MARKOV CHAIN MONTE CARLO AND ITS APPLICATION TO TRAVELLING SALESMAN PROBLEM

Course Instructor: Prof. Jaideep Mulherkar

Team Members:

Nikunj Amipara (201001199)

Parth Shah (201001200)

Om Thakkar (201001203)

Theory

Markov Chains:

A Markov chain is a mathematical system that undergoes transitions from one state to another, among a finite or countable number of possible states. It is a random process usually characterized as memory less: the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of "memorylessness" is called the Markov property. Markov chains have many applications as statistical models of real-world processes.

A sequence {Xn: n>=0} of random variables taking values in a countable state space S is called a Marcov Chain if,

$$P\{X_n = \alpha_n | X_{n-1}=\alpha_{n-1}, X_{n-2}=\alpha_{n-2} \ldots X_0=\alpha_0\} = P\{X_n=\alpha_n \mid X_{n-1}=\alpha_{n-1}\}$$

i.e. the probability that the chain will be in a certain state α_n at time n given all its past history depends only on its previous state at time n-1.

Usually we also impose the condition of homogeneity

$$P\{X_{n+1}=j \mid X_n=i\} = P\{X_1=j \mid X_0=i\}$$

Markov Chain Monte Carlo:

Markov chain Monte Carlo (MCMC) methods are a class of algorithms for sampling from probability distributions based on constructing a Markov chain that has the desired distribution as its equilibrium distribution.

Simulated Annealing:

Simulated annealing (SA) is a generic probabilistic heuristic for the global optimization problem of locating a good approximation to the global optimum of a given function in a large search space. It is often used when the search space is discrete. For certain problems, simulated annealing may be more efficient than exhaustive enumeration, provided that the goal is merely to find an acceptably good solution in a fixed amount of time, rather than the best possible solution. It is generally used for NP-hard problems.

The name and inspiration come from annealing in metallurgy, a technique involving heating and controlled cooling of a material to increase the size of its crystals and reduce their defects, both are attributes of the material that depend on its thermodynamic free energy. Heating and cooling the material affects both the temperature and the thermodynamic free energy. While the same amount of cooling brings the same amount of decrease in temperature it will bring a bigger or smaller decrease in the thermodynamic free energy depending on the rate that it occurs, with a slower rate producing a bigger decrease.

This notion of slow cooling is implemented in the Simulated Annealing algorithm as a slow decrease in the probability of accepting worse solutions as it explores the solution space. Accepting worse solutions is a fundamental property of heuristics because it allows for a more extensive search for the optimal solution.

The method is an adaptation of the Metropolis-Hastings algorithm, a Monte Carlo method to generate sample states. It begins with exploring the problem on global spectrum and slowly moves towards the approximation of global optimum.

Travelling Salesman Problem:

The travelling salesman problem (TSP) asks the following question: Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city? It is an NP-hard problem. It is used as a benchmark for many optimization methods.

The TSP has several applications even in its purest formulation, such as planning, logistics, and the manufacture of microchips. Slightly modified, it appears as a sub-problem in many areas, such as DNA sequencing. In these applications, the concept city represents, for example, customers, soldering points, or DNA fragments, and the concept distance represents travelling times or cost, or a similarity measure between DNA fragments.

Project Description:

As an application of Markov Chain Monte Carlo we have chosen to solve Travelling salesman problem using simulated annealing. We have taken 20 cities as our sample dataset. We will start with a random tour and using simulated annealing we move towards as much better solution as possible. The solution we get may not be the global optimum answer but it is one of the better solutions.

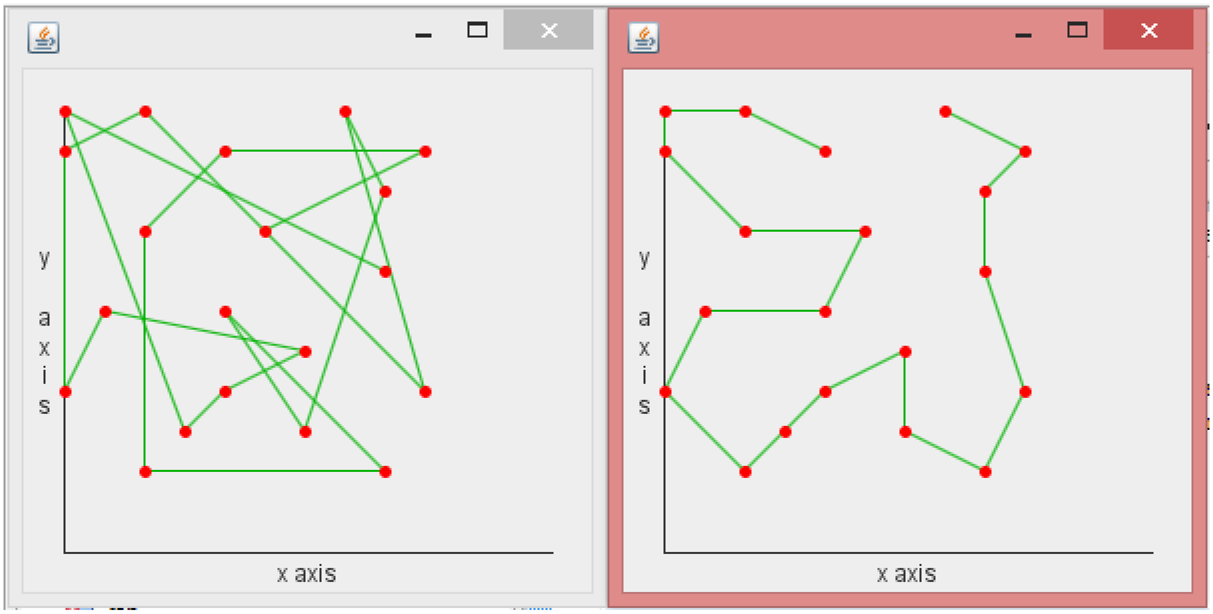
We have divided our project in four major sections.

- Generating and managing cities
- Tour management
- Simulated annealing
- Plotting the results

Results:

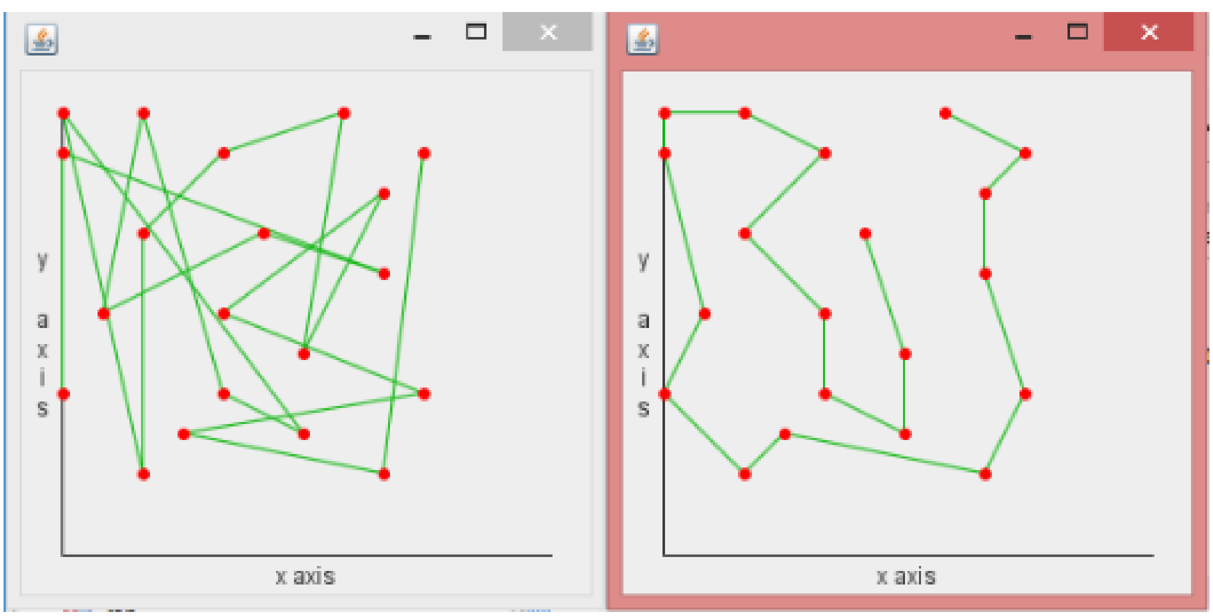
Here, we have displayed results of two different simulations.

Figure has initial tour and the best possible tour that we got after first simulation.



Initial tour: 2452 Final Tour: 909

This tour shows same results for another simulation.



Initial: 2371 Final: 987

Work Distribution:

1. Nikunj Amipara (201001199) – Research on Simulated Annealing, Coding Simulated Annealing part of code
2. Parth Shah (201001200) – Research on Markov Chains, Coding City and Tour part of code
3. Om Thakkar (201001203) – Research on Monte Carlo and applications of MCMC, Coding graphical part of code

References:

- Markov Chain Monte Carlo Simulation Made Simple by Alastair Smith
- Introduction to Markov Chain Monte Carlo by Charles J. Geyer
- Markov Chain Monte Carlo Method and Its Application by Stephen P. Brooks