STOCHASTIC PROCESS

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STOCHASTIC PROCESS

Seminar

Submitted in fulfillment of the requirements

For the degree of

Bachelor of Technology in Information Technology

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CERTIFICATE

This is to certify that the Seminar entitled "**STOCHASTIC PROCESS**" submitted by **BHAUTIK AMIPARA (14BIT006)**, towards the partial fulfillment of the requirements for the degree of Bachelor of Technology in Computer Engineering/ Information Technology of Nirma University is the record of work carried out by him/her under my supervision and guidance. In my opinion, the submitted work has reached a level required for being accepted for examination.

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ABSTRACT

This report give an overview of stochastic processes and how we interact with those in real life situations and tried to figure out the solution of some complicated problems. It includes the stochastic modelling, some popular and vary generous in use stochastic model like random walk, markov chain etc. Hence the basic idea behind this report is to understand the various aspect of randomness, stochastic process and its applications.

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List of figures

Chapter 1
Introduction

1.1 General:

- A stochastic process is just a likelihood process: that is any procedure in nature whose development we can investigate effectively as far as likelihood.
- In probability hypothesis, a stochastic process, or frequently random process, is a gathering of random variables speaking to the development of some arrangement of random qualities after some time. This is the probabilistic partner to a deterministic process (or deterministic framework).
- We often use this hypothesis to determine or forecast the result and outcome of various stochastic process.

1.2 Objective:

- To study and understand the behavior of various real life stochastic or random process.
- To study the modeling of stochastic process and to understand how its turn to useful in find solution of some complex problems.

1.3 Scope:

- **Work:** A detailed analysis of various stochastic models and its behavior over different situation. To finding the solution of famous travelling salesman problem. A Study of different application of stochastic theory.
- **Limitation:** this paper does not talk about narrow mathematics about various theory and does not get into prove of any theory.

Chapter 2

Stochastic process

2.1What is stochastic process?

2.1.1 Definition:

A stochastic process can be considered in one of numerous proportional ways. We can start with a basic probability space (Ω,Σ,P) and a genuine esteemed stochastic process can be defined as a collection of random variables {x(t,w)} filed by the parameter set T.[1]

2.1.2 Example of stochastic process:

- Well known samples of stochastic processes incorporate securities exchange and swapping scale changes; flags, for example, discourse; sound and video; therapeutic information, for example, a patient's EKG,EEG, circulatory strain or temperature; and random development, for example, Brownian movement or random strolls.
- Exchange Rate:

  Figure 2.1 shows the plot of price, dividends and earnings which follows the stochastic path.

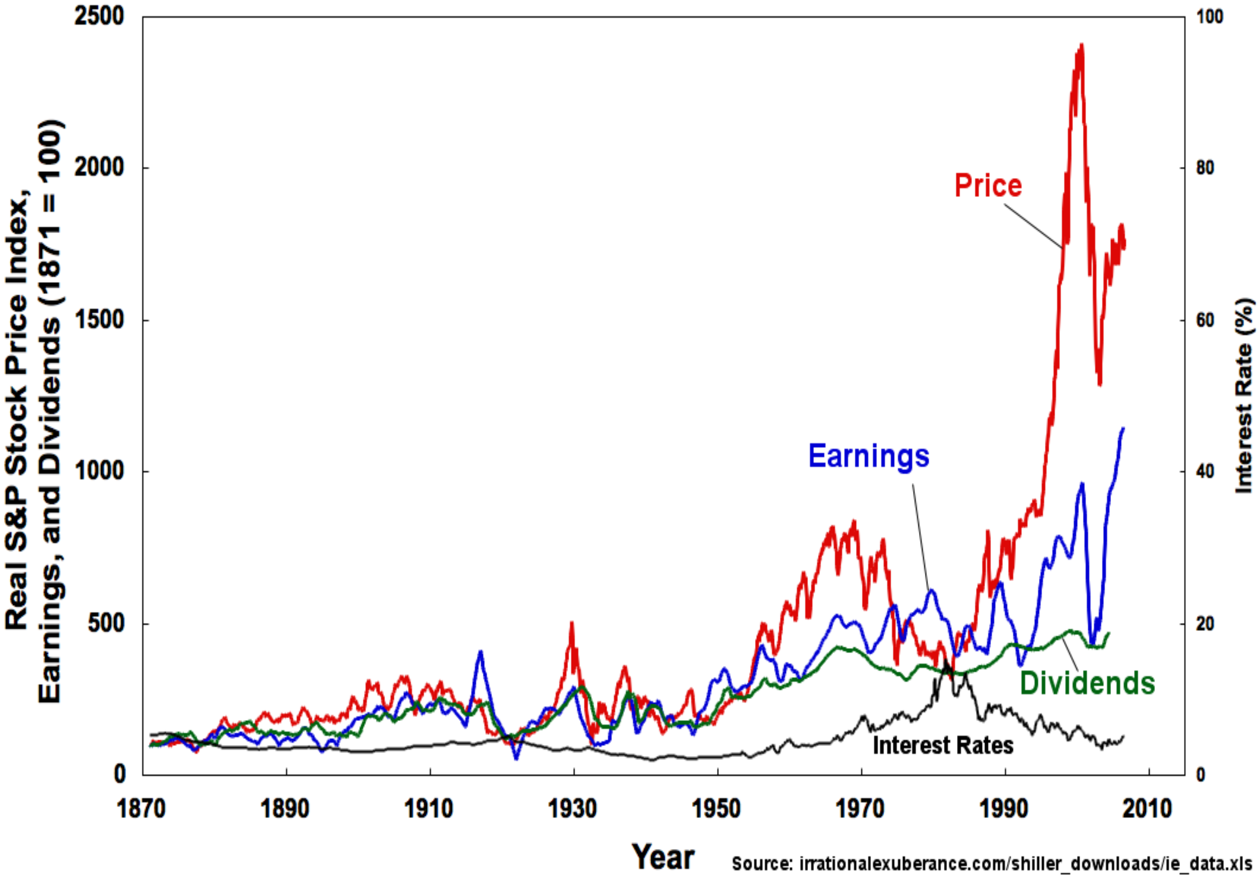

Figure 2.1 Stock market fluctuations have been modeled by stochastic processes

- Budding Yeast:

    The yeast species utilized as a part of blending and heating, Saccharomyces cerevisiae imitates by growing. The grown-up cell, which we call the mother cell, creates a bud which develops and in the long run isolates from the mother to deliver another daughter cell. The mother cell then delivers another bud, after a random time M. The girl cell, then again, first needs to increment in size until it achieves development, which takes a random time D. At that point it acts like a mother cell, delivering a bud after random time M. Inside and out, accordingly, the time from when the bud was delivered until it creates its own particular posterity bud is D + M.

- Photon emission:

  Photons are minute particles of light. Under a few circumstances, a light source will discharge photons at random (as indicated by what is known as a Poisson process, which we will ponder in area 5). Singular photons are too black out to possibly be distinguished by the human eye, however electronic photon indicators can identify single photons. Be that as it may, there is an issue with these machines. Promptly after they have distinguished a photon, there is a brief timeframe period, known as the dead time, amid which no new photons can be detected.

2.2 classification:

- Stochastic processes can be grouped by cardinality of its record set (generally deciphered as time) and state space.
- As the time and state space variables are of two type discrete and continues there will be four type of stochastic process.

1. Discrete time discrete state space stochastic process.
2. Discrete time Continues state space stochastic process.
3. Continues time discrete state space stochastic process.
4. Continues time Continues state space stochastic process.[1]

2.3 Random variables:

- An antiquated yet exceptionally valuable and exceedingly natural definition portrays a random variable as a variable that tackles its qualities by possibility. The more established definition simply given serves enough, in any case, in for all intents and purposes all occurrences of stochastic modeling. In fact, this more seasoned definition was the main methodology accessible for well over a century of important advancement in probability hypothesis and stochastic processes.[1]

- More often than not we hold fast to the tradition of utilizing capital letters for example, X, Y, Z to indicate random variables, and lowercase letters, for example x, y, z for genuine numbers. The expression $(X \leq x)$ is the occasion that the random variable X expect a worth that is not exactly or equivalent to the genuine number x. This occasion could possibly happen, contingent upon the result of the analysis or wonder that decides the worth for the random variable X. The probability that the occasion happens is composed $Pr\{X \leq x\}$. Permitting x to fluctuate, this probability characterizes a capacity

$$F(x) = Pr\{X \ x\}, -\infty < x < +\infty,$$

Called the circulation capacity of the random variable X.

Chapter 3

Stochastic Modeling

3.1 What is stochastic modeling?

- "Stochastic" means being or having a random variable. A stochastic model is a device for evaluating probability dispersions of potential results by taking into consideration random variety in one or more inputs after some time. The random variety is normally in light of changes saw in recorded information for a chose period utilizing standard time-arrangement methods. Circulations of potential results are gotten from countless (stochastic projections) which mirror the random variety in the input(s).[2]

- At the point when Real-Life circumstances in which vulnerability is available comes to worry to model this kind of processes we can utilize Stochastic Modeling with the utilization of probability.

- For instance in betting Personal Finances ,Disease Treatment Options, Economic Forecasting , Product Demand , Call Center Provisioning , Product Reliability and Warranty Analysis ,Population Growth, and so on.

- Modeling is a center actuarial strategy. Verifiably statisticians took after deterministic methodologies while anticipating future results from a given set of information. Stochastic modeling considers the random way of the information and afterward creates yields that are comparably random in nature. As of now stochastic modeling is generally utilized as a part of the fields of wellbeing, short term protection, disaster protection, speculations and resource obligation coordinating.

- Stochastic modeling is with the end goal of assessing the probability of results inside a gauge to foresee what conditions may resemble under various circumstances. The random variables are normally compelled by recorded information.

3.2 The Basic Steps of Stochastic Modeling:

- **I.** Identifying the sample space.
- **II.** Assigning probabilities to the elements of the sample space
- **III.** Identifying the events of interest.
- **IV.** Computing the desired probabilities

3.3 Stochastic model of population growth:

- Consider how to comprehend the outcomes of a population that is developing at a rate of 5% for each year. For this situation we can simply record a mathematical algebraic expression: call the underlying population P0, the population at era t Pt , then

$$P_1 = P_0\,(1+.05)$$

$$P_2 = P_1\,(1+.2) = P_0\,(1+.05)^2$$

and in general

$$P_t = P_0\,(1+.05)^t$$

giving us a general answer for at whatever time t later on. For instance in 500 years or 25 eras an underlying population of 100 individuals would develop to P25 = 100(1.05)25 or 339 individuals. Luckily for this situation the arrangement is simple and we can get the answer with a mini-computer.[b]

- Be that as it may, imagine a scenario in which the normal number of girls per lady in this population were 1.2 however singular ladies fluctuated at random in their real yield of little girls. On the off chance that births of girls strike ladies as a process with a steady risk for every unit of time then the subsequent number takes after what is known as a Poisson appropriation with a mean of 1.2. We can without much of a stretch reproduce such a situation on a PC, for instance with a spreadsheet. We begin with 100 ladies, draw the quantity of little girls she has from a Poisson circulation, and the aggregate number of little girls now is our people to come. We straightforward rehash this process 25 times. Since we are occupied with the variety starting with one trial then onto the next it is intriguing to plot 20 such trials, 20 conceivable population histories, alongside the aftereffect of the arithmetical arrangement. The substantial dark line in figure 3.1 is the mathematical arrangement of the model while the lighter lines are results of our reenactments with randomness fused. The arithmetical result is known as the deterministic model while the reproductions the join randomness are results of the stochastic variant of the model.
- Stochastic equation for population can be written as

$$P(t+1) = P(t)\alpha(t)$$

Where $\alpha(t)$ is randomized in discrete time scale.

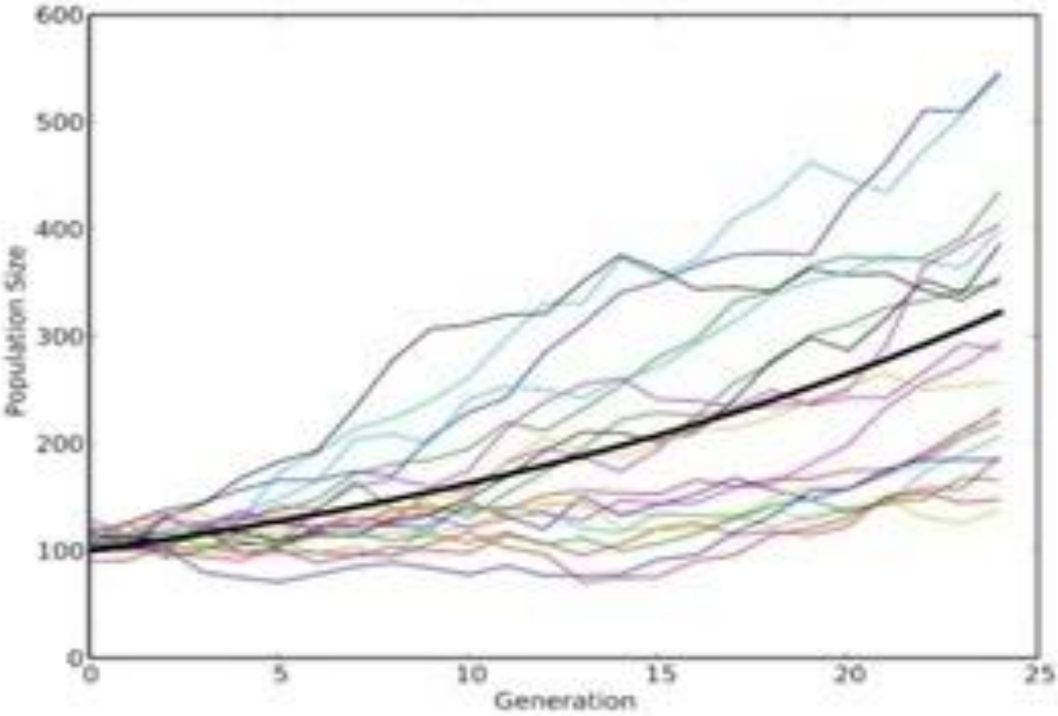

Figure 3.1 Model outcomes of population growth of 5% per generation. The heavy line is the deterministic outcome while the others are various simulated histories with the number of daughters per woman random.

Chapter 4
Simple Random Walk

4.1 What is random walk?

- A random walk is a scientific formalization of a way that comprises of a progression of random steps. For instance, the way followed by an atom as it goes in a fluid or a gas, the pursuit way of a searching creature, the cost of a fluctuating stock and the money related status of a player would all be able to be modeled as random strolls, in spite of the fact that they may not be genuinely random as a general rule. The term random walk was initially presented by Karl Pearson in 1905. Random strolls have been utilized as a part of numerous fields: nature, financial matters, brain research, software engineering, material science, science, and science. Random strolls clarify the watched practices of numerous processes in these fields, and in this manner serve as a basic model for the recorded stochastic movement.[2]

- In other words, A random walk is characterized as a process where the present estimation of a variable is made out of the past quality besides a mistake term characterized as a background noise (ordinary variable with zero mean and difference one). Algebraically a random walk is spoken to as takes after:

$$y_t = y_{t-1} + \varepsilon_t$$

- The implication of a process of this type is that the best prediction of y for next period is the current value, or in other words the process does not allow to predict the change (yt − yt−1). That is, the change of y is absolutely random.

- It can be shown that the mean of a random walk process is constant but its variance is not. Therefore random walk process is nonstationary, and its variance increases with t. In practice, the presence of a random walk process makes the forecast process very simple since all the future values of yt+s for s > 0, is simply yt.

- Suppose that U = (U1, U2,…) is a sequence of independent random variables, each taking values 1 and−1with probability p∈[0,1] and 1-p respectively. Let X=(X0,X1,X2,…) be the partial sum process associated with U, so that

$$\boldsymbol{Xn} = \sum_{k=0}^{n} Ui \quad ,n \in N$$

  The sequence **Xn** is simple random walk with parameter p.[2]

- Figure 4.1 illustrates of eight random walk in one measurement beginning at 0. The plot demonstrates the present position on hold (vertical axis) versus the time steps (level pivot).

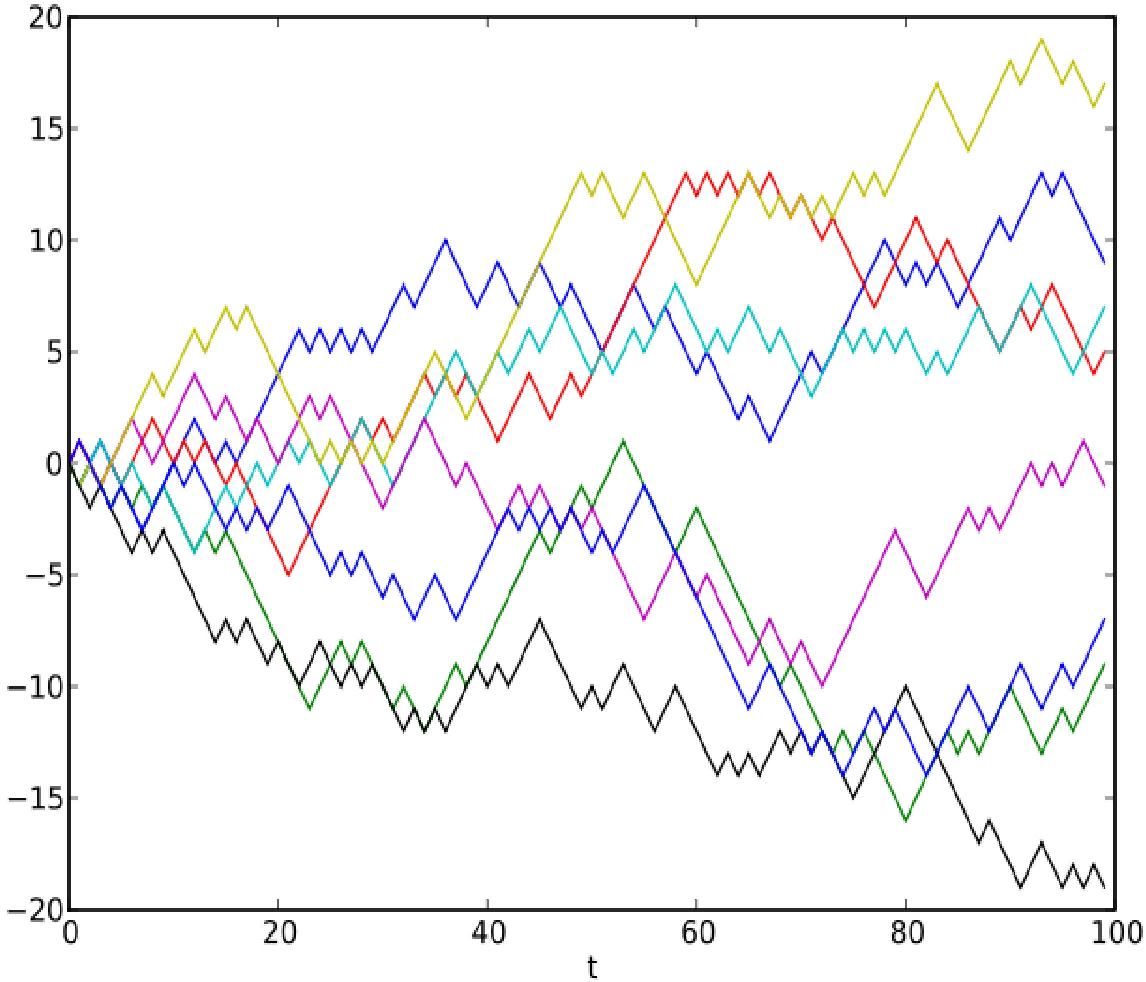

Figure 4.1 Example of eight random walks in one dimension starting at 0. The plot shows the current position on the line (vertical axis) versus the time steps (horizontal axis)

Chapter 5

Markov chains and Monte Carlo method

5.1 What is markov chain?

- A Markov chain is a stochastic process with the Markov property. The expression "Markov chain" alludes to the arrangement of random variables such a process travels through, with the Markov property characterizing serial reliance just between adjoining periods (as in a "chain"). It can in this manner be utilized for portraying frameworks that take after a chain of connected occasions, where what happens next depends just on the present condition of the framework.[1]

- The simple random walk is markov chain.
- All effects of past on future is contain in a current value, so that this are more manageable than regular stochastic processes.

- A discrete-time Markov chain with finite or countable state space X is a sequence $X0,X1,\ldots$ of X −valued random variables such that for all states $i, j, k0, k1, \cdots$ and all times $n = 0, 1, 2, \ldots,$

$$\mathbf{P(Xn+1 = j \mid Xn = i, Xn-1 = kn-1, \ldots) = p(i , j)}$$ [1]

- Transition property:
    - if Xi have values in finite set S with Cardinality M then

$$\mathbf{Pij = P(Xt+1=j|Xt=i) , i,j \in S .}$$

$$\sum_{j \in S} Pij = 1$$

- Transition matrix:

$$\begin{bmatrix} P11 & \cdots & P1j \\ \vdots & \ddots & \vdots \\ Pi1 & \cdots & Pij \end{bmatrix}$$

5.2 Monte Carlo method:

- Monte Carlo methods (or Monte Carlo analyses) are a wide class of computational algorithms that depend on rehashed random inspecting to get numerical results. They are regularly utilized as a part of physical and numerical issues and are most valuable when it is troublesome or difficult to utilize other scientific methods. Monte Carlo methods are essentially utilized as a part of three particular issue classes streamlining, numerical coordination, and producing draws from a probability conveyance.[4]

- In physics-related problems, Monte Carlo methods are quite useful for simulating systems with many coupled degrees of freedom, such as fluids, disordered materials, strongly coupled solids, and cellular structures.
- On a basic level, Monte Carlo methods can be utilized to take care of any issue having a probabilistic understanding. By the law of substantial numbers, integrals depicted by the normal

estimation of some random variable can be approximated by taking the empirical mean (a.k.a. the specimen mean) of autonomous examples of the variable. At the point when the probability dissemination of the variable is parameterized, mathematicians often utilize a Markov Chain Monte Carlo (MCMC) sampler. The focal thought is to outline a prudent Markov chain model with a recommended stationary probability conveyance. By the ergodic theorem, the stationary dissemination is approximated by the empirical measures of the random states of the MCMC sampler.[3]

- Monte Carlo methods are mostly utilized as a part of three particular issue classes: optimization, numerical combination, and producing draws from probability dissemination.

5.3 markov chain Monte Carlo:

- In statistics, Markov chain Monte Carlo (MCMC) methods are a class of calculations for testing from a probability appropriation in light of building a Markov chain that has the coveted dissemination as its harmony circulation. The condition of the chain after various steps is then utilized as an example of the coveted dispersion. The nature of the example enhances as an element of the quantity of steps.[3]

- Markov chain Monte Carlo (MCMC) methods are a class of calculations for inspecting from probability appropriations in light of building a Markov chain that has the sought dissemination as its harmony circulation. It is for the most part utilized when the random vector to be mimicked is made out of an extensive number of autonomous variables to produce Markov chain whose balance conveyance is equivalent to required circulation.

Chapter 6

Monte Carlo Simulation for Travelling Salesmen Problem

6.1 Simulated Annealing:

- Simulated Annealing is a bland probabilistic meta-algorithm used to locate an estimated answer for worldwide advancement issues. It is enlivened by toughening in metallurgy which is a system of controlled cooling of material to diminish imperfections.

- The simulated annealing algorithm takes random walks through the issue space. It searches for the focuses with low energies; in these random walks, the probability of making a stride is controlled by the Boltzmann distribution,

$$p = e^{-(E_{i+1}-E_i)/T} \quad if\ E_{i+1} > E_i$$
$$p = 1 \qquad\qquad if\ E_{i+1} < E_i$$

  Here p is probability of taking next step, E_i is energy of current step. E_{i+1} is energy of next step .[6]

- In other words, a stage will happen if the new energy is lower. On the off chance that the new energy is higher, the move can in any case happen with some probability which is relative to the temperature T and conversely corresponding to the energy distinction $E_{i+1} - E_i$. The temperature T is at first set to a high esteem, and a random walk is completed at that temperature. Then the temperature is brought somewhat agreeing down to a cooling rate c, for a sample: T changes to T*(1-c). The slight probability of making a stride that gives higher energy is the thing that permits simulated annealing to every now and again escape neighborhood minima.

6.2 Travelling Salesman Problem:

- The traveling salesman problem (TSP) asks the accompanying question: Given a rundown of cities and the separations between every pair of cities, what is the briefest conceivable course that visits every city precisely once and comes back to the inception city? As it is a NP-difficult problem straightforward savage power don't work when problem space is substantial. By and large it requires a great deal of time to locate an optimum solution for this problem and given today's calculation capacities we for the most part settle for the solution which is closer to the worldwide optimum solution the length of the time taken to register such solution is inside adequate range and that is the reason it is utilized as a benchmark for some enhancement methods.

6.2.1 Solution approach:

- The most direct solution would be to try all permutations and see which one is cheapest using brute force search.

- The running time for this approach lies within a polynomial factor of O(n!), the factorial of the number of cities, so this solution becomes impractical even for only 20 cities.

6.2.2 Simulated Annealing (SA) for Travelling Salesman problem:

I took 20 cities for the simulation purposes so our state space is set of unique the permutation of those 20 cities and these permutation is called tour. The size of our state space is 20! (Finite).

The algorithm starts with a tour which is just a random sequence of 20 cities and will improve gradually along the time. Our algorithm's next state which will be compared and evaluated with current state is obtained by randomly switching two cities in the current tour.

We took total distance that needs to be travelled for certain tour to be energy of the state and our goal is to minimize the energy of the state.

Temperature is the key to SA's ability to find global optimums, and what differentiates it from simple hill-climbing algorithms, is that at each iteration it has a (decreasing) probability of moving to more energy having states. It always moves to more less energy having states but, when next state is more energy having state, it will move to next state when $e^{(energy-newenergy)/T}$ is grater then random (0,1) variable, where T is temperature. Introduction of random variable randomizes the next state which is the key differentiator of simulated annealing from simple annealing.

Theoretical analysis of SA indicates a cooling rate will determine the probability of escaping from local optimums and eventually finding the global optimum because as time passes based on cooling rate with the time temperature decreases reducing the probability of moving to the more energy having states. In our simulation the temperature is $\mathbf{10^4}$ and cooling rate is **0.01%.**

We have divided our project following major sections.

- Generating and managing cities
- Tour management
- Simulated Annealing
- Plotting the results

6.2.3 Results:

- Here, I have displayed results of two different simulations. The best possible solution is a tour having a distance equal to 863 which we got by running the simulation for more than 100 times.

- Figure 6.1 and 6.2 shows the result of first and second simulation.

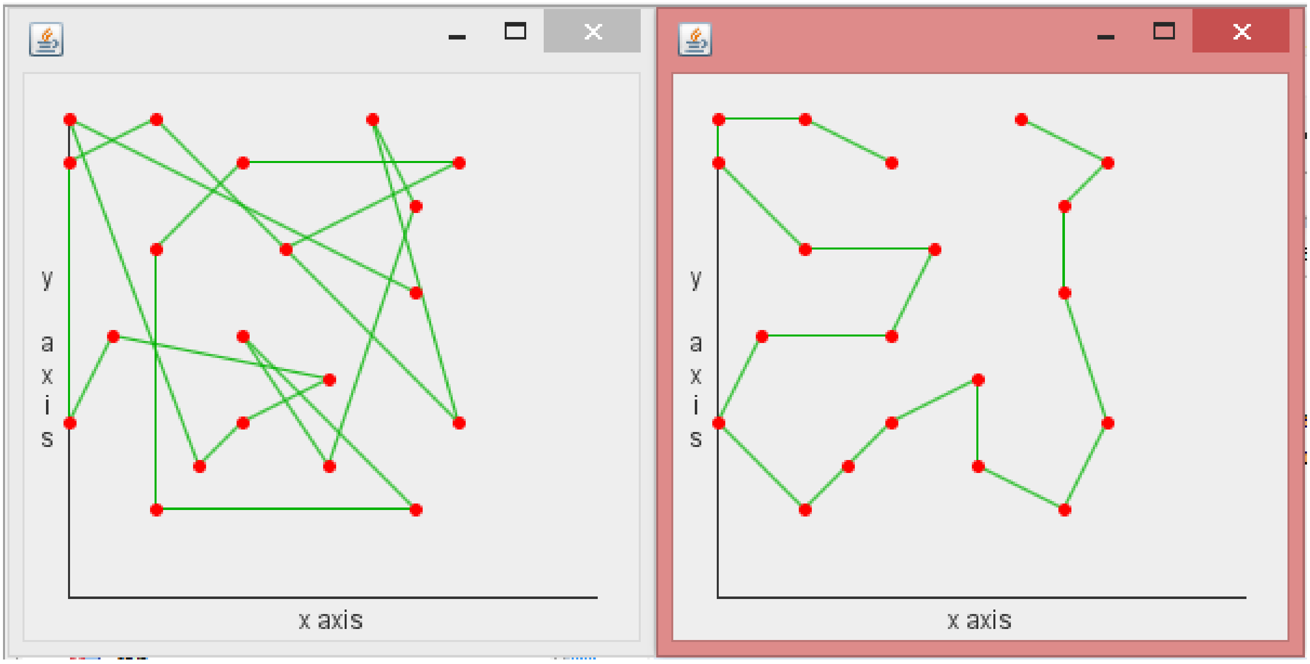

Figure 6.2 Initial and final tour after first simulation

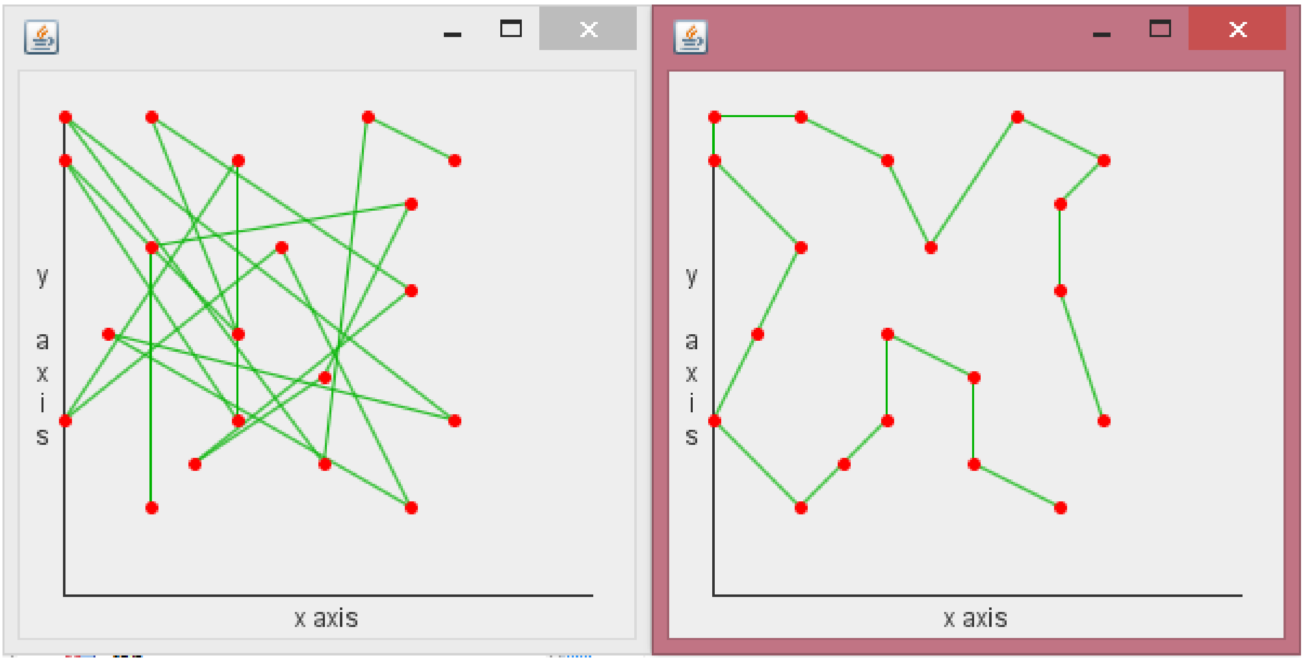

Figure 6.2 Initial and final tour after second simulation.

Simulation No	Initial Distance of the Tour	Best tour distance
1	2616	863
2	2467	863
3	1840	926
4	2011	881
5	2346	916
6	2064	863
7	2579	903
8	2583	947
9	2371	987
10	2452	909

Table 6.1

The data of sample 10 simulation that we ran.

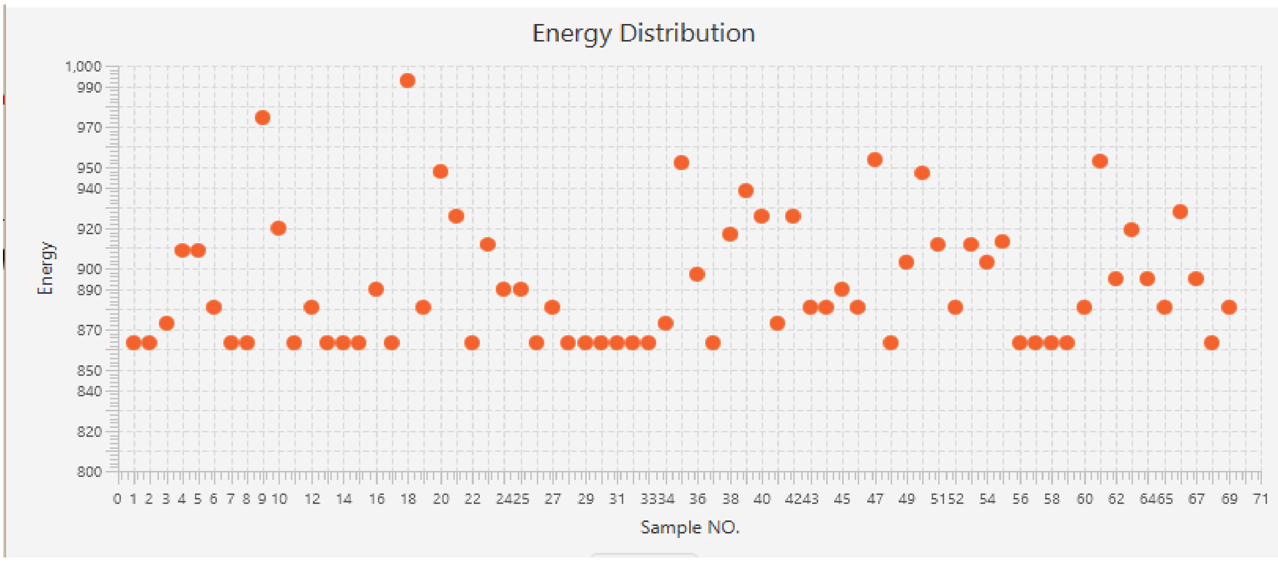

Figure 6.3 Scatter plot of the data of 70 simulations

Chapter 7
Application of markov chain in google PageRank algorithm

7.1 PageRank Algorithm:

- PageRank is an algorithm utilized by Google Search to rank sites in their web index results. PageRank was named after Larry Page, one of the originators of Google. PageRank is a method for measuring the significance of site pages.

- As per Google:

    > PageRank works by checking the number and nature of connections to a page to decide a harsh evaluation of how critical the site is. The basic suspicion is that more essential sites are liable to get more connections from other sites.[c]

- The PageRank algorithm yields a probability distribution used to speak to the probability that a man randomly tapping on connections will touch base at a specific page. PageRank can be ascertained for accumulations of archives of any size. It is expected in a few exploration papers that the distribution is equally separated among all archives in the gathering toward the start of the computational process. The PageRank calculations require a few passes, called "cycles", through the accumulation to modify inexact PageRank qualities to all the more nearly mirror the theoretical genuine worth.[7]

7.1.1 The Google matrix:

- With a specific end goal to produce the Google matrix G, we should first create an adjacency matrix A which speaks to the relations between pages or hubs.

  Accepting there are N pages, we can round out A by doing the accompanying:

1. A matrix component $A_{i,j}$ is loaded with 1 if page j has a connection to page i, and 0 otherwise; this is the adjacency matrix of connections.[c]
2. A related matrix S comparing to the moves in a Markov chain of given system is developed from A by separating the components of segment "j" by various k_j where k_j is the aggregate number of active connections from hub j to every single other hub. The sections having zero matrix components, comparing to dangling hubs, are supplanted by a steady esteem 1/N. Such a methodology includes a connection from each sink, dangling express a to each other hub.[c]

3. Presently by the development the whole of all components in any section of matrix S is equivalent to solidarity. Along these lines the matrix S is mathematically very much characterized and it has a place with the class of Markov chains and the class of Perron-Frobenius administrators. That makes S appropriate for the PageRank algorithm.[c]

- Then the final Google matrix G can be expressed via *S* as:

$$G_{ij} = \alpha S_{ij} + (1-\alpha)\frac{1}{N} \qquad (1)$$

- By the construction the sum of all non-negative elements inside each matrix column is equal to unity. The numerical coefficient α is known as a damping factor.

- The damping factor is used to dump the undesirable outcome due to page with no outgoing link.[7]
- The following figure explains that how solution is carried out by iterative method.
- Here there are 4 pages which are interconnected and V indicates the initial state of PageRank vector, which is ¼ for each page. A is matrix representation of graph of the connection of this pages.[7]

$$v = \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{pmatrix}, \quad Av = \begin{pmatrix} 0.37 \\ 0.08 \\ 0.33 \\ 0.20 \end{pmatrix}, \quad A^2 v = A\,(Av) = A \begin{pmatrix} 0.37 \\ 0.08 \\ 0.33 \\ 0.20 \end{pmatrix} = \begin{pmatrix} 0.43 \\ 0.12 \\ 0.27 \\ 0.16 \end{pmatrix}$$

$$A^3 v = \begin{pmatrix} 0.35 \\ 0.14 \\ 0.29 \\ 0.20 \end{pmatrix}, \quad A^4 v = \begin{pmatrix} 0.39 \\ 0.11 \\ 0.29 \\ 0.19 \end{pmatrix}, \quad A^5 v = \begin{pmatrix} 0.39 \\ 0.13 \\ 0.28 \\ 0.19 \end{pmatrix}$$

$$A^6 v = \begin{pmatrix} 0.38 \\ 0.13 \\ 0.29 \\ 0.19 \end{pmatrix}, \quad A^7 v = \begin{pmatrix} 0.38 \\ 0.12 \\ 0.29 \\ 0.19 \end{pmatrix}, \quad A^8 v = \begin{pmatrix} 0.38 \\ 0.12 \\ 0.29 \\ 0.19 \end{pmatrix}$$

Figure 7.1 Finding the page rank vector via iteration method

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Appendix A

List of reference links

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