Practice 1

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1. What is the gravitational flux through the surface of the earth? The mass of the earth is $M = 5.97 \times 10^{24}$ Kg and the radius is $R = 6.38 \times 10^{3}$ Km. What is the flux through a surface of radius 2R? Without calculation, what would the flux therefore be through a surface of radius 3R? What does this tell you about the change in gravitational field strength as a function of distance?

2. A thin unit-circular disk has a non-uniform density function

$$\rho(x, y) = (x + 1)^2 + (y + 1)^2$$

compute the coordinates for the center of mass. (Is it easier to use cartesian coordinates or polar coordinates?)

3. Consider a lattice of an arbitrary number of points of an arbitrary mass. State the sum used to compute a coordinate of the lattice's center of mass. How does this sum relate to the arithmetic mean of a discrete data set? Can this be extended to a continuous data set? If so, does this also hold for the center of mass calculation?

4. Derive the moment of inertia of a thin ring of radius R, with constant density ρ_0. Recall that the moment of inertia is given as

$$I = \int r^2 dm$$

where $dm = \rho_0 dV$

Extra: By considering the conservation of angular momentum, derive the moment of inertia of a rotating point mass connected by a massless, non extendable string to the origin, the path (x, y) of the mass is given as the real and imaginary parts of the complex number

$$z = re^{i\theta}$$

Are these results the same? If so, why?

5. A thin disk of uniform density on the xy-plane is rotating at constant angular velocity. What happens to this angular velocity when mass is added to the origin (axis of rotation)?

6. An electron is ejected from a metal plate with work function $W_0 = 2.2$ eV by a photon of wavelength $3\mathring{A}$. How fast is the electron moving? Is it relativistic? Justify your answer.

7. Consider yourself in a linear magnetic field of strength B into the page, and a square coil of wire with equal side lengths a localised on the page. Using Lenz's law, find an expression for the induced current if you pull a side of the coil at speed v, such that the coil maintains its square shape. What direction is this current? Without referencing your previous answer, why is the current in this direction? Suppose the field was turned off, how would the feeling differ of pulling the side of the coil, assuming no material resistance from the coil itself?

8. By differentiating implicitly, show that the derivative of

$$y = e^x$$

is itself. Given that

$$\int \sinh(x)dx = \cosh(x)$$

show that the integral of e^x is also itself.

Extra:

E1. The Lorentz transformation can be thought of as the hyperbolic rotation matrix given as

$$L(v) = \begin{pmatrix} \cosh(\phi) & -\sinh(\phi) \\ -\sinh(\phi) & \cosh(\phi) \end{pmatrix}$$

where $\tanh(\phi) = \frac{v}{c}$ and

$$\cosh(\phi) = \frac{1}{\sqrt{1 - \tanh(\phi)^2}} \text{ and } \sinh(\phi) = \frac{\tanh(\phi)}{\sqrt{1 - \tanh(\phi)^2}}$$

Show that light speed is an eigenvector of this matrix, what about this particular vector makes it light speed? How does this relate to one of the fundamental postulates of special relativity?