Logistic Regression:

- from Linear Regression: $y = \beta_0 + \beta_1 X$
- In logistic regression we have to convert it to probability.

So we can consider ratio of odds

$$= \frac{\text{prob of success}}{\text{prob of failure}} = \frac{p}{1-p}$$

$\rightarrow$ $\frac{p}{1-p} = \beta_0 + \beta_1 X$ $\Rightarrow$ ranges $0 - \infty$

$\rightarrow$ to make the range $-\infty$ to ∞

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X$$

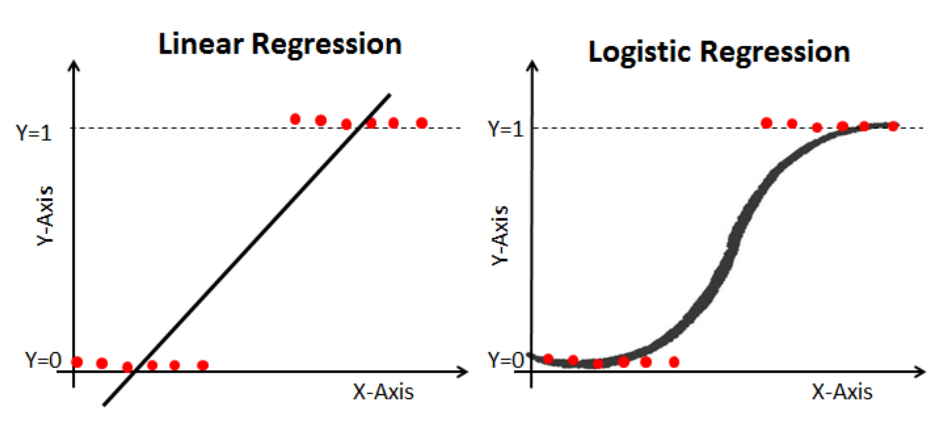
$$\Rightarrow \frac{p}{1-p} = \exp(\beta_0 + \beta_1 X)$$

$$\Rightarrow \frac{1-p}{p} = \exp(-z) \quad [z = \beta_0 + \beta_1 X]$$

$$\Rightarrow \frac{1-p}{p} + 1 = 1 + \exp(-z)$$

$$\Rightarrow \frac{1}{p} = 1 + \exp(-z)$$

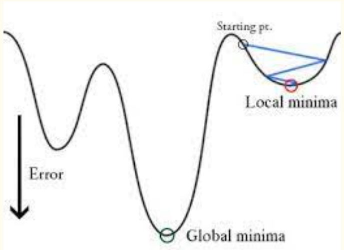
$$\Rightarrow \boxed{p = \frac{1}{1 + \exp(-z)}}$$



→ if we use MSE for Logistic Regression then we will have multiple local minima.

Linear Regression
Cost Function

$$J = \frac{\sum_{i=1}^{n} (\hat{Y}_i - Y_i)^2}{n}$$



→ To find the loss function for Logistic regression, we will see MLE:

$Y \sim Ber(p)$ where,

$$p = \sigma(\beta_0 + \beta_1 X)$$

$$f(x) = p^x (1-p)^{1-x}$$

$$= (\sigma(z))^x (1 - \sigma(z))^{1-x}$$

$$\sigma = \frac{1}{1+e^{-z}}$$

$$z = \beta_0 + \beta_1 X$$

→ Now if we consider MLE, which tries to find the parameter for which likelihood of getting the data (observed) will be maximum.

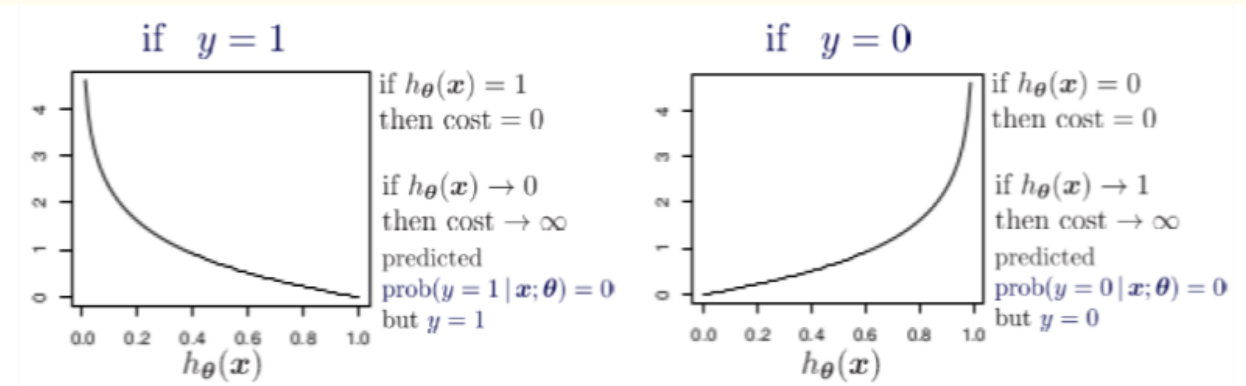
$$L(\theta) = \prod_{i=1}^{n} \sigma(\theta^T x_i)^y \left(1 - \sigma(\theta^T x_i)\right)^{1-y}$$

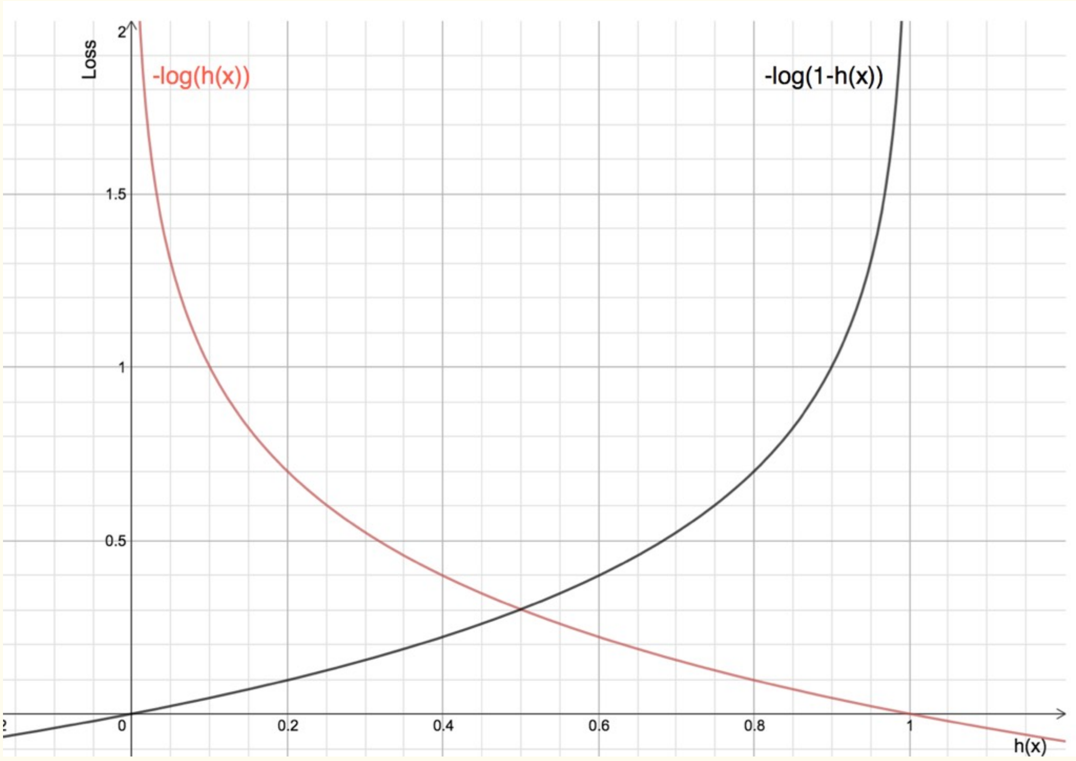
$$\Rightarrow \log(L(\theta)) = \sum y \log(\sigma(\theta^T x_i)) + \sum (1-y) \log(1 - \sigma(\theta^T x_i))$$

→ now maximizes $\log(L(\theta))$ is equivalent to minimizing $-\log(L(\theta))$

$$\therefore -\log(L(\theta)) = -\sum y \log p + (1-y)\log(1-p)$$

- p = probability of 1.
- $(1-p)$ = probability of 0.





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