Radial and Halo Acceleration Relations in MaNGA BCGs

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Abstract—We reproduce the results of the recent study by Tian et al. (2024) by examining the Radial Acceleration Relation (RAR) in a sample of 50 MaNGA Brightest Cluster Galaxies (BCGs). Our analysis reveals a characteristic acceleration scale of $a_0 = 1.10 \times 10^{-9}, \text{m/s}^2$, approximately ten times larger than the scale found in galaxies. This finding suggests that there is no universal acceleration scale associated with MOND. In addition, we explore the Halo Acceleration Relation (HAR) in BCGs as a potential means of distinguishing between different MOND interpolating functions and identifying the best fit. While our results clearly indicate the presence of a HAR in BCGs, the large scatter observed in the data prevents us from definitively distinguishing between the interpolating functions visually. To enhance our findings, we propose methodological changes for future analyses and intend to expand our study to include galaxy clusters. This extension could provide further insights into the HAR and its potential to constrain theoretical models.

Contents

1. Introduction

The current concordance model of cosmology ΛCDM, asserting that the energy budget of the universe is dominated by dark entities as opposed to normal baryonic matter, can explain a wealth of observations such as mass discrepancies in galaxies and clusters, the flatness of galactic rotation curves, difference in the cosmological density parameters Ω_m and Ω_b in addition to a bunch of *a priori* predictions- baryon acoustic oscillations, fluctuations in the Cosmic Microwave Background(CMB) Power Spectrum and the accelerating expansion of the universe. For a detailed review, see [2].

However, its fallacies encompass a bunch of unobserved predictions, such as the missing satellite problem, the core-cusp problem, and a missing baryon problem in galaxies. More importantly, it cannot explain several empirical observations such as the Baryonic Tully-Fisher Relation, Mass Discrepancy Acceleration Relation(MDAR), and Renzo's Rule[3]. Most of these manifest as fine-tuning problems and the supposedly coincidental appearance of an acceleration scale $a_0 \approx 10^{-10} m/s^2$ in these scaling relations.

A relatively new theory that has successfully explained these without invoking dark matter is Modified Newtonian Dynamics(MOND). At the heart of MOND lies Milgrom's Law[1], which seeks to modify Newton's Second Law in the low acceleration limit:

$$g_N = \mu\left(\frac{g}{a_0}\right) g \tag{1}$$

g is the observed acceleration, g_N is the predicted Newtonian Acceleration, and $a_0 \approx 10^{-10} m/s^2$ plays the role of a fundamental constant in the universe. $\mu\left(\frac{g}{a_0}\right)$ is called the MOND interpolating function, which obeys $\mu(x) \to 1$ as $x >> 1$ and $\mu(x) \to x$ as $x << 1$ so that the equation reduces to Newton's Law when accelerations are much greater than a_0. When the accelerations are sufficiently low, it becomes $g = \sqrt{g_N a_0}$.

The relation can also be expressed as follows:

$$g = \nu(y) g_N \tag{2}$$

where $y = g_N/a_0$ behaving asymptotically as: $\nu(y) \approx 1$ as $y >> 1$ and $\nu(y) \approx y^{-1/2}$ as $y << 1$. Yet another challenge for ΛCDM is the Radial Acceleration Relation(RAR)[5],[6], an extremely tight correlation between the observed acceleration and baryonic acceleration:

$$g = \frac{g_N}{1 - e^{\sqrt{(g_N/a_0)}}}, a_0 = (1.2 \pm 0.24) \times 10^{-10} m/s^2 \tag{3}$$

RAR is a direct consequence of Milgrom's Law and a special case of the MDAR. The scatter is almost entirely systematic. Although ΛCDM is able to give the RAR, the tight scatter cannot be trivially reproduced[4]. In addition, the problem of fine-tuning model parameters persists[7].

First observed in galaxies, there have been attempts to test the universality(which it must be if Milgrom's Law is correct)[9][10][11] look for a similar relation in galaxy clusters. Although MOND works poorly on BCG and cluster scale, a hint of the RAR could be detectable, given the tight scatter with which it manifests in galaxies. The key differences in the results in these works compared to those on galaxies were:(i)The estimated value of a_0 was about ten times higher, and (ii)the estimated residual scatter was about 0.2 dex, much higher than galaxies. These results significantly challenge modified gravity theories, suggesting that no universal acceleration scale exists, as indicated by Milgrom's Law.

A separate relation called the Halo Acceleration Relation(HAR) has recently appeared in literature[8]. HAR expresses a relation between the halo acceleration g_h defined as $g - g_N$ and g_N. In passing, $\nu(y)$ can, in principle, take several different mathematical forms that satisfy the constraints. HAR is of interest because while the RAR cannot distinguish between forms of $\nu(y)$, the HAR can, particularly in the high-acceleration regime. The authors determine the best-fitting form by fitting galaxy data to the HAR. However, the HAR, if it exists, has not been analyzed for BCGs/clusters before.

The objectives of our project are the following:

1. To test the RAR on Brightest Cluster Galaxy(BCG) scale
2. To look for a HAR on BCGs and attempt to distinguish between the different forms of the interpolating function.

2. Data

For BCG data, we analyze a sample of 50 galaxies from the Mapping Nearby Galaxies at Apache Point Observatory (MaNGA) survey as compiled in Tian et al 2024[12](hereafter T24). Data is tabulated as $log(g_{obs}), log(g_{bar})$ and their corresponding errors. Errors in $log(g_h) = log(g_{obs} - g_{bar})$ are calculated by error propagation.

In the next section, we describe the data analysis techniques we leveraged throughout the project.

3. Methodology

3.1. MCMC

MCMC, a popular method in Bayesian Parameter Estimation, involves sampling from a probability distribution and leveraging these samples to estimate a desired quantity.

The cornerstone of Bayesian Techniques in statistics is the Bayes' Theorem:

$$P(M,\vec{\theta}|D) = \frac{P(D|M,\vec{\theta})P(M,\vec{\theta})}{P(D)} \tag{4}$$

$P(M,\theta|D)$ is the probability of a model with a parameter vector $\vec{\theta}$ being correct, called the posterior, $L = P(D|M,\theta)$ the likelihood, $P = P(M,\theta)$ the prior(quantifying any presumptions one might have regarding the model parameters) and $P(D)$ the evidence. A Bayesian Parameter Estimation algorithm aims to maximize the likelihood times the prior.

MCMC, stemming from the broader Metropolis-Hastings Algorithm, operates in the following manner:

1. Begin by selecting a random point θ_0 within the initial parameter range provided, along with its corresponding posterior probability $P(\theta_0|D)$
2. Evaluate the posterior probability $P(\theta_2|D)$ at another point within the proposal distribution.
3. Move to θ_2 with a probability $min(\frac{P(\theta_2|D)}{P(\theta_0|D)}, 1)$
4. Generate a random number u uniformly distributed between 0 and 1, then accept the candidate sample if u is less than α.
5. Repeat steps 2 through 4 until the user-specified steps are exhausted. Intuitively, the algorithm eventually lands on the $\vec{\theta}$ with highest posterior probability.

3.2. Orthogonal Distance Regression

In contrast to conventional regression methodologies, where errors are typically assessed vertically concerning the fitted line, Orthogonal Distance Regression (ODR) entails computing the orthogonal distance of data points relative to the fitted line. This approach affords the incorporation of measurement errors associated with both independent and dependent variables along the x and y axes, thereby enhancing the robustness of the regression analysis. This makes it a valuable tool for astrophysical data analysis.

With ODR, we try to minimize the following likelihood function

$$\sum_{i=1}^{n}\left[\frac{(y_i - f(x_i))^2}{n} + (x_i - X_i)^2\right] \tag{5}$$

$$\eta = \frac{\sum \sigma_\varepsilon^2}{\sum \sigma_\mu^2} \tag{6}$$

Here, ε and μ are errors in the measured value, and σ denotes the variance of errors.

4. Data Analysis

4.1. Fitting RAR

We fit the RAR to a straight line using MCMC, $y = mx + c$, taking into account an intrinsic scatter σ_{int}, which takes care of unaccounted astrophysics associated with the RAR. We set non-informative flat priors on m, c as $[-10, 10]$, and flat priors on $log\sigma_{int}$ as $[-0.5, 0.5]$. Our expression for log-likelihood incorporates errors in both x and y:

$$lnL = -0.5\left(\sum(2\pi\sigma^2) + \sum\left(\frac{(y-(mx+c))^2}{\sigma^2}\right)\right) \tag{7}$$

where $\sigma^2 = m^2\sigma_x^2 + \sigma_y^2 + \sigma_{int}^2$

We estimate an intercept of $-4.48^{+0.86}_{-0.89}$ and a slope of $0.49^{+0.09}_{-0.09}$. Note that results might vary slightly based on the fitting technique, but our results are consistent with uncertainties. Residual scatter is lognormal, within a range of about 0.2 dex around 0.

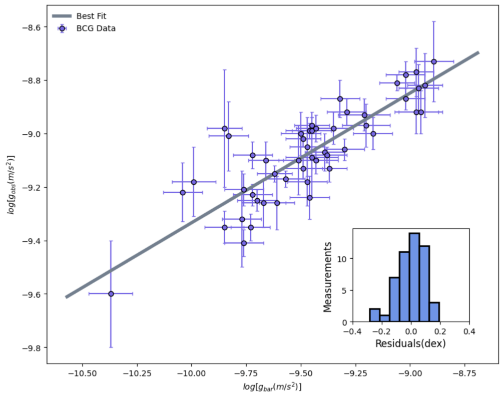

Figure 1. Best fit line and residual distribution for the RAR on BCGs.

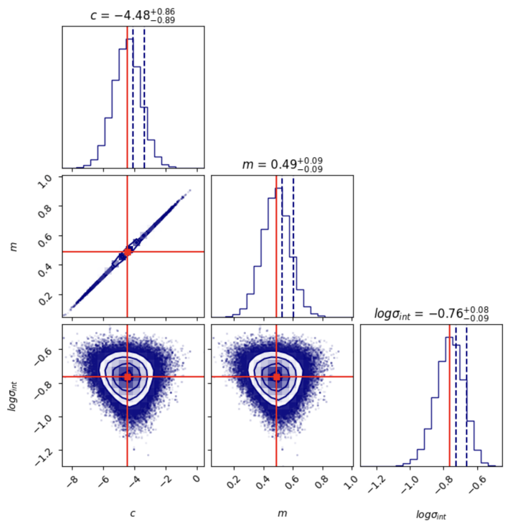

Figure 2. Posterior distributions of parameters estimated for RAR

From the intercept, we infer the value of a_0 to be $1.10 \times 10^{-9} m/s^2$.

4.2. Fitting HAR

For fitting the HAR, we use six commonly used interpolation functions that have already been used in T24:

1. n form

$$v_n(y) = \left[1 + \frac{\left(1+\frac{4}{y^n}\right)^{\frac{1}{2}}}{2}\right]^{\frac{1}{n}} \tag{8}$$

2. β form

$$v_\beta(y) = (1 - e^{-y})^{-\frac{1}{2}} + \beta \cdot e^{-y} \tag{9}$$

3. γ form

$$v_\gamma(y) = (y\left(1 - e^{-\left(y^{\frac{\gamma}{2}}\right)}\right)^{-\frac{1}{\gamma}} + \left(1 - \frac{1}{\gamma}\right)e^{-\left(y^{\frac{\gamma}{2}}\right)} \tag{10}$$

4. δ form

$$v_\delta(y) = \left(1 - e^{-\left(y^{\frac{\delta}{2}}\right)}\right)^{-\frac{1}{\delta}} \tag{11}$$

5. ϵ form

$$v_\epsilon(y) = \tanh^{-\epsilon}\left(y^{\frac{1}{2\epsilon}}\right) \tag{12}$$

6. $\alpha\eta$ form

$$v_{\alpha\eta}(y) = \left[1 + \frac{1}{2}\left(4y^{-\alpha} + \eta^2\right)^{\frac{1}{2}}\right]^{\frac{1}{\alpha}} \tag{13}$$

The fitting method in this case is Frequentist Orthogonal Distance Regression. Given the complexity of these functions, our Bayesian Parameter Estimation code took a long time to compute. Since `scipy.odr` requires to guess values on the estimated parameters; we initiate with guesses the same as the best-fit parameters in T24. However, we keep a_0 as a free parameter with a guess of $2 \times 10^{-9} m/s^2$, the exact estimate obtained in T24.

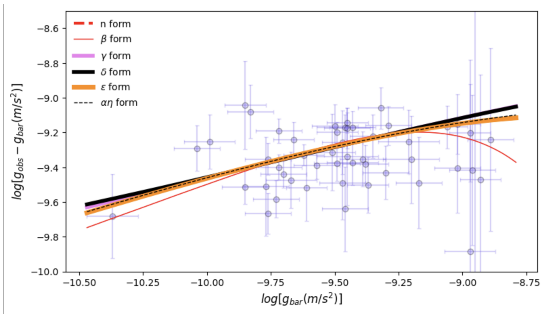

Figure 3. Fitting of different interpolating functions.

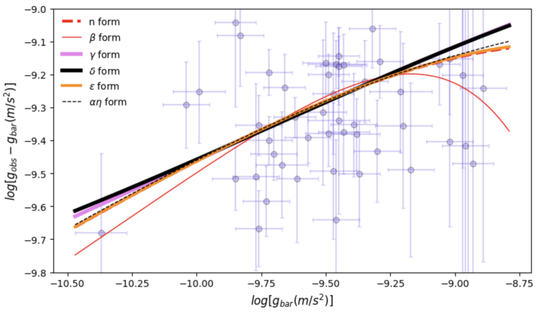

Figure 4. Fitting of different interpolating functions, but with reduced y-range for more clarity.

5. Results and Discussion

Model	Best fit parameters(index, a_0)
n form	$1.34 \pm 0.60, (1.77 \pm 0.49) \times 10^{-9}$
β form	$1.42 \pm 0.70, (8.03 \pm 3.20) \times 10^{-10}$
γ form	$(1.45 \pm 0.39) \times 10^{-1}, (5.28 \pm 16.87) \times 10^{-14}$
δ form	$(2.33 \pm 0.57) \times 10^{-1}, (2.16 \pm 3.86) \times 10^{-12}$
ϵ form	$1.32 \pm 0.57, (1.79 \pm 0.48) \times 10^{-9}$
$\alpha\eta$ form	$1.92 \pm 0.73, 2.65 \times 10^{-4}, (1.87 \pm 0.40) \times 10^{-9}$

Table 1. Best fit parameters for different models

Model	χ^2/dof
n form	1.19
β form	1.16
γ form	1.20
δ form	1.02
ϵ form	0.15
$\alpha\eta$ form	1.22

Table 2. Reduced χ^2 values for different models

Visually, all the functions fit the data reasonably well due to the high scatter in data. The models' reduced χ^2 values are close to 1, except for the ϵ form, which is disfavoured strongly relative to others. They indicate δ to be the best fit. We also compare the Akaike Information Criteria(AIC) and Bayesian Information Criteria(BIC) for the different models.

Model	$\triangle$ AIC	$\triangle$ BIC
n form	0.65	0.65
β form	2.34	2.34
δ form	8.75	8.75
ϵ form	50.55	50.55
$\alpha\eta$ form	2.41	4.33

Table 3. $\triangle$ AIC and $\triangle$ BIC values for the models

The $\triangle$AIC and $\triangle$BIC values for ϵ form are very high, and therefore, there is solid evidence against this model.
While the $\triangle$AIC and $\triangle$BIC values for δ model are lower than the values of the ϵ model (which essentially makes it better), it still has considerably less evidence. The $\alpha\eta$ form and β form are quite similar. Both models are decently good, and there is substantial evidence for them. According to AIC/BIC, the n form is the best model of all, which has the lowest $\triangle$AIC and $\triangle$BIC value among these functions. In this case, we would go with n form as the best form of the interpolation function because the reduced χ^2 of nearly all models are close to 1. T24 states γ as the best form, but note that we have kept a_0 as a free parameter, wherein lies the difference in methodologies and, possibly, the difference in results. We repeated the analysis by segregating the data into bins to compensate for the high scatter. However, the reduced χ^2 values for all the interpolation functions were in the range of 0.2-0.35, which means that binning produces overfitting in all cases. We don't state binning results here, but they are available in the code repository.

6. Conclusion

We analyzed a sample of 50 MaNGA Brightest Cluster Galaxies for radial and halo acceleration relations. We found that the RAR applied for two orders of magnitude in baryonic acceleration. Our resultant value of a_0 was $1.1 \times 10^{-9} m/s^2$, the same order of magnitude obtained for tests on galaxy cluster samples. We used the BCG data to distinguish between MOND Interpolating Functions employing the Halo Acceleration Relation. We found that the n form best fits the given data. However, the scatter in the data makes our results less reliable compared to T24.

6.1. Scope

For BCGs, we would like to redo the analysis of interpolation functions while keeping a_0 fixed, as we determined from the RAR. We used Huber Regression as a better way to distinguish the functions, however one reduced χ^2 value is close to 10^{-16}, although we are not sure why this happened. The code is in the repository mentioned, and the image showing the fits is shown in the appendix.Also, we are currently developing the code to extend the analysis to clusters, which is the project's eventual goal. We have also finished part of the analysis for the X-COP sample. However, we have yet to find a suitable regression method because the results are highly dependent on the initial guess parameters. Although the large scatter in data renders the tests less conclusive at the BCG Scale, it is worth looking at cluster data for both scientific and learning purposes.

7. Data Availability

All codes, figures, and data used in this project can be found in this repository: https://github.com/ambicagovind31/EP4130-RAR-HAR

8. Appendix

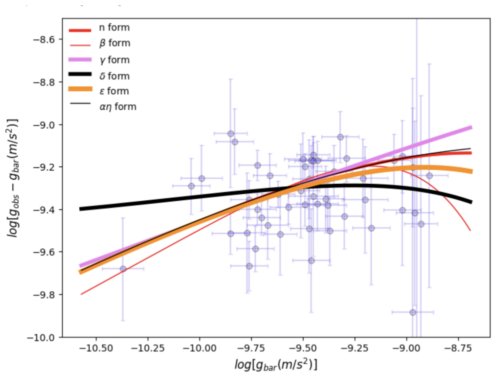

Figure 5. Halo Acceleration Relations determined by interpolation functions computed via Huber Regression

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