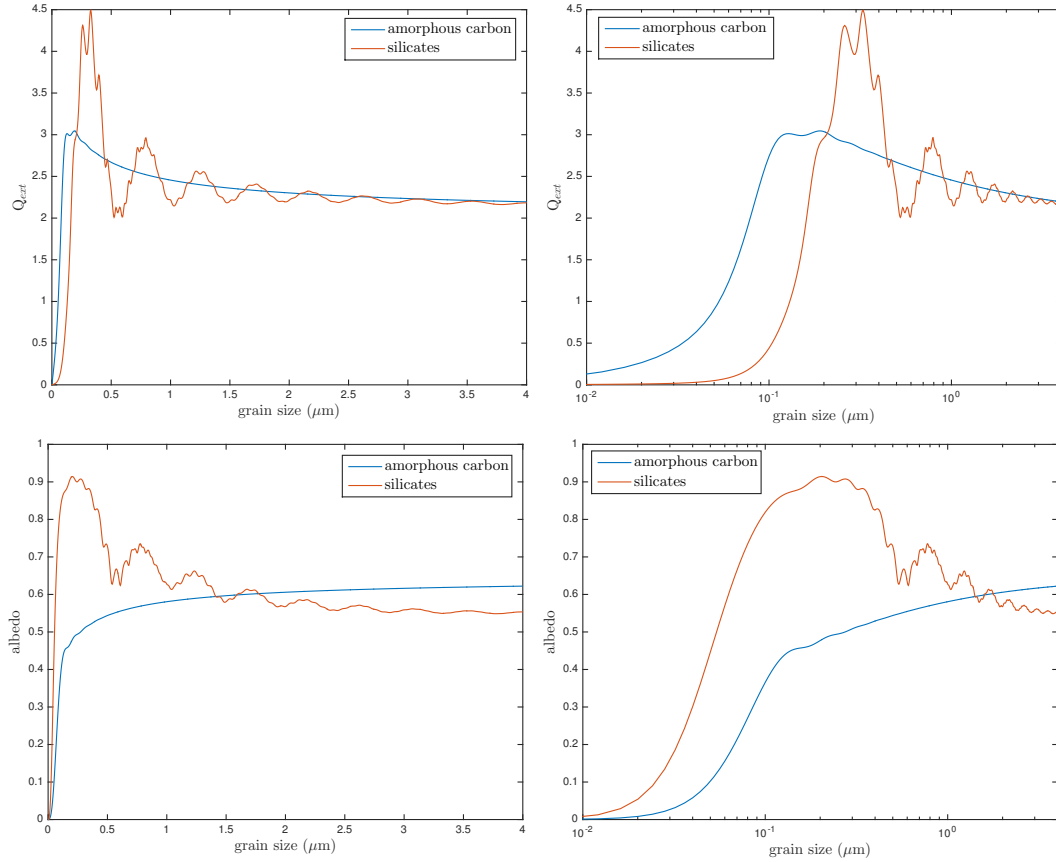


**Figure 7.** Set of models with  $i(r) \propto r^{-2\beta}$  for  $\beta = 1.0$  (left),  $\beta = 1.5$  (middle) or  $\beta = 2.0$  (right),  $\omega = 0$ ,  $R_{in}/R_{out} = 0.2$ ,  $v(r) \propto r$  and  $v_{max} = 1$  illustrating the effects of varying  $\tau$ . Peak fluxes are scaled to unity.

effects of the asymmetric absorption can be seen in different sections of the profiles and the profiles therefore tend to exhibit more features. The region of the profile that is most clearly affected by dust absorption is the flat-topped region. A small amount of absorption in this region results in a skewed profile, with a fraction of the flat-topped section removed. The peak becomes blue-shifted as a result, but only to the original value of  $-V_{min}$ , the minimum velocity corresponding to  $R_{in}$ . In addition to the attenuation in this region, the red wing of the profile is also somewhat reduced, and the blue wing somewhat increased relative to their original symmetric positions. The result is a relatively “jagged” looking profile, often with sharp changes at  $\pm V_{min}$ . The profile is generally asymmetric, although the degree of absorption in the flat-topped region may sometimes make it

seem as though the profile is in fact symmetric and uniformly blue-shifted (see Section 4.5 for further discussion). Observationally, these sharp features might become smoothed due to insufficient spectral resolution.

At high dust optical depths or when the ratio of the inner and outer radii is small, the entire profile is shifted to the blue and the peak moves beyond  $-V_{min}$  further into the blue. The profiles also tend to become more smooth and featureless. A set of models showing the effects of varying optical depths for different density profiles and dust albedos are presented in Figures 7 and 6 with  $R_{in}/R_{out} = 0.2$ .



**Figure 8.** Variation of albedo and extinction efficiency ( $Q_{ext}$ ) with grain size for amorphous carbon and silicates using Mie theory at  $\lambda = 658\mu\text{m}$ . Optical constants are from Zubko et al. (1996) and Draine & Lee (1984) respectively. A linear grain size scale is presented on the left and a log scale on the right.

#### 4.3.4 The dust albedo $\omega$

In the past, there has often been a focus on the effects of absorption by dust on the shapes of line profiles and less attention has been paid to the potential effects of scattering by dust. In fact, line profiles can be significantly affected by scattering of radiation. The greater attenuation of radiation received from the receding portion of the ejecta results in an asymmetry of the line profile whereby the majority of the observed emission is located bluewards of the peak. However, the effects of repeated dust scattering events within the ejecta can substantially alter the shape of a line profile and potentially can act to counter the blue-shifted asymmetry.

Not only does repeated scattering of photons increase the number of potential opportunities for a given photon to be absorbed but it also results in continuous shifting of the frequency of the photon to the red. The photon must do work on the expanding shell of dust in order to escape and thus many of the photons are reprocessed beyond the theoretical maximum velocity on the red side of the profile. Even in the case of dust grains with a relatively low albedo, a surprisingly persistent wing on the red side of the profile is seen, generally beyond the maximum theoretical velocity of the emitting region. In the case of strong dust scattering and high dust optical depths, this can actively result in a shift in the overall asymmetry of the profile, with the majority of

the emission being emitted redwards of the peak. The peak however, remains blue-shifted (for example the bottom left panel of Figure 6) or central (for example the bottom right panel of Figure 6). For the line profile to exhibit this feature requires the dust to be a nearly perfect scatterer and it is therefore unlikely that profiles of this sort will be frequently observed. See Figure 6 for a fuller illustration of the effects of varying  $\omega$  and  $\tau$ , and Figure 8 for plots of the relationship between albedo and dust grain size.

The combination of relatively low dust optical depths, initially flat-topped profiles, greater attenuation on the blue side with increased flux on the red side due to scattering can result in a profile that sometimes ends up appearing almost symmetrical, particularly if contaminants, such as narrow lines or blending with other broad lines, are present or if the resolution of the data is low. The potential for apparently symmetrical profiles that appear to have been uniformly blue-shifted should be noted (see Figures 7 and 6 for examples of this).

#### 4.3.5 Density profile $\rho \propto r^{-\beta}$

Whilst the density profile of the dust may have some effect on the resulting profiles, it is the initial emissivity profile (dependent on the gas density profile) that has the greatest effect on the shape of the line profile. In general, the steeper

the emissivity distribution, the narrower the line profile becomes. The sides of the line profile may become almost vertical for a very steep distribution since the majority of the emission then comes from a very narrow velocity range.

The dependence of the shape of the line profile on the emissivity distribution is described analytically in Section 4.1 for the case of very optically thin dust. However, for even fairly low dust optical depths, the density profile plays a significant role in determining the contour of the line profile where it is affected by dust absorption. As previously discussed, at relatively small optical depths for reasonable  $R_{in}/R_{out}$ , a section of the flat-topped region is removed resulting in a peak at  $-V_{min}$ . The shape of the profile in this region is significantly affected by the density profile. Shallow density profiles (low  $\beta$ ) produce a virtually linear variation in flux between  $-V_{min}$  and  $+V_{min}$  (for example the profiles in the left column of Figure 7). For a fixed dust optical depth, the steeper the distribution becomes, the more concave the profile becomes between  $-V_{min}$  and  $+V_{min}$ , ultimately resulting in a clear shoulder to the profile at  $+V_{min}$  (for example the profiles in the right column of Figure 7). For extremely steep density distributions this can result in a double peaked profile with a trough to the red of  $V = 0$ . A illustration of the effects on the line profiles of varying  $\beta$  and  $\tau$  is shown in Figure 7. As previously noted, these features may not be apparent in observed line profiles with poor spectral resolution due to blending.

#### 4.4 Inferring properties of the dust from the models

The presence of an extended red wing at large positive velocities in combination with increased extinction on the red side at smaller positive velocities can allow the values of  $\tau$  and  $\omega$  to be well constrained. Where this occurs it is possible to translate these values into a dust mass and grain size for a given species or combination of species using grain optical properties and Mie theory (see Figure 8).

For amorphous carbon, albedo generally increases with grain size. The presence and extent of any scattering wing on the red side of the observed profile can therefore help to place limits on the grain radius. However, the greater the grain radius used the smaller the available cross-section for interaction. Larger masses of dust are therefore required to fit the same degree of absorption if a larger grain size is used. This is in contrast to SED radiative transfer modelling where larger grain sizes generally result in less dust being required to fit the IR portion of the SED (W15). These two techniques in tandem may therefore provide limits on grain sizes for different species or combinations thereof.

It is known that the use of different optical properties may substantially alter dust masses derived using SED fitting for a given species of specific grain size (e.g. Owen & Barlow (2015)). However, the use of different sets of optical properties in our models seems to have only a minor effect on our results.

#### 4.5 The wavelength dependence of dust absorption

The greater the dust optical depth, the more attenuation of the line there is. As expected, the red side of the profile

suffers a greater degree of absorption than the blue side. The resulting asymmetry is somewhat more complex than perhaps previously thought however. Dust has repeatedly been cited as the agent responsible for the apparent blue-shifting of line profiles in supernovae in the manner of the profiles presented in Figure 5; that is, relatively high optical depths result in an overall shift of the entire profile towards the blue. The relationship between the blueshifting of the peaks of profiles and their wavelength has been discussed by several authors in relation to dust formation (Smith et al. 2012; Fransson et al. 2014; Gall et al. 2014).

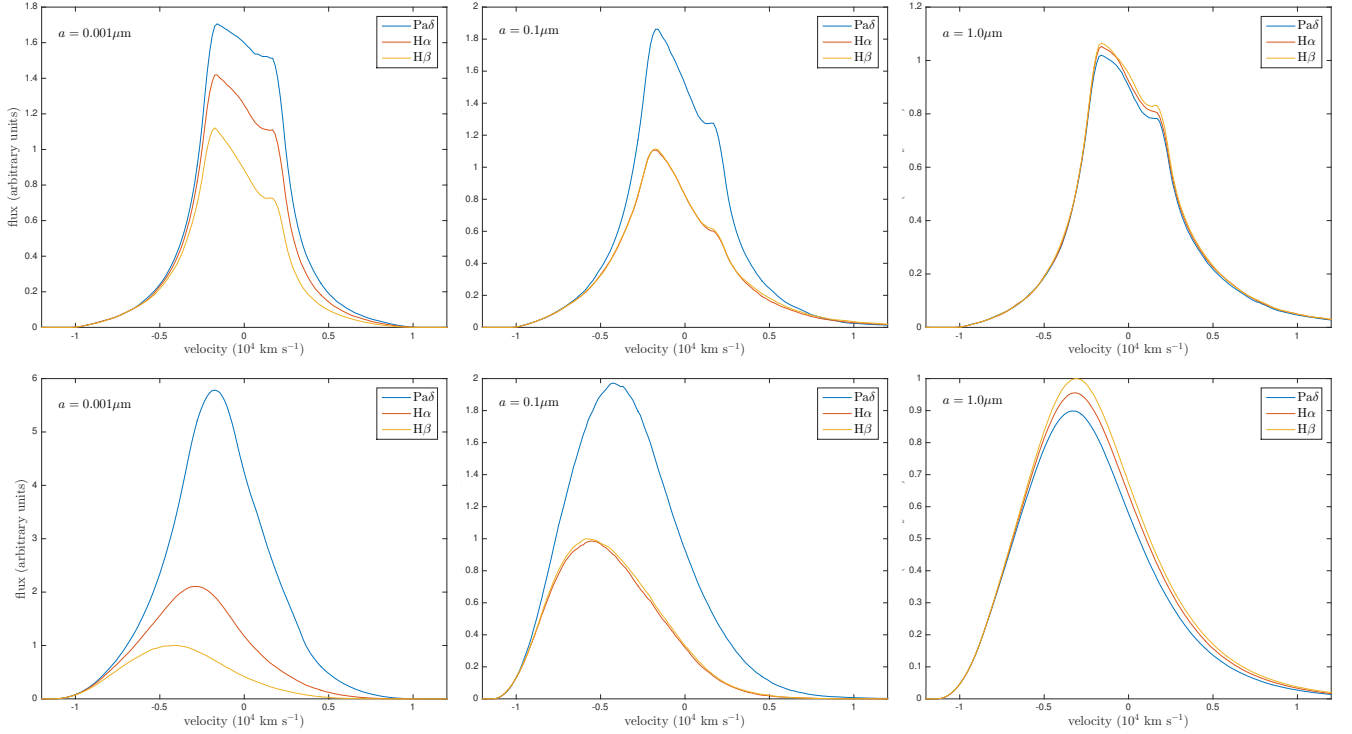
In practice a relatively large dust optical depth is required to actively shift the peak of the profile bluewards of its natural  $V_{min}$  (corresponding to the velocity at the inner radius of the shell) unless this value is very small in comparison to  $V_{max}$  i.e. the profile originally had a very narrow flat top. In many cases it seems likely that the dust may not be optically thick and the blue-shifting of the peak of the profile is a result of attenuation in the flat-topped section (close to  $R_{in}$ ). The peak would then tend to be located at  $-V_{min}$ .

Since dust absorption is wavelength dependent for  $2\pi a < \lambda$ , one might expect the position of the peak line flux to be dependent on the wavelength of the line being considered. We note here that whilst peak velocity variations as a function of line wavelength may occur in cases of high dust optical depths or small  $R_{in}/R_{out}$ , this may not be the case for many lines emitted from the ejecta of supernovae with low dust optical depths. The wavelength-dependence of dust absorption instead can result in differing degrees of extinction in the flat-topped region of each profile but still leave the peak at its blue-shifted position of  $-V_{min}$ . If this is the case then there would be no reason to expect a variation in the position of the peaks of profiles to be correlated with the wavelength dependence of dust absorption. Rather one would expect it potentially to trace the location of different ions within the ejecta, possibly with different  $V_{min}$  observed for different species.

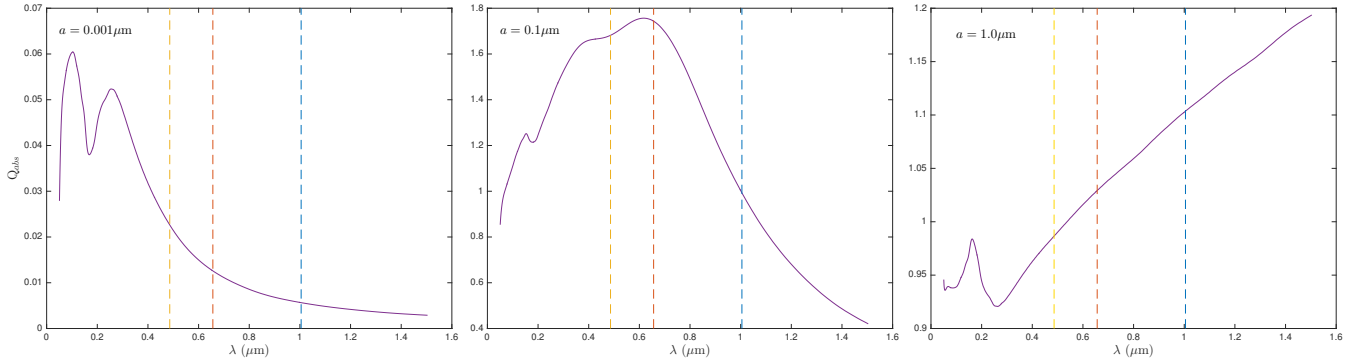
For lines from the same ion, for example the Balmer and Paschen lines of H I, we might expect to see peaks at the same position but differing degrees of absorption. At high spectral resolutions, it might be possible to detect differences in the shape of the line profiles, particularly between  $-V_{min}$  and  $+V_{min}$  where the steepness of the incline traces the degree of dust absorption. This can be seen in Figure 9 where we illustrate the effects of the wavelength dependence of dust absorption for three lines, H $\alpha$  (6563Å), H $\beta$  (4861Å) and Pa $\delta$  (10049Å). All lines were modelled using three different grain sizes and for both optically thin and thick dust cases. We also show the variation of the absorption efficiency with wavelength at three different amorphous carbon grain sizes in Figure 10.

## 5 RESULTS FOR SN 1987A

We have modelled the H $\alpha$  line of SN 1987A at days 714, 806, 1862, 2211, 2875, 3500 and 3604, and the [O I]  $\lambda$ 6300,6363 Å doublet at days 714, 806, 1054 and 1478. After day 3604 the H $\alpha$  profile begins to become dominated by emission from the reverse shock and the structure of the emitting region may no longer be approximated by a single shell model as we do



**Figure 9.** Model line profiles for H $\alpha$  (6563Å in red), H $\beta$  (4861Å in yellow) and Pa $\delta$  (10049Å in blue) for optically thin (*upper*) and optically thick (*lower*) cases respectively. All models adopted density profile  $\rho(r) \propto r^{-4}$  (i.e.  $\beta = 2$ ), velocity profiles  $v(r) \propto r$  and radii ratio  $R_{in}/R_{out} = 0.2$ . The grain radii used were  $a = 0.001 \mu\text{m}$  (left),  $a = 0.1 \mu\text{m}$  (middle) and  $a = 1.0 \mu\text{m}$  (right). All the above models used amorphous carbon.



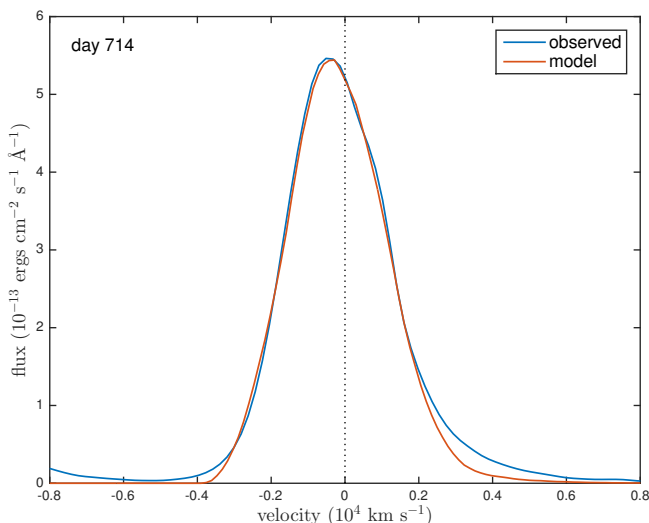
**Figure 10.** The variation of amorphous carbon dust absorption efficiency with grain size. The grain radii plotted are  $a = 0.001 \mu\text{m}$  (left),  $a = 0.1 \mu\text{m}$  (middle) and  $a = 1.0 \mu\text{m}$  (right). The vertical lines mark the wavelengths of H $\alpha$  (6563Å in red), H $\beta$  (4861Å in yellow) and Pa $\delta$  (10049Å in blue).

here (Fransson et al. 2014). The [O I]  $\lambda 6300, 6363 \text{ Å}$  doublet becomes too weak to model after day 1478 (see Figure 1). We continue to adopt a velocity profile  $V(r) = \frac{V_{max}}{R_{max}} r$  and treat the variable parameters listed at the start of Section 4.3. Whilst the albedo and optical depth are not varied directly, they are altered by adjusting the dust mass,  $M_{dust}$ , and the grain size,  $a$ , which together determine the albedo and optical depth via Mie theory and the optical properties of the dust.

In all models, the ejecta occupies a shell with inner radius  $R_{in}$  and outer radius  $R_{out}$ . Packets are emitted accord-

ing to a smooth density profile assuming recombination or collisional excitation such that  $i(r) \propto \rho(r)^2 \propto r^{-2\beta}$ . Initially the dust is considered to have a smooth density distribution and is assumed to be coupled to the gas so as to follow the same radial profile. A clumped distribution of dust is considered later (see Section 5.2).

We estimate the electron scattering optical depths at a temperature of 10,000K between  $R_{in}$  and  $R_{out}$  based on the observed fluxes of the H $\alpha$  line. A temperature of 10,000K is likely too high at the epochs considered but we adopt it in order to be sure not to underestimate electron scattering



**Figure 11.** MRN smooth dust fit to the day 714 H $\alpha$  line of SN 1987A illustrating the underestimation of the red scattering wing for small grain sizes. Model parameters are the same as the smooth dust fit for day 714 (Table 3) except for the grain size distribution and dust mass:  $M_{\text{dust}} = 8.0 \times 10^{-6} M_{\odot}$ ,  $a_{\text{min}} = 0.005 \mu\text{m}$ ,  $a_{\text{max}} = 0.25 \mu\text{m}$  and  $n(a) \propto a^{-3.5}$ .

**Table 2.** Observed luminosities of the H $\alpha$  line and estimated electron scattering optical depths from  $R_{\text{in}}$  to  $R_{\text{out}}$  for the radii detailed in Tables 3 to 4 based on an assumed temperature of 10,000K.

day	H $\alpha$ luminosity ( $10^{37} \text{ erg s}^{-1}$ )	$\tau_e$
714	1.36	$1.44 \times 10^{-2}$
806	0.57	$8.40 \times 10^{-3}$
1862	0.063	$1.59 \times 10^{-3}$
2875	0.0019	$2.19 \times 10^{-4}$
3604	0.0010	$1.49 \times 10^{-4}$

optical depths. The values we calculate and the observed H $\alpha$  luminosities are listed in Table 2. The electron scattering optical depths are negligibly small and we therefore do not include electron scattering in these models.

There is rarely a unique set of parameters that provide the best fit to the data. However, the majority of the parameters of interest can be well constrained by our modelling by considering different elements of the shape of the profile. In particular, by constructing fits to the data using extremising limits for the grain size, credible lower and upper bounds on the possible dust mass formed within the ejecta may be derived. We present here fits to the data obtained using both small and large values of the grain radius  $a$  since it is the grain size which has the most significant effect on the overall dust mass required to reproduce the line profile (see Section 4).

All of our models are of a dusty medium composed solely of amorphous carbon grains. We use the optical constants from the BE sample presented in Zubko et al. (1996). Although previous SED modelling of SN 1987A has limited

the fraction of silicates present in the dusty ejecta to a maximum of 15% (Ercolano et al. (2007), W15), it is useful to consider the effects on our models of using silicate dust. We discuss this in detail in Section 5.5.

For each profile, the maximum velocity is identified from the data as the point where the line vanishes on the blue side. The equivalent point on the red side is indeterminate from observations due to the effects of dust scattering. Similarly, the “corner” of the flat topped section of the profile on the blue side allows the minimum velocity at radius  $R_{\text{in}}$  to be ascertained. As discussed in Section 4, this allows the ratio of the inner and outer radii of the supernova ejecta to be determined since  $R_{\text{in}}/R_{\text{out}} = V_{\text{min}}/V_{\text{max}}$ . The outer radius is calculated from the epoch and maximum velocity.

The only parameters that then remain to be determined are the exponent of the density profile  $\beta$ , the mean grain size radius and the total dust mass. The shape of the blue wing is solely a product of the density profile and the dust mass; the height and shape of the red wing is a product of these and also of the scattering efficiency of the grains (the albedo  $\omega$ ); and the extent and shape of the asymmetry in the flat-topped portion of the profile is a function of only the total dust optical depth determined by the dust mass and the grain size. By iterating over these three parameters therefore, an excellent fit to the data can usually be obtained.

Models are produced in the same manner for the [O I]  $\lambda 6300, 6363 \text{ \AA}$  doublet as for the single H $\alpha$  line, with each component of the doublet being modelled independently and the resulting profiles added according to a specified ratio. Although the theoretical intrinsic flux ratio is 3.1 for optically thin emission, the actual ratio between the two components can be affected by self-absorption (Li & McCray 1992) and we therefore left it as a free parameter. The deduced doublet ratios are listed in Tables 3, 4 and 5.

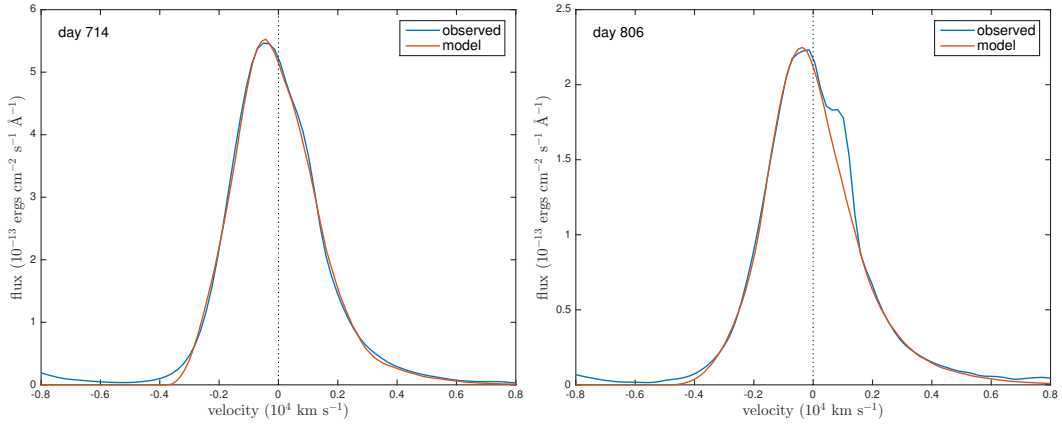
For all lines, though particularly at later epochs, even small fluctuations in the adopted value of the continuum level can have a substantial effect on the fit to the resulting profile. Since it is not feasible to establish the level of the continuum so precisely, the value of the continuum has been left as a free parameter that may be adjusted (to within sensible margins) in order to allow for the widest possible dust mass range to be determined. We generally find it is necessary to assume a continuum level that is slightly lower where the adopted dust mass is higher.

Fits to the H $\alpha$  line profile at days 2211 and 3500 are omitted for the sake of space but are very similar in nature to those of days 1862 to 3604. Parameters for the models at all epochs including these are detailed in Tables 3 to 5.

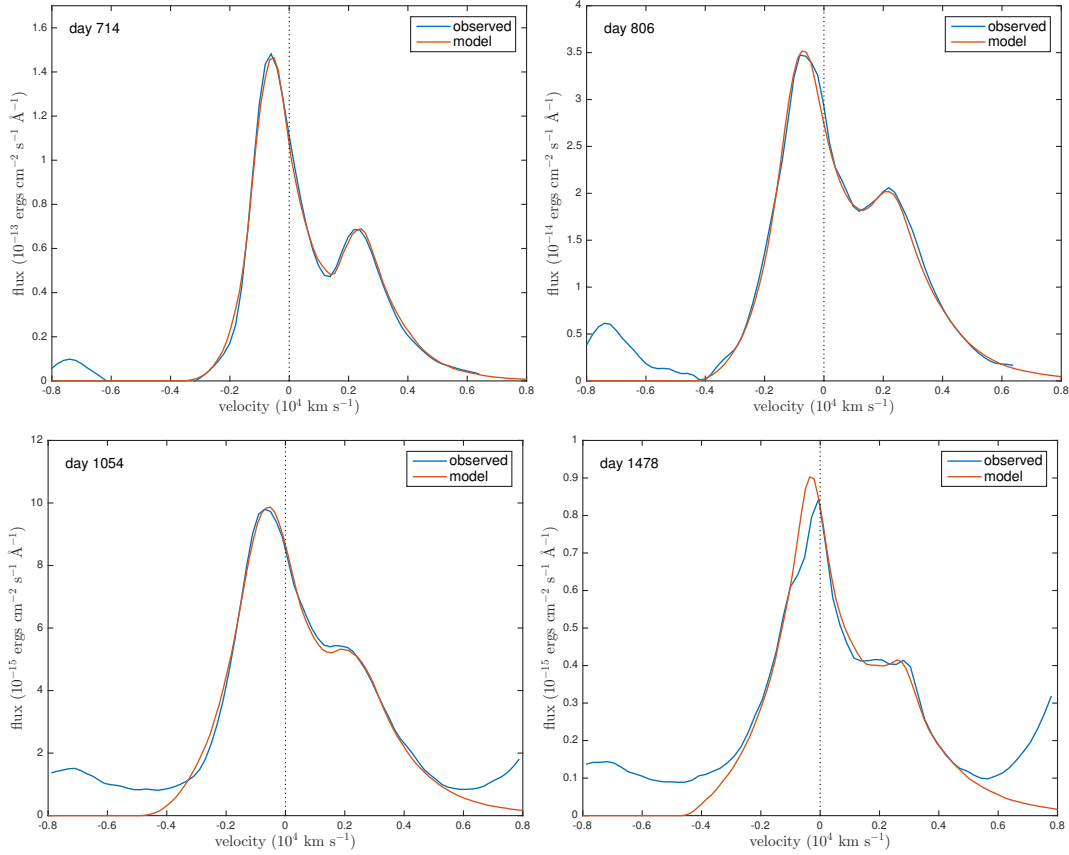
## 5.1 Smooth Density Models for SN 1987A

Even at the earliest epochs there is a substantial wing on the red side of the H $\alpha$  line profile that cannot be fitted by scattering from moving grains with a low albedo. The minimum required albedo is approximately  $\omega \approx 0.5$  implying relatively large grain radii. As previously discussed, the larger the grain size the larger the mass of dust required to reproduce the same optical depth. Figure 11 illustrates the a fit for the day 714 H $\alpha$  profile for the case where a classic MRN (Mathis et al. 1977) grain size distribution is adopted, with  $a_{\text{min}} = 0.005 \mu\text{m}$ ,  $a_{\text{max}} = 0.25 \mu\text{m}$  and  $n(a) \propto a^{-3.5}$ . It can be seen clearly that the extended red wing is signifi-





**Figure 12.** Best smooth dust fit to the SN 1987A H $\alpha$  line at day 714 (*left*) and day 806 (*right*) for the parameters detailed in Table 3 with amorphous carbon grains of radius  $a = 0.35\mu\text{m}$ .



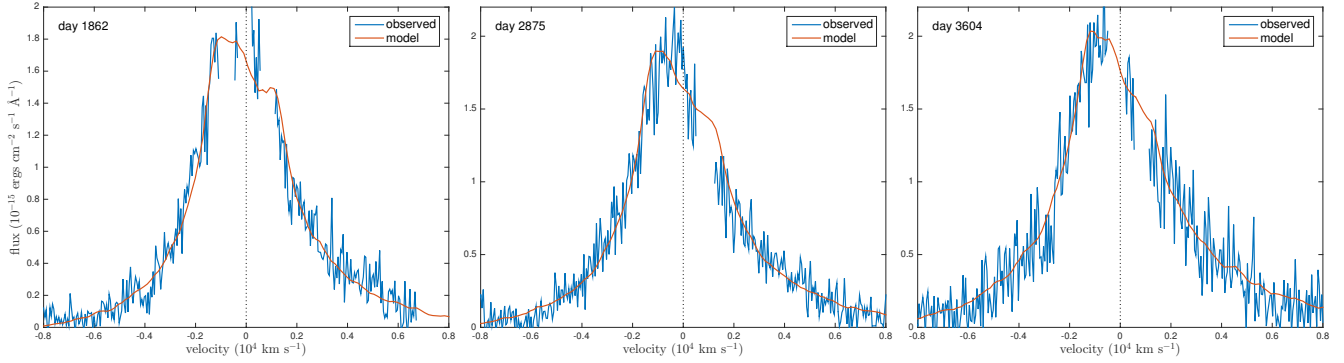
**Figure 13.** Best smooth dust fit to the SN 1987A [O I]  $\lambda 6300,6363 \text{ \AA}$  doublet at day 714 (*upper left*), day 806 (*upper right*), day 1054 (*lower left*) and day 1478 (*lower right*) for the parameters detailed in Table 3 with amorphous carbon grains of radius  $a = 0.35\mu\text{m}$ .

cantly underestimated. Since the albedo of amorphous carbon grains varies significantly with grain radius (see Figure 8) we can establish a strong lower bound to the mean dust grain radius, which we estimate to be  $a \geq 0.35\mu\text{m}$ . This is the smallest grain size that is still capable of reproducing the red scattering wing at all epochs and we therefore use this lower limit value throughout our smooth density modelling.

The ejecta inner and outer radii are calculated at each

epoch from the maximum velocity used, the day number and the specified ratio  $R_{in}/R_{out}$ . The radii generated are consistent with those used in previous models of SN 1987A (Ercolano et al. 2007; Wesson et al. 2015). Figures 12 to 14 show the best fits to the data for days 714 to 3604 whilst Table 3 details the parameters used.

It can be seen that, in order to reproduce the blueshifts seen in the [O I]  $\lambda 6300,6363 \text{ \AA}$  doublet, considerably larger



**Figure 14.** Best smooth model fit to the SN 1987A H $\alpha$  line at days 1862, 2875 and 3604 for parameters detailed in Table 3 with amorphous carbon grains of radius  $a = 0.35\mu\text{m}$ .

**Table 3.** Details of the parameters used for the best fitting smooth models of SN 1987A with amorphous carbon grains of radius  $a = 0.35\mu\text{m}$ . Optical depths are given from  $R_{in}$  to  $R_{out}$ .

	day	$V_{max}$ (km s $^{-1}$ )	$R_{in}/R_{out}$	$\beta$	$M_{dust}$ ( $M_{\odot}$ )	$R_{out}$ (cm)	$R_{in}$ (cm)	[O I] ratio	$\tau_{H\alpha}$	$\tau_V$
[O I]	714	3250	0.07	2.9	$1.15 \times 10^{-4}$	$2.00 \times 10^{16}$	$1.40 \times 10^{15}$	2.6	3.60	7.20
[O I]	806	4000	0.06	2.4	$1.80 \times 10^{-4}$	$2.79 \times 10^{16}$	$1.67 \times 10^{15}$	2.3	2.86	5.71
[O I]	1054	4300	0.05	2.1	$2.80 \times 10^{-4}$	$3.92 \times 10^{16}$	$1.96 \times 10^{15}$	2.7	2.23	4.45
[O I]	1478	4500	0.04	1.7	$3.50 \times 10^{-4}$	$5.75 \times 10^{16}$	$2.30 \times 10^{15}$	3.0	1.30	2.60
H $\alpha$	714	3250	0.25	1.2	$2.50 \times 10^{-5}$	$2.00 \times 10^{16}$	$5.01 \times 10^{15}$		0.61	1.23
H $\alpha$	806	4000	0.22	1.9	$4.50 \times 10^{-5}$	$2.79 \times 10^{16}$	$6.13 \times 10^{15}$		0.52	1.05
H $\alpha$	1862	8500	0.15	1.9	$6.00 \times 10^{-4}$	$1.37 \times 10^{17}$	$2.05 \times 10^{16}$		0.35	0.70
H $\alpha$	2211	9000	0.14	1.9	$1.10 \times 10^{-3}$	$1.72 \times 10^{17}$	$2.41 \times 10^{16}$		0.42	0.83
H $\alpha$	2875	9500	0.14	1.9	$1.80 \times 10^{-3}$	$2.36 \times 10^{17}$	$3.30 \times 10^{16}$		0.36	0.72
H $\alpha$	3500	10000	0.14	1.9	$4.00 \times 10^{-3}$	$3.02 \times 10^{17}$	$4.23 \times 10^{16}$		0.49	0.98
H $\alpha$	3604	10250	0.13	1.9	$5.00 \times 10^{-3}$	$3.19 \times 10^{17}$	$4.15 \times 10^{16}$		0.55	1.10

**Table 4.** Details of the parameters used for the best fitting clumped models of SN 1987A with amorphous carbon grains of radius  $a = 0.6\mu\text{m}$ . Optical depths are given from  $R_{in}$  to  $R_{out}$ .

	day	$V_{max}$ (km s $^{-1}$ )	$R_{in}/R_{out}$	$\beta$	$M_{dust}$ ( $M_{\odot}$ )	$R_{out}$ (cm)	$R_{in}$ (cm)	[O I] ratio	$\tau_{H\alpha}$	$\tau_V$
[O I]	714	3250	0.07	2.7	$4.50 \times 10^{-4}$	$2.00 \times 10^{16}$	$1.20 \times 10^{15}$	2.3	5.09	10.19
[O I]	806	4000	0.06	2.3	$7.00 \times 10^{-4}$	$2.79 \times 10^{16}$	$1.67 \times 10^{15}$	2.1	4.53	9.07
[O I]	1054	4300	0.05	2.3	$1.30 \times 10^{-3}$	$3.92 \times 10^{16}$	$1.96 \times 10^{15}$	2.3	4.33	8.67
[O I]	1478	4500	0.04	2.0	$1.60 \times 10^{-3}$	$5.75 \times 10^{16}$	$2.30 \times 10^{15}$	3.1	2.74	5.48
H $\alpha$	714	3250	0.25	1.4	$7.00 \times 10^{-5}$	$2.00 \times 10^{16}$	$5.01 \times 10^{15}$		0.87	1.74
H $\alpha$	806	4000	0.22	1.8	$1.20 \times 10^{-4}$	$2.79 \times 10^{16}$	$6.13 \times 10^{15}$		0.79	1.59
H $\alpha$	1862	8500	0.14	1.9	$1.65 \times 10^{-3}$	$1.37 \times 10^{17}$	$1.91 \times 10^{16}$		0.48	0.96
H $\alpha$	2211	9000	0.14	1.9	$4.00 \times 10^{-3}$	$1.72 \times 10^{17}$	$2.41 \times 10^{16}$		0.74	1.49
H $\alpha$	2875	9500	0.12	2	$1.00 \times 10^{-2}$	$2.36 \times 10^{17}$	$2.83 \times 10^{16}$		0.96	1.93
H $\alpha$	3500	10000	0.12	2	$1.80 \times 10^{-2}$	$3.02 \times 10^{17}$	$3.63 \times 10^{16}$		1.08	2.16
H $\alpha$	3604	10250	0.12	2	$2.30 \times 10^{-2}$	$3.19 \times 10^{17}$	$3.83 \times 10^{16}$		1.21	2.42

dust masses are required than to fit the H $\alpha$  line. Although the same maximum velocities and therefore outer radii are used in our [O I] and H $\alpha$  models, the inner radii are significantly smaller and the density distribution much steeper for the [O I] models. This implies that the [O I] is steeper towards the centre of the ejecta whereas the H $\alpha$  is more diffuse. This is broadly in agreement with 3D explosion dynamics models that suggest that a few hours after the explosion

the heavier elements will, in comparison to H I, be located more centrally in the ejecta with “bullets” of heavier material reaching the outer edges (Hammer et al. 2010). If dust is forming in the inner regions of the ejecta then the majority of the [O I] emission must travel through the newly formed dust whereas the more diffuse H $\alpha$  emission has a greater chance of escaping unaffected. This may explain the differ-

**Table 5.** Details of the parameters used for the best fitting clumped models of SN 1987A with amorphous carbon grains of radius  $a = 3.5\mu\text{m}$ . Optical depths are given from  $R_{in}$  to  $R_{out}$ .

	day	$V_{max}$ ( $\text{km s}^{-1}$ )	$R_{in}/R_{out}$	$\beta$	$M_{dust}$ ( $M_{\odot}$ )	$R_{out}$ (cm)	$R_{in}$ (cm)	[O I] ratio	$\tau_{H\alpha}$	$\tau_V$
[O I]	714	3250	0.07	2.9	$3.50 \times 10^{-3}$	$2.00 \times 10^{16}$	$1.40 \times 10^{15}$	2.5	5.32	10.64
[O I]	806	4000	0.06	2.3	$5.00 \times 10^{-3}$	$2.79 \times 10^{16}$	$1.67 \times 10^{15}$	2.1	4.72	9.45
[O I]	1054	4300	0.05	2.3	$8.00 \times 10^{-3}$	$3.92 \times 10^{16}$	$1.96 \times 10^{15}$	2.5	3.89	7.78
[O I]	1478	4500	0.04	1.9	$1.10 \times 10^{-2}$	$5.75 \times 10^{16}$	$2.30 \times 10^{17}$	2.8	2.77	5.54
H $\alpha$	1862	8500	0.14	1.9	$2.00 \times 10^{-2}$	$1.37 \times 10^{17}$	$1.91 \times 10^{16}$		0.85	1.70
H $\alpha$	2211	9000	0.14	1.9	$3.30 \times 10^{-2}$	$1.72 \times 10^{17}$	$2.41 \times 10^{16}$		0.89	1.78
H $\alpha$	2875	9500	0.12	2	$8.00 \times 10^{-2}$	$2.36 \times 10^{17}$	$2.83 \times 10^{16}$		1.15	2.30
H $\alpha$	3500	10000	0.12	2	$1.50 \times 10^{-1}$	$3.02 \times 10^{17}$	$3.63 \times 10^{16}$		1.31	2.62
H $\alpha$	3604	10250	0.12	2	$1.70 \times 10^{-1}$	$3.19 \times 10^{17}$	$3.83 \times 10^{16}$		1.33	2.67

ence between the dust masses needed for the [O I] and H $\alpha$  models.

## 5.2 Clumped Models for SN 1987A

It has been shown through the modelling of optical-IR SEDs that when dust is assumed to have a clumped distribution the derived dust masses can be significantly larger than if the dust is distributed smoothly between the inner and outer radii (e.g. Owen & Barlow (2015)). We present two sets of fits to the line profile based on the clumped dust modelling of W15, one set with a minimum grain size and one set with a maximum grain size. Each fit is based on the best fitting smooth model such that the photon packets are emitted assuming a smooth radial density profile. However, the dust is no longer coupled to the gas but instead is located entirely in clumps of size  $R_{out}/50$ . The clumps are distributed stochastically between  $R_{in}$  and  $R_{out}$  with the probability of a given grid cell being a clump proportional to  $r^{-\beta}$  where  $i(r) \propto r^{-2\beta}$ . The number of clumps used is determined by the clump filling factor  $f$  which is kept constant at  $f = 0.1$ . All properties are fixed from the smooth models with the exception of the grain radius, density profile exponent ( $\beta$ ) and the total dust mass.

Models were again constructed using the smallest possible grain size ( $a=0.6\mu\text{m}$  in the clumped case) in order to derive minimum dust masses for clumped distributions. By considering the extent of the red scattering wing, upper limits to the grain size were also derived with the purpose of limiting the maximum dust mass at each epoch. By steadily reducing the grain size from an initial value of  $5\mu\text{m}$  (motivated by the maximum possible grain size derived by W15 for their day 8515 model), we produced a set of models with a maximum grain radius of  $a = 3.5\mu\text{m}$ .

For all but the H $\alpha$  line at days 714 and 806 a similar fit could be obtained with either grain size (see Figures 15 to 19). However, for H $\alpha$  at days 714 and 806 even a small change to the grain radius resulted in a significantly poorer fit, either over- or under-estimating the red wing. We therefore conclude that the dust mass estimates produced for the H $\alpha$  lines at days 714 and 806 for a grain radius of  $a = 0.6\mu\text{m}$  are the best estimates of the dust mass at this epoch. Details of the parameters used in all clumped models are presented in Tables 4 and 5 and fits are presented in Figures 15 to 19.

## 5.3 The effect of clumping

As in the case of SED radiative transfer models, the dust masses required to reproduce the observations in the clumped scenario are considerably higher than for the smooth scenario. The dust masses differ between our smooth and clumped models by a factor of approximately 3. This does not take into account the increase in grain radius between the two cases however. This increase accounts for a reasonable fraction of this difference. We estimate the effects of clumping alone to increase the required dust mass by a factor of approximately 1.5-2.0 from the smooth case.

The increase in grain size from the smooth case to the clumped case is necessary in order to have a slightly larger albedo. Grains of radius  $a = 0.35\mu\text{m}$  do not reproduce the red side of the profiles well for a clumped medium. This is because when the dust is located in clumps the radiation is subject to less scattering as well as to less absorption. The reduction in scattering appears not to be compensated for by the increased dust mass and a larger grain radius is therefore required, particularly at day 714. A grain radius of  $a = 0.6\mu\text{m}$  is therefore used throughout the clumped models as the smallest possible grain size capable of reproducing the observed profiles, although this still underestimates the red wing in some instances.

## 5.4 The effect of a grain size distribution

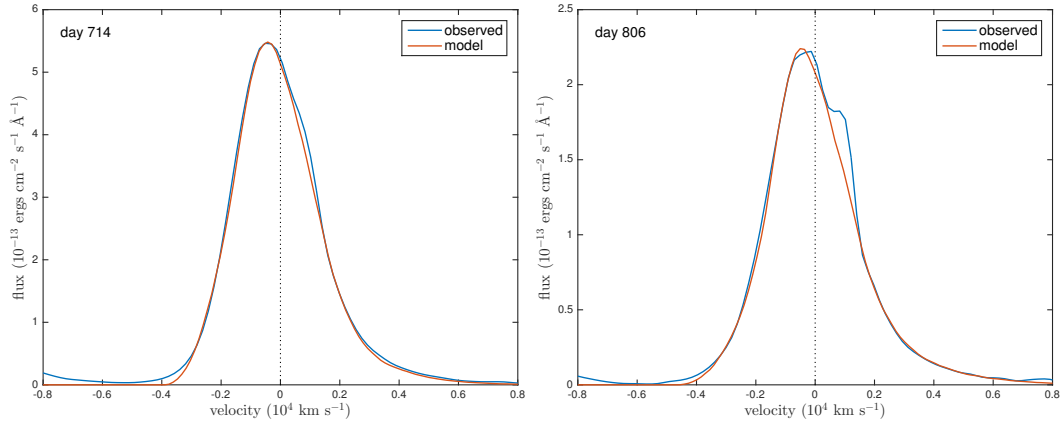
It is important to consider the potential effect on the dust mass of modelling a grain size distribution instead of a single grain size. For a grain size distribution the overall extinction cross section,  $C_{ext}$ , at a given wavelength is

$$C_{ext} = \int_{a_{min}}^{a_{max}} Q_{ext}(a)n(a)\pi a^2 da \quad (13)$$

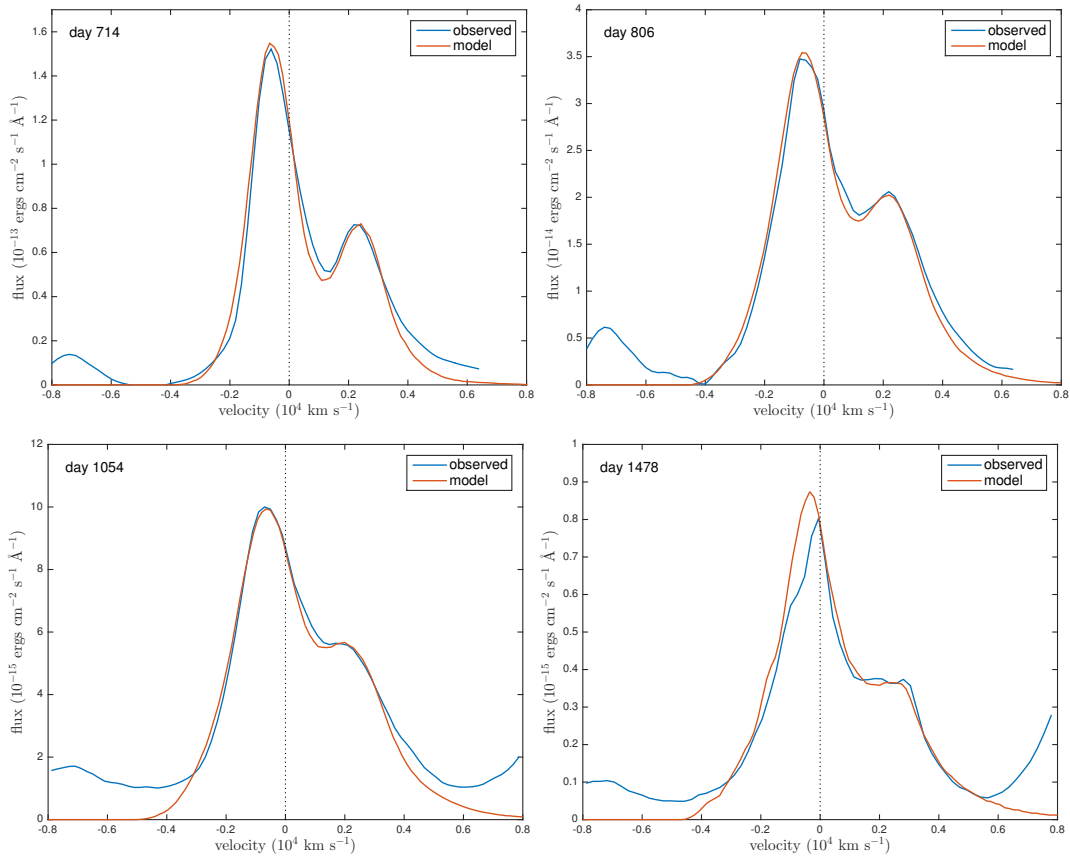
where  $Q_{ext}(a)$  is the extinction efficiency for a grain size  $a$  and  $n(a)$  is the number of grains with size  $a$ . The overall extinction efficiency is then

$$Q_{ext} = \frac{C_{ext}}{\int_{a_{min}}^{a_{max}} n(a)\pi a^2 da} \quad (14)$$

The scattering cross-section  $Q_{sca}$  is similarly calculated. As a result of these calculations, there is rarely a single grain size that has the same albedo and extinction efficiency as a



**Figure 15.** Best clumped dust fit to the SN 1987A H $\alpha$  line at day 714 (*left*) and day 806 (*right*) for the parameters detailed in Table 4 with amorphous carbon grains of radius  $a = 0.6\mu\text{m}$ .

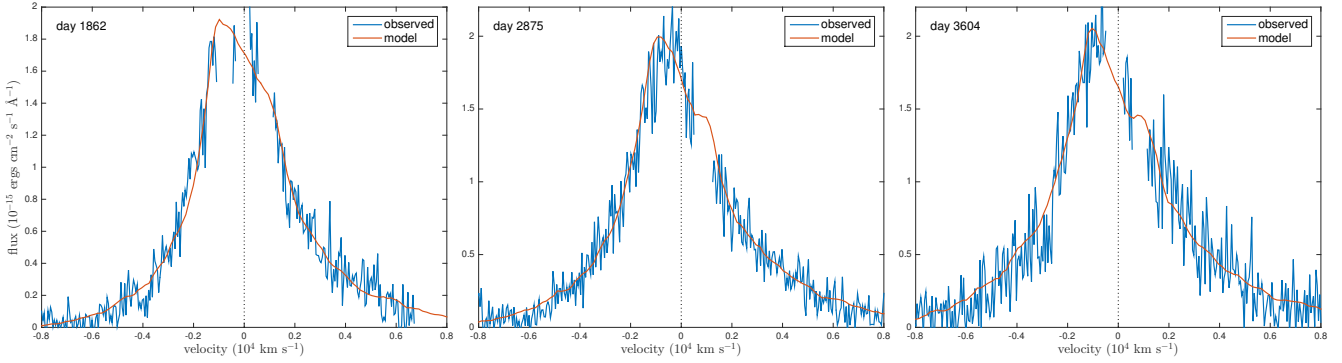


**Figure 16.** Best clumped dust fit to the SN 1987A [O I]  $\lambda 6300,6363 \text{ \AA}$  doublet at day 714 (*upper left*), day 806 (*upper right*), day 1054 (*lower left*) and day 1478 (*lower right*) for the parameters detailed in Table 4 with amorphous carbon grains of radius  $a = 0.6\mu\text{m}$ .

size distribution. Modelling a size distribution may therefore alter the deduced dust mass. Since the models are only sensitive to the overall optical depth and albedo, it is not possible to deduce the grain size range or distribution and only single grain sizes are investigated (as presented above).

Whilst this apparently limits the scope of the results, it is important to consider the extent to which considering grain size distributions would alter the derived dust masses.

By considering a number of grain size ranges and adopting a power law distribution with a variable exponent, we may gain some insight into the effects of adopting a distribution rather than a single size. For the classical MRN power law ( $n(a) \propto a^{-3.5}$ ) with a wide grain size range ( $a_{\min} = 0.001\mu\text{m}$  to  $a_{\max} = 4.0\mu\text{m}$ ) the derived albedo is much too small to reproduce the required wing seen at early epochs. We therefore adopt an approach whereby, for



**Figure 17.** Best clumped dust fit to the SN 1987A H $\alpha$  line at days 1862, 2875 and 3604 for the parameters detailed in Table 4 with amorphous carbon grains of radius  $a = 0.6\mu\text{m}$ .

a number of grain size ranges, we adjust the exponent of the distribution until the overall albedo is the same as that seen for the best fitting single grain size for the clumped distributions. We may then approximately calculate the required dust mass as

$$M_d = \frac{M_s Q_{ext,s}(a_s)}{a_s} \times \frac{\int_{a_{min}}^{a_{max}} n(a) a^3 da}{\int_{a_{min}}^{a_{max}} Q_{ext}(a) n(a) a^2 da} \quad (15)$$

where the subscript  $s$  represents the single grain size quantities and the  $d$  subscript represents quantities for the grain size distribution.

We calculate the required dust masses for the clumped H $\alpha$  model on day 714 for a selection of distributions with varying  $a_{min}$ . These are presented in Table 6. It can be seen that in all cases, a larger dust mass is required in order to reproduce the same profile as a single grain size. The conversion factors presented in the table are valid for any model with grain size  $a = 0.6\mu\text{m}$  and may therefore also be applied to the models for day 806. We repeated the process for  $a = 3.5\mu\text{m}$  but found that, in order to reproduce the required albedo, the distribution had to be heavily weighted towards the larger grains and that the value of  $a_{min}$  had no effect on the required dust mass. Increasing the value of  $a_{min}$  to larger values ( $> 2\mu\text{m}$ ) does not have a significant effect either. This is because both extinction efficiency and albedo tend to a constant value with increasing grain radius and the adoption of different grain size ranges and distributions above a certain threshold results in only insignificant variations in these quantities.

Equation 15 holds only for a single wavelength and therefore is not exact for our models which obviously transport radiation over a range of wavelengths. However, the dust masses derived using the above formula produce almost identical fits to the data as for the single grain size case and therefore should give an excellent representation of the dust mass required when using a distribution.

We conclude that if a distribution of grain sizes is indeed present, the deduced dust masses are likely to underestimate the true mass of newly formed dust.

### 5.5 The effect of different grain species

In all of our analyses so far, we have considered only amorphous carbon as the species of interest. This is largely moti-

**Table 6.** Dust masses for day 714 clumped models of the H $\alpha$  line using different grain size distributions and 100% amorphous carbon.  $f$  is factor of increase over the dust mass for the single size model ( $M = 7 \times 10^{-5} M_\odot$  with  $a = 0.6\mu\text{m}$ ) and  $p$  is the exponent of the grain size distribution  $n(a) \propto a^{-p}$ .

$a_{min}$ ( $\mu\text{m}$ )	$a_{max}$ ( $\mu\text{m}$ )	$p$	$M$ ( $M_\odot$ )	$f$
0.001	4.0	2.45	$1.93 \times 10^{-4}$	2.76
0.01	4.0	2.45	$1.93 \times 10^{-4}$	2.76
0.05	4.0	2.52	$1.84 \times 10^{-4}$	2.62
0.1	4.0	2.72	$1.61 \times 10^{-4}$	2.3
0.5	4.0	8.20	$7.23 \times 10^{-4}$	1.03

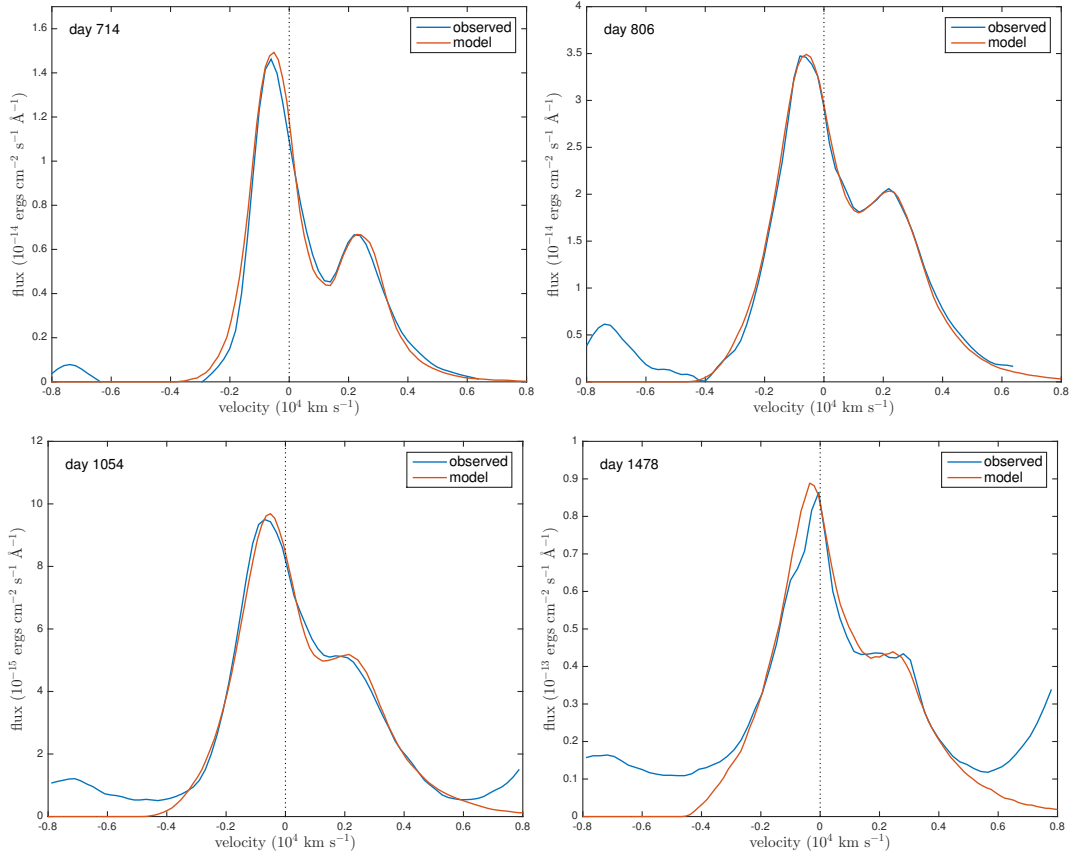
vated by previously published optical and IR analyses that have found that, if silicates contribute a fraction of the total dust mass, it is likely that this fraction is limited to approximately 15% or less (Ercolano et al. (2007), W15). The parameters that affect the quantity of dust required in our models are the mean albedo and the optical depth. There might be many possible combinations of grain species and sizes that result in a good fit to the data.

We consider the change in dust mass when a medium of 100% silicates is used instead of amorphous carbon. We use the astronomical silicate optical constants presented by Draine & Lee (1984). In a similar manner to the approach detailed in Section 5.4, we may calculate the mass of silicates that gives a fit equivalent to that for a single carbon grain size. We consider the albedo for the original carbon grain size, calculate the equivalent grain size for silicates that results in the same albedo and then calculate the new dust mass by considering the change in the extinction cross-section:

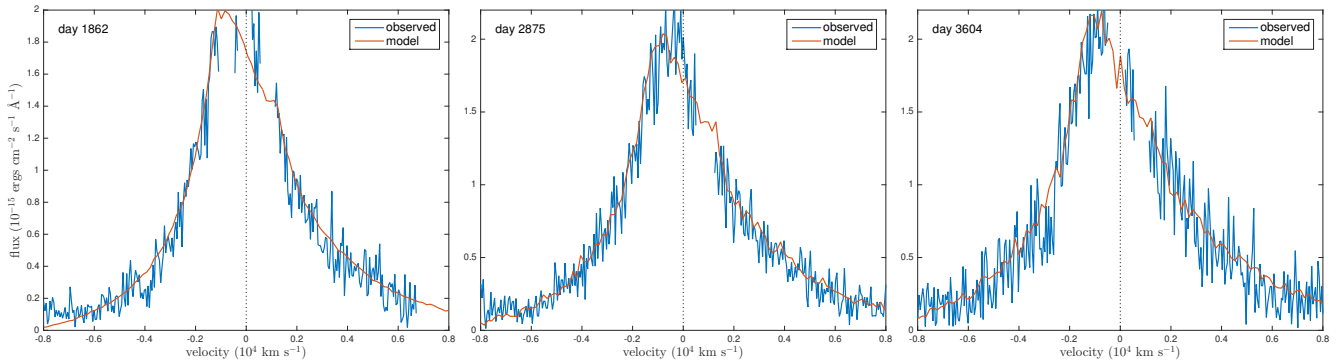
$$M_{sil} = M_{amc} \left( \frac{Q_{amc}}{Q_{sil}} \right) \left( \frac{a_{sil}}{a_{amc}} \right) \left( \frac{\rho_{sil}}{\rho_{amc}} \right) \quad (16)$$

Because of the nature of the variation of albedo with grain size for silicates (see Figure 8), there is often more than one silicate grain size that will give rise to the same albedo at a given wavelength. We consider some of the possibilities and the resulting mass conversion factors in Table 7. In our best fitting amorphous carbon models with  $a = 0.6\mu\text{m}$  (the





**Figure 18.** Best clumped dust fit to the SN 1987A [O I]  $\lambda 6300,6363$  Å doublet at day (upper left), day 806 (upper right), day 1054 (lower left) and day 1478 (lower right) for the parameters detailed in Table 5 with amorphous carbon grains of radius  $a = 3.5\mu\text{m}$ .



**Figure 19.** Best clumped dust fit to the SN 1987A H $\alpha$  line at days 1862, 2875 and 3604 for the parameters detailed in Table 5 with amorphous carbon grains of radius  $a = 3.5\mu\text{m}$ .

first two entries in Table 7), using any fraction of silicates of either grain size would serve to increase the dust mass. However, for the larger amorphous carbon grain radius of  $a = 3.5\mu\text{m}$  (the last three entries), using silicate dust would reduce the dust mass to about half of our amorphous carbon values. These factors produce dust masses that are still approximately within our predicted minima and maxima.

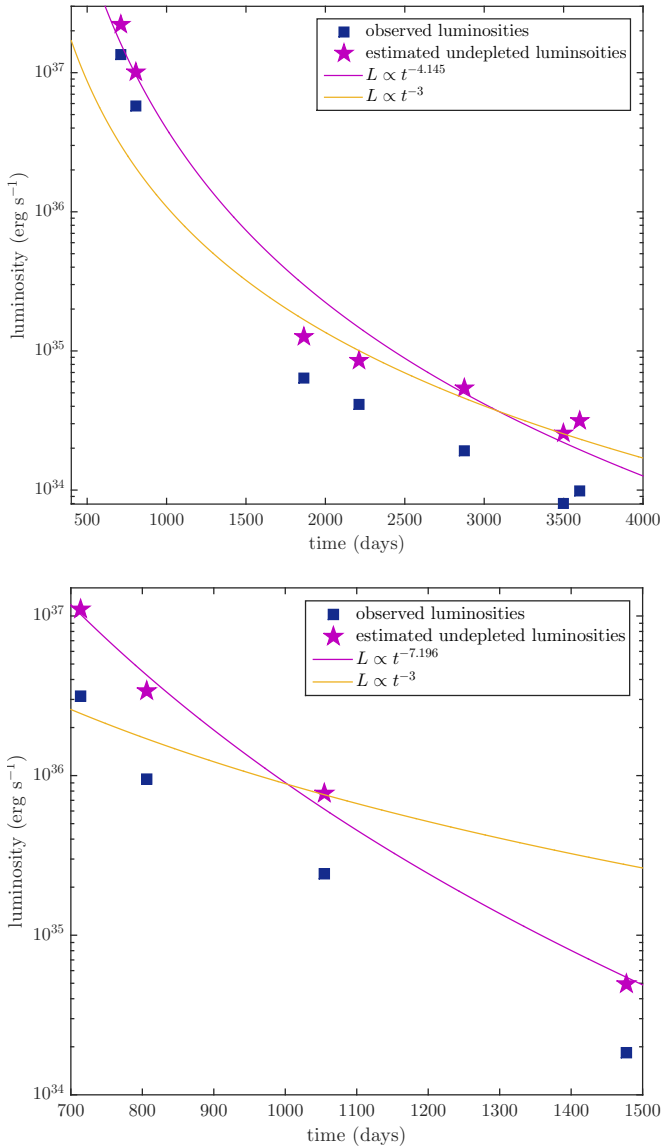
## 5.6 Predicting unattenuated fluxes

## 6 DISCUSSION

We have collated a range of optical archival spectral data and by modelling the evolution of the H $\alpha$  and [O I]  $\lambda 6300,6363$  Å lines have placed constraints on the evolution of newly formed dust in the ejecta of SN 1987A. We have done this using Monte Carlo models that consider both the absorbing and scattering effects of dust.

**Table 7.** Dust masses conversion factors for single size models using different grain species of 100% amorphous carbon or 100% silicates.  $f$  is the fractional change from the single size amorphous carbon dust mass to the single size silicates dust mass.

carbon			silicates			$f = \frac{M_{sil}}{M_{amc}}$
$a$ ( $\mu\text{m}$ )	$\omega$	$Q_{ext}$	$a$ ( $\mu\text{m}$ )	$\omega$	$Q_{ext}$	
0.6	0.56	2.61	0.0583	0.58	0.08	5.37
0.6	0.56	2.61	4.00	0.56	2.18	13.0
3.5	0.62	2.21	0.0641	0.64	0.10	0.65
3.5	0.62	2.21	1.020	0.63	2.15	0.49
3.5	0.62	2.21	1.376	0.62	2.35	0.61

**Figure 20.** Predicted undepleted luminosities for the H $\alpha$  line (*above*) and [O I] $\lambda$ 6300,6363 Å doublet (*below*) presented with the best power-law fit to the data.

Lucy et al. (1989) analysed the [O I]  $\lambda$ 6300,6363 Å doublet for SN 1987A and found optical depths for a range of epochs. They translated these into dust masses for day 775 only. From our smooth modelling of the [O I] doublets we obtain  $\tau_V = 7.20$  at day 714 and  $\tau_V = 5.71$  at day 806. These values are considerably higher than the values given by Lucy et al. (1989) who derived  $\tau_V = 1.19$  at day 725 and  $\tau_V = 1.25$  at day 775. The value of the assumed albedo accounts for the majority of this discrepancy. Lucy et al. (1989) considered line profiles before and after dust condensation and concluded that any evidence of an extended red scattering wing was unconvincing. Accordingly, they adopted a model with perfectly absorbing dust ( $\omega = 0$ ). For our amorphous carbon models of [O I]  $\lambda$ 6300,6363 Å with grain radius  $a = 0.35\mu\text{m}$ , we obtain an albedo of approximately  $\omega = 0.5$  at  $\lambda = 6300$  Å.

The Lucy et al. (1989) model also assumed that both gas and dust were uniformly distributed throughout the ejecta in contrast to our models which adopt steep profiles with the majority of the emission stepped towards the centre of the ejecta. Since the gas and dust density distributions are coupled in our models, the dust follows the same steep density distribution. These additional factors may account for the remainder of the discrepancy between our values of  $\tau_V$  and those of Lucy et al. (1989).

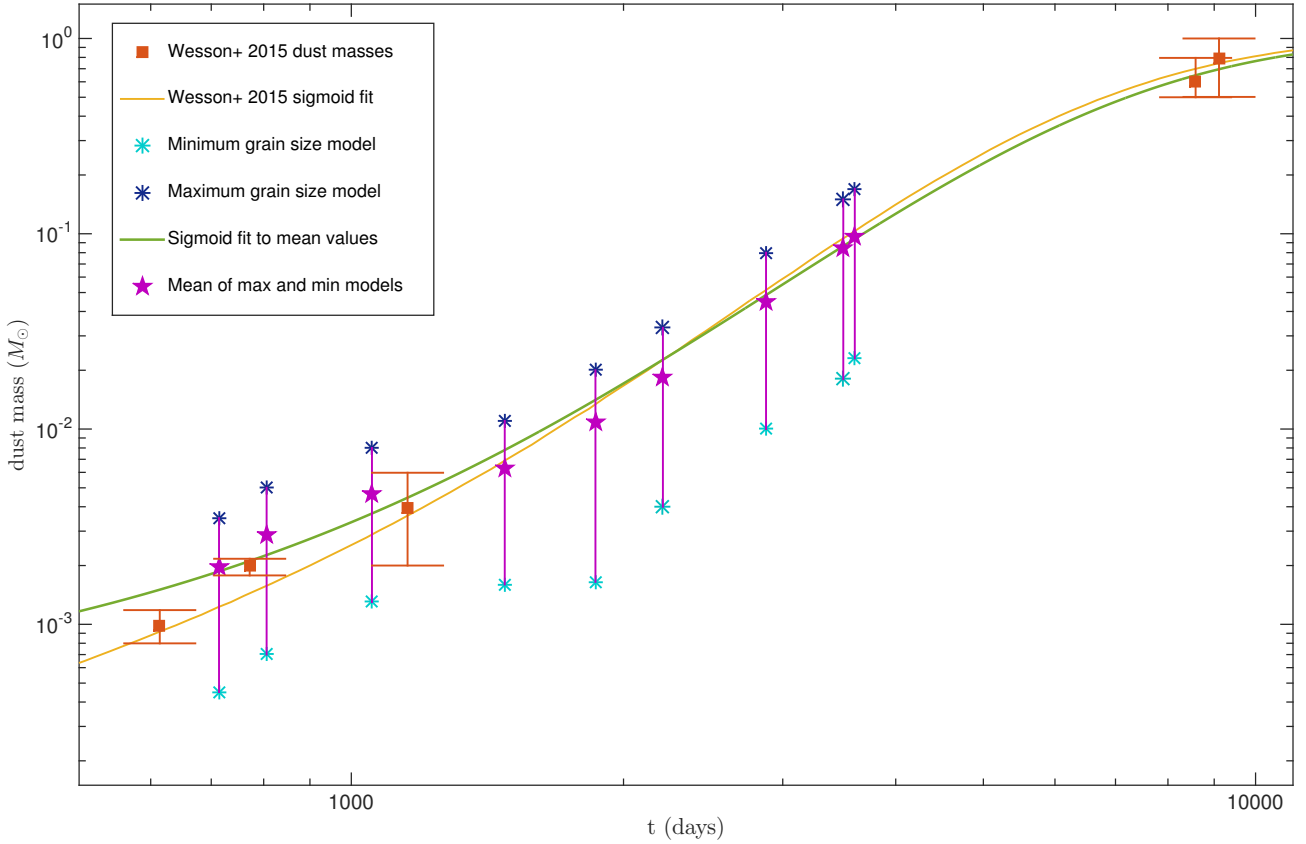
Lucy et al. (1989) also noted that the dust optical depth increased rapidly after day 580 and then the rate of increase of the dust optical depth appeared to slow between day 670 and day 775, the latest day that they considered. Our results, for both clumped and smooth models, suggest that the dust optical depth actually drops between day 714 and day 806 before starting to increase again at later epochs. This is consistent with the results of Lucy et al. (1989) where the slowing rate of increase of dust optical depth could imply an impending turning point subsequent to day 775.

The dust masses derived by Lucy et al. (1989) at day 775 ( $M_{dust} = 4.4 \times 10^{-6} M_\odot$  for amorphous carbon) are significantly different to those obtained from our smooth dust modelling of the [O I]  $\lambda$ 6300,6363 Å doublet at day 806 ( $M_{dust} = 1.8 \times 10^{-4} M_\odot$  for amorphous carbon). There are three main reasons for the discrepancy. Firstly, the albedo is significantly larger in our modelling as already discussed. A larger dust mass is therefore required to produce the same amount of absorption. Secondly, the grain size is considerably larger which reduces the total cross-section of interaction and so a greater dust mass must be present to compensate for this. Finally, the adopted maximum velocity ( $4000 \text{ km s}^{-1}$ ) in our model is larger than the value adopted by Lucy et al. (1989) ( $1870 \text{ km s}^{-1}$ ). The larger value of  $V_{max}$  increases the total volume of the ejecta significantly and therefore significantly more dust is required to produce the same optical depth.

We now consider the comparison with the results of W15 (see Figure 21). W15 used a sigmoid fit to their dust mass evolution, of the form

$$M_d(t) = ae^{be^{ct}} \quad (17)$$

where they obtain values of  $a = 1.0 M_\odot$  (representing the maximum dust mass),  $b = -8.53$  and  $c = -0.0004$ . Both their dust masses and this sigmoid fit are shown in Figure 21. This exhibits an initial period of slow growth followed by



**Figure 21.** Derived dust masses for SN 1987A as a function of epoch. *Dark and light blue asterisks* - maximum ( $a = 3.5\mu\text{m}$ ) and minimum ( $a = 0.6\mu\text{m}$ ) dust masses respectively for the [O I] models for  $t \leq 1478$  days and for the H $\alpha$  models for  $t \geq 1862$  days. *Purple stars* - predicted dust masses calculated as the mean of the maximum and minimum dust masses. *Green line* - sigmoid fit to our predicted dust masses. *Red squares* - dust masses derived by W15 in their photometric modelling of SN 1987A. *Yellow line* - sigmoid fit to the W15 values.

an intermediary period of acceleration followed by another slowing until a plateau is ultimately reached. In this sense it may be relatively representative of the process of dust formation whereby initial conditions appropriate for grain growth gradually develop until optimal conditions are reached at an intermediate epoch when grain growth is at its fastest before conditions once again deteriorate and the rate slows again (as discussed by W15). Performing a least-squares regression to this function using our predicted dust masses, we derive a sigmoid fit with coefficients  $a = 1.007M_{\odot}$ ,  $b = -8.005$  and  $c = -0.0003$ . These values are remarkably similar to those derived by W15. This sigmoid fit is also plotted in Figure 21.

We find that at all epochs the dust masses derived by W15 are entirely within the dust mass ranges predicted by our models.

It should be noted that our sigmoid fit is a simple fit to the mean of the maximum and minimum dust masses. It does not take into account grain growth. It seems likely that at earlier epochs, whilst grains are still small relative to later epochs, the lower end of our dust mass estimates are more representative than the upper end; the reverse would be true at later epochs.

This is in contrast to the sigmoid fit of W15. W15 generate fits to their early data using an MRN distribution between radii of  $0.005\mu\text{m}$  and  $0.25\mu\text{m}$  whilst fits to their last two epochs used a similar distribution between  $3.005\mu\text{m}$  and  $3.25\mu\text{m}$ . Their sigmoid fit therefore takes into account some degree of grain growth between the earlier and later epochs.

As mentioned, we could not fit the red wing of the profiles at early epochs using an MRN distribution. W15 found that they could not obtain a fit with grains of  $\sim 1.0\mu\text{m}$  in size at earlier epochs. However, they did not consider values in between these size points such as grains of  $a \approx 0.6\mu\text{m}$  that we find at earlier epochs. For SED modelling it is generally the case that the larger the grain size used, the less dust is required to produce the same level of flux. This may account for the differences between the W15 dust masses and our minimum dust mass estimates at earlier epochs. The models of W15 also adopt a medium which is 15% silicate dust in contrast to our models which are 100% amorphous carbon. This could also contribute to the differences, as could the use of different sets of optical constants - we have used the optical constants of Zubko et al. (1996) for their BE sample whereas W15 use constants from Hanner (1988). W15 state that in order to fit their data at early epochs (day 615) with

the Zubko ACH2 constants smaller inner and outer radii are needed and half as much dust ( $5.0 \times 10^{-4} M_{\odot}$ ) is required.

W15 also derived a maximum possible grain size at late epochs. They concluded that the grains could not be larger than  $\sim 5 \mu\text{m}$  by day 8515. This is directly in line with the maximum grain sizes we derive at slightly earlier epochs. We find that grain radii likely cannot have exceeded  $\sim 3.5 \mu\text{m}$  at day 3604 - the dust mass that we generate using this grain radius is similar to the value of the W15 sigmoid fit at this epoch.

Determining the relationship between the radii of dust grains in the ejecta and the post-explosion time is important for understanding the likelihood of dust surviving the passage of the reverse shock travelling back through the ejecta. By the time the reverse shock begins to appear in the line profiles (around day 5000), our models predict that the grains could already be as large as several microns in radius but are likely larger than  $\sim 0.6 \mu\text{m}$ . Grains larger than  $\sim 0.2 \mu\text{m}$  are more likely to survive. It has recently been suggested that very large grains (up to  $4.2 \mu\text{m}$ ) may have formed in the ejecta of SN 2010jl within a few hundred days after the explosion [Gall et al. \(2014\)](#). Whilst the grain radii we obtain for SN 1987A are not as large as found by [Gall et al. \(2014\)](#) for SN 2010jl, both results suggest that grains large enough to survive the destructive force of the reverse shock have formed by a few hundred days post-explosion.

There is now an observational consensus that a very large quantity of dust has formed in SN 1987A between the time of the original explosion and the present day. Perhaps more important therefore is the manner of its evolution. We have shown that dust masses have reached the order of  $0.1 M_{\odot}$  by day 3604. However, it is known that values several times as large as this are ultimately expected and thus a substantial fraction of the dust is likely to have formed after this epoch. This is in strong agreement with the results produced by W15.

Our modelling concurs with the suggestion of W15 that even after  $\sim 3000$  days the dust mass is only a fraction of its final value. This is in contrast to [Sarangi & Cherchneff \(2015\)](#) whose chemistry models predict that the evolution of dust formation will have reached its plateau by around 5 years after the explosion first occurred.

Ideally, our models would cover the entire evolution of SN 1987A right up to the present day. However, the excitation of gas in the outer edges of the ejecta by the reverse shock after  $\sim$  day 5000 results in a significant, broad and asymmetric emission that dominates the original line profile. In addition to this, the narrow lines from the equatorial ring start to become so significant relative to the original broad H $\alpha$  profile that, post-removal, there is not enough of the broad profile remaining to be able reliably infer information from its features. These are factors that are likely to be common to some other core collapse supernovae and could have an impact on the wider applicability of this particular technique at very late epochs. Care should also be taken to ensure that observed line profiles are the temporally appropriate profile and not in fact a product of a light echo representing the state of the ejecta at some previous epoch. Nonetheless, this technique has proved effective in determining dust masses formed in the ejecta of SN 1987A at multiple epochs through the detailed modelling of asymmetric

line profiles and so the method clearly has wider application to multiple supernovae and supernova remnants.

## 7 CONCLUSIONS

We have discussed the various effects of dust scattering and absorption on supernova line profile shapes and the different features that may be produced. In particular, attention is drawn to the fact that a classical blue-shifted peak and asymmetric profile with most flux on the blue side is not the only profile type that can signify the presence of dust. Profiles may also have the majority of their flux on the red side in the case of strong dust scattering. They can often exhibit an extended red scattering wing although care should of course be taken to ascertain that this cannot be accounted for by electron scattering. A pronounced shoulder or corner may be present on the red side of the profile located at the minimum (inner) velocity of the shell.

We have modelled the H $\alpha$  and [O I]  $\lambda 6300, 6363 \text{ \AA}$  line profiles from SN 1987A over a range of epochs and have obtained dust masses of the order of  $0.1 M_{\odot}$  by day 3604. We derive a sigmoid fit to this data that predicts a current dust mass of  $0.67 M_{\odot}$ , in line with current SED-based dust mass estimates for SN 1987A. We conclude that large grains are necessary in order to reproduce the extended red scattering wing and the asymmetry seen in several of the lines and that grains larger than  $0.6 \mu\text{m}$  have formed by day 714.

We have demonstrated the efficacy of the DAMOCLES code for determining dust masses and established its potential for application to the spectra of other supernovae.

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## APPENDIX A: APPENDIX A

Since the outflow velocities in supernovae are high, the photon packets are subject to Doppler shifting upon emission and at each scattering event. When the packet is initially emitted, it has a frequency and a trajectory in the rest frame of the emitter. Both of these must be transformed to the observer's frame in order for the packet to be propagated through the grid. The new direction and frequency in the observer's frame may be simply found by transforming the momentum 4-vector  $\mathbf{P}$  which is defined as

$$\mathbf{P} = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} h\nu \\ h\nu x \\ h\nu y \\ h\nu z \end{pmatrix} \quad (\text{A1})$$

We may then derive  $\mathbf{P}'$ , the momentum 4-vector in the observer's frame using the relation

$$\mathbf{P}' = \Lambda \mathbf{P} \quad (\text{A2})$$

where

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta_x & -\gamma\beta_y & -\gamma\beta_z \\ -\gamma\beta_x & 1 + (\gamma - 1)\frac{\beta_x^2}{\beta^2} & (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2} & (\gamma - 1)\frac{\beta_x\beta_z}{\beta^2} \\ -\gamma\beta_y & (\gamma - 1)\frac{\beta_y\beta_x}{\beta^2} & 1 + (\gamma - 1)\frac{\beta_y^2}{\beta^2} & (\gamma - 1)\frac{\beta_y\beta_z}{\beta^2} \\ -\gamma\beta_z & (\gamma - 1)\frac{\beta_z\beta_x}{\beta^2} & (\gamma - 1)\frac{\beta_z\beta_y}{\beta^2} & 1 + (\gamma - 1)\frac{\beta_z^2}{\beta^2} \end{pmatrix}$$

and  $\beta = \frac{v}{c} = (\beta_x, \beta_y, \beta_z)$ ,  $\beta = |\beta|$  and  $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ .

In practice, the velocities considered are low enough that it is unnecessary to consider terms of order  $O(\frac{v^2}{c^2})$  and thus  $\Lambda$  may be reduced to

$$\Lambda = \begin{pmatrix} 1 & -\beta_x & -\beta_y & -\beta_z \\ -\beta_x & 1 & 0 & 0 \\ -\beta_y & 0 & 1 & 0 \\ -\beta_z & 0 & 0 & 1 \end{pmatrix} \quad (\text{A3})$$

The new direction of travel and frequency in the observer's frame are therefore given by

$$\nu' = \nu(1 - x\beta_x - y\beta_y - z\beta_z) \quad (\text{A4})$$

$$x' = \frac{\nu}{\nu'}(x - \beta_x)$$

$$y' = \frac{\nu}{\nu'}(y - \beta_y)$$

$$z' = \frac{\nu}{\nu'}(z - \beta_z)$$

For each scattering event, the packet must be transformed both into and out of the comoving frame. The reverse transform is applied by using the inverse Lorentz matrix  $\Lambda^{-1}$  which is obtained by reversing the sign of  $\mathbf{v}$ . Positive  $\mathbf{v}$  is defined for frames moving away from each other and thus  $\mathbf{v}$  is defined to be negative in the direction of the observer.



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